MEC3705 – DYNAMICS

PART I

KINETICS OF SYSTEMS OF PARTICLES

4 Kinetics of Systems of Particles

Chapter Outline

4/1 Introduction

In the previous two chapters, we have applied the principles of dynamics to the motion of a particle. Although we focused primarily on the kinetics of a single particle in Chapter 3, we mentioned the motion of two particles, considered together as a system, when we discussed work-energy and impulse-momentum.

Our next major step in the development of dynamics is to extend these principles, which we applied to a single particle, to describe the motion of a general system of particles. This extension will unify the remaining topics of dynamics and enable us to treat the motion of both rigid bodies and nonrigid systems.

Recall that a rigid body is a solid system of particles wherein the distances between particles remain essentially unchanged. The overall motions found with machines, land and air vehicles, rockets and spacecraft, and many moving structures provide examples of rigid-body problems. On the other hand, we may need to study the time-dependent changes in the shape of a nonrigid, but solid, body due to elastic or inelastic deformations. Another example of a nonrigid body is a defined mass of liquid or gaseous particles flowing at a specified rate. Examples are the air and fuel flowing through the turbine of an aircraft engine, the burned gases issuing from the nozzle of a rocket motor, or the water passing through a rotary pump.

4/2 Generalized Newton's Second Law

We now extend Newton's second law of motion to cover a general mass system which we model by considering n mass particles bounded by a closed surface in space, Fig. 4/1. This bounding envelope, for example, may be the exterior surface of a given rigid body, the bounding surface of an arbitrary portion of the body, the exterior surface of a rocket containing both rigid and flowing particles, or a particular volume of fluid particles. In each case, the system to be considered is the mass within the envelope, and that mass must be clearly defined and isolated.

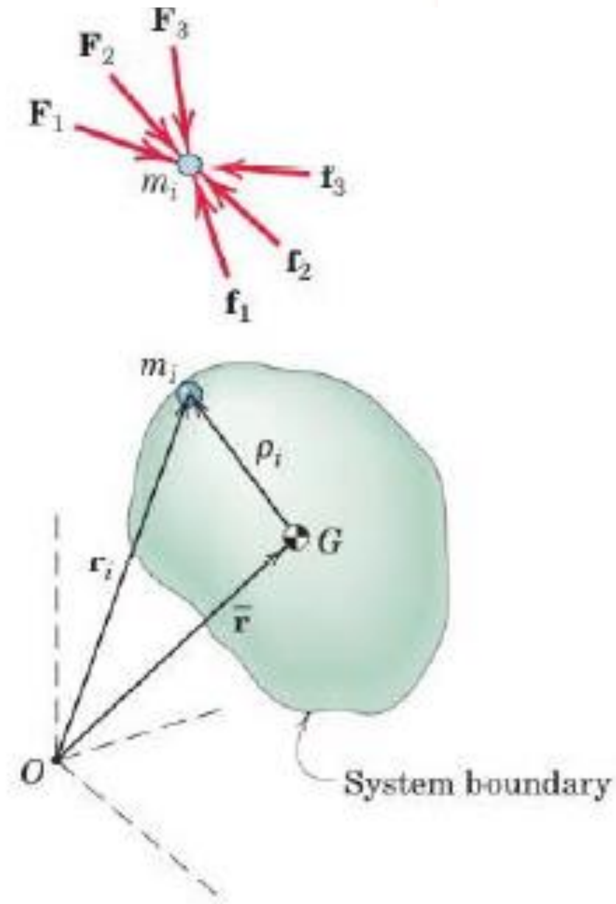

Figure 4/1 shows a representative particle of mass m_i of the system isolated with forces $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, . . . acting on m_i from sources *external* to the envelope, and forces $\mathbf{f}_1$, $\mathbf{f}_2$, $\mathbf{f}_3$, . . . acting on m_i from sources *internal* to the system boundary. The external forces are due to contact with external bodies or to external gravitational, electric, or magnetic effects. The internal forces are forces of reaction with other mass particles within the boundary. The particle of mass m_i is located by its position vector $\mathbf{r}_i$ measured from the nonaccelerating origin O of a Newtonian set of reference axes.* The center of mass G of the isolated system of particles is located by the position vector $\bar{\mathbf{r}}$ which, from the definition of the mass center as covered in statics, is given by

$$m\bar{\mathbf{r}} = \Sigma m_i \mathbf{r}_i$$

where the total system mass is $m = \Sigma m_i$. The summation sign Σ represents the summation $\Sigma_{i=1}^{n}$ over all n particles.

