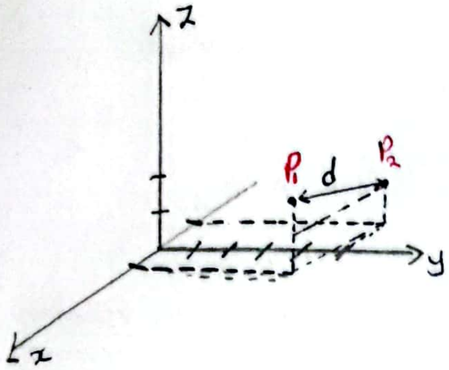
MAT 2110 TUTORIAL SHEET 6 SOLUTIONS (VECTORS)

Q1

a) $P_1(1,4,2)$ $P_2(-1,5,1)$



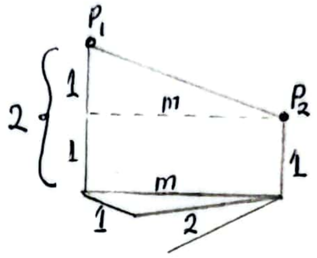
distance between P_1 and P_2

$$\sqrt{(1-(-1))^2+(4-5)^2+(2-1)^2}$$

$$\sqrt{4+1+1}$$

$$P_1P_2=\sqrt{6} = 2.45 \text{ units}$$

By use of the graph



by pythagoras theorem

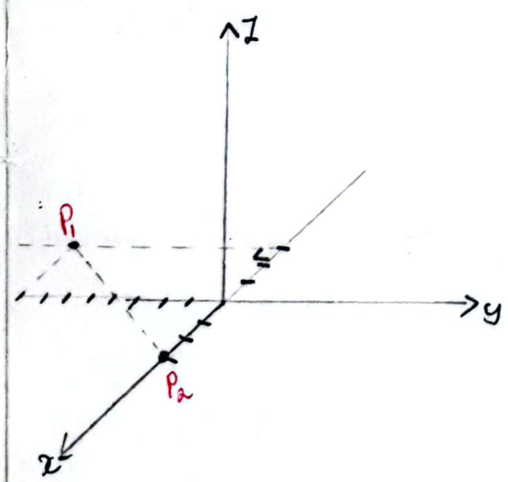
$$m=\sqrt{2^2+1^2} = \sqrt{5} = 2.24$$

$$P_1P_2=\sqrt{(2.24)^2+1^2}$$

$$P_1P_2=\sqrt{6.0176}$$

$$P_1P_2 = 2.45 \text{ units}$$

b) $P_1(-3,-8,0)$ $P_2(3,0,0)$



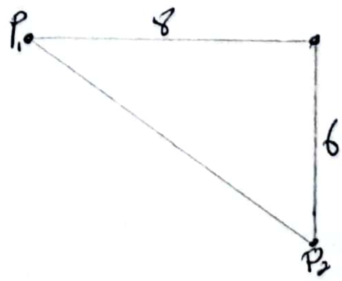
distance between P_1 and P_2

$$\sqrt{(-3-3)^2+(-8-0)^2+(0-0)^2}$$

$$\sqrt{36+64}$$

$$P_1P_2=\sqrt{100} = 10 \text{ units}$$

By use of the graph



Applying pythagoras theorem

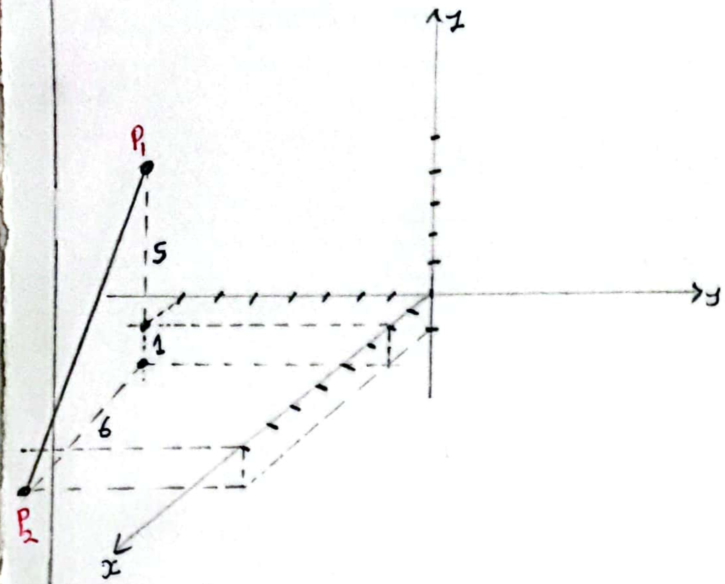
$$P_1P_2 = \sqrt{8^2+6^2}$$

$$= \sqrt{100}$$

$$= 10 \text{ units.}$$

Q1

a) $P_1(2,-7,5)$: $P_2(8,-7,-1)$



distance between P_1 and P_2

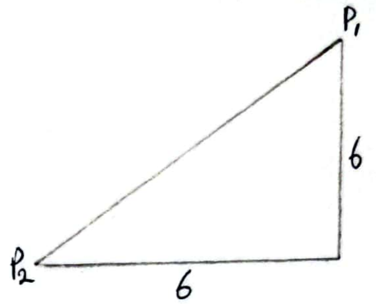
$$\sqrt{(2-8)^2+(-7-(-7))^2+(5-(-1))^2}$$

