The University of Zambia
Department of Mathematics and Statistics
MAT 3110 - Engineering Mathematics II

Tutorial Sheet 2 - Power Series Method, Euler equations and systems of Linear ODEs April, 2024

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1. Apply the power series method to solve the following differential equations.

   (a) $y' = 2y$ (b) $y'' + y = 0$ (c) $y' = ky \quad k$ us a constant
   (d) $(1-x)y' = y$ (e) $(x+1)y' = 3y$ (f) $(1+x)y' + y = 0$
   (g) $y' + 2xy = 0$ (h) $y' = 3x^2y$ (i) $y'' - y = 0$
   (j) $y'' + 2xy = 0$ (k) $y'' - y' = 0$ (l) $y'' - 9y = 0$

2. Show that

$$\sum_{m=2}^{\infty} m(m-1)a_m x^{m-2} = \sum_{j=1}^{\infty}(j+1)ja_{j+1}x^{j-1} = \sum_{s=0}^{\infty}(s+2)(s+1)a_{s+2}x^s$$

3. For each of the series below, shift the index so that the power under the summation sign is x^m.

   (a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3n} x^{n+2}$ (b) $\sum_{s=1}^{\infty} \frac{s(s+1)}{s^2+1} x^{s-1}$ (c) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{6^k} x^{k-3}$

4. Find the general solution of the following differential equations.

   (a) $xy' = 3y + 3$ (b) $(x-3)y' - xy = 0$ (c) $y' = 2xy$
   (d) $(1-x^4)y' = 4x^3y$ (e) $(x+1)y' - (2x+3)y = 0$ (f) $(1+x)y'' - y = x$
   (g) $y'' - 3y' + 2y = 0$ (h) $y'' - 4xy' + (4x^2 - 2)y = 0$ (i) $(1-x^2)y'' - 2xy + 2y = 0$
   (j) $y'' - xy + y = 0$

5. Find the general solution of the following differential equations.

   (a) $x^2y'' - 6y = 0$ (b) $x^2y'' + 4xy' = 0$ (c) $x^2y'' - 2xy' + 2y = 0$
   (d) $x^2y'' + 9xy' + 16y = 0$ (e) $x^2y'' + xy' - y = 0$ (f) $x^2y'' + 3xy' + y = 0$
   (g) $x^2y'' + 3xy' + 5y = 0$ (h) $x^2y'' + xy' + y = 0$

6. Solve the following initial value problems.

   (a) $x^2y'' - 4xy' + 4y = 0,\ y(1) = 4,\ y'(1) = 13$
   (b) $4x^2y'' - 4xy' - y = 0,\ y(4) = 2,\ y'(4) = \frac{1}{4}$
   (c) $x^2y'' - 5xy' + 8y = 0,\ y(1) = 5,\ y'(1) = 18$

7. Convert each of the following linear ordinary differential equation into a system of first linear ordinary differential equations

   (a) $y'' - 4y' + 5y = 0$
   (b) $y''' - 5y'' + 9y = t\cos 2t$
   (c) $y'''' = 3y''' - \pi y'' + 2\pi y' - 6y = 11$

8. Rewrite each of the systems you found in the question above into a matrix-vector form

9. Find the eigenvalues and corresponding eigenvectors of the following matrices.

   (a) $\begin{pmatrix} 5 & -2 \\ 9 & -6 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 3 \\ -3 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$

   (e) $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

10. Find the general solution of each system below.

   (a) $\mathbf{x}' = \begin{pmatrix} 2 & 7 \\ -5 & -10 \end{pmatrix} \mathbf{x}$ (b) $\mathbf{x}' = \begin{pmatrix} -3 & 6 \\ -3 & 3 \end{pmatrix} \mathbf{x}$ (c) $\mathbf{x}' = \begin{pmatrix} 8 & -4 \\ 1 & 4 \end{pmatrix} \mathbf{x}$

   (d) $\mathbf{x}' = \begin{pmatrix} -3 & 2 \\ -1 & -5 \end{pmatrix} \mathbf{x}$ (e) $\mathbf{x}' = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \mathbf{x}$ (f) $\mathbf{x}' = \begin{pmatrix} 2 & -5 \\ 1 & -4 \end{pmatrix} \mathbf{x}$

End of Tutorial Sheet