# The University of Zambia
# Department of Mathematics and Statistics
# MAT 3110 - Engineering Mathematics II

**Tutorial Sheet 3 - Fourier Series** **June, 2024**

1. Find the fundamental period of the following trigonometric functions
   (a) $\cos x$ (b) $\sin x$ (c) $\cos(2x)$
   (d) $\sin(2x)$ (e) $\cos(\pi x)$ (f) $\sin(\pi x)$
   (g) $\cos(nx)$ (h) $\sin(nx)$ (i) $\cos\left(\frac{2\pi x}{k}\right)$
   (j) $\sin\left(\frac{2\pi x}{k}\right)$ (k) $\cos\left(\frac{2\pi x}{k}\right)$ (l) $\sin\left(\frac{2\pi x}{k}\right)$

2. If $f(x)$ and $g(x)$ have period $p$, show that $h(x) = af(x) + bg(x)$ has period $p$. $a$ and $b$ are real constants.

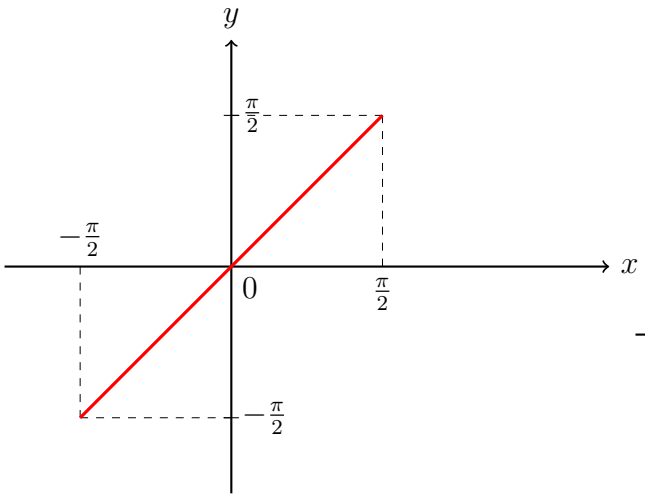
3. If $f(x)$ has period $p$, show that $f(ax)$, $a \neq 0$, and $f\left(\frac{x}{b}\right)$, $b \neq 0$, are periodic functions of $x$ of periods $\frac{p}{a}$ and $bp$, respectively. Give examples.

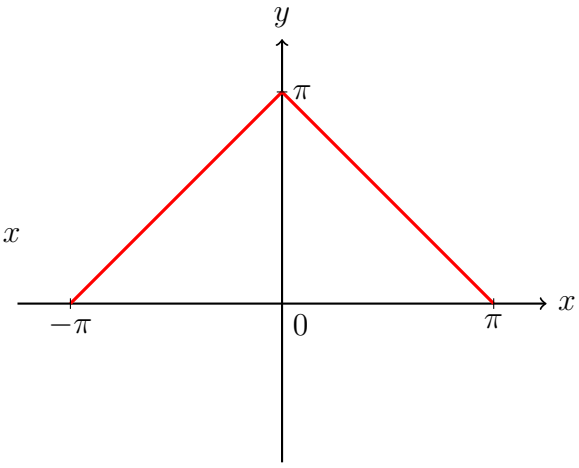
4. Show that the constant function $f(x) = c$ is periodic with any period but has no fundamental period.

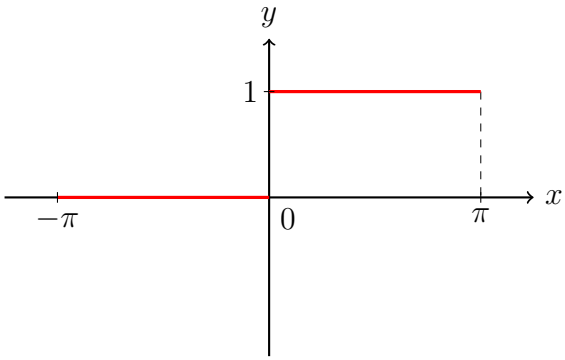
5. Sketch the graph of the following function in the interval $\pi < x < \pi$.
   (a) $f(x) = |x|$ (b) $f(x) = |\sin x|$
   (c) $f(x) = \sin|x|$ (d) $f(x) = e^{-|x|}$
   (e) $f(x) = |e^{-x}|$ (f) $f(x) = \begin{cases} x, & : -\pi < x < 0 \\ \pi - x, & : 0 < x < \pi. \end{cases}$
   (g) $f(x) = \begin{cases} -\cos^2 x, & : \text{ if } -\pi < x < 0 \\ \cos^2 x, & : \text{ if } 0 < x < \pi. \end{cases}$