$$\sqrt{(-6)^2+(6)^2}$$

$$\sqrt{36+36}$$

$$P_1P_2=\sqrt{72}=8.49 \text{ units}$$

by use of the graph



by pythagoras theorem

$$P_1P_2=\sqrt{6^2+6^2}$$

$$P_1P_2=\sqrt{36+36}$$

$$P_1P_2=\sqrt{72}$$

$$=8.49 \text{ units}$$

Q2 a) $x^2+y^2=0$

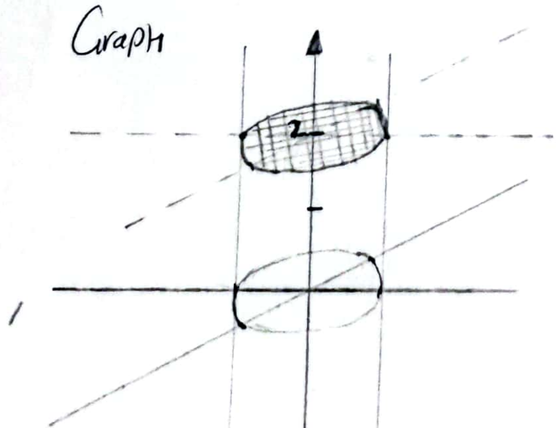
x and y intersect at 0 when $x=0$ and $y=0$, the graph is the z axis of points $(0,0\,z)$ contains all points of $(0,0z)$

b) $x^2+y^2=1$ and $z=2$.

$x^2+y^2=1$ is a cylinder of radius $(r)=\underline{\underline{1}}$

Intersection is between the circle of $r=1$ and the $z=2$ that is parallel to the xy-plane. (above)

Graph



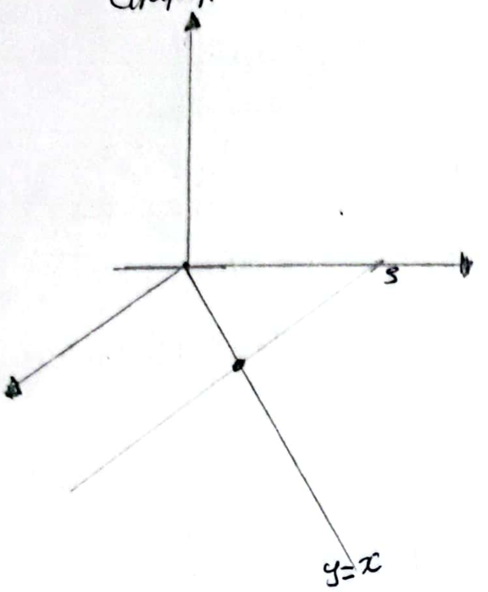
P2

c) Intersection of $y = x$ and $y = 5$

$y = x$ is a plane parallel /perpendicular to the xy-plane, $y = 5$ is parallel to the xz plane

the Intersection is at $(5, 5, 0)$

Graph.



if $y = 5$

$y = x \quad x = 5$

$5 = x$

$\underline{(5, 5, 0)}$

Q3 ⓐ $u = \langle 1, 2, 3 \rangle$ and $v = \langle 2, -3, -1 \rangle$

$$u \cdot v = (1 \times 2) + (2 \times (-3)) + (3 \times (-1)) = 2 - 6 - 3 = -7$$

$$|u| = \sqrt{1^2 + 2^2 + 3^2} = 3.7417$$

$$|v| = \sqrt{2^2 + (-3)^2 + (-1)^2} = 3.7417$$

$$\theta = \cos^{-1}\left[\frac{u \cdot v}{|u||v|}\right] = \cos^{-1}\left[\frac{-7}{[3.7417](3.7417)}\right] = \cos^{-1}\left[\frac{-7}{14.0003}\right]$$

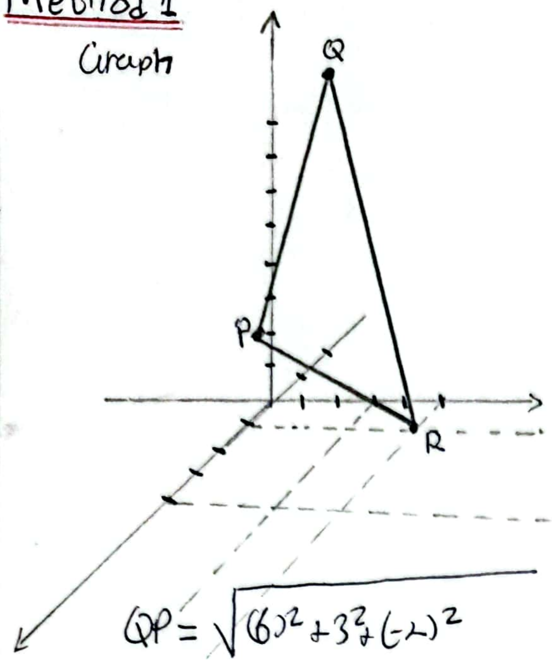
$$\theta = \frac{119.9993}{2} = 60^\circ = \frac{\pi}{3}$$

in ~~radians~~ $= \frac{119.9993\pi}{180} = \underline{2.0947}$ radians

(Q3(b)) $P=(4,3,6)$ $Q=(-2,0,8)$ and $R=(1,5,0)$

Method 1

Graph



Assume PQ & PR are perpendicular

then by pythagoras theorem

$(QR)^2=(QP)^2+(PR)^2$

$QR=\sqrt{(-2-1)^2+(0-5)^2+(8-0)^2}$

$QR=\sqrt{9+25+64}$

$QR=\sqrt{98}=9.8995$ units

$QP=\sqrt{(6)^2+3^2+(-2)^2}$

$=\sqrt{49}=\underline{\underline{7}}$

$PR=\sqrt{(3)^2+(-2)^2+6^2}$

$=\sqrt{49}=\underline{\underline{7}}$

$(QR)^2=(QP)^2+(PR)^2$

$(\sqrt{98})^2=(\sqrt{49})^2+(\sqrt{49})^2$

$98=49+49$

$98=98$

$\therefore$ the triangle with vertices P, Q and R is a right angled triangle.