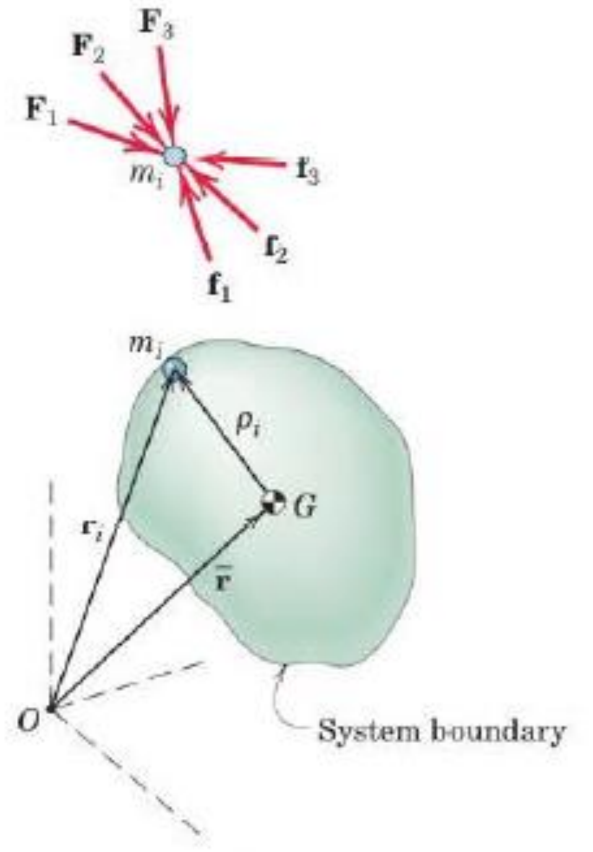

Figure 4/1

Newton's second law, Eq. 3/3, when applied to m_i gives

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots + \mathbf{f}_1 + \mathbf{f}_2 + \mathbf{f}_3 + \cdots = m_i\ddot{\mathbf{r}}_i$$

where $\ddot{\mathbf{r}}_i$ is the acceleration of m_i. A similar equation may be written for each of the particles of the system. If these equations written for *all* particles of the system are added together, the result is

$$\Sigma\mathbf{F} + \Sigma\mathbf{f} = \Sigma m_i\ddot{\mathbf{r}}_i$$

The term $\Sigma\mathbf{F}$ then becomes the vector sum of *all* forces acting on all particles of the isolated system from sources external to the system, and

$\Sigma\mathbf{f}$ becomes the vector sum of all forces on all particles produced by the internal actions and reactions between particles. This last sum is identically zero since all internal forces occur in pairs of equal and opposite actions and reactions. By differentiating the equation defining $\bar{\mathbf{r}}$ twice with time, we have $m\ddot{\bar{\mathbf{r}}} = \Sigma m_i\ddot{\mathbf{r}}_i$ where m has a zero time derivative as long as mass is not entering or leaving the system.* Substitution into the summation of the equations of motion gives

$$\Sigma\mathbf{F} = m\ddot{\bar{\mathbf{r}}} \quad \text{or} \quad \Sigma\mathbf{F} = m\bar{\mathbf{a}} \qquad \textbf{(4/1)}$$

where $\bar{\mathbf{a}}$ is the acceleration $\ddot{\bar{\mathbf{r}}}$ of the center of mass of the system.

Equation 4/1 is the generalized Newton's second law of motion for a mass system and is called the *equation of motion of m*. The equation states that the resultant of the external forces on *any* system of masses equals the total mass of the system times the acceleration of the center of mass. This law expresses the so-called *principle of motion of the mass center*.

Observe that $\bar{\mathbf{a}}$ is the acceleration of the mathematical point which represents instantaneously the position of the mass center for the given n particles. For a nonrigid body, this acceleration need not represent the acceleration of any particular particle. Note also that Eq. 4/1 holds for each instant of time and is therefore an instantaneous relationship. Equation 4/1 for the mass system had to be proved, as it cannot be inferred directly from Eq. 3/3 for a single particle.

Equation 4/1 may be expressed in component form using x-y-z coordinates or whatever coordinate system is most convenient for the problem at hand. Thus,

$$\Sigma F_x = m\bar{a}_x \qquad \Sigma F_y = m\bar{a}_y \qquad \Sigma F_z = m\bar{a}_z \qquad \textbf{(4/1}\boldsymbol{a}\textbf{)}$$

Although Eq. 4/1, as a vector equation, requires that the acceleration vector $\bar{\mathbf{a}}$ have the same direction as the resultant external force $\Sigma\mathbf{F}$, it does not follow that $\Sigma\mathbf{F}$ necessarily passes through G. In general, in fact, $\Sigma\mathbf{F}$ does not pass through G, as will be shown later.

