THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS & STATISTICS
MAT 2110–Engineering Mathematics I

Tutorial Sheet 7 **October 2024**

Assignment: Submit solutions to questions labeled * **Due Date: 25/10/2024**

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1. Solve the following differential equations:
   (a) $\dfrac{dy}{dx} = \dfrac{\sec^2 y}{1+x^2}$
   (b) $\dfrac{dy}{dx} = 3x^2\left(1+y^2\right)^{\frac{3}{2}}$
   (c)* $\frac{1}{y}dy + y\, e^{\cos x} \sin x dx = 0$
   (d) $t^{-1}\,\dfrac{dy}{dx} = 2\cos^2 y, \quad y(0) = \dfrac{\pi}{4}$
2. Show that each of the following differential equations is homogeneous. Then, solve the equations:
   (a) $(x+y)dx - xdy = 0$
   (b)* $\left(x^3 + y^2\,\sqrt{x^2+y^2}\right)dx - xy\,\sqrt{x^2+y^2}dy = 0$
   (c)* $\left(3x^2+9xy+5y^2\right)dx - \left(6x^2+4xy\right)dy = 0, \quad y(2) = -6$
3. Determine whether each of the following equations is exact. If not, find the integrating factor. Then, find the solution:
   (a) $(1+ye^x+xye^x)\,dx + (xe^x+2)\,dy = 0, \;\; y(0) = 1$
   (b) $\left(x+3x^3\sin y\right)dx + \left(x^4\cos y\right)dy = 0$
   (c)* $\left(2xy^2+y\right)dx + \left(2y^3-x\right)dy = 0$
   (d)* $(2x+y\cos(xy))dx + (x\cos(xy)-2y)dy = 0$
4. Given the ordinary differential equation $ydx - xdy = 0$,
   (a) show that $I(x,y) = -\dfrac{1}{x^2+y^2}$ is an integrating factor.
   (b) Hence find the general solution to the equation.
5. Solve the following linear ordinary differential equations:
   (a)* $\cos x\frac{dy}{dx} + y\sin x = 2x\cos^2 x, \quad y\left(\frac{\pi}{4}\right) = \frac{-15\sqrt{2}\;\pi^2}{32}$
   (b) $\left(t^2+1\right)\frac{dy}{dt} = yt - t$
   (c)* $\left(1-x^2\right)y' - x^2 y = (1+x)\sqrt{1-x^2}$
6. Solve each of the following ordinary differential equations:
   (a) $\dfrac{dy}{dx} + 2y = xy^{-2}$
   (b)* $y' + xy = xy^2, \quad y(0) = -1$
7. Find the solution to each of the following differential equations:
   (a)* $y'' - y' - 2y = 0$
   (b)* $y'' + \frac{1}{2}y' - \frac{1}{2}y = 0, \quad y(0) = 6, \quad y(\ln 4) = -\frac{1}{4}$
   (c) $y'' - 8y' + 16y = 0$
   (d) $y'' - 6y' + 25y = 0$
   (e)* $\dfrac{d^2y}{dx^2} + 4\pi^2 y = 0, \quad y(1) = 2, \quad y'(1) = 2\pi$
8. Use the method of undetermined coefficients to solve each of the following differential equations:
   (a)* $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 2y = 4x^2, \quad y(0) = 2, \quad y'(0) = 0$
   (b) $y'' - 2y' - 3y = 2e^x - 10\sin x$
   (c) $2\dfrac{d^2y}{dx^2} - 7\dfrac{dy}{dx} + 3y = x - 3e^x$
   (d)* $y'' - 3y' + 2y = x^2e^x$
9. Use the method of variation of parameters to find the general solution to the following differential equations:
   (a) $\dfrac{d^2y}{dx^2} + y = \dfrac{1}{1+\sin x}$
   (b)* $y'' + y = \sec x \csc x$
   (c)* $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 5y = e^x\tan 2x$
10. A metal object has a mass, $m = 2kg$ and a specific heat, $c = 500J/(kg.^oC)$. The object is initially at a temperature of 100^oC and is placed in an environment with a temperature of $T_0 = 25^oC$. The heat transfer coefficient is $h = 10W/(m^2.^oC)$ and the surface area of the object is $A = 0.1m^2$. The equation governing the temperature, $T(t)$ of the object is given by
$$\frac{dT}{dt} = -\frac{hA}{mc}\left(T - T_0\right).$$
    Find the temperature of the object after 20 seconds.
11. * In the study of electric circuits, an application of Kirchhoff's laws leads to the differential equation
$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E(t),$$
    where L is the inductance, R is the resistance, C is the capacitance, $E(t)$ is the electromotive force, $q(t)$ is the charge on the capacitor, and t is the time. If a circuit has in series $E = 100\sin(60t)\;V$, $R = 2\Omega$, $C = \frac{1}{260}$ farads and $L = \frac{1}{10}\;H$, and if the initial charge on the capacitor are both zero, find the charge on the capacitor at any time $t > 0$.