Method 2

determine $\vec{PQ}$ and $\vec{PR}$, then apply dot product to determine whether or not $\vec{PQ}$ and $\vec{PR}$ are perpendicular.

$\vec{PQ}=((-2-4),(0-3),(8-6))$

$=(-6,-3,2)$

$\vec{PR}=((1-4),(5-3),(0-6))$

$=(-3,2,-6)$

$\vec{PQ}\cdot\vec{PR}=((-3)(-6)+(-3\times2)+(2\times(-6)))$

$\vec{PQ}\cdot\vec{PR}=18-6-12=\underline{\underline{0}}$

$\therefore$ the dot product of $\vec{PQ}$ & $\vec{PR}=0$

then $\vec{PQ}$ and $\vec{PR}$ are perpendicular.

$\therefore$ the triangle with vertices P, Q & R is a right angled triangle

13b

Area

$PQ = 7 \qquad QP = 7$

Area of right angled triangle $= \frac{1}{2} bh = \frac{1}{2} \times 7 \times 7 = \frac{49}{2} \text{(units)}^2$

Q4

a) $(1,-1,1)$ and $(-1,1,-1)$

Vector equation

$\vec{r_0} = \langle 1,-1,1 \rangle$

$\vec{r}(t) = (x_0, y_0, z_0) + t(a,b,c) = \vec{r_0} + t\vec{v}$

$\vec{v} = \langle -1-1, 1-(-1), -1-1 \rangle$

$\vec{v} = \langle -2, 2, -2 \rangle$

$\vec{r}(t) = \langle 1,-1,1 \rangle + t\langle -2,2,-2 \rangle$

$\vec{r}(t) = \langle 1-2t, 2t-1, 1-2t \rangle$

Parametric equation

$x = 1-2t \quad , \quad y = -1+2t \quad , \quad z = 1-2t$

Symmetric equation

$$\frac{x-1}{-2} = \frac{y+1}{2} = \frac{z-1}{-2}$$

$\vec{r_0}$

$\vec{v} = P - \vec{r_0}$

b) $(1,0,3)$ parallel to $2\hat{i} - \hat{j} - 1k$

$(2,-1,-1)$

Vector equation

$\vec{r}(t) = \langle 1,0,3 \rangle + t\langle 2,-1,-1 \rangle$

$\vec{r}(t) = \langle (1+2t), -t, 3-t \rangle$

Symmetric equation

$$\frac{x-1}{2} = \frac{y}{-1} = \frac{z-3}{-1}$$

Parametric equation

$x = 1+2t \,,\, y = -t \,,\, z = 3-t$

Q4 c) $(-1,1,0)$ parallel to $0i + 0j - 7k$

$(0,0,-7)$

Vector equation

$\vec{r}(t) = \langle -1, 1, 0 \rangle + t\langle -7 \rangle$

$\vec{r}(t) = \langle -1, 1, -7t \rangle$

Parametric equations

$x = -1,\ y = 1\ z = -7t$

Symmetric equations

$x = -1,\ y = 1$ and $\frac{z+7}{1}$

d) $(-7, \sqrt{2}, 1)$ L_1: $x = 7 + 2n,\ y = 2 + n,\ z = -6 - 3n$

L_2: $\frac{x-3}{1} = \frac{y-9}{5} = \frac{z-13}{5}$

L_2 in parametric form

$x - 3 = n_2$ $\quad y - 9 = 5n_2$ $\quad z - 13 = 5n_2$

$x = 3 + n_2$ $\quad y = 9 + 5n_2$ $\quad z = 13 + 5n_2$

L_1 $\quad L_2$

$x = 7 + 2n$ --- $x = 3 + n_2$ ----- (i)

$y = 2 + n$ ---- $y = 9 + 5n_2$ ----- (ii)

$z = -6 - 3n$ --- $z = 13 + 5n_2$ ---- (iii)

