MEC3705 – DYNAMICS

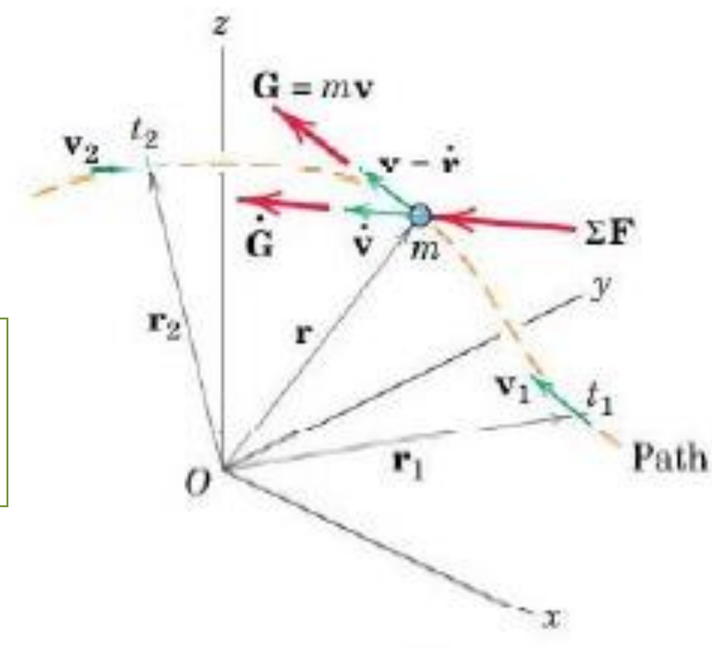
PART I

KINETICS OF PARTICLES – IMPULSE AND MOMENTUM

- In this lesson we shall look at the integration of the equation of motion with respect to time rather than displacement

- This approach leads to the equations of **impulse** and **momentum**

Linear Impulse and linear Momentum



$$\Sigma \mathbf{F} = m\dot{\mathbf{v}} = \frac{d}{dt}(m\mathbf{v}) \qquad or \qquad \boxed{\Sigma \mathbf{F} = \dot{\mathbf{G}}} \qquad \mathbf{(3/25)}$$

We now write the three scalar components of Eq. 3/25 as

$$\Sigma F_x = \dot{G}_x \qquad \Sigma F_y = \dot{G}_y \qquad \Sigma F_z = \dot{G}_z \qquad \mathbf{(3/26)}$$

These equations may be applied independently of one another.

The Linear Impulse-Momentum Principle

$$\Sigma \mathbf{F}\, dt = d\mathbf{G},$$

$$\int_{t_1}^{t_2} \Sigma \mathbf{F}\, dt = \mathbf{G}_2 - \mathbf{G}_1 = \Delta \mathbf{G} \qquad \textbf{(3/27)}$$

Alternatively, we may write Eq. 3/27 as

$$\mathbf{G}_1 + \int_{t_1}^{t_2} \Sigma \mathbf{F}\, dt = \mathbf{G}_2 \qquad \textbf{(3/27}a\textbf{)}$$

Under these conditions, it will be necessary to express $\Sigma\mathbf{F}$ and $\mathbf{G}$ in component form and then combine the integrated components. The components of Eq. 3/27a are the scalar equations

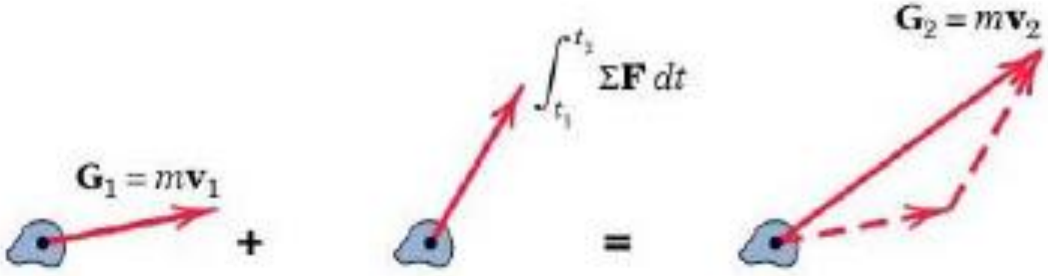
$$m(v_1)_x + \int_{t_1}^{t_2} \Sigma F_x \, dt = m(v_2)_x$$

$$m(v_1)_y + \int_{t_1}^{t_2} \Sigma F_y \, dt = m(v_2)_y \qquad \textbf{(3/27}\boldsymbol{b}\textbf{)}$$

$$m(v_1)_z + \int_{t_1}^{t_2} \Sigma F_z \, dt = m(v_2)_z$$

These three scalar impulse-momentum equations are completely independent.

We now introduce the concept of the *impulse-momentum diagram*. Once the body to be analyzed has been clearly identified and isolated, we construct three drawings of the body as shown in Fig. 3/12. In the first drawing, we show the initial momentum $m\mathbf{v}_1$, or components thereof. In



the second or middle drawing, we show all the external linear impulses (or components thereof). In the final drawing, we show the final linear momentum $m\mathbf{v}_2$ (or its components). The writing of the impulse-momentum equations 3/27*b* then follows directly from these drawings, with a clear one-to-one correspondence between diagrams and equation terms.

