The University of Zambia
Department of Mathematics and Statistics
MAT 3110 - Engineering Mathematics II

Tutorial Sheet 5 - Double Integrals **August, 2024**

1. Sketch the region of integration and evaluate the integral.

(a) $$\int_0^1 \int_0^2 \left(4 - y^2\right)\ \mathrm{d}y\mathrm{d}x$$

(b) $$\int_0^3 \int_{-2}^0 \left(x^2 y - 2xy\right)\ \mathrm{d}y\mathrm{d}x$$

(c) $$\int_{-1}^0 \int_{-1}^1 (x + y + 1)\ \mathrm{d}x\mathrm{d}y$$

(d) $$\int_{\pi}^{2\pi} \int_0^{\pi} (\sin x + \cos y)\ \mathrm{d}x\mathrm{d}y$$

(e) $$\int_0^{\pi} \int_0^{x} x \sin y\ \mathrm{d}y\mathrm{d}x$$

(f) $$\int_0^{\pi} \int_0^{\sin x} y\ \mathrm{d}y\mathrm{d}x$$

(g) $$\int_1^{\ln 8} \int_0^{\ln y} e^{x+y}\ \mathrm{d}x\mathrm{d}y$$

(h) $$\int_1^2 \int_y^{y^2} \mathrm{d}x\mathrm{d}y$$

(i) $$\int_0^1 \int_0^{y^2} 3y^3 e^{xy}\ \mathrm{d}y\mathrm{d}x$$

(j) $$\int_0^1 \int_0^2 \left(4 - y^2\right) \mathrm{d}y\mathrm{d}x$$

2. Perform a double integral over the given function f over the given region

(a) $f(x,y) = \frac{x}{y}$ over the region in the first quadrant bounded by the lines $y = x$, $y = 2x$, $x = 1$ and $x = 2$.

(b) $f(x,y) = \frac{1}{xy}$ over the square $1 \leq x \leq 2, 1 \leq y \leq 2$.

(c) $f(x,y) = x^2 + y^2$ over the triangular region with vertices $(0,0)$, $(1,0)$ and $(0,1)$.

(d) $f(x,y) = y\cos(xy)$ over the rectangle $0 \leq x \leq \pi$, $0 \leq y \leq 1$.

(e) $f(x,y) = y - \sqrt{x}$ over the triangular region cut from the first quadrant of the xy-plane by the line $x + y = 1$.

(f) $f(x,y) = e^x \ln y$ over the region in the first quadrant of the xy-plane that lies above the curve $x = \ln y$ from $y = 1$ and $y = 2$.

3. Sketch the region of integration and write an equivalent double integral with the order of integration reversed.

(a) $$\int_0^1 \int_2^{4-2x} \mathrm{d}y\mathrm{d}x$$

(b) $$\int_0^2 \int_{y-2}^0 \mathrm{d}x\mathrm{d}y$$

(c) $$\int_0^1 \int_y^{\sqrt{y}} \mathrm{d}x\mathrm{d}y$$

(d) $$\int_0^1 \int_{1-x}^{1-x^2} \mathrm{d}y\mathrm{d}x$$

(e) $$\int_0^1 \int_0^{e^x} \mathrm{d}y\mathrm{d}x$$

(f) $$\int_0^{\ln 2} \int_{e^y}^2 \mathrm{d}x\mathrm{d}y$$

(g) $$\int_0^{\frac{3}{2}} \int_0^{9-4x^2} 16x\ \mathrm{d}y\mathrm{d}x$$

(h) $$\int_0^2 \int_0^{4-y^2} y\ \mathrm{d}x\mathrm{d}y$$

(i) $$\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y\ \mathrm{d}x\mathrm{d}y$$

(j) $$\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 6x\ \mathrm{d}y\mathrm{d}x$$

4. Sketch the region of integration, determine the order of integration, and evaluate the integral.

(a) $$\int_0^{\pi}\int_x^{\pi}\frac{\sin y}{y}\ \mathrm{d}y\mathrm{d}x$$

(b) $$\int_0^2\int_x^2 2y^2\sin(xy)\ \mathrm{d}y\mathrm{d}x$$

(c) $$\int_0^1\int_y^1 x^2e^{xy}\ \mathrm{d}x\mathrm{d}y$$

(d) $$\int_0^2\int_0^{4-x^2}\frac{xe^{2y}}{4-y}\ \mathrm{d}y\mathrm{d}x$$

(e) $$\int_0^{2\sqrt{\ln 3}}\int_{\frac{y}{2}}^{\sqrt{\ln 3}} e^{x^2}\ \mathrm{d}y\mathrm{d}x$$

(f) $$\int_0^3\int_{\sqrt{\frac{x}{3}}}^1 e^{y^3}\ \mathrm{d}y\mathrm{d}x$$

(g) $$\int_0^{\frac{1}{16}}\int_{y^{\frac{1}{4}}}^{\frac{1}{2}}\cos\left(16\pi x^5\right)\ \mathrm{d}x\mathrm{d}y$$

(h) $$\int_0^8\int_{\sqrt[3]{x}}^2\frac{1}{y^4+1}\ \mathrm{d}x\mathrm{d}y$$

5. Evaluate $\int\int_R \left(y-2x^2\right)\ \mathrm{d}A$ where R is the region inside $|x|+|y|=1$

6. Evaluate $\int\int_R xy\ \mathrm{d}A$ where R is the region bounded by the lines $y=x$, $y=2x$, and $x+y=2$.

7. Find the volume of the region that lies under $z=x^2+y^2$ and above the triangle enclosed by the lines $y=x$, $x=0$, and $x+y=2$ in the xy-plane.

8. Find the volume of the solid that is bounded above by $z=x^2$ and below by the region enclosed by $y=2-x^2$ and the line $y=x$ in the xy-plane.

9. Find the volume of the solid whose base is the region in the xy-plane that is bounded by $y=4-x^2$ and the line $y=3x$, while the top of the solid is bounded by the plane $z=x+4$.

10. Find the volume of the solid of the solid in the first octant bounded by the coordinate planes, $x^2+y^2=4$, and $z+y=3$.

11. Find the volume of the solid in the first octant bounded by the coordinate planes, $x=3$, and $z=4-y^2$.

12. Find the volume of the solid cut from the first octant by the surface $z=4-x^2-y^2$.

13. Find the volume of the wedge cut from the first octant by $z=12-3y^2$ and the plane $x+y=2$.

14. Find the volume of the solid cut from the square column $|x|+|y|\le 1$ by $z=0$ and $3x+z=3$.

15. Find the volume of the solid that is bounded on the front and back by the planes $x=2$ and $x=1$, on the sides by $y=\pm\frac{1}{x}$, and above and below by $z=x+1$ and $z=0$.

16. Find the volume of the solid that is bounded on the front and back by $x=\pm\frac{\pi}{3}$, on the sides by $y=\pm\sec x$, above by $z=1+y^2$, and below by the xy-plane.

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