4/3 WORK-ENERGY

In Art. 3/6 we developed the work-energy relation for a single particle, and we noted that it applies to a system of two joined particles. Now consider the general system of Fig. 4/1, where the work-energy relation for the representative particle of mass m_i is $(U_{1\text{-}2})_i = \Delta T_i$. Here $(U_{1\text{-}2})_i$ is the work done on m_i during an interval of motion by all forces $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots$ applied from sources external to the system and by all forces $\mathbf{f}_1 + \mathbf{f}_2 + \mathbf{f}_3 + \cdots$ applied from sources internal to the system. The kinetic energy of m_i is $T_i = \frac{1}{2} m_i v_i^2$, where v_i is the magnitude of the particle velocity $\mathbf{v}_i = \dot{\mathbf{r}}_i$.

Work-Energy Relation

For the entire system, the sum of the work-energy equations written for all particles is $\Sigma(U_{1\text{-}2})_i = \Sigma\Delta T_i$, which may be represented by the same expressions as Eqs. 3/15 and 3/15*a* of Art. 3/6, namely,

$$U_{1\text{-}2} = \Delta T \qquad \text{or} \qquad T_1 + U_{1\text{-}2} = T_2 \tag{4/2}$$

where $U_{1\text{-}2} = \Sigma(U_{1\text{-}2})_i$, the work done by all forces, external and internal, on all particles, and ΔT is the change in the total kinetic energy $T = \Sigma T_i$ of the system.

For a rigid body or a system of rigid bodies joined by ideal frictionless connections, no net work is done by the internal interacting forces or moments in the connections. We see that the work done by all pairs of internal forces, labeled here as $\mathbf{f}_i$ and $-\mathbf{f}_i$, at a typical connection, Fig. 4/2, in the system is zero since their points of application have identical displacement components while the forces are equal but opposite. For this situation $U_{1\text{-}2}$ becomes the work done on the system by the external forces only.

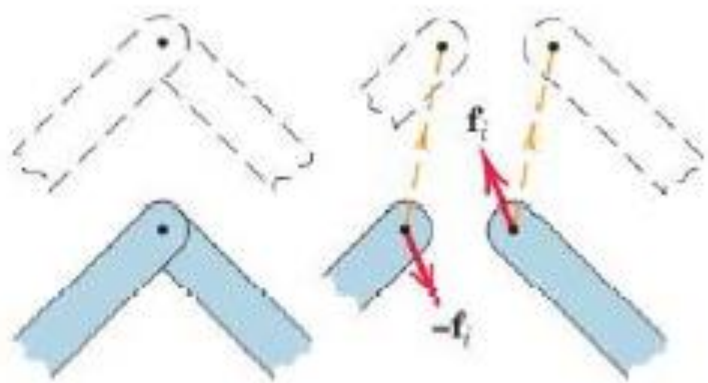

Figure 4/2

For a nonrigid mechanical system which includes elastic members capable of storing energy, a part of the work done by the external forces goes into changing the internal elastic potential energy V_e. Also, if the work done by the gravity forces is *excluded* from the work term and is accounted for instead by the changes in gravitational potential energy V_g, then we may equate the work $U'_{1\text{-}2}$ done on the system during an interval of motion to the change ΔE in the total mechanical energy of the system. Thus, $U'_{1\text{-}2} = \Delta E$ or

$$U'_{1\text{-}2} = \Delta T + \Delta V \qquad \textbf{(4/3)}$$

or

$$T_1 + V_1 + U'_{1\text{-}2} = T_2 + V_2 \qquad \textbf{(4/3}\boldsymbol{a}\textbf{)}$$

which are the same as Eqs. 3/21 and 3/21*a*. Here, as in Chapter 3, $V = V_g + V_e$ represents the total potential energy.