Conservation of Linear Momentum

If the resultant force on a particle is zero during an interval of time, we see that Eq. 3/25 requires that its linear momentum **G** remain constant. In this case, the linear momentum of the particle is said to be *conserved*. Linear momentum may be conserved in one coordinate direction, such as x, but not necessarily in the y- or z-direction. A careful examination of the impulse-momentum diagram of the particle will disclose whether the total linear impulse on the particle in a particular direction is zero. If it is, the corresponding linear momentum is unchanged (conserved) in that direction.

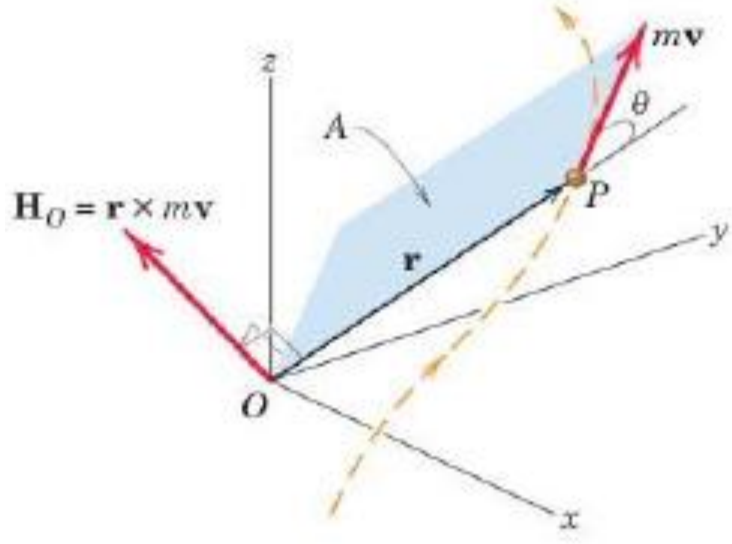
Conservation of Linear Momentum

Consider now the motion of two particles a and b which interact during an interval of time. If the interactive forces $\mathbf{F}$ and $-\mathbf{F}$ between them are the only unbalanced forces acting on the particles during the interval, it follows that the linear impulse on particle a is the negative of the linear impulse on particle b. Therefore, from Eq. 3/27, the change in linear momentum $\Delta\mathbf{G}_a$ of particle a is the negative of the change $\Delta\mathbf{G}_b$ in linear momentum of particle b. So we have $\Delta\mathbf{G}_a = -\Delta\mathbf{G}_b$ or $\Delta(\mathbf{G}_a + \mathbf{G}_b) = \mathbf{0}$. Thus, the total linear momentum $\mathbf{G} = \mathbf{G}_a + \mathbf{G}_b$ for the system of the two particles remains constant during the interval, and we write

$$\Delta\mathbf{G} = \mathbf{0} \qquad \text{or} \qquad \mathbf{G}_1 = \mathbf{G}_2 \tag{3/28}$$

Equation 3/28 expresses the *principle of conservation of linear momentum*.

3/10 ANGULAR IMPULSE AND ANGULAR MOMENTUM

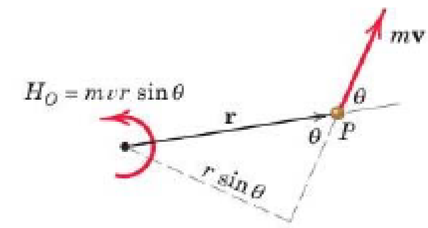


(*a*)

Consider fixed coordinates *x*-*y*-*z*. The velocity of the particle is $\mathbf{v} = \dot{\mathbf{r}}$, and its linear momentum is $\mathbf{G} = m\mathbf{v}$. The *moment* of the *linear momentum vector* $m\mathbf{v}$ about the origin *O* is defined as the *angular momentum* $\mathbf{H}_O$ of *P* about *O* and is given by the cross-product relation for the moment of a vector

$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v} \tag{3/29}$$

The angular momentum then is a vector perpendicular to the plane A defined by $\mathbf{r}$ and $\mathbf{v}$. The sense of $\mathbf{H}_O$ is clearly defined by the right-hand rule for cross products.



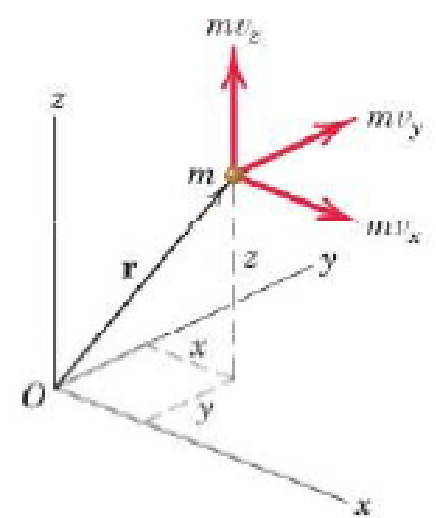
View in plane A

(b)

The scalar components of angular momentum may be obtained from the expansion

$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v} = m(v_z y - v_y z)\mathbf{i} + m(v_x z - v_z x)\mathbf{j} + m(v_y x - v_x y)\mathbf{k}$$

$$\mathbf{H}_O = m \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & z \\ v_x & v_y & v_z \end{vmatrix} \qquad \textbf{(3/30)}$$



so that

$$H_x = m(v_z y - v_y z) \qquad H_y = m(v_x z - v_z x) \qquad H_z = m(v_y x - v_x y)$$

