The University of Zambia
Department of Mathematics and Statistics
MAT 3110 - Engineering Mathematics II

Tutorial Sheet 11 - Surface Integrals **October, 2024**

1. Find a parametrization of the following surfaces. (There are many correct ways to do this)
   (a) The paraboloid $z = x^2 + y^2$, $z \leq 4$.
   (b) The paraboloid $z = 9 - x^2 - y^2$, $z \geq 0$.
   (c) The first-octant portion of the cone $z = \frac{\sqrt{x^2+y^2}}{2}$ between $z = 0$ and $z = 3$.
   (d) The cap cut from the sphere $x^2 + y^2 + z^2 = 9$ by the cone $z = \sqrt{x^2 + y^2}$.
   (e) The portion of the sphere $x^2 + y^2 + z^2 = 4$ in the first octant between the xy-plane and the cone $z = \sqrt{x^2 + y^2}$.
   (f) The portion of the sphere $x^2 + y^2 + z^2 = 3$ between the plane $z = \frac{\sqrt{3}}{2}$ and $z = \frac{-\sqrt{3}}{2}$.
   (g) The upper portion cut from the sphere $x^2 + y^2 + z^2 = 8$ by the plane $z = -2$.
   (h) The surfac cut from the parabolic cylinder $z = 4 - y^2$ by the planes $x = 0$, $x = 2$, and $z = 0$.
   (i) The surface cut from the parabolic cylinder $y = x^2$ by the planes $z = 0$, $z = 3$, and $y = 2$.
   (j) The portion of the cylinder $y^2 + z^2 = 9$ between the planes $x = 0$ and $x = 3$.
   (k) The portion of the cylinder $x^2 + z^2 = 4$ above the xy-plane between the planes $y = -2$ and $y = 2$.
   (l) The portion of the plane $x + y + z = 1$.
      i. inside the cylinder $x^2 + y^2 = 9$
      ii. inside the cylinder $y^2 + z^2 = 9$
   (m) The portion of the plane $x - y + 2z = 2$
      i. inside the cylinder $x^2 + z^2 = 3$
      ii. inside the cylinder $y^2 + z^2 = 2$
   (n) The portion of the cylinder $(x - 2)^2 + z^2 = 4$ between the planes $y = 0$ and $y = 3$
   (o) The portion of the cylinder $y^2 + (z - 5)^2 = 25$ between the planes $x = 0$ and $x = 10$

2. Use a parametrization to express the area of each of the following surfaces as a double integral. The evaluate the integral. (There are many correct ways to set up the integrals).
   (a) The portion of the plane $y + 2z = 2$ inside the cylinder $x^2 + y^2 = 1$
   (b) The portion of the plane $z = -x$ inside the cylinder $x^2 + y^2 = 4$
   (c) The portion of the cone $z = 2\sqrt{x^2 + y^2}$ between the planes $z = 2$ and $z = 6$

(d) The portion of the cone $z = \frac{\sqrt{x^2+y^2}}{3}$ between the planes $z = 1$ and $z = \frac{4}{3}$

(e) The portion of the cylinder $x^2 + y^2 = 1$ between the planes $z = 1$ and $z = -4$

(f) The portion of the cylinder $x^2 + z^2 = 10$ between the planes $y = -1$ and $y = 1$

(g) The cap cut from the paraboloid $z = 2 - x^2 - y^2$ by the cone $z = \sqrt{x^2 + y^2}$

(h) The portion of the paraboloid $z = x^2 + y^2$ between the planes $z = 1$ and $z = 4$

(i) The lower portion cut from the sphere $x^2 + y^2 + z^2 = 2$ by the cone $z = \sqrt{x^2 + y^2}$

(j) The portion of the sphere $x^2 + y^2 + z^2 = 4$ between the planes $z = -1$ and $z = \sqrt{3}$

3. Use a parametrization to find the flux

$$\int\int_S \vec{\mathbf{F}} \cdot \mathrm{d}\vec{S}$$

across the surface in the given direction.

(a) $\vec{F} = z^2\ \mathbf{i} + x\ \mathbf{j} - 3z\ \mathbf{k}$ outward (normal away from the x-axis) through the surface cut from the parabolic cylinder $z = 4 - y^2$ by the planes $x = 0$, $x = 1$, and $z = 0$.

(b) $\vec{F} = x^2\ \mathbf{i} - xz\ \mathbf{k}$ outward (normal away from the yz-plane) through the surface cut from the parabolic cylinder $y = x^2$, $-1 \leq x \leq 1$, by the planes $z = 0$ and $z = 2$.

(c) $\vec{F} = z\ \mathbf{k}$ across the portion of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant in the direction away from the origin.

(d) $\vec{F} = x\ \mathbf{i} + y\ \mathbf{j} + z\ \mathbf{k}$ across the sphere $x^2 + y^2 + z^2 = 1$ in the direction away from the origin.

(e) $\vec{F} = 2xy\ \mathbf{i} + 2yz\ \mathbf{j} + 2xz\ \mathbf{k}$ upward across the portion of the plane $x + y + z = 2$ that lies above the square $0 \leq x \leq 1$, $0 \leq y \leq 1$ in the xy-plane.

(f) $\vec{F} = x\ \mathbf{i} + y\ \mathbf{j} + z\ \mathbf{k}$ outward through the portion of the cylinder $x^2 + y^2 = 1$ cut by the planes $z = 0$ and $z = 1$.

(g) $\vec{F} = xy\ \mathbf{i} - z\ \mathbf{k}$ outward (normal away from the z-axis) through the cone $z = \sqrt{x^2 + y^2}$, $0 \leq z \leq 1$.

(h) $\vec{F} = y^2\ \mathbf{i} + xz\ \mathbf{j} -\ \mathbf{k}$ outward (normal away from the z-axis) through the cone $z = 2\sqrt{x^2 + y^2}$, $0 \leq z \leq 2$.

(i) $\vec{F} = -x\ \mathbf{i} - y\ \mathbf{j} + z^2\ \mathbf{k}$ outward (normal away from the z-axis) through the portion of the cone $z = \sqrt{x^2 + y^2}$ between the planes $z = 1$ and $z = 2$.

(j) $\vec{F} = 4x\ \mathbf{i} + 4y\ \mathbf{j} + 2\ \mathbf{k}$ outward (normal away from the z-axis) through the surface cut from the botton of the paraboloid $z = x^2 + y^2$ by the plane $z = 1$.

End of Tutorial Sheet