We now examine the expression $T = \Sigma \frac{1}{2} m_i v_i^2$ for the kinetic energy of the mass system in more detail. By our principle of relative motion discussed in Art. 2/8, we may write the velocity of the representative particle as

$$\mathbf{v}_i = \overline{\mathbf{v}} + \dot{\boldsymbol{\rho}}_i$$

where $\overline{\mathbf{v}}$ is the velocity of the mass center G and $\dot{\boldsymbol{\rho}}_i$ is the velocity of m_i with respect to a translating reference frame moving with the mass center G. We recall the identity $v_i^2 = \mathbf{v}_i \cdot \mathbf{v}_i$ and write the kinetic energy of the system as

$$\begin{aligned} T = \Sigma \frac{1}{2} m_i \mathbf{v}_i \cdot \mathbf{v}_i &= \Sigma \frac{1}{2} m_i (\overline{\mathbf{v}} + \dot{\boldsymbol{\rho}}_i) \cdot (\overline{\mathbf{v}} + \dot{\boldsymbol{\rho}}_i) \\ &= \Sigma \frac{1}{2} m_i \bar{v}^2 + \Sigma \frac{1}{2} m_i |\dot{\boldsymbol{\rho}}_i|^2 + \Sigma m_i \overline{\mathbf{v}} \cdot \dot{\boldsymbol{\rho}}_i \end{aligned}$$

Because $\boldsymbol{\rho}_i$ is measured from the mass center, $\Sigma m_i \boldsymbol{\rho}_i = \mathbf{0}$ and the third term is $\overline{\mathbf{v}} \cdot \Sigma m_i \dot{\boldsymbol{\rho}}_i = \overline{\mathbf{v}} \cdot \frac{d}{dt} \Sigma(m_i \boldsymbol{\rho}_i) = 0$. Also $\Sigma \frac{1}{2} m_i \bar{v}^2 = \frac{1}{2} \bar{v}^2 \Sigma m_i = \frac{1}{2} m \bar{v}^2$. Therefore, the total kinetic energy becomes

$$T = \frac{1}{2} m \bar{v}^2 + \Sigma \frac{1}{2} m_i |\dot{\boldsymbol{\rho}}_i|^2 \qquad \textbf{(4/4)}$$

This equation expresses the fact that the total kinetic energy of a mass system equals the kinetic energy of mass-center translation of the system as a whole plus the kinetic energy due to motion of all particles relative to the mass center.

4/4 IMPULSE-MOMENTUM

We now develop the concepts of momentum and impulse as applied to a system of particles.

Linear Momentum

From our definition in Art. 3/8, the linear momentum of the representative particle of the system depicted in Fig. 4/1 is $\mathbf{G}_i = m_i\mathbf{v}_i$ where the velocity of m_i is $\mathbf{v}_i = \dot{\mathbf{r}}_i$.

The linear momentum of the system is defined as the vector sum of the linear momenta of all of its particles, or $\mathbf{G} = \Sigma m_i\mathbf{v}_i$. By substituting the relative-velocity relation $\mathbf{v}_i = \overline{\mathbf{v}} + \dot{\boldsymbol{\rho}}_i$ and noting again that $\Sigma m_i\boldsymbol{\rho}_i = m\overline{\boldsymbol{\rho}} = \mathbf{0}$, we obtain

$$\mathbf{G} = \Sigma m_i(\overline{\mathbf{v}} + \dot{\boldsymbol{\rho}}_i) = \Sigma m_i\overline{\mathbf{v}} + \frac{d}{dt}\Sigma m_i\boldsymbol{\rho}_i$$

$$= \overline{\mathbf{v}}\,\Sigma m_i + \frac{d}{dt}(\mathbf{0})$$

or

$$\boxed{\mathbf{G} = m\overline{\mathbf{v}}} \qquad \textbf{(4/5)}$$

Thus, the linear momentum of any system of constant mass is the product of the mass and the velocity of its center of mass.

The time derivative of $\mathbf{G}$ is $m\dot{\bar{\mathbf{v}}} = m\bar{\mathbf{a}}$, which by Eq. 4/1 is the resultant external force acting on the system. Thus, we have

$$\Sigma \mathbf{F} = \dot{\mathbf{G}} \tag{4/6}$$

which has the same form as Eq. 3/25 for a single particle. Equation 4/6 states that the resultant of the external forces on any mass system equals the time rate of change of the linear momentum of the system. It is an alternative form of the generalized second law of motion, Eq. 4/1. As was noted at the end of the last article, $\Sigma\mathbf{F}$, in general, does not pass through the mass center G. In deriving Eq. 4/6, we differentiated with respect to time and assumed that the total mass is constant. Thus, the equation does not apply to systems whose mass changes with time.