Rate of Change of Angular Momentum

is the vector cross product

$$\Sigma \mathbf{M}_O = \mathbf{r} \times \Sigma \mathbf{F} = \mathbf{r} \times m\dot{\mathbf{v}}$$

the moment $\mathbf{M}_O$

Obtained by differentiation

$$\dot{\mathbf{H}}_O = \dot{\mathbf{r}} \times m\mathbf{v} + \mathbf{r} \times m\dot{\mathbf{v}} = \mathbf{v} \times m\mathbf{v} + \mathbf{r} \times m\dot{\mathbf{v}}$$

The term $\mathbf{v} \times m\mathbf{v}$ is zero since the cross product of parallel vectors is identically zero. Substitution into the expression for $\Sigma \mathbf{M}_O$ gives

$$\Sigma \mathbf{M}_O = \dot{\mathbf{H}}_O \qquad \textbf{(3/31)}$$

Rate of Change of Angular Momentum

Equation 3/31 is a vector equation with scalar components

$$\Sigma M_{O_x} = \dot{H}_{O_x} \qquad \Sigma M_{O_y} = \dot{H}_{O_y} \qquad \Sigma M_{O_z} = \dot{H}_{O_z} \qquad \textbf{(3/32)}$$

The Angular Impulse-Momentum Principle

Equation 3/31 gives the instantaneous relation between the moment and the time rate of change of angular momentum. To obtain the effect of the moment $\Sigma\mathbf{M}_O$ on the angular momentum of the particle over a finite period of time, we integrate Eq. 3/31 from time t_1 to time t_2. Multiplying the equation by dt gives $\Sigma\mathbf{M}_O\,dt = d\mathbf{H}_O$, which we integrate to obtain

$$\int_{t_1}^{t_2} \Sigma\mathbf{M}_O\,dt = (\mathbf{H}_O)_2 - (\mathbf{H}_O)_1 = \Delta\mathbf{H}_O \qquad \textbf{(3/33)}$$

where $(\mathbf{H}_O)_2 = \mathbf{r}_2 \times m\mathbf{v}_2$ and $(\mathbf{H}_O)_1 = \mathbf{r}_1 \times m\mathbf{v}_1$. The product of moment and time is defined as *angular impulse*, and Eq. 3/33 states that the *total angular impulse on m about the fixed point O equals the corresponding change in angular momentum of m about O.*

Alternatively, we may write Eq. 3/33 as

$$(\mathbf{H}_O)_1 + \int_{t_1}^{t_2} \Sigma \mathbf{M}_O \, dt = (\mathbf{H}_O)_2 \qquad \mathbf{(3/33}a\mathbf{)}$$

which states that the initial angular momentum of the particle plus the angular impulse applied to it equals its final angular momentum. The units of angular impulse are clearly those of angular momentum, which are N·m·s or kg·m^2/s in SI units and lb-ft-sec in U.S. customary units.

As in the case of linear impulse and linear momentum, the equation of angular impulse and angular momentum is a vector equation where changes in direction as well as magnitude may occur during the interval of integration. Under these conditions, it is necessary to express $\Sigma \mathbf{M}_O$

and $\mathbf{H}_O$ in component form and then combine the integrated components. The x-component of Eq. 3/33a is

$$(H_{O_x})_1 + \int_{t_1}^{t_2} \Sigma M_{O_x}\, dt = (H_{O_x})_2$$

or

$$m(v_z y - v_y z)_1 + \int_{t_1}^{t_2} \Sigma M_{O_x}\, dt = m(v_z y - v_y z)_2 \qquad \textbf{(3/33b)}$$

where the subscripts 1 and 2 refer to the values of the respective quantities at times t_1 and t_2. Similar expressions exist for the y- and z-components of the angular impulse-momentum equation.

Conservation of Angular Momentum

If the resultant moment about a fixed point O of all forces acting on a particle is zero during an interval of time, Eq. 3/31 requires that its angular momentum $\mathbf{H}_O$ about that point remain constant. In this case, the angular momentum of the particle is said to be *conserved*. Angular momentum may be conserved about one axis but not about another axis. A careful examination of the free-body diagram of the particle will disclose whether the moment of the resultant force on the particle about a fixed point is zero, in which case, the angular momentum about that point is unchanged (conserved).

Conservation of Angular Momentum

Consider now the motion of two particles a and b which interact during an interval of time. If the interactive forces $\mathbf{F}$ and $-\mathbf{F}$ between them are the only unbalanced forces acting on the particles during the interval, it follows that the moments of the equal and opposite forces about any fixed point O not on their line of action are equal and opposite. If we apply Eq. 3/33 to particle a and then to particle b and add the two equations, we obtain $\Delta\mathbf{H}_a + \Delta\mathbf{H}_b = \mathbf{0}$ (where all angular momenta are referred to point O). Thus, the total angular momentum for the system of the two particles remains constant during the interval, and we write

$$\Delta\mathbf{H}_O = \mathbf{0} \quad \text{or} \quad (\mathbf{H}_O)_1 = (\mathbf{H}_O)_2 \qquad \textbf{(3/34)}$$

which expresses the *principle of conservation of angular momentum*.

SECTION D. SPECIAL APPLICATIONS

3/12 IMPACT

The principles of impulse and momentum have important use in describing the behavior of colliding bodies. *Impact* refers to the collision between two bodies and is characterized by the generation of relatively large contact forces which act over a very short interval of time. It is important to realize that an impact is a very complex event involving material deformation and recovery and the generation of heat and sound.

