THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS & STATISTICS
MAT 2110–Engineering Mathematics I

Tutorial Sheet 5 **August 2024**

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1. Describe the sets of points in space whose coordinates satisfy the given inequalities or combinations of equations and inequalities:
   (a) $x \geq 0, \quad y \geq 0, \quad z = 0$
   (b) $0 \leq x \leq 1$
   (c) $x^2 + y^2 \leq 1, \quad z = 3$
2. Given that $u = 2i - 3j + 4k, \quad v = -2i - 3j + 5k$ and $w = i - 7j + 3k$, find
   (a) $u \cdot v$ (b) $(u + v) \cdot w$ (c) $Proj_w u$ (d) the angle between u and w
3. Find the unit vector(s) whose direction angles are equal.
4. Use the dot product to find two unit vectors perpendicular to the vectors $-2i + 4k$ and $3i - 2j - k$.
5. Find an equation for the line:
   (a) containing the point $(3, 1, -2)$ and parallel to $\frac{x-2}{3} = \frac{y+1}{6} = \frac{z-5}{2}$
   (b) in the $xy-$plane that passes through the points $(1, 3, 0)$ and $(2, 4, 0)$
   (c) perpendicular to, and passing through the point of intersection of the lines
   $x = -1 + 2t, \quad y = 1 - t, \quad z = 7 + 3t$ and $\frac{x-1}{-2} = \frac{y+14}{8} = \frac{z-6}{-1}$
6. Discuss the intersection of the following pair of lines:
   (a) $\mathcal{L}_1 : \dfrac{x-2}{3} = \dfrac{y-5}{2} = \dfrac{z-1}{-1}$

   $\mathcal{L}_2 : \dfrac{x-4}{-4} = \dfrac{y-5}{4} = \dfrac{z+2}{1}$

   (b) $\mathcal{L}_1 : x = -2 - 5t, \quad y = -3 - 2t, \quad z = 1 + 4t$
   $\mathcal{L}_2 : x = 2 + 3n, \quad y = -1 + n, \quad z = 3n$

   (c) $\mathcal{L}_1 : x = 1 - 2t, \quad y = 3t, \quad z = 0$
   $\mathcal{L}_2 : x = -2 + 3m, \quad y = 3 - 6m, \quad z = 1 + m$

   (d) $\mathcal{L}_1 : \langle x, y, z\rangle = \langle 7, -1, 1\rangle + t\langle 2, -1, 3\rangle$
   $\mathcal{L}_2 : \dfrac{x-3}{-2} = \dfrac{y+2}{1} = \dfrac{z-4}{-3}$

   (e) $\mathcal{L}_1 : \dfrac{x+1}{20} = \dfrac{y}{-5} = \dfrac{z-1}{10}$

   $\mathcal{L}_2 : x = 11 - 4t, \quad y = -3 + t, \quad z = 7 - 2t$
7. Find the area of a parallelogram with adjacent vertices at $(-2, 1, 0), \quad (1, 4, 2)$ and $(-3, 1, 5)$.
8. Calculate the area of a triangle with vertices at $(3, 1, 7), \quad (2, -1, 4)$ and $(7, -2, 4)$.
9. Calculate the volume of a parallelepiped determined by the points $P(2, 1, -1), \quad Q(-3, 1, 4)$, $R(-1, 0, 2)$ and $S(-3, -1, 5)$.
10. Find an equation for the plane:
    (a) through the points $P(1, -1, 2), \quad Q(2, 1, 3)$, and $R(-1, 2, -1)$
    (b) through the point $(-1, 6, 0)$ perpendicular to the line $x = -1 + t, \quad y = 6 - 2t, \quad z = 3t$
    (c) containing the line $\langle x, y, z\rangle = \langle 1, 2, 3\rangle + t\langle -1, -4, 1\rangle$ and parallel to $u = 2i + 3j + k$ and $v = i - j + 2k$.