Angular Momentum

We now determine the angular momentum of our general mass system about the fixed point O, about the mass center G, and about an arbitrary point P, shown in Fig. 4/3, which may have an acceleration $\mathbf{a}_P = \ddot{\mathbf{r}}_P$.

About a Fixed Point O. The angular momentum of the mass system about the point O, fixed in the Newtonian reference system, is defined as the vector sum of the moments of the linear momenta about O of all particles of the system and is

$$\mathbf{H}_O = \Sigma(\mathbf{r}_i \times m_i\mathbf{v}_i)$$

The time derivative of the vector product is $\dot{\mathbf{H}}_O = \Sigma(\dot{\mathbf{r}}_i \times m_i\mathbf{v}_i) + \Sigma(\mathbf{r}_i \times m_i\dot{\mathbf{v}}_i)$. The first summation vanishes since the cross product of two parallel vectors $\dot{\mathbf{r}}_i$ and $m_i\mathbf{v}_i$ is zero. The second summation is $\Sigma(\mathbf{r}_i \times m_i\mathbf{a}_i) = \Sigma(\mathbf{r}_i \times \mathbf{F}_i)$, which is the vector sum of the moments about O of all forces acting on all particles of the system. This moment sum $\Sigma\mathbf{M}_O$ represents only the moments of forces external to the system, since the internal forces cancel one another and their moments add up to zero. Thus, the moment sum is

$$\Sigma\mathbf{M}_O = \dot{\mathbf{H}}_O \qquad \textbf{(4/7)}$$

which has the same form as Eq. 3/31 for a single particle.

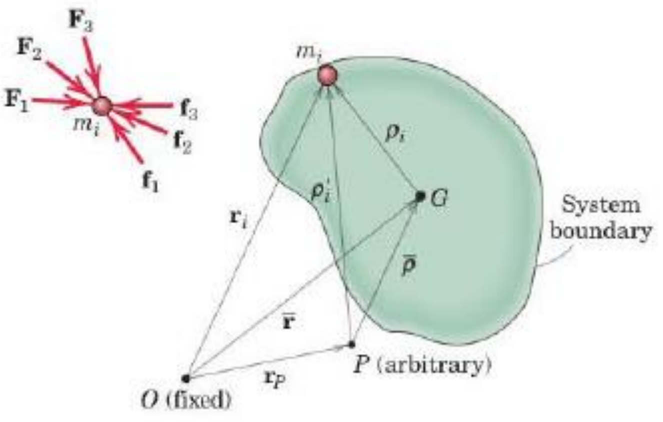

Figure 4/3

Equation 4/7 states that the resultant vector moment about any fixed point of all external forces on any system of mass equals the time rate of change of angular momentum of the system about the fixed point. As in the linear-momentum case, Eq. 4/7 does not apply if the total mass of the system is changing with time.

About the Mass Center G. The angular momentum of the mass system about the mass center G is the sum of the moments of the linear momenta about G of all particles and is

$$\mathbf{H}_G = \Sigma \boldsymbol{\rho}_i \times m_i \dot{\mathbf{r}}_i \tag{4/8}$$

We may write the absolute velocity $\dot{\mathbf{r}}_i$ as $(\dot{\bar{\mathbf{r}}} + \dot{\boldsymbol{\rho}}_i)$ so that $\mathbf{H}_G$ becomes

$$\mathbf{H}_G = \Sigma \boldsymbol{\rho}_i \times m_i(\dot{\bar{\mathbf{r}}} + \dot{\boldsymbol{\rho}}_i) = \Sigma \boldsymbol{\rho}_i \times m_i \dot{\bar{\mathbf{r}}} + \Sigma \boldsymbol{\rho}_i \times m_i \dot{\boldsymbol{\rho}}_i$$

The first term on the right side of this equation may be rewritten as $-\dot{\bar{\mathbf{r}}} \times \Sigma m_i \boldsymbol{\rho}_i$, which is zero because $\Sigma m_i \boldsymbol{\rho}_i = \mathbf{0}$ by definition of the mass center. Thus, we have

$$\mathbf{H}_G = \Sigma \boldsymbol{\rho}_i \times m_i \dot{\boldsymbol{\rho}}_i \tag{4/8a}$$

The expression of Eq. 4/8 is called the *absolute* angular momentum because the absolute velocity $\dot{\mathbf{r}}_i$ is used. The expression of Eq. 4/8*a* is called the *relative* angular momentum because the relative velocity $\dot{\boldsymbol{\rho}}_i$ is used. With the mass center G as a reference, the absolute and relative angular momenta are seen to be identical. We will see that this identity does not hold for an arbitrary reference point P; there is no distinction for a fixed reference point O.

