THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS & STATISTICS
MAT 2110–Engineering Mathematics I

Tutorial Sheet 6 **September 2024**

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1. Find the domain and range of each of the following functions:

   (a) $f(x,y) = \ln\left|36 - 4x^2 + 9y^2\right|$ **(b)** $f(x,y) = \dfrac{1}{\sqrt{16 - x^2 - y^2}}$

   (c) $g(x,y) = \dfrac{\sqrt{y - x^2}}{1 - x^2}$ **(d)** $h(x,y) = \sin^{-1}(y - x)$

2. Evaluate each of the following limits:

   (a) $\displaystyle\lim_{\substack{(x,y)\to(4,3)\\ x\neq y+1}} \frac{\sqrt{x} - \sqrt{y+1}}{x - y - 1}$ **(b)** $\displaystyle\lim_{(x,y,z)\to(\pi,0,3)} ze^{-2y}\cos(2x)$

   (c) $\displaystyle\lim_{(x,y)\to(0,0)} -\frac{x}{\sqrt{x^2+y^2}}$ **(d)** $\displaystyle\lim_{(x,y)\to(0,0)} \ln\left(\frac{3x^2 - x^2y^2 + 3y^2}{x^2+y^2}\right)$

3. Determine the continuity of each of the following:

   (a) $f(x,y) = \begin{cases} \ln\left(\frac{3x^2 - x^2y^2 + 3y^2}{x^2+y^2}\right), & (x,y) \neq (0,0) \\ \ln 3, & (x,y) = (0,0) \end{cases}$ at $(0,0)$

   (b) $g(x,y) = \dfrac{x+y}{2 + \cos x}$ on $\mathbb{R}^2$

   (c) $f(x,y) = \dfrac{1}{x^2 + z^2 - 1}$ at $(-1,1,1)$

   (d) $h(x,y) = \begin{cases} \frac{xy}{|xy|}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ at $(0,0)$

4. Find the indicated partial derivatives in each of the following:

   (a) $f(x,y) = x^y$: $f_x(-2,2)$, $\frac{\partial^2 f}{\partial x \partial y}$

   (b) $f(x,y,z) = \sinh(xy - z^2)$: $\frac{\partial f}{\partial y}$, $\frac{\partial^3 f}{\partial x \partial z^2}$

   (c) $W(P,V,\delta,v,g) = PV + \frac{V\delta v^2}{2g}$: $\frac{\partial W}{\partial V}$

5. If resistors of R_1, R_2, and R_3 ohms are connected in parallel to make an $R-$ohms resistor, the value of R can be found from the equation

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

   Find the value of $\frac{\partial R}{\partial R_2}$ when $R_1 = 30$, $R_2 = 45$, and $R_3 = 90$ ohms.

6. Each of the following equations defines z as a function of the independent variables x and y. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$:
   (a) $xz + y\ln x = x^2 - 4$ **(b)** $x^2 + y^2 + z^2 + \sin(xyz) = 1$

7. Find $\frac{dw}{dt}$, $\frac{\partial w}{\partial t}$, $\frac{\partial w}{\partial s}$, $\frac{\partial w}{\partial r}$, and $\frac{\partial w}{\partial u}$ in each of the following:

   (a) $w = z - \sin(xy)$, where $x = t$, $y = \ln t$, and $z = e^{t-1}$

   (b) $w = \ln(x^2 + y^2 + z^2)$, where $x = t + s$, $y = ts$, and $z = \ln\left(\frac{t}{s}\right)$

   (c) $w = \frac{1}{16}x^4 y + \arctan y$, where $x = t^2 + s + u^3$ and $y = tr^3 + s^2 - 5u$

8. If $z = f(x,y)$, where $x = t + s$ and $y = t - s$. Show that

$$\frac{\partial^2 z}{\partial t \partial s} = \frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2}.$$

9. Given that $z = f(x,y) = x^2 y - 3y$, find Δz and dz if x changes from 4 to 3.99 and y changes from 3 to 3.02.
10. The power P dissipated in a resistor is given by $P = \frac{E^2}{R}$. If $E = 200$ volts and $R = 8$ ohms, find the change in P resulting from a drop of 5 volts in E and an increase of 0.2 ohms in R.
11. Find equations for the tangent plane and normal line at the point P on the given surface:

    (a) $x^2 + 2xy - y^2 + z^2 = 7$, $P(1,-1,3)$

    (b) $z = \ln\left(x^2 + y^2\right)$, $P(1,0,0)$

    (c) $\cos(\pi x) - x^2 y + e^{xz} + yz - 4 = 0$, $P(0,1,2)$

12. Determine the nature of the critical points of each of the following:

    (a) $f(x,y) = x^3 - 3xy + y^3 - 2$

    (b) $f(x,y) = \sin x \cos y$, $0 \le x \le 2\pi$, $0 \le y \le 2\pi$

    (c) $h(x,y) = x^4 - 4x^2 + y^4 - 4y^2$

    (d) $g(x,y) = 1 - x^2 y^2$

13. In each of the following, use the method of **Lagrange multipliers** to find

    (a) the extreme values of the function $f(x,y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$
    (b) the maximum and minimum of $f(x,y,z) = xyz$ if (x,y,z) lies on the ellipsoid $x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$
    (c) the extreme values of $f(x,y,z) = xy + z^2$ on the circle in which the plane $y - x = 0$ intersects the sphere $x^2 + y^2 + z^2 = 4$
    (d) the point closest to the origin on the curve of intersection of the plane $2y + 4z = 5$ and the cone $z^2 = 4x^2 + 4y^2$.