The University of Zambia
Department of Mathematics and Statistics
MAT 3110 - Engineering Mathematics II

Tutorial Sheet 7 - Triple Integrals in Cartesian Coordinates **Sept, 2024**

1. Write six different triple integrals for the volume of the rectangular solid in the first octant bounded by the coordinate planes and the planes $x = 1$, $y = 2$, and $z = 3$. Evaluate each of the six integrals to check if you get the same value.

2. Write six different triple integrals for the volume of the tetrahedron cut from the first octant by the plane $6x + 3y + 2z = 6$. Evaluate each of the six integrals to check if you get the same value.

3. Write six different triple integrals for the volume of the region in the first octant enclosed by the cylinder $x^2 + z^2 = 4$ and the plane $y = 3$. Evaluate each of the six integrals to check if you get the same value.

4. Let D be the region bounded by the paraboloids $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$. Write six different triple integrals for the volume of D. Evaluate each of the six integrals to check if you get the same value.

5. Let D be the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 2y$. Write six different triple integrals for the volume of D. Evaluate each of the six integrals to check if you get the same value.

6. Evaluate the following integrals.

   (a) $\displaystyle\int_0^1\int_0^1\int_0^1 \left(x^2+y^2+z^2\right)\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$

   (b) $\displaystyle\int_0^{\sqrt{2}}\int_0^{3y}\int_{x^2+3y^2}^{8-x^2-y^2} \mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y$

   (c) $\displaystyle\int_1^e\int_1^e\int_1^e \frac{1}{xyz}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$

   (d) $\displaystyle\int_0^1\int_0^{3-3x}\int_0^{3-3x-y} \mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y$

   (e) $\displaystyle\int_0^1\int_0^{\pi}\int_0^{\pi} y\sin z\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$

   (f) $\displaystyle\int_{-1}^1\int_{-1}^1\int_{-1}^1 (x+y+z)\,\mathrm{d}y\,\mathrm{d}x\,\mathrm{d}z$

   (g) $\displaystyle\int_0^3\int_0^{\sqrt{9-x^2}}\int_0^{\sqrt{9-x^2}} \mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$

   (h) $\displaystyle\int_0^2\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_0^{2x+y} \mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y$

   (i) $\displaystyle\int_0^1\int_0^{2-x}\int_0^{2-x-y} \mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$

   (j) $\displaystyle\int_0^1\int_0^{1-x^2}\int_3^{4-x^2-y} x\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$

   (k) $\displaystyle\int_0^{\pi}\int_0^{\pi}\int_0^{\pi} \cos\left(x+y+z\right)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$

   (l) $\displaystyle\int_1^e\int_1^e\int_1^e \ln z\ln y\ln x\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$

   (m) $\displaystyle\int_0^{\frac{\pi}{4}}\int_0^{\ln\sec z}\int_{-\infty}^{2y} e^x\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$

   (n) $\displaystyle\int_0^7\int_0^2\int_0^{\sqrt{4-x^2}} \frac{x}{z+1}\,\mathrm{d}y\,\mathrm{d}x\,\mathrm{d}z$

7. Find the volume of the following solids using triple integrals.
   (a) The solid that is bounded on top by $y + z = 1$, on the side by $y = x^2$ and on the bottom by $z = 0$.
   (b) The solid that is bounded by the planes $z = 0$, $y = -1$, $x = 1$ and $x = 0$, bounded by the surface $z = y^2$.
   (c) The solid between the cylinder $z = y^2$ and the xy-plane that is bounded by the planes $x = 0$, $x = 1$, $y = -1$ and $y = 1$.
   (d) The solid in the first octant bounded by the coordinate planes and the planes $x + z = 1$, $y + 2z = 2$.
   (e) The solid in the first octant bounded by the coordinate planes, the plane $y + z = 2$, and the cylinder $x = 4 - y^2$.
   (f) The solid cut from the cylinder $x^2 + y^2 = 1$ by the planes $z = -y$ and $z = 0$.
   (g) The tetrahedron in the first octant bounded by the coordinate planes and the plane $x + \frac{y}{2} + \frac{z}{3} = 1$.
   (h) The solid in the first octant bounded by the coordinate planes, the plane $y = 1 - x$, and the surface $z = \cos\left(\frac{\pi x}{2}\right)$, $0 \leq x \leq 1$.
   (i) The solid that is inside the cylinder $x^2 + y^2 = 1$ and inside the cylinder $x^2 + z^2 = 1$.
   (j) The solid in the first octant bounded by the coordinate planes the surface $z = 4 - x^2 - y^2$.
   (k) The solid in the first octant bounded by the coordinate planes, the plane $x + y = 4$, and the cylinder $y^2 + 4z^2 = 16$.
   (l) The solid cut from the cylinder $x^2 + y^2 = 4$ by the plane $z = 0$ and the plane $x + z = 3$.
   (m) The solid between the planes $x + y + 2z = 2$ and $2x + 2y + z = 4$ in the first octant.
   (n) The solid bounded by the planes $z = x$, $x + z = 8$, $z = y$, $y = 8$, and $z = 0$.
   (o) The solid cut from the solid elliptical cylinder $x^2 + 4y^2 \leq 4$ by the xy-plane and the plane $z = x + 2$.
   (p) The solid bounded in the back by the plane $x = 0$, on the front and sides by the parabolic cylinder $x = 1 - y^2$, on the top by the paraboloid $z = x^2 + y^2$, and on the bottom by the xy-plane.

8. Evaluate the following integrals.
   (a) $\displaystyle\int_0^4 \int_0^1 \int_{2y}^2 \frac{4\cos(x^2)}{2\sqrt{z}}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$
   (b) $\displaystyle\int_0^1 \int_0^1 \int_{x^2}^1 12xze^{zy^2}\,\mathrm{d}y\,\mathrm{d}x\,\mathrm{d}z$
   (c) $\displaystyle\int_0^1 \int_{\sqrt[3]{z}}^1 \int_0^{\ln 3} \frac{\pi e^{2x}\sin(\pi y^2)}{y^2}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$
   (d) $\displaystyle\int_0^2 \int_0^{4-x^2} \int_0^x \frac{2\sin(2z)}{4-z}\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}x$

End of Tutorial Sheet