Direct Central Impact

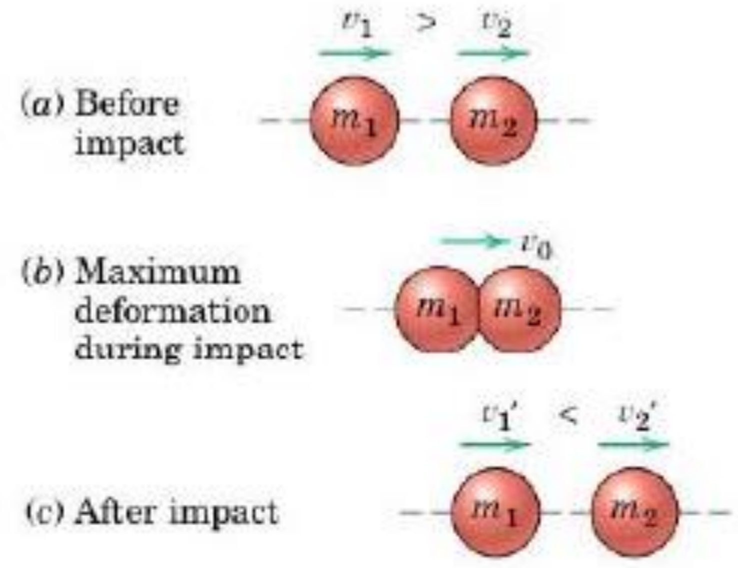


Figure 3/17

As an introduction to impact, we consider the collinear motion of two spheres of masses m_1 and m_2, Fig. 3/17a, traveling with velocities v_1 and v_2. If v_1 is greater than v_2, collision occurs with the contact forces directed along the line of centers. This condition is called *direct central impact*.

Direct Central Impact

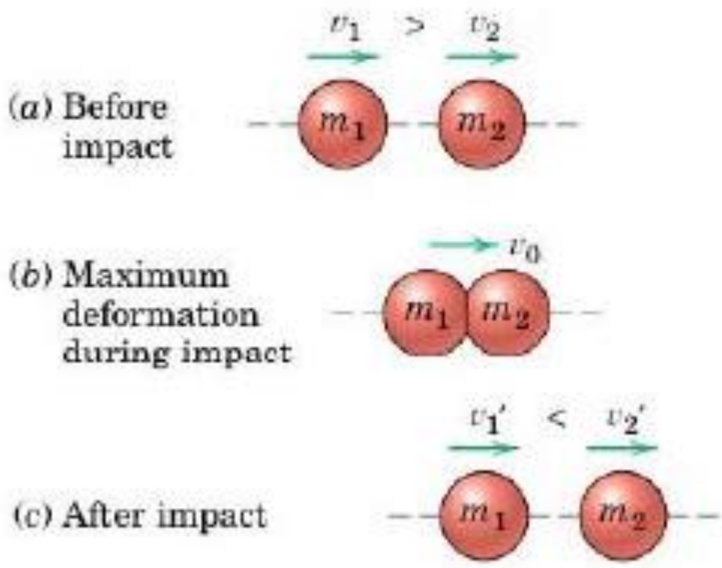


Figure 3/17

Following initial contact, a short period of increasing deformation takes place until the contact area between the spheres ceases to increase. At this instant, both spheres, Fig. 3/17*b*, are moving with the same velocity v_0. During the remainder of contact, a period of restoration occurs during which the contact area decreases to zero. In the final condition shown in part *c* of the figure, the spheres now have new velocities v_1' and v_2', where v_1' must be less than v_2'. All velocities are arbitrarily assumed positive to the right, so that with this scalar notation a velocity to the left would carry a negative sign.

Direct Central Impact

Because the contact forces are equal and opposite during impact, the linear momentum of the system remains unchanged, as discussed in Art. 3/9. Thus, we apply the law of conservation of linear momentum and write

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2' \qquad \textbf{(3/35)}$$

Direct Central Impact

Coefficient of Restitution

For given masses and initial conditions, the momentum equation contains two unknowns, v_1' and v_2'. Clearly, we need an additional relationship to find the final velocities. This relationship must reflect the capacity of the contacting bodies to recover from the impact and can be expressed by the ratio e of the magnitude of the restoration impulse to the magnitude of the deformation impulse. This ratio is called the *coefficient of restitution*.

Direct Central Impact

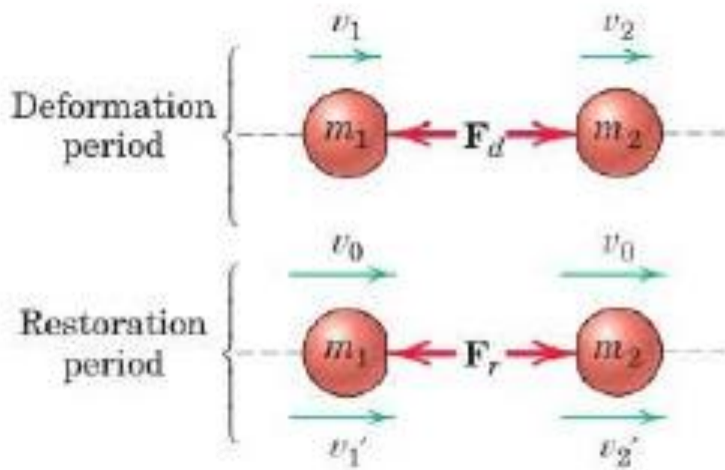


Figure 3/18

Direct Central Impact

Let F_r and F_d represent the magnitudes of the contact forces during the restoration and deformation periods, respectively, as shown in Fig. 3/18. For particle 1 the definition of e together with the impulse-momentum equation give us

$$e = \frac{\int_{t_0}^{t} F_r\,dt}{\int_{0}^{t_0} F_d\,dt} = \frac{m_1[-v_1' - (-v_0)]}{m_1[-v_0 - (-v_1)]} = \frac{v_0 - v_1'}{v_1 - v_0}$$