11. Find the distance between:
    (a) the point $(2, 2, 0)$ and the line $x = -t, \quad y = t, \quad z = -1 + t$
    (b) the lines $\frac{x+2}{-5} = \frac{y+3}{-2} = \frac{z-1}{4}$ and $\frac{x-2}{3} = \frac{y+1}{1} = \frac{z}{3}$
    (c) the point $(3, 0, 10)$ and the plane $2x + 3y + z = 2$
12. (a) Show that the distance between the parallel planes $Ax + By + Cz = D_1$ and $Ax + By + Cz = D_2$ is
    $$d = \frac{|D_1 - D_2|}{\sqrt{A^2 + B^2 + C^2}}.$$
    (b) Hence find an equation for the plane parallel to the plane $2x - y + 2z = -4$ if the point $(3, 2, -1)$ is equidistant from the two planes.
13. Discuss the intersection of each of the following pair of a lines and a plane:
    (a) $x = 3 + 2t, \quad y = 2t, \quad z = t$ and $x + 3y - z + 4 = 0$
    (b) $\dfrac{x+1}{3} = \dfrac{z}{5}, \ y = -2$ and $2x - 3z = 7$
14. Discuss the intersection of following planes:
    (a) $x - 2y + 4z = 2; \quad x + y - 2z = 5$
    (b) $3x - 3y + 21z = 4; \quad 7x - 7y + 49z = 5$
    (c) $-2x + 4y - 12z = 5; \quad 3x - 6y + 18z = 7.5$
15. In each of the following, $R(t)$ is the position of a particle in space at time t. Find the particle's velocity, acceleration, speed and acceleration scalar:
    (a) $R(t) = (\sec t)i + (\tan t)j + \frac{4}{3}tk, \quad t = \frac{\pi}{6}$
    (b) $(2\ln(t+1))\,i + t^2 j + \frac{t^2}{2}k$
16. Find the length of the indicated portion of the curve:
    (a) $R(t) = \left(\frac{t^2}{2}\right) i + \left(\frac{t^3}{3}\right) k, \quad 0 \le t \le \sqrt{8}$
    (b) $R(t) = (t\cos t)\,i + (t\sin t)\,j + \left(\frac{2\sqrt{2}}{3}\right) t^{\frac{3}{2}} k, \quad 0 \le t \le \pi$
    (c) $R(t) = (1 + 2t)\,i + (1 + 3t)\,j + (6 - 6t)\,k, \quad$ from $\quad (-1, -2, 12) \quad$ to $\quad (1, 1, 6)$
17. Find the point on the curve
    $$R(t) = (12\sin t)\,i - (12\cos t)\,j + 5tk$$
    at a distance 13π units along the curve from the origin in the direction opposite to the direction of increasing arc length.
18. For each given space curve $R(t)$, find the unit tangent vector, $\mathbf{T}$, the principal unit normal vector, $\mathbf{N}$, the Binormal vector, $\mathbf{B}$, and the curvature, κ :
    (a) $R(t) = \left(\frac{t^3}{3}\right) i + \left(\frac{t^2}{2}\right) j, \quad t > 0$
    (b) $R(t) = (\cos t + t\sin t)\,i + (\sin t - t\cos t)\,j + 3k$
    (c) $R(t) = (\cos t)i + (\sin t)j + tk, \quad t = 0$
    (d) $R(t) = (\cosh t)i - (\sinh t)j + tk$
19. In each of the following, write $\mathbf{a} = a_{\mathrm{T}}\,\mathbf{T} + a_{\mathrm{N}}\,\mathbf{N}$ without finding $\mathbf{T}$ and $\mathbf{N}$:
    (a) $R(t) = (1 + 3t)\,i + (t - 2)\,j - 3tk$
    (b) $R(t) = (t + 1)i + (2t)j + t^2k, \quad t = 1$
    (c) $R(t) = (e^t\cos t)i + (e^t\sin t)j + \sqrt{2}\,e^t k, \quad t = 0$