solving (ii) & (iii) simultaneously

$2 + n = 9 + 5n_2 \Rightarrow n - 5n_2 = 7$

$-6 - 3n = 13 + 5n_2 \Rightarrow -3n - 5n_2 = 1$

$n - 5n_2 = 7$

$-3n - 5n_2 = 19$

$4n = -12$

$n = -3$

$7 + 2(-3) = 3 + n_2$

$1 = 3 + n_2$

$n_2 = -2$

$x = 7 + 2(-3)$ $\quad y = 2 - 3$ $\quad z = -6 - 3(-3)$

$x = 1$ $\quad y = -1$ $\quad z = 3$

coordinates of intersection $= (1, -1, 3)$

Q4 (2) (

Vector equation

$(-7, \sqrt{2}, 1)$ and $(1, -1, 3)$

$\vec{r_0} = \langle -7, \sqrt{2}, 1 \rangle$

$\vec{v} = \langle 1-(-7), -1-\sqrt{2}, 3-1 \rangle$

$\vec{v} = \langle 8, -1-\sqrt{2}, 2 \rangle$

$\vec{r}(t) = \vec{r_0} + t\vec{v}$

$= \langle -7, \sqrt{2}, 1 \rangle + t\langle 8, -1-\sqrt{2}, 2 \rangle$

$\vec{r}(t) = \langle -7+8t, \sqrt{2}+t(-1-\sqrt{2}), 1+2t \rangle$

Parametric equation

$x = 8t - 7$, $y = \sqrt{2} + t(-1-\sqrt{2})$, $z = 1+2t$

Symmetric equation

$$\frac{x+7}{8} = \frac{y-\sqrt{2}}{-1-\sqrt{2}} = \frac{z-1}{2}$$

Q5 If two vectors are parallel, then they're scalar multiples of each other (one being a scaler multiple of the other)

Or

If the changes in x, y & z is equal in both vectors.

$L_1 : x = 1 - at$, $y = 5t$, $z = -1 - 14t$, $L_2 : \frac{2(x+7)}{-1} = \frac{2(y-100)}{5} = \frac{z+19}{-b}$

L_2 as a parametric equation $\Rightarrow x = -\frac{t}{2} - 7$, $y = \frac{5t}{2} + 100$, $z = -bt - 19$

L_1	L_2
$x = 1 - at$	$x = -\frac{t}{2} - 7$
$y = 5t$	$y = \frac{5t}{2} + 100$
$z = -1 - 14t$	$z = -bt - 19$

$\langle -a, 5, -14 \rangle$ $\langle -\frac{1}{2}, \frac{5}{2}, -b \rangle$

$\therefore$ $\underline{\underline{a = 1}}$ $\underline{\underline{b = 7}}$

Q6

$L_1: \frac{x-3}{2} = \frac{y+1}{4} = \frac{z-2}{-1}$ $L_2 = \frac{x-3}{r} = \frac{y+1}{-2} = \frac{z-2}{2}$

parametric equation

L_1

$x = 2t+3$

$y = 4t-1$

$z = -t+2$

L_2

$x = rt+3$

$y = -2t-1$

$z = 2t+2$

L_1 L_2

$\langle 2, 4, -1 \rangle$ $\langle r, -2, 2 \rangle$

let $L_1 = \bar{A}$ and $L_2 = \bar{B}$

dot product of A and B

$$\cos\theta = \left| \frac{U \cdot V}{\|U\|\|V\|} = \frac{A \cdot B}{\|A\|\|B\|} \right.$$

if the two vectors are orthogonal, then they are perpendicular to each other. (They are at a 90° angle)

$\theta = 90°$

$$\cos(90) = \frac{2r + (4)(-2) + (-1)(2)}{\|\sqrt{2^2+4^2+(-1)^2}\| \, \|\sqrt{r^2+(-2)^2+2^2}\|}$$

$2r - 8 - 2 = 0$

$2r = 10$ ~~$\frac{10}{2}$~~

$\therefore r = \underline{\underline{5}}$

$\underline{\underline{r = 5}}$

Q7

a) L_1: ~~x=~~ $\quad L_2$

$x = 1+t \quad x = 17+3r \quad 1+t = 17+3r$ ------ (i)

$y = -3+2t \quad y = 4+r \quad -3+2t = 4+r$ ------ (ii)

$z = -2-t \quad z = -8-r \quad -2-t = -8-r$ ------ (iii)

solving for t & r from eq (i) & (ii)

$t - 3r = 16$ ----> $2t - 6r = 32$

$2t - r = 7$ ----> $2t - r = 7$

$-5r = 25$

$\underline{r = -5}$

$1+t = 17+3(-5)$

$t = \underline{\underline{1}}$

$-2-t = -8-(-5)$

$-t = 2-3$

$t = 1$

$x = 1+1$

$x = \underline{\underline{2}}$

$y = -3+2(1)$

$y = \underline{\underline{-1}}$

$z = -2-1$

$z = \underline{\underline{-3}}$

$x = 17+3(-5)$

$x = \underline{\underline{2}}$

$y = 4-5$

$y = \underline{\underline{-1}}$

$z = -8+5$

$z = \underline{\underline{-3}}$

$\therefore$ point of intersection is

$\langle 2, -1, -3 \rangle$

b) $L_1 \quad L_2$

$x = 1+t \quad x = 3n \quad 1+t = 3n$ ------ (i)

$y = 2-t \quad y = 2-n \quad 2-t = 2-n$ ------ (ii)

$z = 3t \quad z = 2+n \quad 3t = 2+n$ ------ (iii)

eq (i) & (ii)

$t - 3n = -1$

$+ \; -t + n = 0$

$n = \frac{1}{2}$

$t - \frac{3}{2} = -1$

$t = \frac{1}{2}$

eq (ii) & (iii)

$-t + n = 0$

$+ \; 3t - n = 2$

$2t = 2$

$t = 1$

<u>NO SOL</u>

$\therefore$ L_1 & L_2 are skew

c) L_1:

$$\frac{x-1}{1} = \frac{y+1}{3} = \frac{z-4}{-1}$$

L_2:

$$\frac{x}{2} = \frac{y-3}{1} = \frac{z+3}{-4}$$

$x = t+1$

$y = 3t-1$

$z = -t+4$

$x = 2n \dashrightarrow t+1 = 2n$

$y = n+3 \dashrightarrow 3t-1 = n+3$

$z = -4n-3 \dashrightarrow -t+4 = -4n-3$

eq (i) & (ii)

