

Q10

Find the rectangular coordinates of the center of the sphere.

$$r^2 + z^2 = 4r \cos \theta + 6r \sin \theta + 2z$$

$$x^2 + y^2 + z^2 = 4x + 6y + 2z \quad \text{completing the square}$$

$$x^2 - 4x + y^2 - 6y + z^2 - 2z = 0$$

$$[x^2 - 4x + (2)^2 - (2)^2] + [y^2 - 6y + (3)^2 - (3)^2] + [z^2 - 2z + (1)^2 - (1)^2] = 0$$

$$[(x-2)^2 - 4] + [(y-3)^2 - 9] + [(z-1)^2 - 1] = 0$$

$$(x-2)^2 + (y-3)^2 + (z-1)^2 = 14 \quad \rightarrow \text{Standard equation of the sphere.}$$

hence center is

$$\underline{\underline{(2, 3, 1)}}$$

Q11

Find the rectangular coordinates of the center

$$\rho = 2 \sin \phi (\cos \theta - 2 \sin \theta)$$

multiply by ρ and expand

$$\rho^2 = 2\rho \sin \phi \cos \theta - 4\rho \sin \phi \sin \theta$$

$$x^2 + y^2 + z^2 = 2x - 4y \rightarrow \text{complete the square}$$

$$x^2 - 2x + y^2 + 4y + z^2 = 0$$

$$[x^2 - 2x + (1)^2 - (1)^2] + [y^2 + 4y + (-2)^2 - (-2)^2] + z^2 = 0$$

$$[(x-1)^2 - 1] + [(y+2)^2 - 4] + z^2 = 0$$

$$(x-1)^2 + (y+2)^2 + z^2 = 5$$

hence center is

$$\underline{\underline{(1, -2, 0)}}$$