

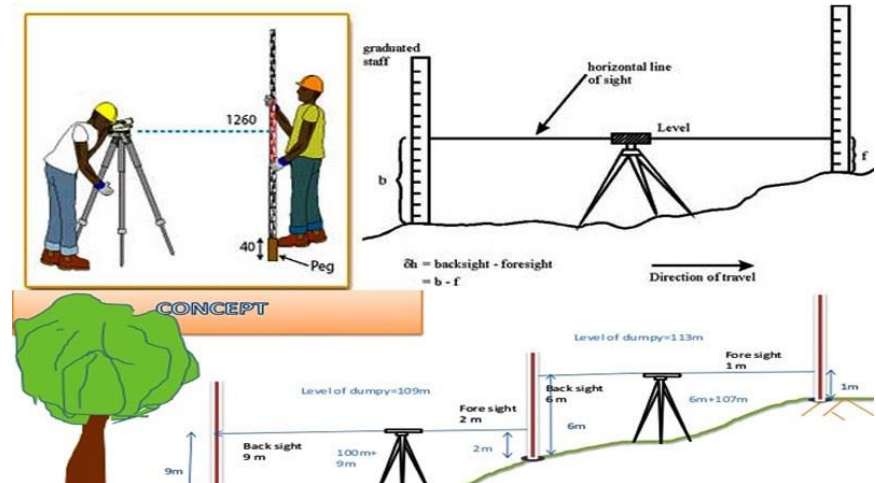


Lecturer's Details
Name: Mr. Bwalya J. Kawimbe
Office: BEng. Main Building, 1st Floor, Former Zagis Offices,
Room 2 - Uhza
Contact Details:
Mobile: +260 968 324 284
Email: bwalya.kawimbe@unza.zm / bkawimbe@gmail.com

Department of Geomatic Engineering School of Engineering

Topic 1: Levelling Contents

- ✓ Definitions
- ✓ Principles of levelling
- ✓ Types
- ✓ Booking Methods
- ✓ Errors and corrections
- ✓ Classes and accuracies
- ✓ Error identification and correction
- ✓ Setting out levels



Brief Introduction – Levelling

LEVELLING

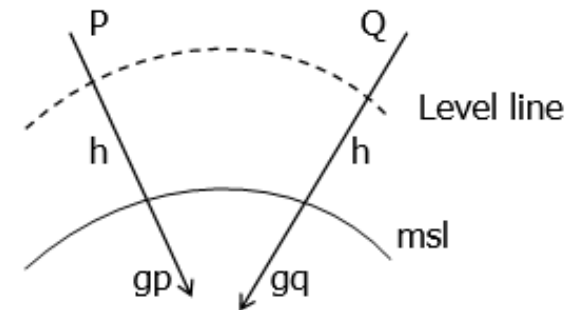
- Levelling is the process of determination of height differences between two or more points. Only relative (vertical distances/height differences) can be observed directly.

Further we can say that levelling is the method used in establishing elevations of points.

Terminologies used in levelling

- **Level line**

Line with a constant height above a datum say mean sea level (msl). Since the earth is curved so is the mean sea level and the level line.



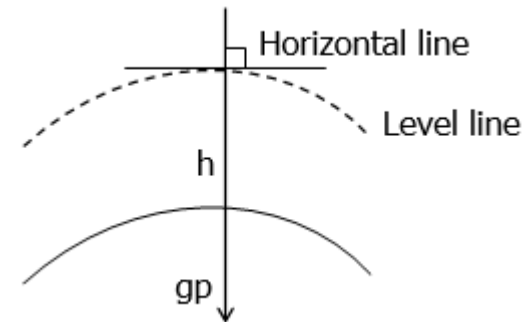
Brief Introduction – Levelling

▪ Datum

This is the surface to which the elevation of points are related/ referenced. The most common surface is the mean sea level.

▪ Horizontal line

Line which is normal to the direction of gravity and tangent to a level line at a particular point



▪ Reduced level (RL)

The height of a point relative to a chosen datum.

▪ Bench mark (BM)

A bench mark is a reference mark or point with known reduced level and position. This can be permanent (PBM) or temporary (TBM).



