THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS & STATISTICS
MAT 2110–Engineering Mathematics I

Tutorial Sheet 3 **May 2024**

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1. Evaluate each of the following integrals:

   (a) $\displaystyle\int \frac{e^{\sqrt{x+1}}}{\sqrt{x+1}}\,dx$ **(b)** $\displaystyle\int \sin\theta\sqrt{7-2\cos\theta}\,d\theta$ **(c)** $\displaystyle\int e^x \sec^2(e^x)\,dx$

   (d) $\displaystyle\int_0^{\sqrt{e-1}} \frac{2x\ln\left(x^2+1\right)}{x^2+1}\,dx$ **(e)** $\displaystyle\int \frac{dx}{\sqrt{e^{2x}-1}}$ **(f)** $\displaystyle\int \frac{3^x}{\sinh(e^x)}\,dx$

   (g) $\displaystyle\int_0^1 x^3\sqrt{1-x^2}\,dx$ **(h)** $\displaystyle\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sqrt{4+4\cot^2 2t}\;dt$ **(i)** $\displaystyle\int \frac{x^5}{\sqrt[3]{1-2x^3}}\,dx$

2. Evaluate each of the following integrals:

   (a) $\displaystyle\int \frac{\sin^2\theta}{\cot\theta\,\cos^3\theta}\,d\theta$ **(b)** $\displaystyle\int \sec^3 t\tan^3 t\,dt$ **(c)** $\displaystyle\int \csc^4 x\cot^4 x\,dx$

   (d) $\displaystyle\int \left(\tan^4\theta-1\right)^2\,d\theta$ **(e)** $\displaystyle\int \sin^7\theta\,d\theta$ **(f)** $\displaystyle\int \sin^4 3x\cos^2 3x\,dx$

   (g) $\displaystyle\int_{-\ln 2}^{\ln 2} \frac{\cosh\theta}{\sinh\theta+\cosh\theta}\,d\theta$ **(h)** $\displaystyle\int \operatorname{sech}^5 5x\,\tanh 5x\,dx$ **(i)** $\displaystyle\int \tanh^3 x\,dx$

3. Use both trigonometric substitution and hyperbolic substitution to evaluate each of the following:

   (a) $\displaystyle\int \frac{x^2}{\sqrt{9-x^2}}\,dx$ **(b)** $\displaystyle\int \frac{x+1}{\sqrt{4-x^2}}$

   (c) $\displaystyle\int_{-1}^{0} \frac{dx}{\sqrt{3-2x-x^2}}$ **(d)** $\displaystyle\int_{-2}^{-1} \frac{x\,dx}{x^2+4x+5}$

4. Evaluate each of the following integrals:

   (a) $\displaystyle\int \frac{49x^2-19x-18}{12x^3+13x^2-20x+4}\,dx$ **(b)** $\displaystyle\int \frac{3x^4+x^3+2x+1}{x^4(2x+1)}\,dx$

   (c) $\displaystyle\int \frac{2x^3-2x^2-12x-17}{(x-3)(x^2+2x+2)}\,dx$ **(d)** $\displaystyle\int \frac{x^4-147x^3-40x^2-3x+1}{(7x+1)^2(x^2+1)^2}\,dx$

5. Evaluate each of the following indefinite integrals:

   (a) $\displaystyle\int \frac{dx}{\sqrt{x}-x\sqrt{x}}$ **(b)** $\displaystyle\int \frac{\sqrt{x}}{2+\sqrt[4]{x}}\,dx$

   (c) $\displaystyle\int \frac{1+\sqrt{2x+1}}{x+1}\,dx$ **(d)** $\displaystyle\int \frac{x^3}{1+\sqrt[5]{2x^2+1}}\,dx$

6. Evaluate the following integrals:

(a) $\displaystyle\int \frac{dx}{1-\sin x}$ (d) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{d\theta}{2+\cos\theta}$ (g) $\displaystyle\int \frac{d\theta}{\sin\theta-\cos\theta}$

(b) $\displaystyle\int \frac{d\theta}{3-4\sin\theta+2\cos\theta}$ (e) $\displaystyle\int \frac{dx}{2\sin x+\cos x}$ (h) $\displaystyle\int \frac{\cos x}{\sin x-1}\,dx$

(c) $\displaystyle\int \frac{dt}{1+3\cos t}$ (f) $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{d\theta}{1+\sin\theta+\cos\theta}$ (i) $\displaystyle\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} \frac{\cos\theta\,d\theta}{\sin\theta\cos\theta+\sin\theta}$

7. Evaluate the following indefinite integrals:

(a) $\displaystyle\int x^2\tan^{-1}x\,dx$ **(b)** $\displaystyle\int \sin\left(\sqrt{x}\right)\,dx$ **(c)** $\displaystyle\int \ln\left(x-\sqrt{x^2-1}\right)\,dx$

8. Derive the reduction formula for each of the following and use it to evaluate the indicated integral:

(a) $\displaystyle\int x^n\sin(ax)\,dx, a\in\mathbb{R};$ $\displaystyle\int x^4\sin\left(\frac{1}{3}x\right)\,dx$

(b) $\displaystyle\int \sinh^n x\,dx;$ $\displaystyle\int_0^{\sinh^{-1}1} \sinh^5 x\,dx$