$t - 2n = -1 \dashrightarrow 3t - 6n = -3$

$3t - n = 4 \dashrightarrow 3t - n = 4$

$-5n = -7$

$n = \frac{7}{5}$

eq ①

$t - 2\left(\frac{7}{5}\right) = -1$

$t = -1 + \frac{14}{5}$

$t = \frac{9}{5}$

eq ②

$3t = \frac{4}{1} - \frac{7}{5}$

$3t = \frac{13}{5}$

$t = \frac{13}{15}$

eq (3)

$-t = -4\left(\frac{7}{5}\right) - 3 - 4$

$-t = -\frac{28}{5} - \frac{7}{1}$

$-t = -\frac{63}{5}$

$t = \frac{63}{5}$

NO SOL

$\therefore$ l_1 & l_2 are skew

d) L_1

$x = 2t+1$

$y = t$

$z = 4t+1$

L_2

$x = n \dashrightarrow 2t+1 = n$ ---- (i)

$y = 2n-2 \dashrightarrow t = 2n-2$ --- (ii)

$z = -3n-2 \dashrightarrow 4t+1 = -3n-2$ --- (iii)

eq ① & ②

$2t - n = -1 \dashrightarrow 2t - n = -1$

$t - 2n = 2 \dashrightarrow 2t - 4n = 4$

$3n = -5$

$n = -\frac{5}{3}$

eq (ii) & (iii)

$t - 2n = 2 \dashrightarrow 4t - 8n = 8$

$4t + 3n = -3 \dashrightarrow 4t + 3n = -3$

$-13n = 11$

$n = -\frac{11}{13}$

NO SOL

$\therefore$ l_1 & l_2 are skew.

P8

7(b)

$\langle 1,-1,3\rangle \quad \langle 3,-1,1\rangle$

$d=\sqrt{(1-3)^2+(-1-(-1))^2+(3-1)^2}$

$d=\sqrt{4+0+4}$

$d=\sqrt{8}$

$d=\underline{\underline{2\sqrt{2}}}$ units

7(c)

$\langle 1,3,-1\rangle \quad \langle 2,1,-4\rangle$

$d=\sqrt{(1-2)^2+(3-1)^2+(-1+4)^2}$

$d=\sqrt{1+4+9}$

$d=\underline{\underline{\sqrt{14}}}$ units

7(d)

$\langle 2,1,4\rangle \quad \langle 1,2,-3\rangle$

$d=\sqrt{(2-1)^2+(1-2)^2+(4+3)^2}$

$d=\sqrt{1+1+49}$

$d=\underline{\underline{\sqrt{51}}}$ units

Q9

(a) $(1,2,3)$ $\qquad \dfrac{x-2}{-5}=\dfrac{y-1}{1}=\dfrac{z-3}{-4}$ $= x=-5t+2,\ y=t+1,\ z=-4t+3$

define the line in terms of vector

$(-5t+2)\hat{i}+(t+1)\hat{j}+(-4t+3)k$

$-5t\hat{i}+2\hat{i}+t\hat{j}+\hat{j}+3k-4tk$

$(2\hat{i}+\hat{j}+3k)+(-5t\hat{i}+t\hat{j}-4tk)$

$(2\hat{i}+\hat{j}+3k)+t(-5\hat{i}+\hat{j}-4k)$

direction values of the line

let $n=\langle -5,1,-4\rangle$

$(2,1,3)$

$m=\overset{Q}{(1,2,3)}-\overset{P}{(2,1,3)}$

$m=\langle -1,1,0\rangle$

$d=\left\|\dfrac{n\times m}{n}\right\|=\dfrac{\|n\times m\|}{\|n\|}$

$n\times m=\begin{vmatrix}\hat{i} & \hat{j} & k\\ -1 & 1 & 0\\ -5 & 1 & -4\end{vmatrix}=-4\hat{i}-4\hat{j}+4k$

$=\langle -4,-4,4\rangle$

$\|n\|=\sqrt{42}$

$\|n\times m\|=\sqrt{48}$

$d = \frac{\sqrt{48}}{\sqrt{42}} = 1.0691$ units

Q9

b) $(0, 1, -1)$ line through points $(2, 1, -4)$ and $(1, 1, 1)$

Let the point $(0, 1, -1) = Q$

we've to determine the point on a lined and set it as P

$\vec{r_0} = \langle 2, 1, -4 \rangle$

$\vec{V} = \langle 1-2, 1-1, 1-(-4) \rangle$

$\vec{V} = \langle -1, 0, 5 \rangle$

Parametric equation

$x = 2 - t$

$y = 1$

$z = -4 + 5t$

$U =$ direction vector $\Rightarrow \langle -1, 0, 5 \rangle$

$P \Rightarrow \langle 2, 1, -4 \rangle$

$\vec{PQ} = \langle 0-2, 1-1, -1-(-4) \rangle$

$\vec{PQ} = \langle -2, 0, 3 \rangle$

$$\vec{PQ} \times U = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 3 \\ -1 & 0 & 5 \end{vmatrix} = 0\hat{i} + 7\hat{j} + 0\hat{k} = \langle 0, 7, 0 \rangle$$

$\|\vec{PQ}\| = \sqrt{7}$

$\|U\| = \sqrt{26}$

$\therefore$ distance $(D) = \frac{\sqrt{7}}{\sqrt{26}} = \sqrt{\frac{7}{26}}$ units

Q9

ⓒ $(1,2,-3)$ $x = 2-t,\ y = 1+t,\ z = -2-t$

$Q = (1,2,-3)$ to find P, set $t=0$

$P = (2,1,-2)$

$\overrightarrow{PQ} = \langle 1-2,\ 2-1,\ -3+2 \rangle$ $u = \langle -1,1,-1 \rangle$

$\overrightarrow{PQ} = \langle -1,1,-1 \rangle$ $\therefore \overrightarrow{PQ} = u$

$\|\overrightarrow{PQ} \times u\| = \begin{vmatrix} i & j & k \\ -1 & 1 & -1 \\ -1 & 1 & -1 \end{vmatrix}$

The point Q and the line are parallel.