Differentiating Eq. 4/8 with respect to time gives

$$\dot{\mathbf{H}}_G = \Sigma\dot{\boldsymbol{\rho}}_i \times m_i(\dot{\bar{\mathbf{r}}} + \dot{\boldsymbol{\rho}}_i) + \Sigma\boldsymbol{\rho}_i \times m_i\ddot{\mathbf{r}}_i$$

The first summation is expanded as $\Sigma\dot{\boldsymbol{\rho}}_i \times m_i\dot{\bar{\mathbf{r}}} + \Sigma\dot{\boldsymbol{\rho}}_i \times m_i\dot{\boldsymbol{\rho}}_i$. The first term may be rewritten as $-\dot{\bar{\mathbf{r}}} \times \Sigma m_i\dot{\boldsymbol{\rho}}_i = -\dot{\bar{\mathbf{r}}} \times \frac{d}{dt}\Sigma m_i\boldsymbol{\rho}_i$, which is zero from the definition of the mass center. The second term is zero because the cross product of parallel vectors is zero. With $\mathbf{F}_i$ representing the sum of all external forces acting on m_i and $\mathbf{f}_i$ the sum of all internal forces acting on m_i, the second summation by Newton's second law becomes $\Sigma\boldsymbol{\rho}_i \times (\mathbf{F}_i + \mathbf{f}_i) = \Sigma\boldsymbol{\rho}_i \times \mathbf{F}_i = \Sigma\mathbf{M}_G$, the sum of all external moments about point G. Recall that the sum of all internal moments $\Sigma\boldsymbol{\rho}_i \times \mathbf{f}_i$ is zero. Thus, we are left with

$$\Sigma\mathbf{M}_G = \dot{\mathbf{H}}_G \tag{4/9}$$

where we may use either the absolute or the relative angular momentum.

Equations 4/7 and 4/9 are among the most powerful of the governing equations in dynamics and apply to any defined system of constant mass—rigid or nonrigid.

About an Arbitrary Point P. The angular momentum about an arbitrary point P (which may have an acceleration $\ddot{\mathbf{r}}_P$) will now be expressed with the notation of Fig. 4/3. Thus,

$$\mathbf{H}_P = \Sigma\boldsymbol{\rho}_i' \times m_i\dot{\mathbf{r}}_i = \Sigma(\bar{\boldsymbol{\rho}} + \boldsymbol{\rho}_i) \times m_i\dot{\mathbf{r}}_i$$

The first term may be written as $\bar{\boldsymbol{\rho}} \times \Sigma m_i\dot{\mathbf{r}}_i = \bar{\boldsymbol{\rho}} \times \Sigma m_i\mathbf{v}_i = \bar{\boldsymbol{\rho}} \times m\bar{\mathbf{v}}$. The second term is $\Sigma\boldsymbol{\rho}_i \times m_i\dot{\mathbf{r}}_i = \mathbf{H}_G$. Thus, rearranging gives

$$\boxed{\mathbf{H}_P = \mathbf{H}_G + \bar{\boldsymbol{\rho}} \times m\bar{\mathbf{v}}} \qquad \textbf{(4/10)}$$

Equation 4/10 states that the absolute angular momentum about any point P equals the angular momentum about G plus the moment about P of the linear momentum $m\bar{\mathbf{v}}$ of the system considered concentrated at G.