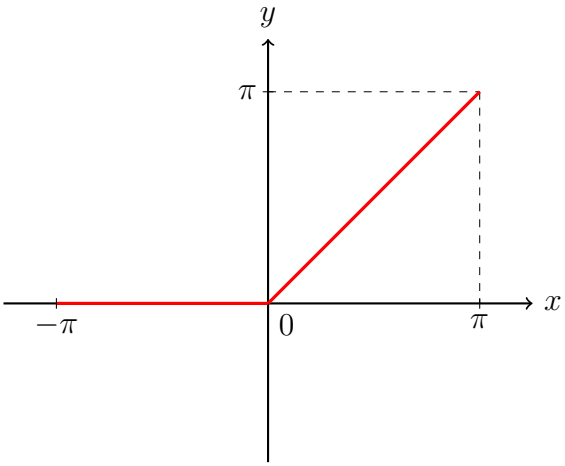
6. Find the Fourier series of the following functions, which are assumed to have period $2\pi$.
   (a) $f(x) = |x|$ (b) $f(x) = \begin{cases} x, & : -\pi < x < 0 \\ \pi - x, & : 0 < x < \pi. \end{cases}$
   (c) $f(x) = x^2 \quad (-\pi < x < \pi)$ (d) $f(x) = x^2 \quad (0 < x < 2\pi)$

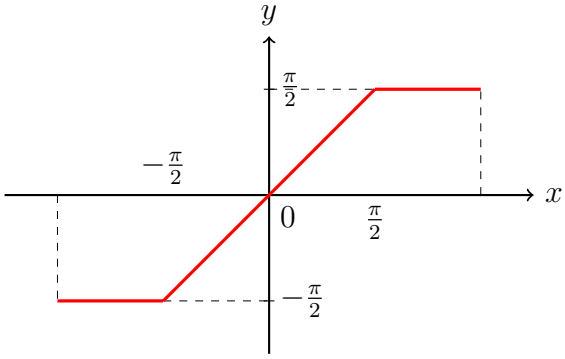
7.

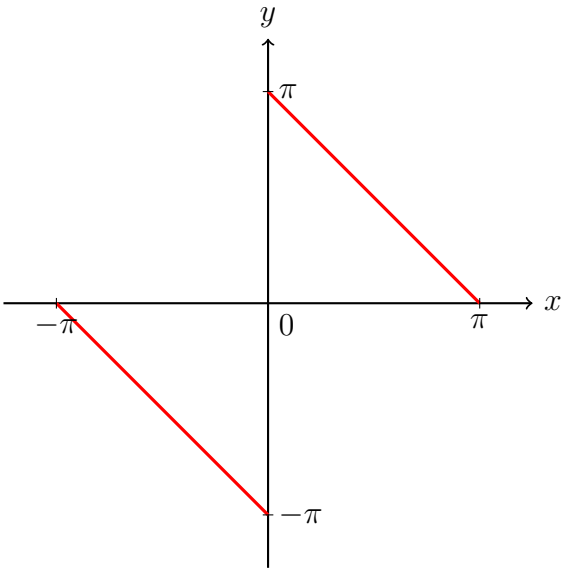


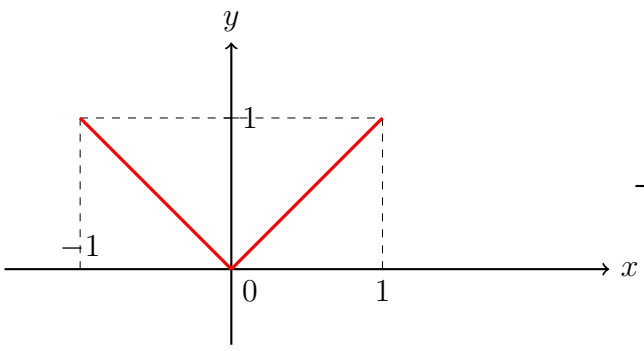

8. Are the following functions even or odd or neither even nor odd?
   (a) $e^x$
   (b) $e^{-|x|}$
   (c) $x^3 \cos(nx)$
   (d) $x^2 \tan(\pi x)$

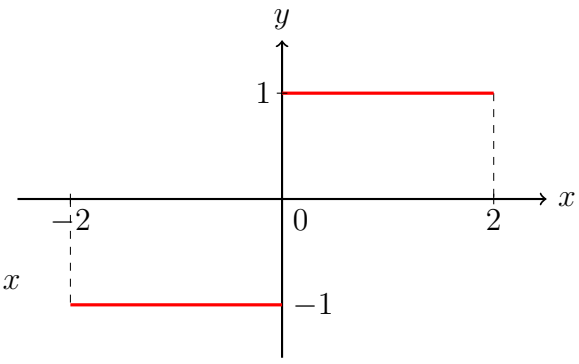
(e) $\sinh x - \cosh x$
(f) $\sin^2 x$
(g) $\sin(x^2)$
(h) $\ln x$
(i) $\frac{x}{x^2+1}$
(j) $x \cot x$

