MAT2110 - Infinite Series

Maclaurin Series

$$\boxed{P_n(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \dots + \frac{f^n(0)x^n}{n!}}$$
at $c=0$

i) $g(x) = \cos(\sqrt{5}x)$

$g'(x) = -\sqrt{5}\sin(\sqrt{5}x)$

$g''(x) = -5\cos(\sqrt{5}x)$

$g'''(x) = 5\sqrt{5}\sin(\sqrt{5}x)$

$g^4(x) = 25\cos(\sqrt{5}x)$

$g^5(x) = -25\sqrt{5}\sin(\sqrt{5}x)$

$g^6(0) = -125\cos(\sqrt{5}x)$

at $c=0$

$g(0) = 1$

$g'(0) = 0$

$g''(0) = -5$

$g'''(0) = 0$

$g^4(0) = 25$

$g^5(0) = 0$

$g^6(0) = -125$

$$P_n(x) = 1 - \frac{5x^2}{2!} + \frac{25x^4}{4!} - \frac{125x^6}{6!} + \dots +$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n 5^n x^{2n}}{2n!}$$

ii) $h(x) = \sinh(x)$ $\quad h(0) = 0$

$h'(x) = \cosh(x)$ $\quad h'(0) = 1$

$h''(x) = \sinh(x)$ $\quad h''(0) = 0$

$h'''(x) = \cosh(x)$ $\quad h'''(0) = 1$

$h^4(x) = \sinh(x)$ $\quad h^4(0) = 0$

$h^5(x) = \cosh(x)$ $\quad h^5(0) = 1$

$h^6(x) = \sinh(x)$ $\quad h^6(0) = 0$

$$P_n(x) = 0 + x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \frac{x^9}{9!} + \dots$$

$$\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$$

Radius & interval of convergence

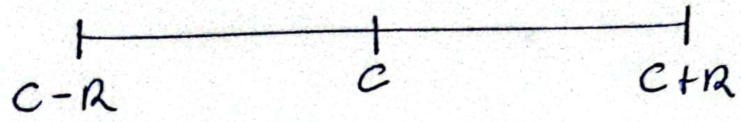
Radius (R) _ by applying the ratio test

$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$, then the series converges

$|x - c| < R$ converges

$|x - c| > R$ diverges

Interval of convergence (I)



NOTE
must check for end points

Summary

1. $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = 0$ then $R = \infty$, $I = (-\infty, \infty)$ → converges for all x

2. $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \infty$ then $R = 0$, $I = \{0\}$

nTh Taylor polynomial

If f has n derivatives at c, then the polynomial

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \cdots + \frac{f^n(c)}{n!}(x-c)^n$$

Taylor polynomial at $c = a$ $\quad a \geq 1$

nTh Maclaurin polynomial

$$P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^n(0)}{n!}x^n$$

To maclaurin polynomial at $c = 0$