Levelling – Methods of levelling

There are several methods you can employ in levelling, some of them are listed below:

Direct leveling – where you use a level and a staff to measure vertical distances directly.

Indirect or trigonometric leveling – where any angular measuring instrument and a distance measuring Instrument is used to measure vertical angles and the distance to the other point(s), respectively.

Stadia levelling – where any angular measuring and levelling instruments and a staff is used.

Barometric levelling – by measuring the differences in atmospheric pressure at various stations by use of a barometer, heights of points can be determined taking into consideration the principle that pressure of the atmosphere varies inversely with the elevation.

Levelling – Effects of Curvature & Refraction

Let Radius of Earth be ≈ 6400 km

$A'B' = D$ km and $B'B'' = c$

Consider a level set up at point A, as shown in figure above, and a staff is held vertical at B. With a properly adjusted level, the line of collimation $A'B'$ will be horizontal and will intersect the staff at B' .

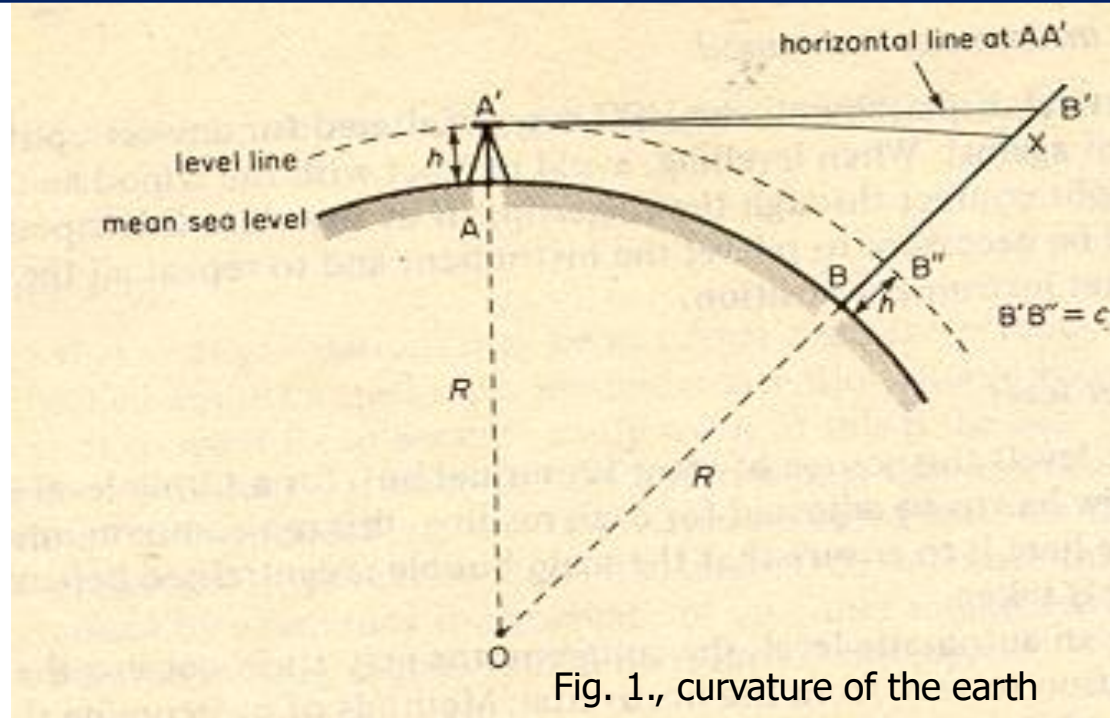


Fig. 1., curvature of the earth

This line of collimation however will be tangential to a level line, which will be at a constant height (h) if we consider the earth as a sphere. As can be seen from the figure above, the level line will intersect the staff at B'' , thus the staff reading will be greater by $B'B''$. This effect is caused by the curvature of the Earth and the correction to be applied to staff readings is called the **Curvature Correction**.