(c) $\displaystyle\int \frac{x^n}{\sqrt{x^2+1}}\,dx;$ $\displaystyle\int \frac{x^4}{\sqrt{x^2+1}}\,dx$

(d) $\displaystyle\int \frac{dx}{(a^2+x^2)^{n+1}}, a\in\mathbb{R};$ $\displaystyle\int \frac{dx}{(4+x^2)^3}$

9. Evaluate the following improper integrals:

(a) $\displaystyle\int_{-\infty}^{0} \frac{dx}{x^2+1}\,dx$ **(b)** $\displaystyle\int_1^{\infty} \frac{\ln x}{x^2}\,dx$ **(c)** $\displaystyle\int_{-\infty}^{\infty} 2xe^{-x^2}\,dx$

(d) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin^3\theta}{\sqrt{\cos\theta}}\,d\theta$ **(e)** $\displaystyle\int_0^2 \frac{x^3}{\sqrt{16-x^4}}\,dx$ **(f)** $\displaystyle\int_{-1}^{1} \frac{dx}{\sqrt{x^7}}$

10. Find the area bounded by the following curves and lines:

(a) finite area between $y=\frac{1}{2}x-2x^2$ and $y=x^2-\frac{35}{2}x+24$

(b) between the $x-$axis and $y=\sqrt{1+\cos 4x},\ 0\le x\le 4$

(c) between $y=\cot\theta$ and $y=\csc\theta$ from $\theta=0$ to $\theta=\frac{\pi}{2}$

11. Find the length of the portion of the curve described below:

(a) $y=\cosh t,\ -\ln 2\le t\le \ln 2$

(b) $x=\frac{2}{3}(y-1)^{\frac{3}{2}},\quad 1\le y\le 4$

(c) $x=2\cos^3 t,\ y=2\sin^3 t,\quad 0\le t\le 2\pi$

(d) $\dfrac{r}{\sqrt{1+\sin 2\theta}}=1,\quad 0\le\theta\le\pi\sqrt{2}$

12. Use the disc or the washer method to find the volume of the solids described below:

(a) The finite region bounded by $y=\sqrt{x}$ and $y=x$ is rotated about

(i) the $x-$axis **(ii)** the $y-$axis **(iii)** the line $x=1$ **(iv)** the line $y=2$

(b) The finite region enclosed by the curve $\frac{y^2}{9}-x^2=1$ and the line $y=\frac{1}{3}x+3$ is rotated about

(i) the $x-$axis **(ii)** the $y-$axis **(iii)** the line $y=3$

(c) The region lying below the $x-$axis of the function

$$f(x) = \begin{cases} -\frac{3}{2}x, & -4 \leq x \leq -2 \\ -x^2 - 2x + 3, & -2 < x \leq 1 \end{cases}$$

is revolved about

(i) the $x-$axis **(ii)** the line $y = -1$

13. Use the shell method to find the volume of the solids described below:

(a) The region under the curve $y = e^x - 1$, $0 \leq x \leq \ln 2$, is rotated about

(i) the $x-$axis **(ii)** the $x-$axis **(iii)** the line $y = -2$ **(iv)** the line $x = 3$

(b) The finite region bounded by $y = \arctan x$, $y = \frac{\pi}{4}$ and the $y-$axis is rotated about

(i) the $y-$axis **(ii)** the line $x = 1$

(c) The region bounded by the lines $y = 0$axis and $y = 4 - x$ and the curve $y = \sqrt{x-2}$ is revolved about

(i) the $x-$axis **(ii)** the $y-$axis **(iii)** the line $y = 1$ **(iv)** the line $x = 2$

14. Find the area of each of the following surface of revolution generated by revolving the given curve about the indicated axis:

(a) $y = \sqrt[3]{x}$, $1 \leq y \leq 2$; $y-$ axis

(b) $x = \frac{1}{3}y^{\frac{3}{2}} - y^{\frac{1}{2}}$, $0 \leq y \leq 9$; $y-$ axis

(c) $y = \sqrt{r^2 - x^2}$, $-r \leq x \leq r$; $x-$ axis

15. For each of the following rods lying on the $x-$axis with varying densities $\delta(x)$, find the moment about the origin and the centre of mass:

(a) $\delta(x) = 1 - \frac{x}{2}$, $0 \leq x \leq 4$

(b) $\delta(x) = 1 + \frac{x}{\sqrt{x}}$, $1 \leq x \leq 4$

(c) $\delta(x) = \begin{cases} 2 - x, & 0 \leq x < 1 \\ x, & 1 \leq x < 2 \end{cases}$

16. Find the centre of mass or centroid of a thin plate covering the region described below:

(a) The region bounded by $y = x - x^2$ and $y = -x$ with density ρ

(b) The region enclosed by the $x-$axis and the curve $y = \cos x$ with density $\delta = \pi$

(c) The region between the curve $y = \frac{2}{x}$ and the $x-$axis from $x = 1$ to $x = 4$ with density $\delta(x) = \sqrt{x}$

(d) The region between the $x-$axis and the curve $y = \sec^2(x^2)$, $\frac{-\pi}{\sqrt{4}} \leq x \leq \frac{\pi}{\sqrt{4}}$ with density $\rho(x) = x$.