The University of Zambia
Department of Mathematics and Statistics
MAT 3110 - Engineering Mathematics II

Tutorial Sheet 1 - Laplace Transforms **March, 2024**

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1. Find the Laplace transforms of the following functions.

   (a) $(1-2t)^2$ (b) $\cos^2\left(\frac{t}{2}\right)$ (c) $e^{2t}\sinh t$

   (d) $e^{-t}\sin 4t$ (e) $\sin(3t+2)$ (f) $5\sin\left(3t-\frac{\pi}{2}\right)$

   (g) t^2e^{-3t} (h) $3e^{-2t}\cos(4t)$ (i) $\sinh t\cos t$

2. Find the inverse Laplace transforms of the following functions.

   (a) $\dfrac{5s+1}{s^2-25}$ (b) $\dfrac{s}{4s^2+\pi^2}$ (c) $\dfrac{12}{s^4}-\dfrac{228}{s^6}$

   (d) $\dfrac{4s+32}{s^2-16}$ (e) $\dfrac{s+10}{s^2-s-2}$ (f) $\dfrac{1}{(s+1)(s+2)}$

   (g) $\dfrac{\pi}{(s+\pi)^2}$ (h) $\dfrac{6}{(s+1)^3}$ (i) $\dfrac{4}{s^2-2s-3}$

   (j) $\dfrac{\pi}{s^2+10\pi s+24\pi^2}$ (k) $\dfrac{2s-1}{s^2-6s+18}$

3. Solve the following initial value problems by using Laplace transforms.

   (a) $y'+2y=0, \quad y(0)=\frac{3}{2}$

   (b) $y''-y'-6y=0, \quad y(0)=11,\ y'(0)=28$

   (c) $y''+9y=10e^{-t}, \quad y(0)=0,\ y'(0)=0$

   (d) $y''-4y'+3y=6t-8, \quad y(0)=0,\ y'(0)=0$

4. Solve the following shifted data initial value problems by using Laplace transforms.

   (a) $y'-6y=0, \quad y(-1)=4.$

   (b) $y''-2y'-3y=0, \quad y(4)=-3,\ y'(4)=-17$

   (c) $y''+2y'+5y=10t-100,\ y(2)=-4,\ y'(2)=14$

5. Sketch or graph the given function, which is assumed to be zero outside the given interval. Represent it, using unit step functions. Find its Laplace transform.

   (a) $t \ \ (0<t<2)$ (b) $t-2 \ \ (t<2)$ (c) $\cos 4t \ \ (0<t<\pi)$

   (d) $e^t \ \ (0<t<\frac{\pi}{2})$ (e) $\sin\pi t \ \ (2<t<4)$ (f) $e^{-\pi t} \ \ (2<t<4)$

   (g) $t^2 \ \ (1<t<2)$ (h) $t^2 \ \ (0<t<\frac{3}{2})$ (i) $\sin t \ \ (\frac{\pi}{2}<t<\pi)$

6. Find and sketch the inverse Laplace transforms of the following functions.

(a) $\dfrac{e^{-3s}}{(s-1)^3}$ (b) $\dfrac{6(1-e^{-\pi s})}{s^2+9}$ (c) $\dfrac{4(e^{-2s}-2e^{-5s})}{s}$

(d) $\dfrac{e^{-3s}}{s^4}$ (e) $\dfrac{2(e^{-s}-e^{-3s})}{s^2-4}$ (f) $\dfrac{1+e^{-2\pi(s+1)}(s+1)}{(s+1)^2+1}$

7. Solve the following initial value problems with a discontinuous force.

(a) $y''+9y=\begin{cases} 8\sin t, & : 0<t<\pi \\ 0, & : t\geq \pi \end{cases}$ $\quad y(0)=0,\ y'(0)=4$

(b) $y''+3y'+2y=\begin{cases} 4t, & : 0<t<1 \\ 8, & : t\geq 1 \end{cases}$ $\quad y(0)=0,\ y'(0)=0$

(c) $y''+y'-2y=\begin{cases} 3\sin t-\cos t, & : 0<t<2\pi \\ 3\sin 2t-\cos 2t, & : t\geq 2\pi \end{cases}$ $\quad y(0)=1,\ y'(0)=0$

(d) $y''+3y'+2y=\begin{cases} 1, & : 0<t<1 \\ 0, & : t\geq \pi \end{cases}$ $\quad y(0)=0,\ y'(0)=0$

(e) $y''+2y'+5y=\begin{cases} 10\sin t, & : 0<t<2\pi \\ 0, & : t\geq 2\pi \end{cases}$ $\quad y(\pi)=1,\ y'(\pi)=2e^{-\pi}-2$

(f) $y''+4y=\begin{cases} 8t^2, & : 0<t<5 \\ 0, & : t\geq 5 \end{cases}$ $\quad y(1)=1+\cos 2,\ y'(1)=4-2\sin 2$

8. Find the following convolutions.

(a) $1*\sin t$ (b) e^t*e^{-t} (c) $\cos(2t)*\cos t$ (d) $\sin t*\cos t$ (e) $t*e^t$

9. Solve the following integral equations using Laplace transforms.

(a) $y(t)+4\displaystyle\int_0^t y(\tau)\ (t-\tau)\mathrm{d}\tau=2t$ (b) $y(t)-\displaystyle\int_0^t y(\tau)\mathrm{d}\tau=1$

(c) $y(t)-\displaystyle\int_0^t y(\tau)\ \sin 2(t-\tau)\mathrm{d}\tau=\sin 2t$ (d) $y(t)+\displaystyle\int_0^t y(\tau)\ \cosh(t-\tau)\mathrm{d}\tau=t+e^t$

(e) $y(t)+2e^t\displaystyle\int_0^t y(\tau)\ e^{-\tau}\mathrm{d}\tau=te^t$ (f) $y(t)-\displaystyle\int_0^t y(\tau)\ (t-\tau)\mathrm{d}\tau=2-\frac{1}{2}t^2$

10. Find the inverse Laplace transform of the following functions. ω is a constant.

(a) $\dfrac{2\pi s}{(s^2+\pi^2)^2}$ (b) $\dfrac{1}{s^2(s^2+4)}$ (c) $\dfrac{18s}{(s^2+36)^2}$

11. Prove the following change of scale result:

$$\mathcal{L}\{f(at)\}=\frac{1}{a}F\left(\frac{s}{a}\right)$$

Hence evaluate the Laplace transforms of the following functions

(a) $t\cos(6t)$ (b) $t^2\cos(7t)$

End of Tutorial Sheet