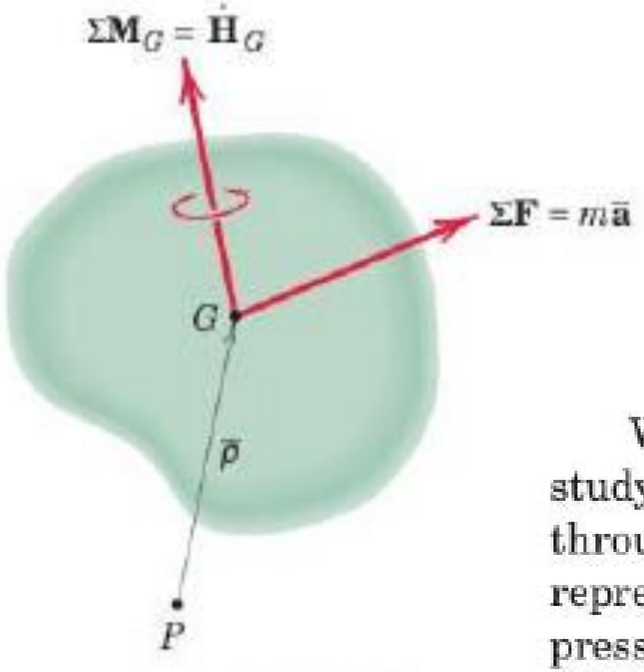

Figure 4/4

We now make use of the principle of moments developed in our study of statics where we represented a force system by a resultant force through any point, such as G, and a corresponding couple. Figure 4/4 represents the resultants of the external forces acting on the system expressed in terms of the resultant force $\Sigma \mathbf{F}$ through G and the corresponding couple $\Sigma \mathbf{M}_G$. We see that the sum of the moments about P of all forces external to the system must equal the moment of their resultants. Therefore, we may write

$$\Sigma \mathbf{M}_P = \Sigma \mathbf{M}_G + \bar{\boldsymbol{\rho}} \times \Sigma \mathbf{F}$$

which, by Eqs. 4/9 and 4/6, becomes

$$\Sigma \mathbf{M}_P = \dot{\mathbf{H}}_G + \bar{\boldsymbol{\rho}} \times m\bar{\mathbf{a}} \qquad \textbf{(4/11)}$$

Equation 4/11 enables us to write the moment equation about any convenient moment center P and is easily visualized with the aid of Fig. 4/4. This equation forms a rigorous basis for much of our treatment of planar rigid-body kinetics in Chapter 6.

We may also develop similar momentum relationships by using the momentum relative to P. Thus, from Fig. 4/3

$$(\mathbf{H}_P)_{\text{rel}} = \Sigma \boldsymbol{\rho}_i' \times m_i \dot{\boldsymbol{\rho}}_i'$$

where $\dot{\boldsymbol{\rho}}_i'$ is the velocity of m_i relative to P. With the substitution $\boldsymbol{\rho}_i' = \bar{\boldsymbol{\rho}} + \boldsymbol{\rho}_i$ and $\dot{\boldsymbol{\rho}}_i' = \dot{\bar{\boldsymbol{\rho}}} + \dot{\boldsymbol{\rho}}_i$ we may write

$$(\mathbf{H}_P)_{\text{rel}} = \Sigma \bar{\boldsymbol{\rho}} \times m_i \dot{\bar{\boldsymbol{\rho}}} + \Sigma \bar{\boldsymbol{\rho}} \times m_i \dot{\boldsymbol{\rho}}_i + \Sigma \boldsymbol{\rho}_i \times m_i \dot{\bar{\boldsymbol{\rho}}} + \Sigma \boldsymbol{\rho}_i \times m_i \dot{\boldsymbol{\rho}}_i$$

The first summation is $\bar{\boldsymbol{\rho}} \times m\bar{\mathbf{v}}_{\text{rel}}$. The second summation is $\bar{\boldsymbol{\rho}} \times \frac{d}{dt} \Sigma m_i \boldsymbol{\rho}_i$ and the third summation is $-\dot{\bar{\boldsymbol{\rho}}} \times \Sigma m_i \boldsymbol{\rho}_i$ where both are zero by definition of the mass center. The fourth summation is $(\mathbf{H}_G)_{\text{rel}}$. Rearranging gives us

$$(\mathbf{H}_P)_{\text{rel}} = (\mathbf{H}_G)_{\text{rel}} + \bar{\boldsymbol{\rho}} \times m\bar{\mathbf{v}}_{\text{rel}} \qquad \textbf{(4/12)}$$

where $(\mathbf{H}_G)_{\text{rel}}$ is the same as $\mathbf{H}_G$ (see Eqs. 4/8 and 4/8a). Note the similarity of Eqs. 4/12 and 4/10.

The moment equation about P may now be expressed in terms of the angular momentum relative to P. We differentiate the definition $(\mathbf{H}_P)_{\text{rel}} = \Sigma\boldsymbol{\rho}_i' \times m_i\dot{\boldsymbol{\rho}}_i'$ with time and make the substitution $\ddot{\mathbf{r}}_i = \ddot{\mathbf{r}}_P + \ddot{\boldsymbol{\rho}}_i'$ to obtain

$$(\dot{\mathbf{H}}_p)_{\text{rel}} = \Sigma\dot{\boldsymbol{\rho}}_i' \times m_i\dot{\boldsymbol{\rho}}_i' + \Sigma\boldsymbol{\rho}_i' \times m_i\ddot{\mathbf{r}}_i - \Sigma\boldsymbol{\rho}_i' \times m_i\ddot{\mathbf{r}}_P$$