Similarly, for particle 2 we have

$$e = \frac{\int_{t_0}^{t} F_r\,dt}{\int_{0}^{t_0} F_d\,dt} = \frac{m_2(v_2' - v_0)}{m_2(v_0 - v_2)} = \frac{v_2' - v_0}{v_0 - v_2}$$

Direct Central Impact

We are careful in these equations to express the change of momentum (and therefore Δv) in the same direction as the impulse (and thus the force). The time for the deformation is taken as t_0 and the total time of contact is t. Eliminating v_0 between the two expressions for e gives us

$$e = \frac{v_2' - v_1'}{v_1 - v_2} = \frac{|\text{relative velocity of separation}|}{|\text{relative velocity of approach}|} \qquad \textbf{(3/36)}$$

If the two initial velocities v_1 and v_2 and the coefficient of restitution e are known, then Eqs. 3/35 and 3/36 give us two equations in the two unknown final velocities v_1' and v_2'.

Direct Central Impact

Energy Loss During Impact

Impact phenomena are almost always accompanied by energy loss, which may be calculated by subtracting the kinetic energy of the system just after impact from that just before impact. Energy is lost through the generation of heat during the localized inelastic deformation of the material, through the generation and dissipation of elastic stress waves within the bodies, and through the generation of sound energy.

According to this classical theory of impact, the value $e = 1$ means that the capacity of the two particles to recover equals their tendency to deform. This condition is one of *elastic impact* with no energy loss. The value $e = 0$, on the other hand, describes *inelastic or plastic impact* where the particles cling together after collision and the loss of energy is a maximum. All impact conditions lie somewhere between these two extremes.

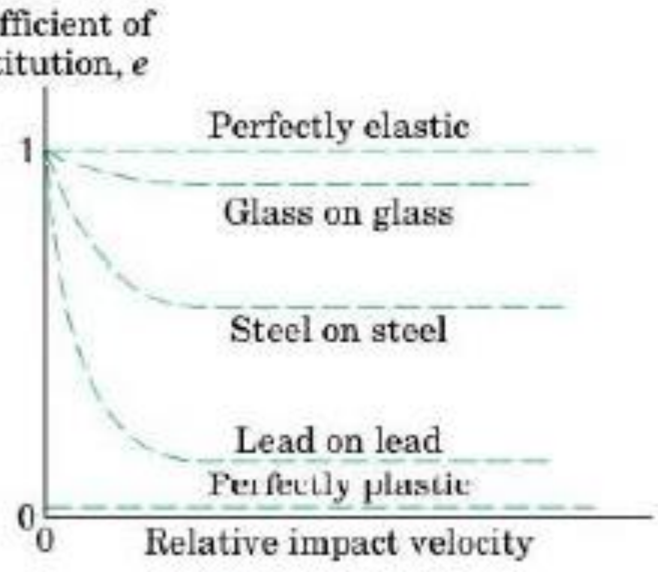


Figure 3/19

Oblique Central Impact

We now extend the relationships developed for direct central impact to the case where the initial and final velocities are not parallel, Fig. 3/20. Here spherical particles of mass m_1 and m_2 have initial velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ in the same plane and approach each other on a collision course, as shown in part *a* of the figure. The directions of the velocity vectors are measured from the direction tangent to the contacting surfaces, Fig. 3/20*b*. Thus, the initial velocity components along the *t*- and *n*-axes are $(v_1)_n = -v_1 \sin\theta_1$, $(v_1)_t = v_1 \cos\theta_1$, $(v_2)_n = v_2 \sin\theta_2$,

