The University of Zambia
Department of Mathematics and Statistics
MAT 3110 - Engineering Mathematics II

Tutorial Sheet 8 - Triple Integrals in Cylindrical Coordinates   Sept, 2024

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1. Evaluate the following cylindrical coordinate integrals.

   (a) $\displaystyle\int_0^{2\pi}\int_0^1\int_r^{\sqrt{2-r^2}} r\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$
   (b) $\displaystyle\int_0^{2\pi}\int_0^3\int_{\frac{r^2}{3}}^{\sqrt{18-r^2}} r\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

   (c) $\displaystyle\int_0^{2\pi}\int_0^{\frac{\theta}{2\pi}}\int_0^{3+24r^2} r\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$
   (d) $\displaystyle\int_0^{\pi}\int_0^{\frac{\theta}{\pi}}\int_{-\sqrt{4-r^2}}^{3\sqrt{4-r^2}} zr\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

   (e) $\displaystyle\int_0^{2\pi}\int_0^1\int_r^{\frac{1}{\sqrt{2-r^2}}} 3r\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$
   (f) $\displaystyle\int_0^{2\pi}\int_0^1\int_{-\frac{1}{2}}^{\frac{1}{2}} \left(r^2\sin^2\theta+z^2\right) r\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

2. Evaluate the following cylindrical coordinate integrals.

   (a)
   $$\int_0^{2\pi}\int_0^3\int_0^{\frac{z}{3}} r^3\,\mathrm{d}r\,\mathrm{d}z\,\mathrm{d}\theta$$
   (b)
   $$\int_{-1}^1\int_0^{2\pi}\int_0^{1+\cos\theta} 4r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}z$$
   (c)
   $$\int_0^1\int_0^{\sqrt{z}}\int_0^{2\pi} \left(r^2\cos^2\theta+z^2\right) r\,\mathrm{d}\theta\,\mathrm{d}r\,\mathrm{d}z$$
   (d)
   $$\int_0^2\int_{r-2}^{\sqrt{4-r^2}}\int_0^{2\pi} \left(r\sin\theta+1\right) r\,\mathrm{d}\theta\,\mathrm{d}z\,\mathrm{d}r$$

3. Let D be the region bounded below by the plane $z=0$, above by the sphere $x^2+y^2+z^2=4$, and on the sides by cylinder $x^2+y^2=1$. Set up the triple integrals in cylindrical coordinates that give the volume of D using the following orders of integration.

   (a) $\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$ (b) $\mathrm{d}r\,\mathrm{d}z\,\mathrm{d}\theta$ (c) $\mathrm{d}\theta\,\mathrm{d}z\,\mathrm{d}r$

4. Let D be the region bounded below by the cone $z=\sqrt{x^2+y^2}$ and above by the paraboloid $z=2=x^2-y^2$. Set up the triple integrals in cylindrical coordinates that give the volume of D using the following orders of integration.

   (a) $\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$ (b) $\mathrm{d}r\,\mathrm{d}z\,\mathrm{d}\theta$ (c) $\mathrm{d}\theta\,\mathrm{d}z\,\mathrm{d}r$

5. Give the limits of integration for evaluating the integral

   $$\int\int\int f\left(r,\theta,z\right) r\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$$

   as an iterated over the region that is bounded below by the plane $z=0$, on the side by the cylinder $r=\cos\theta$, and on top by the paraboloid $z=3r^2$.

6. Convert the integral
$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{0}^{x}\left(x^2+y^2\right)\,\mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y$$
to an equivalent integral in cylindrical coordinates and evaluate the result.

7. Find the volume of the following solids using a triple integral in cylindrical coordinates.
   (a) The cylinder whose base is the circle $r = 2\sin\theta$ in the xy-plane and whose top lies in the plane $z = 4 - y$.
   (b) The cylinder whose base is the circle $r = 3\cos\theta$ in the xy-plane and whose top lies in the plane $z = 5 - x$.
   (c) The cylinder whose base is the region in the xy-plane that lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$ and whose top lies in the plane $z = 4$.
   (d) The cylinder whose base is the region in the xy-plane that between the circles $r = \cos\theta$ and $r = 2\cos\theta$, and whose top lies in the plane $z = 3 - y$.
   (e) The prism whose base is the triangle in the xy-plane bounded by the x-axis and the lines $y = x$ and $x = 1$ and whose top lies in the plane $z = 2 - y$.
   (f) The prism whose base is the triangle in the xy-plane bounded by the y-axis and the lines $y = x$ and $y = 1$ and whose top lies in the plane $z = 2 - x$.

End of Tutorial Sheet