Q8

A function f defined on a symmetric interval $[-L, L]$ is

i) Even if $f(-t) = f(t)$ for $-L \leq t \leq L$

ii) Odd if $f(-t) = -f(t)$ for $-L \leq t \leq L$

(a) $f(x) = e^x$

$f(-x) = e^{-x}$

Clearly $e^{-x} \neq e^x$

$\therefore f(x) = e^x$ is not even

$-f(t) = -e^x$

Clearly $f(-x) \neq -f(t)$

$\therefore f(x) = e^x$ is not odd

(b) $f(x) = e^{-|x|}$

$f(-x) = e^{-|-x|} = e^{-|x|}$

Clearly $f(-x) = f(x)$

$\therefore f(x) = e^{-|x|}$ is even

(c) $x^3 \cos(\pi x) = f(x)$

$f(-x) = (-x)^3 \cos(-\pi x) = -x^3 \cos(\pi x)$

Clearly

$f(-x) \neq f(x)$

but $f(-x) = -f(x)$

hence odd

(d) $f(x) = x^2 \tan(\pi x)$

$f(-x) = (-x)^2 \tan(-\pi x)$

$f(-x) = -x^2 \tan(\pi x)$

Odd not even

(e) $f(x) = \sinh(x) - \cosh(x)$

$f(-x) = -\sinh(x) - \cosh(x)$

$-f(x) = -\sinh(x) + \cosh(x)$

Neither even nor odd

(f) $f(x) = \sin^2 x$

$f(-x) = \sin^2(-x)$

$f(-x) = \sin^2(x)$

hence even

(g) $\sin(x^2) = f(x)$

$f(-x) = \sin[(-x)^2] = \sin(x^2)$

hence even

(h) $f(x) = \ln(x)$

(i) $f(x) = \dfrac{x}{x^2+1}$

$f(-x) = -\dfrac{x}{x^2+1}$

hence ~~even~~ odd