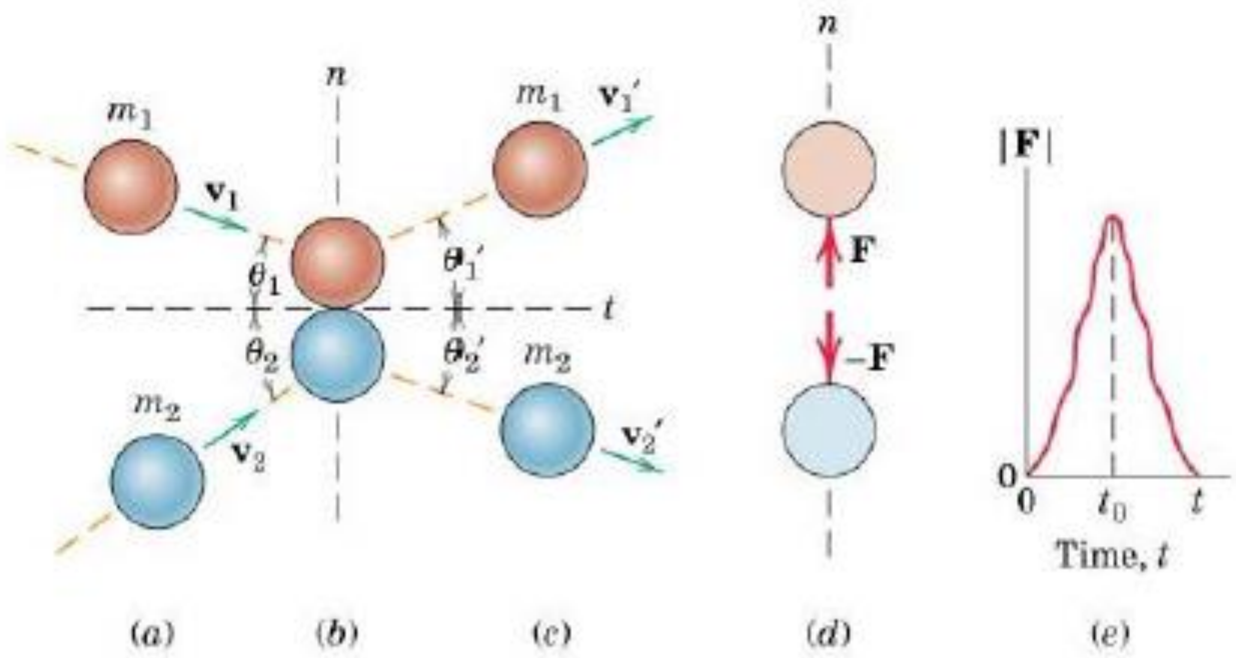
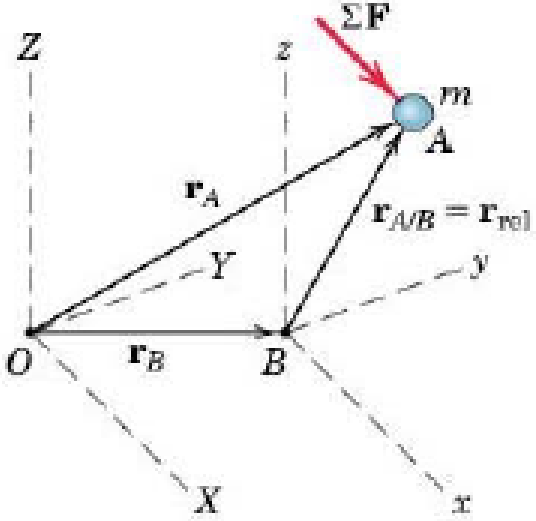


Figure 3/20



Relative-Motion Equation

We now consider a particle A of mass m, Fig. 3/25, whose motion is observed from a set of axes x-y-z which translate with respect to a fixed reference frame X-Y-Z. Thus, the x-y-z directions always remain parallel to the X-Y-Z directions. We postpone discussion of motion relative to a rotating reference system until Arts. 5/7 and 7/7. The acceleration of the origin B of x-y-z is $\mathbf{a}_B$. The acceleration of A as observed from or relative to x-y-z is $\mathbf{a}_{rel} = \mathbf{a}_{A/B} = \ddot{\mathbf{r}}_{A/B}$, and by the relative-motion principle of Art. 2/8, the absolute acceleration of A is

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{rel}$$

Thus, Newton's second law $\Sigma\mathbf{F} = m\mathbf{a}_A$ becomes

$$\Sigma\mathbf{F} = m(\mathbf{a}_B + \mathbf{a}_{rel}) \qquad \textbf{(3/50)}$$

D'Alembert's Principle

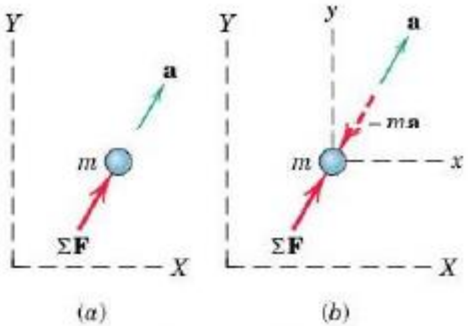


Figure 3/26

The particle acceleration we measure from a fixed set of axes X-Y-Z, Fig. 3/26a, is its absolute acceleration $\mathbf{a}$. In this case the familiar relation $\Sigma\mathbf{F} = m\mathbf{a}$ applies. When we observe the particle from a moving system x-y-z attached to the particle, Fig. 3/26b, the particle necessarily appears to be at rest or in equilibrium in x-y-z. Thus, the observer who is accelerating with x-y-z concludes that a force $-m\mathbf{a}$ acts on the particle to balance $\Sigma\mathbf{F}$. This point of view, which allows the treatment of a dynamics problem by the methods of statics, was an outgrowth of the work of D'Alembert contained in his *Traité de Dynamique* published in 1743.

This approach merely amounts to rewriting the equation of motion as $\Sigma\mathbf{F} - m\mathbf{a} = \mathbf{0}$, which assumes the form of a zero force summation if $-m\mathbf{a}$ is treated as a force. This fictitious force is known as the *inertia force*, and the artificial state of equilibrium created is known as *dynamic equilibrium*. The apparent transformation of a problem in dynamics to one in statics has become known as *D'Alembert's principle*.

Constant-Velocity, Nonrotating Systems

In discussing particle motion relative to moving reference systems, we should note the special case where the reference system has a constant velocity and no rotation. If the *x-y-z* axes of Fig. 3/25 have a constant velocity, then $\mathbf{a}_B = \mathbf{0}$ and the acceleration of the particle is $\mathbf{a}_A = \mathbf{a}_{rel}$. Therefore, we may write Eq. 3/50 as

$$\Sigma\mathbf{F} = m\mathbf{a}_{rel} \qquad (3/51)$$

which tells us that Newton's second law holds for measurements made in a system moving with a constant velocity. Such a system is known as an inertial system or as a Newtonian frame of reference. Observers in the moving system and in the fixed system will also agree on the designation of the resultant force acting on the particle from their identical free-body diagrams, provided they avoid the use of any so-called "inertia forces."