The first summation is identically zero, and the second summation is the sum $\Sigma\mathbf{M}_P$ of the moments of all external forces about P. The third summation becomes $\Sigma\boldsymbol{\rho}_i' \times m_i\mathbf{a}_P = -\mathbf{a}_P \times \Sigma m_i\boldsymbol{\rho}_i' = -\mathbf{a}_P \times m\bar{\boldsymbol{\rho}} = \bar{\boldsymbol{\rho}} \times m\mathbf{a}_P$. Substituting and rearranging terms give

$$\Sigma\mathbf{M}_P = (\dot{\mathbf{H}}_P)_{\text{rel}} + \bar{\boldsymbol{\rho}} \times m\mathbf{a}_P \qquad \textbf{(4/13)}$$

The form of Eq. 4/13 is convenient when a point P whose acceleration is known is used as a moment center. The equation reduces to the simpler form

$$\Sigma \mathbf{M}_P = (\dot{\mathbf{H}}_P)_{\text{rel}} \quad \text{if} \quad \begin{cases} \text{1. } \mathbf{a}_P = \mathbf{0} \text{ (equivalent to Eq. 4/7)} \\ \text{2. } \bar{\boldsymbol{\rho}} = \mathbf{0} \text{ (equivalent to Eq. 4/9)} \\ \text{3. } \bar{\boldsymbol{\rho}} \text{ and } \mathbf{a}_P \text{ are parallel } (\mathbf{a}_P \text{ directed} \\ \quad \text{toward or away from } G) \end{cases}$$

4/5 Conservation of Energy and Momentum

Under certain common conditions, there is no net change in the total mechanical energy of a system during an interval of motion. Under other conditions, there is no net change in the momentum of a system. These conditions are treated separately as follows.

Conservation of Energy

A mass system is said to be *conservative* if it does not lose energy by virtue of internal friction forces which do negative work or by virtue of inelastic members which dissipate energy upon cycling. If no work is done on a conservative system during an interval of motion by external forces (other than gravity or other potential forces), then none of the energy of the system is lost. For this case, $U'_{1\text{-}2} = 0$ and we may write Eq. 4/3 as

$$\Delta T + \Delta V = 0 \tag{4/14}$$

or

$$T_1 + V_1 = T_2 + V_2 \tag{4/14a}$$

which expresses the *law of conservation of dynamical energy*. The total energy $E = T + V$ is a constant, so that $E_1 = E_2$. This law holds only in the ideal case where internal kinetic friction is sufficiently small to be neglected.

Conservation of Momentum

If, for a certain interval of time, the resultant external force $\Sigma\mathbf{F}$ acting on a conservative or nonconservative mass system is zero, Eq. 4/6 requires that $\dot{\mathbf{G}} = \mathbf{0}$, so that during this interval

$$\mathbf{G}_1 = \mathbf{G}_2 \tag{4/15}$$

which expresses the *principle of conservation of linear momentum*. Thus, in the absence of an external impulse, the linear momentum of a system remains unchanged.

Similarly, if the resultant moment about a fixed point O or about the mass center G of all external forces on any mass system is zero, Eq. 4/7 or 4/9 requires, respectively, that

$$(\mathbf{H}_O)_1 = (\mathbf{H}_O)_2 \qquad \text{or} \qquad (\mathbf{H}_G)_1 = (\mathbf{H}_G)_2 \tag{4/16}$$

These relations express the *principle of conservation of angular momentum* for a general mass system in the absence of an angular impulse. Thus, if there is no angular impulse about a fixed point (or about the mass center), the angular momentum of the system about the fixed point (or about the mass center) remains unchanged. Either equation may hold without the other.

We proved in Art. 3/14 that the basic laws of Newtonian mechanics hold for measurements made relative to a set of axes which translate with a constant velocity. Thus, Eqs. 4/1 through 4/16 are valid provided all quantities are expressed relative to the translating axes.

Equations 4/1 through 4/16 are among the most important of the basic derived laws of mechanics. In this chapter we have derived these laws for the most general system of constant mass to establish the generality of these laws. Common applications of these laws are specific mass systems such as rigid and nonrigid solids and certain fluid systems, which are discussed in the following articles. Study these laws carefully and compare them with their more restricted forms encountered earlier in Chapter 3.

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The principles of particle-system kinetics form the foundation for the study of the forces associated with the water-spraying equipment of these firefighting boats.

References

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