Levelling – Effects of Curvature & Refraction

$$\text{Then } (R + h + c)^2 = (R + h)^2 + D^2$$

$$c(2R + 2h + c) = D^2 \text{ after simplification}$$

$$\text{so that } c = D^2 / (2R + 2h + c)$$

$$c = D^2 / 2R \text{ since } h \text{ and } c \text{ are small compared to } R.$$

$$= D^2 / 12800$$

$$\underline{c \approx 0.078 D^2 \text{ (metres)}}$$

where D (Km)

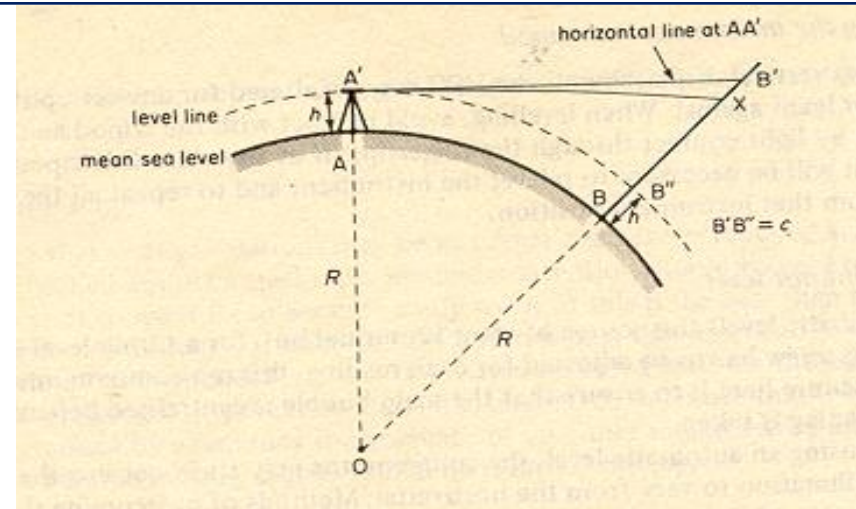


Fig. 1., curvature of the earth

The effect of *refraction* is such that the line of sight is bend towards the Earth to follow line A'X (see figure). Refraction is a variable effect depending on the atmospheric conditions but ordinary work the line A'X can be considered to be circular with a radius seven (7) times that radius of the Earth. In magnitude, therefore, refraction has a value of 1/7 that of curvature but is of opposite sign (effect). Therefore the combined correction is:

Curvature and Refraction correction = $\frac{6}{7}(0.078) D^2$.

$$\underline{\approx 0.067 D^2 \text{ (metres)}}$$

Levelling – Effects of Curvature & Refraction

$$\text{Curvature and Refraction correction} = \frac{6}{7}(0.078) D^2.$$

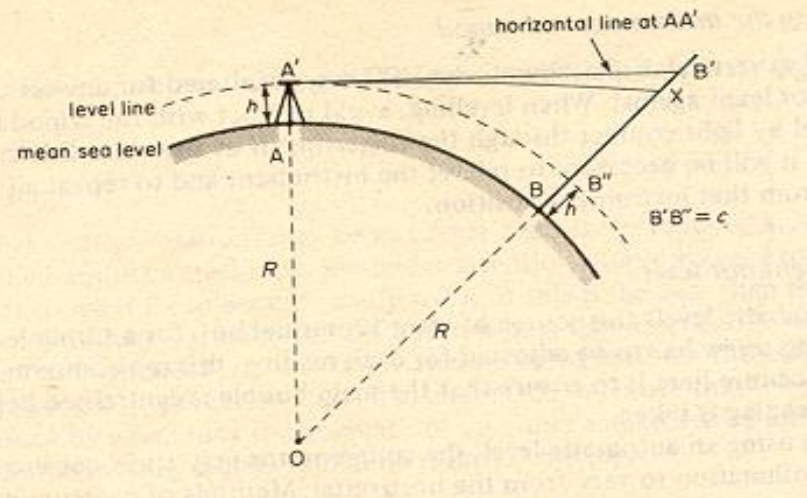
$$\approx \mathbf{0.067 D^2 \text{ (metres)}}$$

Thus, for $D = 100 \text{ m}$ (0.1 km), the error = $0.0007 \approx 1 \text{ mm}$