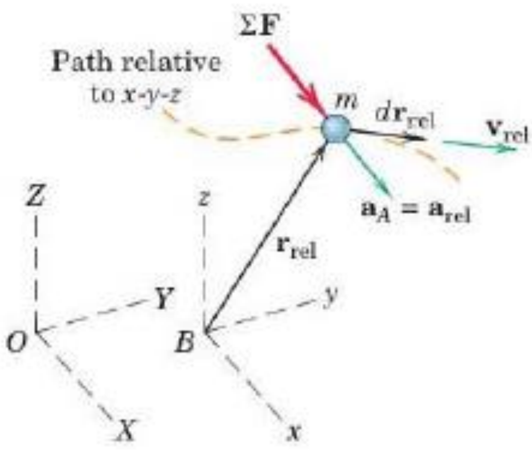


Figure 3/28

We will now examine the parallel question concerning the validity of the work-energy equation and the impulse-momentum equation relative to a constant-velocity, nonrotating system. Again, we take the x-y-z axes of Fig. 3/25 to be moving with a constant velocity $\mathbf{v}_B = \dot{\mathbf{r}}_B$ relative to the fixed axes X-Y-Z. The path of the particle A relative to x-y-z is governed by $\mathbf{r}_{rel}$ and is represented schematically in Fig. 3/28. The work done by $\Sigma\mathbf{F}$ relative to x-y-z is $dU_{rel} = \Sigma\mathbf{F}\cdot d\mathbf{r}_{rel}$. But $\Sigma\mathbf{F} = m\mathbf{a}_A = m\mathbf{a}_{rel}$ since $\mathbf{a}_B = \mathbf{0}$. Also $\mathbf{a}_{rel}\cdot d\mathbf{r}_{rel} = \mathbf{v}_{rel}\cdot d\mathbf{v}_{rel}$ for the same reason that $a_t\,ds = v\,dv$ in Art. 2/5 on curvilinear motion. Thus, we have

$$dU_{rel} = m\mathbf{a}_{rel}\cdot d\mathbf{r}_{rel} = mv_{rel}\,dv_{rel} = d(\tfrac{1}{2}mv_{rel}{}^2)$$

We define the kinetic energy relative to x-y-z as $T_{rel} = \frac{1}{2}mv_{rel}{}^2$ so that we now have

$$dU_{rel} = dT_{rel} \quad \text{or} \quad U_{rel} = \Delta T_{rel} \qquad \textbf{(3/52)}$$

which shows that the work-energy equation holds for measurements made relative to a constant-velocity, nonrotating system.

Relative to x-y-z, the impulse on the particle during time dt is $\Sigma\mathbf{F}\,dt = m\mathbf{a}_A\,dt = m\mathbf{a}_{rel}\,dt$. But $m\mathbf{a}_{rel}\,dt = m\,d\mathbf{v}_{rel} = d(m\mathbf{v}_{rel})$ so

$$\Sigma\mathbf{F}\,dt = d(m\mathbf{v}_{rel})$$

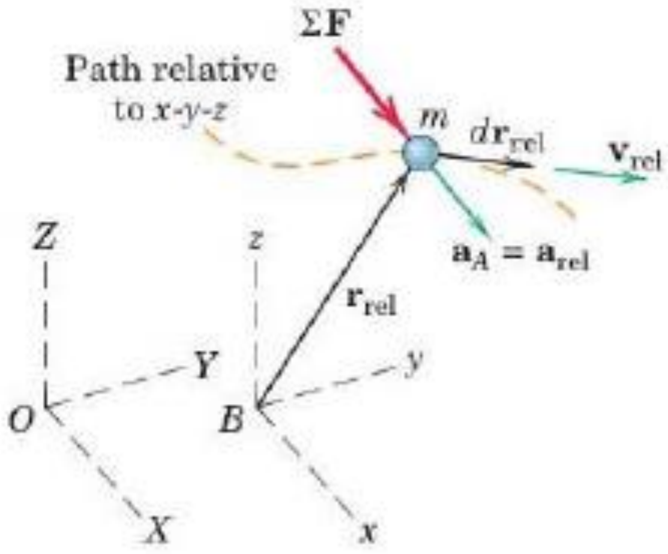


Figure 3/28

We define the linear momentum of the particle relative to x-y-z as $\mathbf{G}_{rel} = m\mathbf{v}_{rel}$, which gives us $\Sigma\mathbf{F}\,dt = d\mathbf{G}_{rel}$. Dividing by dt and integrating give

$$\Sigma\mathbf{F} = \dot{\mathbf{G}}_{rel} \quad \text{and} \quad \int \Sigma\mathbf{F}\,dt = \Delta\mathbf{G}_{rel} \tag{3/53}$$

Thus, the impulse-momentum equations for a fixed reference system also hold for measurements made relative to a constant-velocity, nonrotating system.

