Q6

(b) $f(x) = \begin{cases} x, & : -\pi < x < 0 \\ \pi - x, & : 0 < x < 2\pi \end{cases}$

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)\,dx = \frac{1}{2\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{2\pi}\int_0^{\pi}(\pi - x)\,dx$$

$$a_0 = \frac{1}{2\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{1}{2\pi}\left[\pi x - \frac{x^2}{2}\right]_0^{\pi} = \frac{1}{2\pi}\left[\frac{0^2}{2} - \frac{\pi^2}{2}\right] + \frac{1}{2\pi}\left[\pi^2 - \frac{\pi^2}{2}\right]$$

$$a_0 = -\frac{\pi}{4} + \frac{\pi}{4} = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0} x\cos(nx)\,dx + \frac{1}{\pi}\int_0^{\pi}(\pi\cos(nx) - x\cos(nx))\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0} x\cos(nx)\,dx + \int_0^{\pi}\cos(nx)\,dx - \frac{1}{\pi}\int_0^{\pi} x\cos(nx)\,dx$$

$U = x \quad dv = \cos(nx)\,dx$

$du = dx \quad v = \frac{\sin(nx)}{n}$

$N = x \quad dm = \cos(nx)\,dx$

$dN = dx \quad m = \frac{\sin(nx)}{n}$

$$a_n = \frac{1}{\pi}\left[\underbrace{\frac{x\sin(nx)}{n}\Big|_{-\pi}^{0}}_{0} - \frac{1}{n}\int_{-\pi}^{0}\sin(nx)\,dx\right] - \frac{1}{\pi}\left[\underbrace{\frac{x\sin(nx)}{n}\Big|_0^{\pi}}_{0} - \frac{1}{n}\int_0^{\pi}\sin(nx)\,dx\right]$$

$$a_n = -\frac{1}{\pi n}\left[-\frac{\cos(nx)}{n}\right]_{-\pi}^{0} + \frac{1}{\pi n}\left[-\frac{\cos(nx)}{n}\right]_0^{\pi}$$

$$a_n = \frac{1}{\pi n^2}\left[1 - \cos(n\pi)\right] + \frac{1}{\pi n^2}\left[-\cos(n\pi) + 1\right]$$

$$a_n = \frac{1}{\pi n^2}\left[1 - (-1)^n - (-1)^n + 1\right] = \frac{1}{\pi n^2}\left[2 - 2(-1)^n\right]$$

$$a_n = \frac{2}{\pi n^2}\left[1 - (-1)^n\right]$$

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$$b_n = \frac{1}{\pi}\int_{-\pi}^{0} x\sin(nx)\,dx + \frac{1}{\pi}\int_{0}^{\pi}(\pi - x)\sin(nx)\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{0} x\sin(nx)\,dx + \frac{1}{\pi}\int_{0}^{\pi}(\pi\sin(nx) - x\sin(nx))\,dx$$

$$b_n = \underbrace{\frac{1}{\pi}\int_{-\pi}^{0} x\sin(nx)\,dx}_{A} + \underbrace{\int_{0}^{\pi}\sin(nx)\,dx}_{B} - \underbrace{\frac{1}{\pi}\int_{0}^{\pi} x\sin(nx)\,dx}_{C}$$

A

$$\left.\begin{array}{ll} u = x & dv = \sin(nx)\,dx \\ du = dx & v = -\dfrac{\cos(nx)}{n} \end{array}\right\} = \frac{1}{\pi}\left[-\frac{x\cos(nx)}{n}\Big|_{-\pi}^{0} + \frac{1}{n}\int_{-\pi}^{0}\cos(nx)\,dx\right]$$

$$= \frac{1}{\pi}\left[\left(-\frac{x\cos(nx)}{n}\right)_{-\pi}^{0} + \frac{1}{n}\underbrace{\left(\frac{\sin(nx)}{n}\right)_{-\pi}^{0}}_{0}\right] = \frac{(-1)^n}{n} = A$$

B

$$= \left[-\frac{\cos(nx)}{n}\right]_{0}^{\pi} = \frac{1-(-1)^n}{n} = B$$

C

$$\left.\begin{array}{ll} u = x & dv = \sin(nx)\,dx \\ du = dx & v = -\dfrac{\cos(nx)}{n} \end{array}\right\} = -\frac{1}{\pi}\left[-\frac{x\cos(nx)}{n}\Big|_{0}^{\pi} + \frac{1}{n}\int_{0}^{\pi}\cos(nx)\,dx\right]$$

$$= -\frac{1}{\pi}\left[\left(-\frac{x\cos(nx)}{n}\right)_{0}^{\pi} + \frac{1}{n}\underbrace{\left(\frac{\sin(nx)}{n}\right)_{-\pi}^{0}}_{0}\right] = \frac{(-1)^n}{n}$$

$$b_n = A + B + C$$

$$b_n = \frac{(-1)^n}{n} + \frac{1}{n} - \frac{(-1)^n}{n} + \frac{(-1)^n}{n}.$$

$$b_n = \frac{1}{n} + \frac{(-1)^n}{n}$$

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$a_0 = 0 \qquad a_n = \frac{2}{\pi n^2}[1-(-1)^n] \qquad b_n = \frac{1+(-1)^n}{n}$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$$

$$f(x) = \sum_{n=1}^{\infty} \left[\frac{2}{\pi n^2}(1-(-1)^n)\cos(nx) + \frac{1+(-1)^n}{n}\sin(nx) \right]$$

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