THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS & STATISTICS
MAT 2110–Engineering Mathematics I

Tutorial Sheet 4 **July 2024**

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1. Determine the convergence or the divergence of the following sequences:

   (a) $\{a_n\} = \left\{\frac{1-(-1)^n}{\sqrt{n}}\right\}$ (b) $\{a_n\} = \left\{\sin\left(\frac{n\pi}{2}\right)\right\}$ (c) $\{a_n\} = \left\{n\left(2^{\frac{1}{n}} - 1\right)\right\}$ (d) $\{a_n\} = \left\{\frac{(-4)^n}{n!}\right\}$

2. Find the n^{th} partial sum of each of the following series. Hence determine whether the series converge or diverge:

   (a) $4 + \frac{4}{3} + \frac{4}{9} + \frac{4}{27} + \cdots + \frac{4}{3^{n-1}} + \ldots$

   (b) $\frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \cdots + \frac{1}{(n+1)(n+2)} + \ldots$

   (c) $\frac{2}{2.4} + \frac{2}{3.5} + \frac{2}{4.6} + \cdots + \frac{2}{(n+1)(n+3} + \ldots$

3. Determine the convergence or the divergence of the following series:

   (a) $\sum_{n=1}^{\infty} \frac{2^n}{3^{n-1}}$ **(b)** $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$ **(c)** $\sum_{n=1}^{\infty} n^{-0.5}$

   (d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$ **(e)** $\sum_{n=1}^{\infty} \frac{e^n}{3}$ **(f)** $\sum_{n=1}^{\infty} \frac{(-1)^n n}{2n+1}$

4. Verify that the Integral Test can be applied to the following series and use it to determine convergence or divergence:

   (a) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ (b) $\sum_{n=1}^{\infty} \frac{\arctan n}{n^2+1}$ (c) $\sum_{n=1}^{\infty} \frac{n}{n^4+1}$

5. Use the Direct Comparison Test to determine the convergence or the divergence of the following series:

   (a) $\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$ (b) $\sum_{n=1}^{\infty} \frac{1}{2n-3}$ (c) $\sum_{n=1}^{\infty} \frac{\ln n}{\ln(\ln n)}$

6. Use the Limit Comparison Test to determine the convergence or the divergence of the following series:

   (a) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$ (b) $\sum_{n=1}^{\infty} \frac{5}{4^n+3}$ (c) $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

7. Determine the convergence or the divergence of the following series using the Ratio or the Root Test:

   (a) $\sum_{n=1}^{\infty} \frac{2^n 3^n}{n^n}$ (b) $\sum_{n=1}^{\infty} \frac{n+1}{n!}$ (c) $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$ (d) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\left(\frac{3}{n}\right)^n}{n^2}$

8. Find the first four non-zero terms of both the Maclaurin series and Taylor series (at $x = c$) generated by the following functions:

(a) $f(x) = \sec x, \quad c = \pi$

(b) $g(x) = \dfrac{x}{x-1}, \quad c = 2$

(c) $h(x) = \sqrt{1+x}, \quad c = 3$

9. Find the radius and interval of convergence of the following series:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+4)^n}{n(3^n)}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-1)^{2n-2}}{(2n-1)!}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(3x-1)^n}{n^2}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^n}$

10. Find the Maclaurin series for the following functions:

(a) $f(x) = \dfrac{2x}{1+x^2}$

(b) $g(x) = \cos\left(\sqrt{5x}\right)$

(c) $h(x) = \ln[(1-x)(1-2x)]$

(d) $m(x) = \sinh x$

(e) $n(x) = \ln(1+x^2)$

(f) $p(x) = \sec x$

11. Use the Maclaurin series for $\ln(1-x)$ to approximate $\ln\left(\frac{3}{2}\right)$ so that the absolute value error is less that 10^{-4}.

12. Given that

$$\arctan x = x - \frac{x^3}{3} + \frac{x^3}{3} - \frac{x^5}{5} + \cdots + (-1)^{n-1}\frac{x^{2n-1}}{2n-1} + \ldots, \quad |x| < 1,$$

evaluate the following integral:

(a) $\displaystyle\int_0^1 \frac{\arctan(x^2)}{x}\,dx$ (b) $\displaystyle\int_0^{\frac{1}{64}} \frac{\arctan x}{\sqrt{x}}\,dx$

giving your answer such that the absolute error does not exceed 0.01.