THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS & STATISTICS
MAT 2110–Engineering Mathematics I

Tutorial Sheet 1 **March 2024**

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1. Find an equation for the circle with the given centre $C(h,k)$ and radius r. Then, sketch the circle.
   (a) $C(0,-1),\ \ r=2$ **(b)** $C(1,1),\ \ r=\sqrt{3}$
   (c) $C\left(-\sqrt{3},\sqrt{2}\right),\ \ r=5$ **(d)** $C\left(\frac{1}{2},-1\right),\ \ r=\sqrt[4]{2}$
2. Find the centre and the radius of each of the following circles:
   (a) $x^2+y^2-8x+4y+16=0$ **(b)** $4x^2+4y^2-16x=9$
   (c) $4x^2+4y^2-8\sqrt{2}\,x+4y+12=0$ **(d)** $x^2+y^2+2y=3$
3. The circle passing through the points $(2,0)$ and $(3,-1)$ has radius 3.
   (a) Find the centre of the circle.
   (b) Hence, or otherwise, write down the standard equation of the circle.
4. Discuss each of the following conic sections and sketch the graph:
   (a) $x=-3y^2$ **(b)** $2x^2+y^2=4$ **(c)** $x^2-y^2=1$
   (d) $169x^2-4225=-25y^2$ **(e)** $28x^2-3y=0$ **(f)** $64x^2-36y^2=2304$
5. Discuss each of the following conic sections and sketch the graph:
   (a) $x^2+8y-6x+13=0$ **(b)** $2x^2+2y^2-28x+12y=-114$
   (c) $y^2-4x^2+16x=24$ **(d)** $4x^2-4x+1+9y^2+0$
   (e) $32y^2+64x^2-32y+16x-7=0$ **(f)** $x^2-3x+4+20y=2y^2+x+50$
   (g) $\sqrt{2}\,x^2-\sqrt{\left(\frac{2}{3}\right)}\,x+2\sqrt{2}\,y+\frac{4\sqrt{2}}{3}=2$ **(h)** $y^2-8y+16=4$
6. Find the equation of a parabola with vertex at $(1,-2)$, horizontal axis of symmetry and $x-$intercept at $x=12$.
7. Two parabolas $x^2=4p(y-a)$ and $x^2=-4p(y-b)$ with $p>0$, intersect at the point $(-2,5)$ and their axis of symmetry is the $y-$axis.
   (a) Find a and b in terms of p.
   (b) Given that the focal chord for the parabola $x^2=4p(y-a)$ passes through the point $(-2,5)$, and that it is parallel to the line $8y+15x-117=0$, find the relationship between p and a.
   (c) Hence, or otherwise, find the value(s) of a and b.
8. A conic section has eccentricity, $e=\frac{3}{2}$, centre $(3,2)$ and one equation of the directrix given by $x=\frac{5}{3}$.
   (a) Identify the conic section.
   (b) Write down the equation of conic section.
   (c) Find the centre, focus or foci, vertex or vertices, directrix or directrices of the conic section.
   (d) Sketch the conic section.

9. The cable of a suspension bridge is suspended between two towers that are 200 metres apart and 75 metres high. If the cable at its lowest is 15 metres above the bridge, what is the height of the cable from the bridge at a point 40 metres from the centre of the bridge?
10. Discuss each of the following conic sections and sketch the graph:

    (a) $3x^2 + 4\sqrt{3}xy - y^2 = 7$ **(b)** $6x^2 + 4\sqrt{3}\,xy + 2y^2 + 40\sqrt{3}\,x + \left(40 - \sqrt{3}\right) y + 201 = 0$

    (c) $400x^2 - 640xy + 256y^2 = 369$ **(d)** $3x^2 + 2\sqrt{3}xy + y^2 - 8x + 8\sqrt{3}y = 0$

    (e) $4x^2 + 12xy + 9y^2 - 6\sqrt{13}\,x + 4\sqrt{13}\,y + 39 = 0$ **(f)** $5x^2 + 5y^2 - 6xy - 10\sqrt{2}\,x + 6\sqrt{2}\,y + 2 = 0$
11. For each of the following parametric equations with corresponding parametric intervals, find the Cartesian equation.

    (a) $x = t^3, \quad y = \frac{t^2}{2}, \quad -\infty < t < \infty$

    (b) $x = \sin(2\pi(1-t)), \quad y = \cos(2\pi(1-t)), \quad 0 \le t \le 1$

    (c) $x = \sin\alpha, \quad y = \cos 2\alpha, \ 0 \le \alpha \le \pi$

    (d) $x = t^2, \quad y = \sqrt{t^4 + 1}, \quad t \ge 0$

    (e) $x = 2t, \quad y = |t - 2|, \quad -\infty < t < \infty$

    (f) $x = e^t, \quad y = e^{3t} + 1, \quad -\infty < t < \infty$
12. Parametrise each of the following Cartesian equations using the given parameter and state the parameter interval:

    (a) $y - 2 = 4(x-1)^2$, parameter: slope $t = \frac{dy}{dx}$

    (b) $16x^2 + 3y^2 - 64x + 64 = 48$, parameter: angle θ

    (c) $-x^2 + 4y^2 + 2x = 5, \ y \le -1$, parameter: angle θ
13. For each of the following given polar equations, find the equivalent Cartesian equation. Then, identify and describe the graph.

    (a) $r = 4\csc\theta$ (b) $r = 4\tan\theta\sec\theta$

    (c) $r\sin\theta = \ln r + \ln\cos\theta$ (d) $r = \dfrac{7}{1 - \cos\theta}$
14. Replace the following Cartesian equations by their equivalent polar equations:

    (a) $x^2 - y^2 = 1$ (b) $xy = 2$

    (c) $(x-5)^2 + (y+1)^2 = 4$ (d) $x^2 + xy + y^2 = 1$
15. Discuss each of the following conics:

    (a) $r = \dfrac{2\sqrt{3}}{1 + \sin\theta}$ (b) $r = \dfrac{27}{3 + 4\cos\theta}$

    (c) $r = \dfrac{12}{4 - 8\sin\theta}$ (d) $r = \dfrac{32 - 32\cos\theta}{3\sin^2\theta}$
16. The orbit of the asteroid Apollo, with the sun at one focus, is approximated by

$$r = \frac{9}{9 + 5\cos\theta},$$

    where r is measured in astronomical units. Find the longest distance of the Apollo from the sun.