Q10 ⓐ $P = (1,0,0)$ has normal vector $N = k$

General Form

$a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$

$x_1 = 1$
$y_1 = 0$
$z_1 = 0$

$\vec{N} = \underset{a}{0i} + \underset{b}{0j} + \underset{c}{1k}$

$\langle 0,0,1 \rangle$

$0(x-1) + 0(y-0) + 1(z-0) = 0$

$\underline{\underline{z=0}}$

b) $(1,2,-4)$ $(2,3,7)$ and $(4,-1,3)$

determine the normal vector from $(1,2,-4)$ to $(2,3,7)$ & $(4,-1,3)$

$u = \langle 1,1,11 \rangle$
$v = \langle 3,-3,7 \rangle$

$n = u \times v = \begin{vmatrix} i & j & k \\ 1 & 1 & 11 \\ 3 & -3 & 7 \end{vmatrix} = \underset{a}{40i} + \underset{b}{26j} - \underset{c}{6k}$

$a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$

$40(x-1) + 26(y-2) - 6(z+4) = 0$

$40x - 40 + 26y - 52 - 6z - 24 =$

$\boxed{40x + 26y - 6z - 116 = 0}$

Q1 $3x - y + 4z - 8 = 0$

ⓐ $\langle x, y, z \rangle = \langle 3, 0, 5 \rangle + t\langle 7, -11, -8 \rangle$

$x = 3 + 7t \quad y = 0 + (-11t) \quad z = 5 - 8t$

$y = -11t$

$3(3+7t) - (-11t) + 4(5-8t) - 8 = 0$

$9 + 21t + 11t + 20 - 32t - 8 = 0$

$32t - 32t = -21$

0

$x = 3 + 7(0) \quad y = -11(0) \quad z = 5 - 8(0)$

$x = 3 \quad y = 0 \quad z = 5$

$P(3, 0, 5)$

ⓑ $-2x + 6y + 4z - 4 = 0$

$\langle x, y, z \rangle = \langle 5, -1, 4 \rangle + t\langle 1, -2, 3 \rangle$

$x = 5 + t \quad y = -1 + (-2t) \quad z = 4 + 3t$

$y = -1 - 2t$

$-2(5+t) + 6(-1-2t) + 4(4+3t) = 4$

$\cancel{-10} - 2t \cancel{-6} \cancel{-12t} \cancel{+16} + \cancel{12t} = 4$

$-2t = 4$

$t = -2$

$x = 5 - 2$

$x = 3$

$y = -1 - 2(-2)$

$y = 3$

$z = 4 + 3(-2)$

$z = -2$

$P(3, 3, -2)$

c) $5x + 3y + 4z - 20 = 0$

$x = 5 + t, \; y = -1 - 2t, \; z = 4 + t$

$5(5+t) + 3(-1-2t) + 4(4+t) - 20 = 0$

$25 + 5t - 3 - 6t + 16 + 4t = 20$

$3t = -18$

$t = -6$

$x = 5 - 6 = -1$

$y = -1 - 2(-6) = 11$

$z = 4 - 6 = -2$

$P(-1, 11, -2)$

pl

d) $9x - 6y + 12z - 24 = 0$

$\langle x, y, z \rangle = \langle 4, 0, -1 \rangle + t\langle 2, 1, -1 \rangle$

$x = 4 + 2t \quad , y = t \quad , z = -1 - t$

$9(4 + 2t) - 6(t) + 12(-1 - t) = 24$

$36 + 18t - 6t - 12 - 12t = 24$

$t = 0$

$x = 4 + 2(0) = 4$

$y = 0$

$z = -1 - 0 = -1$

$P(4, 0, -1)$

e) $6x - 2y + 3z + 6 = 0$

$\frac{x - 4}{2} = \frac{y - 12}{-3} = \frac{z + 19}{5}$

$x = 2t + 4 \quad , y = -3t + 12 \quad , z = 5t - 19$

$6(2t + 4) - 2(-3t + 12) + 3(5t - 19) = -6$

$12t + \cancel{24} + 6t - \cancel{24} + 15t - 57 = -6$

$33t = 51$

$t = \frac{51}{33}$

$x = 2\left(\frac{51}{33}\right) = 3.09$

$y = -3\left(\frac{51}{33}\right) + 12 = 7.36$

$z = 5\left(\frac{51}{33}\right) - 19 = -11.27$

$P(3.09, 7.36, -11.27)$

Q12

a) $Q = (-1, 0, 5)$, $\overset{a}{2x} + \overset{b}{6y} - \overset{c}{3z} - \overset{d}{6} = 0$ $\quad n = \langle 2, 6, -3 \rangle$

$$d = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|(2)(-1) + (6)(0) + (-3)(5) + (-6)|}{\sqrt{2^2 + 6^2 + (-3)^2}}$$