$D = 1 \text{ km}$ the error = 0.067 m . Therefore, for short sight distances these lines almost coincide but for long distances as seen in example the effect should be taken into account.

$$\text{Angular Curvature correction (c)} = \frac{0.078 \text{ m / Km}}{1000 \text{ m} * \text{arc}1''} = 16.192'' / \text{Km}$$

$$\text{Angular Curvature and refraction correction (c \& r)} = \frac{0.068 \text{ m / Km}}{1000 \text{ m} * \text{arc}1''} = 14.026'' / \text{Km}$$





Levelling – Direct Levelling

Method of Booking and Calculation

In each step the readings of the observer are recorded on the booking form. The booker carries out field checks and calculates the distances. There are two methods of booking and carrying out calculations of the reduced levels (RLs).

Rise and Fall method

Between each pair of subsequent points the rise or fall is calculated and the initial(unadjusted) RLs are derived from this, i.e. $RLA = RLB + \Delta H_{AB}$, ΔH_{AB} could (+) Rise or (-) Fall (see figure)

Example to explain the method – NEXT SLIDE



Levelling – Direct Levelling

Height of Instrument (Height of Collimation) method

In this method the Reduced level of the line of sight is calculated in each step by adding the staff reading to a point with the known Reduced level. The RLs of the remaining points in that step are obtained by subtracting their respective readings from the RL of the line of sight (usually indicated by H.I., the height of Instrument).

See the example: NEXT SLIDE!



Levelling

Height of Instrument (Height of Collimation) method

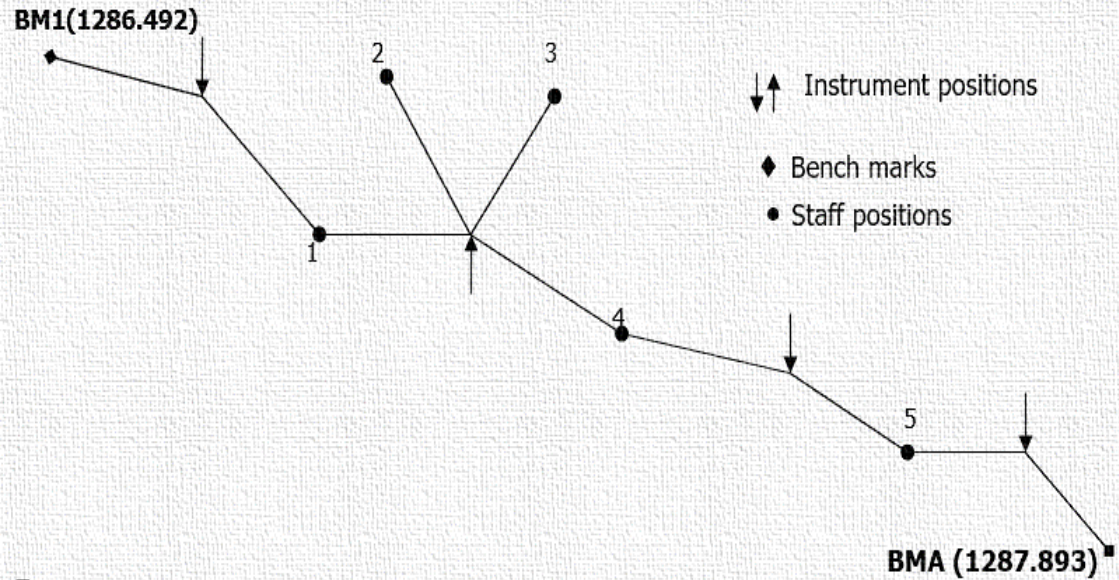
Checks

$$\Sigma BS - \Sigma FS = \Sigma RISE - \Sigma FALL = RLLAST - RLFIRST = 1.416$$

If these checks result in some discrepancy a calculation error has been made.

Note that in all cases (including IS) height differences are calculated.

'CP' in the remarks' column indicates that the instrument position was changed at that staff reading.