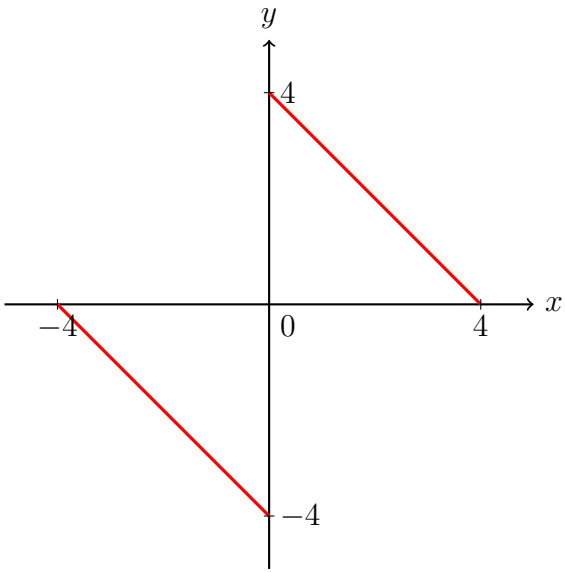
9. Are the following functions even or odd or neither even nor odd?
   (a) Sums and products of even functions
   (b) Sums and products of odd functions
   (c) Absolute values of odd functions
   (d) Product of an odd and an even function

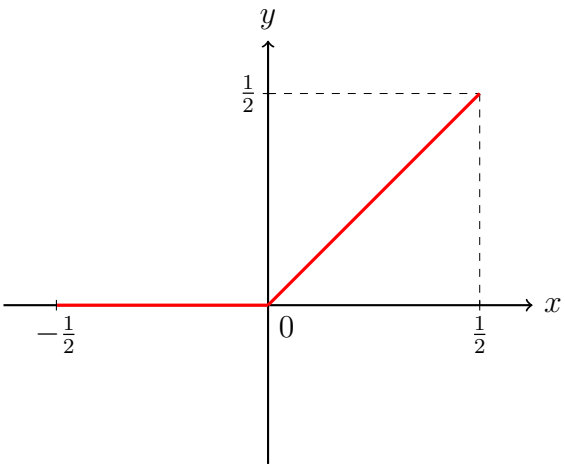
10. Consider the following functions. Are the functions even, odd, or neither even nor odd. Find their Fourier series.
   (a) $f(x) = x^2 \quad (-1 < x < 1),$ period $= 2$
   (b) $f(x) = 1 - \frac{x^2}{4} \quad (-2 < x < 2),$ period $= 4$
   (c) $f(x) = \cos(\pi x) \quad \left(-\frac{1}{2} < x < \frac{1}{2}\right),$ period $= 1$
   (d) $f(x) = x|x| \quad (-1 < x < 1),$ period $= 2$

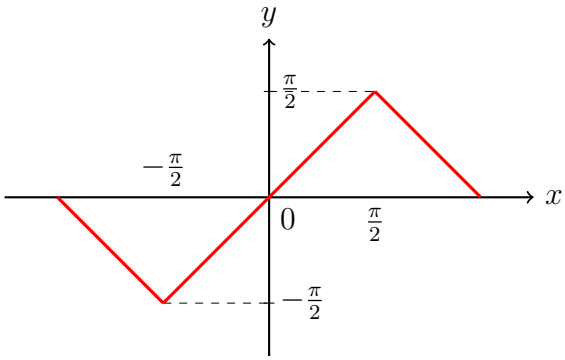
11. Consider the graphs of functions given below. Are the functions even, odd, or neither even nor odd. Find their Fourier series.

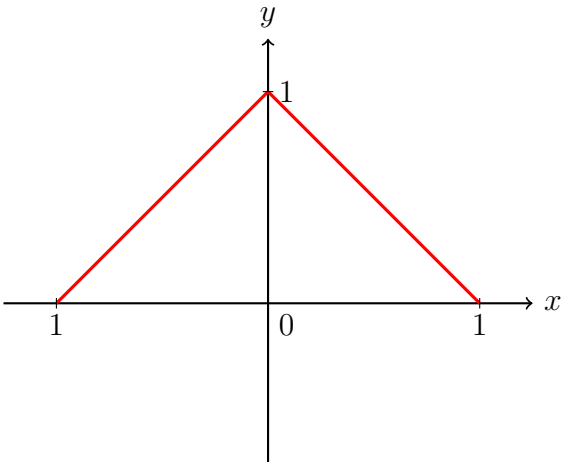





**End of Tutorial Sheet**