$$d = \frac{|-2 + 0 - 15 - 6|}{\sqrt{49}} = \frac{23}{7} \text{ units}$$

b) $Q = (0, -1, 0)$, $5x + 3y + 4z - 20 = 0$ $\quad n = \langle 5, 3, 4 \rangle$

$$d = \frac{|(5)(0) + (3)(-1) + (0)(4)|}{\sqrt{5^2 + 3^2 + 4^2}} = \frac{23}{\sqrt{50}} = \frac{23}{5\sqrt{2}} = \frac{23}{5\sqrt{2}} \text{ units}$$

c) $Q = (3, 1, -1)$, $x + y + z - 3 = 0$ $\quad n = \langle 1, 1, 1 \rangle$

$$d = \frac{|(1)(3) + (1)(1) + (-1)(1)|}{\sqrt{3}} = \frac{3}{\sqrt{3}} \text{ units}$$

Q13

a) $2x + 3y - 7z - 2 = 0$; $4x + 6y - 14z - 8 = 0$

1st determine if they're parallel

$\vec{v}_{1\perp} = 2\hat{i} + 3\hat{j} - 7k$ $\qquad \vec{v}_{1\perp} = \frac{1}{2}\vec{v}_{2\perp}$ $\qquad a_1 = \frac{1}{2}a_2$

$\vec{v}_{2\perp} = 4\hat{i} + 6\hat{j} - 14k$ $\qquad\qquad b_1 = \frac{1}{2}b_2$

$\qquad\qquad c_1 = \frac{1}{2}c_2$

meaning $\vec{v}_{1\perp} \parallel \vec{v}_{2\perp}$

do not intersect.

Q13

b) $3x + 9y - 6z - 24 = 0$; $4x + 12y - 8z - 32 = 0$

solve y-interms of x by firstly eliminating z

$$\begin{array}{l|l} 8 & 3x + 9y - 6z - 24 = 0 \longrightarrow 24x + 72y - 48z - 192 = 0 \\ 6 & 4x + 12y - 8z - 32 = 0 \longrightarrow 24x + 72y - 48z - 192 = 0 \end{array}$$

$$0 + 0 + 0 + 0 = 0$$

$\therefore$ the two plans do not interseat

they are parallel

c) $4x - 12y - 16z - 52 = 0$; $x + y - 4z - 8 = 0$

solve y-interms of x by eliminating z

$$\begin{array}{l|l} \frac{1}{4} & 4x - 12y - 16z - 52 = 0 \longrightarrow x - 3y - 4z - 13 = 0 \\ 1 & x + y - 4z - 8 = 0 \longrightarrow x + y - 4z - 8 = 0 \end{array}$$

$$0 \quad -4y - 0 - 5 = 0$$

$$-4y = 5$$

$$y = -\frac{5}{4}$$

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

Q13

d) $x - 2y + 2.5z - 1 = 0$; $3x - 6y + 7.5z - 3 = 0$ (A, B)

multiply eq A by 3 to try and eliminate z

$$3(3x - 2y + 2.5z - 1 = 0) = 3x - 6y + 7.5z - 3 = 0 \text{ ---- (ii)}$$
$$- \; (3x - 6y + 7.5z - 3 = 0 \text{ ---- (i)})$$
$$0$$

plane A and plane B arent intersecting.

Q14

ⓐ $R(t) = (e^{-t})\hat{i} + (2\cos 3t)\hat{j} + (2\sin 3t)k$; $t = 0$ Valued func. $R(t)$

find $\frac{dR}{dt} = R't$

$$R'(t) = -(e^{-t})\hat{i} - (6\sin 3t)\hat{j} + (6\cos 3t)k$$

$$R'(0) = -\hat{i} + 0\hat{j} + 6k$$

ⓑ $R(t) = \frac{4}{9}(1+t)^{3/2}\hat{i} + \frac{4}{9}(1-t)^{3/2}\hat{j} + (\sqrt{t^2-1})k$; $t=0$

$$R'(t) = \frac{2}{3}(1+t)^{1/2}\hat{i} - \frac{2}{3}(1-t)^{1/2}\hat{j} + \left(\frac{t}{\sqrt{t^2-1}}\right)k$$

$$R'(0) = \frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} + 0k$$

Q|5

(a) $R(t) = 3(\sin 2t)\,i + 3(\cos 2t)\,j + (t^2)\,k; \quad 0 \le t \le 5.$

$$L = \int_a^b |R'(t)|\,dt \qquad \begin{array}{l} b = 5 \\ a = 0 \end{array}$$

$$R'(t) = 6(\cos 2t)\,i - 6(\sin 2t)\,j + (2t)\,k$$

$$L = \int_0^5 \sqrt{(6\cos 2t)^2 + (-6\sin 2t)^2 + (2t)^2}\,dt$$

$$L = \int_0^5 \sqrt{36\cos^2 2t + 36\sin^2 2t + 4t^2}\,dt$$

$$L = \int_0^5 \sqrt{36(\cos^2 2t + \sin^2 2t) + 4t^2}\,dt$$

$$L = \int_0^5 \sqrt{36 + 4t^2}\,dt = \int_0^5 \sqrt{4(9 + t^2)}\,dt$$

$$L = 2\int_0^5 \sqrt{9 + t^2}\,dt \qquad \begin{array}{l} dt = 3\tan\theta \\ dt = 3\sec^2\theta\,d\theta \end{array}$$