Pnt	Readings			Distance		Height of Instrument	RLs	Adj. RLs	REM.
	BS	IS	FS	BS	FS				
BM1	0.831			32.1		1287.323	1286.492	1286.492	BM
1	0.952		0.221	28.0	34.1	1288.054	1287.102	1287.098	CP
2		1.576					1286.478	1286.471	
3		0.897					1287.157	1287.150	
4	1.594		0.578	40.5	26.8	1289.070	1287.476	1287.469	CP
5	1.532		1.633	41.2	39.9	1288.969	1287.437	1287.426	CP
BMA			1.061		42.3		1287.908	1287.893	BM
Σ	4.909	2.473	3.493	141.8	143.1		1287.908		
	3.493						1286.492		
	1.416						1.416		



Levelling – Adjustment to the loop

In any levelling loop, the measured height difference between the starting and the ending point will most probably not be equal to the known height difference. When this discrepancy, usually known as the **misclosure** exceeds a certain limit, the survey becomes unacceptable and has to be repeated. On the other hand when the misclosure is acceptable, it can be adjusted to eliminate the misclosure.

The misclosure ϵ in a loop is calculated as follows:

$$\begin{aligned}\epsilon &= (\text{Measured } \Delta H) - (\text{Known } \Delta H) \\ &= (\text{Measured RL of closing point}) - (\text{Known RL of closing point})\end{aligned}$$



Levelling – Adjustment to the loop

A general formula for the allowable misclosure is

$$\epsilon \text{ allowable} = \pm \sigma \cdot \sqrt{D} \text{ (Km) mm}$$

where σ = constant depending on type of instrument and type of survey, in most cases
 σ is 20 - 30mm

D = length of the levelling loop in Km, equal to the sum of all backsight and foresight distances

The correction per station is calculated as:

$$\text{Correction} = - \epsilon / (\text{number of stations})$$

In a series levelling, the corrections are incremental per instrument station. Intermediate stations get the same correction as the foresight in a particular step, which means that the correction is increased after every change point. After applying the corrections, you get the adjusted reduced levels. For benchmark levels, corrections should not be applied since these are adopted.



Levelling – Adjustment to the loop

In the example given in the two booking methods above, the adjustment is in the same way,

Take $\sigma = 30 \text{ mm}$

Total distance $D = 141.8 + 143.1 = 284.9 \text{ m}$

Number of stations = 4; $\epsilon(\text{Allowable misclosure}) = 30\sqrt{.2849} = 16 \text{ mm.}$

Measured $\Delta H = (1287.908 - 1286.492) = 1.416 \text{ m}$

Known $\Delta H = (1287.893 - 1286.492) = 1.401 \text{ m}$

Misclosure $\epsilon = 1.416 - 1.401 = +0.015 \text{ m} = +15 \text{ mm.}$

Since the resulting misclosure of the above levelling loop (15 mm) falls within the allowable misclosure, we can now adjust the reduced levels. This could have been caused by random errors.



Levelling – Adjustment to the loop

Point	Initial RLs	Corr.(m)	Adjusted RL	Remarks
BM1	1286.492	0	1286.492	BM
1	1287.102	0.0037	1287.098	CP
2	1286.478	0.0075	1286.471	
3	1287.157	0.0075	1287.150	
4	1287.476	0.0075	1287.469	CP
5	1287.437	0.0112	1287.426	CP
BMA	1287.908	0.015	1287.893	BM



Levelling – Trigonometric Levelling (Heighting)

If the reduced levels of several points some distance apart in hilly terrain are required then the trigonometric levelling is employed. This method is faster than direct levelling. It involves the measurement of distances and vertical angles from known points to the points to be heighted. This can be done by the use of any angular and distance measuring instrument e.g. Theodolite and EDM or Total Station.

With this method of heighting there are three approaches:

1. *Single ended trigonometric heighting* – the observations are taken from one end of the line only and curvature and refraction must be allowed for in the calculations.
2. *Reciprocal trigonometric heighting* - the observations are taken from each end of the line but not at the same time and curvature and refraction must be allowed for in the calculations.
3. *Simultaneous reciprocal trigonometric heighting* - the observations are taken from each end of the line at exactly the same time in order that the curvature and refraction effects will cancel each other out in the calculations