1 Given that $f(x) = 4\cos x$ and $g(x) = e^x$,

(a) Sketch the graphs $y = 4\cos x$ and $y = e^x$ on the same coordinate system. Hence, state the number of solutions to the equation

$$f(x) = 4e^{-x}\cos x - 1 = 0$$

in the interval $[-2\pi, \pi]$ [6]

(b) Show that one zero of $f(x) = 0$ lies on the interval $[0.5, 1]$. [4]

(c) Use bisection method to perform five iterations to approximate the root of $f(x) = 0$ in the interval $[0.5, 1]$. [5]

2 Given the function $f(x) = 2\sin x - 2^{\frac{x}{4}} - 1$

(a) Use the Newton-Raphson method to find the solution accurate to within 10^{-4} for the equation $f(x) = 0$, using the initial point $x_0 = -5$. [5]

(b) Use the Secant method to find the solution accurate to within 10^{-4} for the equation $f(x) = 0$, using the initial point $x_0 = -5$. [5]

(c) Comment by comparing the rate of convergence for the two methods. [2]

3 The fixed-point iteration method is to be applied for approximating a root of the non-linear equation $f(x) = x^4 - x - 10 = 0$.

(a) How many solutions of the non-linear equation $f(x) = 0$ are there in the interval $[1, \infty)$? Are they simple [5]

(b) Find an interval $[1, b]$ that contains the smallest positive solution of the nonlinear equation $f(x) = 0$. [3]

(c) Estimate the zero of $f(x) = 0$ using fixed point iteration method by computing 5 iterations in each case for $g(x) = x$ when;

(i) $g(x) = x^4 - 10$ [5]

(ii) $g(x) = \frac{x^4 - x - 10}{2x^6 + 5} + x$ [5]