$$L = 2\int_0^5 \sqrt{9 + 9\tan^2\theta} \cdot 3\sec^2\theta\,d\theta$$

$$L = 2\int_0^5 \sqrt{9(1 + \tan^2\theta)} \cdot 3\sec^2\theta\,d\theta$$

Q5 a) cont.d

Reduction Formula

$$L = 18\int_0^5 \sec^3\theta \, d\theta$$

$$\int \sec^n x\,dx = \frac{\sec^{n-2}x\tan x + (n-2)\int \sec^{n-2}x\,dx}{n-1}$$

$$L = 18\left[\frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\tan(x) + \sec(x)\right|_0^5$$

$$\theta = \tan^{-1}\left[\frac{t}{3}\right]$$

$$L = 18\left[\frac{1}{2}\times\frac{1}{\cos(\tan^{-1}(\frac{t}{3}))}\cdot\tan(\tan^{-1}(\frac{t}{3})) + \frac{1}{2}\ln\left|\tan(\tan^{-1}(\frac{t}{3})) + \frac{1}{\cos(\tan^{-1}(\frac{t}{3}))}\right]_0^5$$

$L = \underline{40.71 \text{ units.}}$

b) $R(t) = (e^t\cos t)\hat{i} + (e^t\sin t)\hat{j} + (e^t)\hat{k} \; ; \; -\ln 4 \le t \le 0$

$R'(t) = (e^t\cos t - e^t\sin t)\hat{i} + (e^t\cos t + e^t\sin t)\hat{j} + e^t\hat{k}$

$$L = \int_{-\ln 4}^{0} \sqrt{(e^t\cos t - e^t\sin t)^2 + (e^t\cos t + e^t\sin t)^2 + (e^t)^2}$$

$$L = \left|3e^t\right|_{-\ln 4}^{0} = \left(3e^0 - 3e^{-\ln 4}\right) = \left(3 - \frac{3}{e^{\ln 4}}\right)$$

$= \underline{\frac{9}{4} \text{ units}}$

Q16 :

ii) $R(t) = (\cos t + t\sin t)\hat{i} + (\sin t - t\cos t)\hat{j} + 3k$

unit tangent vector $T(t)$

$$T(t) = \frac{r'(t)}{\|r'(t)\|} \qquad r'(t) \neq 0$$

$$R'(t) = (-\sin t + t\cos t + \sin t)\hat{i} + (\cos t - t(-\sin t) + \cos t)\hat{j}.$$

$$R'(t) = (t\cos t)\hat{i} + (2\cos t + t\sin t)\hat{j}$$

$$T(t) = \frac{(t\cos t)\hat{i} + (2\cos t + t\sin t)\hat{j}}{\sqrt{(t\cos t)^2 + (2\cos t + t\sin t)^2}}$$

Principle unit Normal vector $N(t)$

$$N(t) = \frac{T(t)}{\|T'(t)\|}$$

$$T'(t) = \frac{\dfrac{\left(\sqrt{(t\cos t)^2 + (2\cos t + t\sin t}\right)\left((\cos t - t\sin t)\hat{i} + (t\cos t - \sin t)\hat{j}\right) - \left((t\cos t)\hat{i} + (2\cos t + t\sin t)\hat{j}\right)\left(\dfrac{1}{2(2t + 4t\cos 2t)^{1/2}}\right)}{\left((t\cos t) + (2\cos t + t\sin t)\right)^{1/2}}}{\left((t\cos t)^2 + (2\cos t + t\sin t)^2\right)^{1/4}}$$

Q17 Velocity Vector

a) $R(t) = (\sin t)\hat{i} + (\cos t)\hat{j} + (\sinh t)k$

$\vec{v} = R'(t) = (\cos t)\hat{i} - (\sin t)\hat{i} + (\cosh t)k$

b) $R(t) = t^2\hat{i} + t^3\hat{j} + (te^t)k$

$\vec{v} = R'(t) = 2t\hat{i} + 3t^2\hat{j} + (te^t + e^t)k$

c) $R(t) = t\hat{i} + (t+t^2)\hat{j} + (\cos \pi t)k$

$\vec{v} = R'(t) = \hat{i} + (1+2t)\hat{j} + (\pi)(-\sin \pi t)k$

$= \hat{i} + (1+2t)\hat{j} - \pi(\sin \pi t)k$

Acceleration vector

a) $\vec{a} = R''(t) = -(\sin t)\hat{i} - (\cos t)\hat{j} + (\sinh t)k$

b) $\vec{a} = R''(t) = 2\hat{i} + 6t\hat{j} + (te^t + 2e^t)k$

c) $\vec{a} = R''(t) = 2\hat{j} - \pi(\pi)(\cos \pi t)k$

$= 2\hat{j} - \pi^2(\cos \pi t)k$

Speed $|\vec{v}|$

a) $|\vec{v}| = \sqrt{(\cos t)^2 + (-\sin t)^2 + (\cosh t)^2} = \sqrt{1 + \cosh^2 t}$

b) $|\vec{v}| = \sqrt{(2t)^2 + (3t^2)^2 + (te^t + e^t)^2} = \sqrt{4t^2 + 9t^4 + t^2e^{2t} + e^{2t} + 2te^{2t}}$

c) $|\vec{v}| = \sqrt{(1)^2 + (1+2t)^2 + (-\pi(\sin \pi t))^2} = \sqrt{2 + 4t^2 + 4t + \pi^2 \sin^2 \pi t}$