3/14 Relative Motion

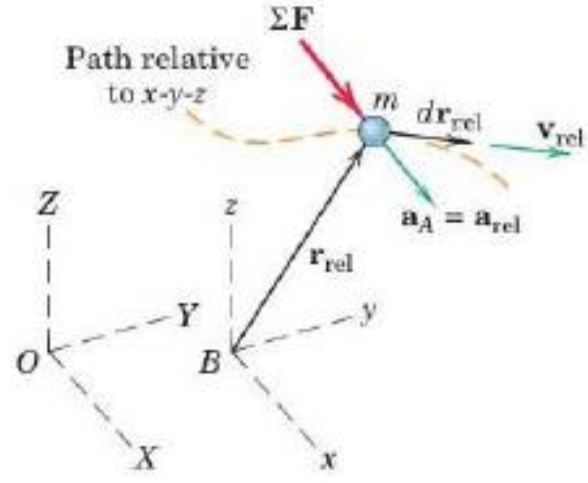


Figure 3/28

Finally, we define the relative angular momentum of the particle about a point in x-y-z, such as the origin B, as the moment of the relative linear momentum. Thus, $(\mathbf{H}_B)_{rel} = \mathbf{r}_{rel} \times \mathbf{G}_{rel}$. The time derivative gives $(\dot{\mathbf{H}}_B)_{rel} = \dot{\mathbf{r}}_{rel} \times \mathbf{G}_{rel} + \mathbf{r}_{rel} \times \dot{\mathbf{G}}_{rel}$. The first term is nothing more than $\mathbf{v}_{rel} \times m\mathbf{v}_{rel} = \mathbf{0}$, and the second term becomes $\mathbf{r}_{rel} \times \Sigma\mathbf{F} = \Sigma\mathbf{M}_B$, the sum of the moments about B of all forces on m. Thus, we have

$$\Sigma\mathbf{M}_B = (\dot{\mathbf{H}}_B)_{rel} \qquad (3/54)$$

which shows that the moment-angular momentum relation holds with respect to a constant-velocity, nonrotating system.

3/14 RELATIVE MOTION

Although the work-energy and impulse-momentum equations hold relative to a system translating with a constant velocity, the individual expressions for work, kinetic energy, and momentum differ between the fixed and the moving systems. Thus,

$$(dU = \Sigma\mathbf{F}\cdot d\mathbf{r}_A) \neq (dU_{\text{rel}} = \Sigma\mathbf{F}\cdot d\mathbf{r}_{\text{rel}})$$

$$(T = \tfrac{1}{2}mv_A{}^2) \neq (T_{\text{rel}} = \tfrac{1}{2}mv_{\text{rel}}{}^2)$$

$$(\mathbf{G} = m\mathbf{v}_A) \neq (\mathbf{G}_{\text{rel}} = m\mathbf{v}_{\text{rel}})$$

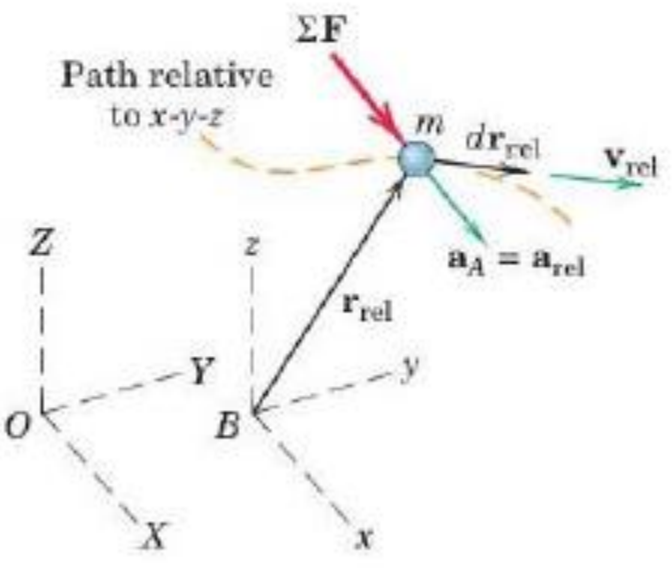


Figure 3/28

Russell Munson/CORBIS

Relative motion is a critical issue during aircraft-carrier landings.

3/15 Chapter Review

In Chapter 3 we have developed the three basic methods of solution to problems in particle kinetics. This experience is central to the study of dynamics and lays the foundation for the subsequent study of rigid-body and nonrigid-body dynamics. These three methods are summarized as follows:

1. Direct Application of Newton's Second Law

First, we applied Newton's second law $\Sigma \mathbf{F} = m\mathbf{a}$ to determine the instantaneous relation between forces and the acceleration they produce. With the background of Chapter 2 for identifying the kind of motion and with the aid of our familiar free-body diagram to be certain that all forces are accounted for, we were able to solve a large variety of problems using x-y, n-t, and r-θ coordinates for plane-motion problems and x-y-z, r-θ-z, and R-θ-ϕ coordinates for space problems.

2. Work-Energy Equations

Next, we integrated the basic equation of motion $\Sigma\mathbf{F} = m\mathbf{a}$ with respect to displacement and derived the scalar equations for work and energy. These equations enable us to relate the initial and final velocities to the work done during an interval by forces external to our defined system. We expanded this approach to include potential energy, both elastic and gravitational. With these tools we discovered that the energy approach is especially valuable for conservative systems, that is, systems wherein the loss of energy due to friction or other forms of dissipation is negligible.

3. Impulse-Momentum Equations

Finally, we rewrote Newton's second law in the form of force equals time rate of change of linear momentum and moment equals time rate of change of angular momentum. Then we integrated these relations with respect to time and derived the impulse and momentum equations. These equations were then applied to motion intervals where the forces were functions of time. We also investigated the interactions between particles under conditions where the linear momentum is conserved and where the angular momentum is conserved.

References

- J.L. Meriam and L. G. Krage, Dynamics 6TH edition