The University of Zambia
Department of Mathematics and Statistics
MAT 3110 - Engineering Mathematics II

Tutorial Sheet 4 - Lines, Planes and Surfaces in 3D-space **August, 2024**

1. Find the parametric equations for the lines in the following.
   (a) The line through $(2, 4, 5)$ perpendicular to the plane $3x + 7y - 5y = 21$
   (b) The line through $(2, 3, 0)$ perpendicular to the vectors $-\mathbf{i}+2\,\mathbf{j}+3\,\mathbf{k}$ and $3\,\mathbf{i}+4\,\mathbf{j}+5\,\mathbf{k}$.

2. Find the equations for each of the following plane.
   (a) The plane through the points $(1, 1, -1)$, $(2, 0, 2)$ and $(0, -2, 1)$.
   (b) The plane through the point $(2, 4, 5)$ perpendicular to the line

$$x = 5 + t,\ y = 1 + 3t,\ z = 4t.$$

3. Find the distance from the line $x = 2 + t,\ y = 1 + t, z = -\frac{1}{2} - \frac{1}{2}t$ to the plane $x + 2y + 6z = 10$.

4. Find the point where the line $x = -1 + 3t,\ y = -2,\ z = 5t$ and plane $2x - 3z = 7$ intersect.

5. Find the equation of a line in which the planes $5x - 2y = 11$ and $4y - 5z = -17$ intersect.

6. Sketch graphs of the following surfaces in 3D-space.

(a) $x^2 + y^2 + z^2 = 4$	(b) $4x^2 + 4y^2 = z^2$	(c) $z = 1 + y^2 - x^2$
(d) $y^2 - z^2 = 4$	(e) $y = -(x^2 + z^2)$	(f) $z^2 - 4x^2 - 4y^2 = 4$
(g) $16x^2 + 4y^2 = 1$	(h) $z = x^2 + y^2 + 1$	(i) $x^2 + y^2 - z^2 = 4$
(j) $x = 4 - y^2$	(k) $x^2 + z^2 = y$	(l) $z^2 - \left(\frac{x^2}{4}\right) - y^2 = 1$
(m) $x^2 + z^2 = 1$	(n) $4x^2 + 4y^2 + z^2 = 4$	(o) $16x^2 + 9z^2 = 4x^2$
(p) $z = x^2 - y^2 - 1$	(q) $9x^2 + 4y^2 + z^2 = 36$	(r) $4x^2 + 9y^2 = y^2$
(s) $x^2 + y^2 - 16z^2 = 16$	(t) $z^2 + 4y^2 = 9$	(u) $z = -(x^2 + y^2)$
(v) $y^2 - x^2 - z^2 = 1$	(w) $x^2 - 4y^2 = 1$	(x) $z = 4x^2 + y^2 - 4$

7. Sketch graphs of the following surfaces in 3D-space.

(a) $4y^2 + z^2 - 4x^2 = 4$	(b) $z = 1 - x^2$
(c) $x^2 + y^2 = z$	(d) $\left(\frac{x^2}{4}\right) + y^2 - z^2 = 1$
(e) $yz = 1$	(f) $36x^2 + 9y^2 + 4z^2 = 36$
(g) $9x^2 + 16y^2 = 4z^2$	(h) $4z^2 - x^2 - y^2 = 4$

8. The following table gives the coordinates of specific points in space in one of the three coordinate systems. Find the coordinates for each point in the other two systems. There may be more than one right answer because the points in cylindrical and spherical coordinates may have more than one coordinate triple.

	Rectangular (x, y, z)	Cylindrical (r, θ, z)	Spherical (ρ, θ, ϕ)
1	$(0, 0, 0)$		
2	$(1, 0, 0)$		
3	$(0, 1, 0)$		
4	$(0, 0, 1)$		
5		$(1, 0, 0)$	
6		$\left(\sqrt{2}, 0, 1\right)$	
7		$\left(1, \frac{\pi}{2}, 1\right)$	
8			$\left(\sqrt{3}, -\frac{\pi}{2}, \frac{\pi}{3}\right)$
9			$\left(2\sqrt{2}, \frac{3\pi}{2}, \frac{\pi}{2}\right)$
10			$\left(\sqrt{2}, \frac{2\pi}{3}, \pi\right)$

9. Translate the equations and inequalities from the given coordinate system (rectangular, cylindrical, spherical) into equations and inequalities in the other two systems. Also, identify the figure being defined.

(a) $r = 0$
(b) $x^2 + y^2 = 5$
(c) $z = 0$
(d) $z = -2$
(e) $z = \sqrt{x^2 + y^2},\ z \leq 1$
(f) $z = \sqrt{x^2 + y^2},\ 1 \leq z \leq 2$
(g) $\rho \sin\phi \cos\theta = 0$
(h) $\tan^2\phi = 1$
(i) $x^2 + y^2 + z^2 = 4$
(j) $x^2 + y^2 + \left(z - \frac{1}{2}\right)^2 = \frac{1}{4}$
(k) $\rho = 5\cos\phi$
(l) $\rho = -6\cos\phi$
(m) $r = \csc\theta$
(n) $r = -3\sec\theta$
(o) $\rho = \sqrt{2}\sec\phi$
(p) $\rho = 9\csc\phi$
(q) $x^2 + y^2 + (z-1)^2 = 1,\ z \leq 1$
(r) $r^2 + z^2 = 4,\ z \leq -\sqrt{2}$
(s) $\rho = 3, \frac{\pi}{3} \leq \phi \leq \frac{2\pi}{3}$
(t) $x^2 + y^2 + z^2 = 3,\ 0 \leq z \leq \frac{\sqrt{3}}{2}$
(u) $z = 4 - 4r^2,\ 0 \leq r \leq 1$
(v) $z = 4 - r,\ 0 \leq r \leq 4$
(w) $\phi = \frac{3\pi}{4},\ 0 \leq \rho \leq \sqrt{2}$
(x) $\phi = \frac{\pi}{2},\ 0 \leq \rho \leq \sqrt{7}$
(y) $z + r^2\cos(2\theta) = 0$
(z) $z^2 - r^2 = 1$

10. Find the rectangular coordinates of the center of the sphere

$$r^2 + z^2 = 4r\cos\theta + 6r\sin\theta + 2z.$$

11. Find the rectangular coordinates of the center of the sphere

$$\rho = 2\sin\phi\,(\cos\theta - 2\sin\theta)\,.$$

End of Tutorial Sheet