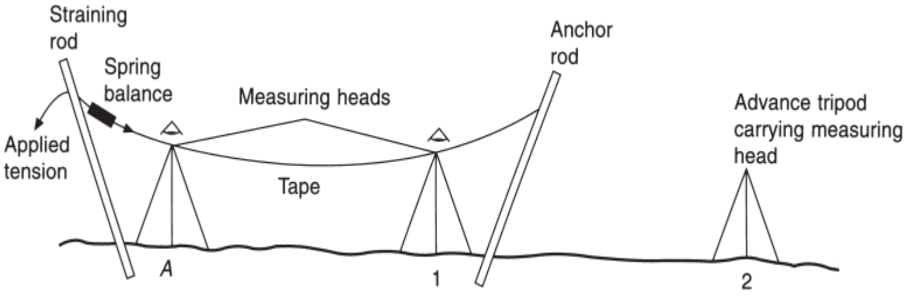
Lecturer's Details
Name: Mr. Bwalya J. Kawimbe
Office: B.Eng. Main Building, 1st Floor, Former Zagis Offices, Room 2 – Unza
Contact Details:
Mobile: +260 968 324 284
Email: bwalya.kawimbe@unza.zm / bkawimbe@gmail.com

B.Eng. - UNZA

Linear Measurements - Tapes

Contents

- ✓ Application and limitations
- ✓ Principles and methods
- ✓ Errors and corrections



Introduction to Taping

General:

Linear measurement is one of the basic quantity of surveying.

Taping refers to the exercise of physically measuring horizontal distances.

Methods

Direct linear measurements can be obtained or estimated through a number of methods:

1) pacing
2) odometer readings
3) stadia (tacheometry)
4) taping
5) electronic distance measurements (EDM)

Introduction to Taping

Taping Equipment

30, 100 metre tape	Steel tapes are manufactured under fixed conditions of temperature and tensile force.
tension handle	Tension handles allow the user to apply a specified tensile force on the tape.
tape grips	These allow the user to firmly grasp the steel tape and resist the pull of the tape from the person located on the other end of the tape.
plumb bobs	These are used to locate the tape precisely over a specified point.
chaining pins	These are used to mark tape lengths.

Introduction to Taping

General Procedures

Person A	Holds the tension handle located at the "zero" end of the tape.
Person B	Holds the tape reel and uses a tape grip to pull the tape.
Person B	Pinches the plumb bob string at a convenient point on the tape.
Person A	Holds the plumb bob string along the edge of the tape above his/her intended mark.
Person B	Holds the plumb bob over his/her mark. Calls out "mark….mark…..mark" to indicate that the plumb bob is being held over the intended mark. Braces for the tension applied by person A.

Introduction to Taping

General Procedure

Person A Does the actual pulling. Reports the tension value and the corresponding tape measurement.

Person B Reports the tape reading at his/her end of the tape.

The measured length is determined by subtracting tape reading A from tape reading B.

Introduction to Taping

The measurement procedure is repeated with one important difference. Person B holds the plumb bob string at a different point on the tape.

The two measured lengths are then compared. If the difference between the measurements is less than the allowable tolerance, then the measured length between the two points can be considered accurate.

If they are not, the length measurements are to be repeated until two successive measurements are obtained within the allowable tolerance.

Introduction to Taping

Recording Your Taping Work

You record your data in the field book. The following are needed in the field book;

1) identify or label the distance being measured

2) record the tape readings and calculate the measured length

3) record the corresponding tension value

4) and identify how the tape was supported.

Introduction to Taping

Leg	Distance (m)	Support	Tension (N)		
A1-A2 #1	28.0-0.131 27.869	2 points	60		
A1-A2 #2	28.2-0.335 27.865	2 points	65		

Taping Errors

In general, the distance measurement obtained in the field will be in error. Errors in the distance measurement can arise from a number of sources:

1) Instrument errors. A tape may be faulty due to a defect in its manufacturing or from kinking.

2) Natural errors. The actual horizontal distance between the ends of the tape can vary due to the effects of:

- *temperature,*
- *elongation due to tension, and*
- *sagging.*

Taping Errors

3) Personal errors. Errors will arise from carelessness by the survey crew:

- poor alignment
- tape not horizontal
- improper plumbing
- faulty reading of the tape

Taping Errors

The true length can be determined by incorporating a series of corrections as shown below:

$$L_T = L_f + C_S + C_P + C_T + C_L$$

where L_T is the true length (m)

L_f is the length measured in the field (m)

and

C_S is a correction for sagging (m)

C_P is a correction for elongation (m)

C_T is a correction for thermal expansion(m)

C_L is a correction for scale (m)

Corrections for tape error measurements

Standardization correction

When a tape is bought and before it is used it is supposed to be standardized. Standardization in this case is the checking of the tape in conformity to the nominal values indicated on it. This also is done after a tape has been in use for some time. Usually this is done by measuring it against a standard tape in a workshop.

$$C_{std} = l_m(l' - l_n)/l_n \text{ where}$$

l_m = measured distance with tape

l' = actual tape length

l_n = nominal length of tape

OR

$$l_{true} = l_m(l'/l_n) \text{ where}$$

l_m = measured distance with tape

l' = actual tape length

l_n = nominal length of tape

Corrections for tape error measurements

Temperature correction

As the tape is being used in the field, it is affected by the change in temperatures. Normally, tapes are designed in such a way that they give the required length at the design temperature (nominal temperature). This affects the tape in two ways (expansion or contraction) depending on the temperatures. To counteract the effects that are brought about due to the tape operating at temperatures other than the nominal, the correction to the measured distance is determined:

$$C_{temp} = \alpha l_m (t_f - t_s)$$

where

l_m = measured distance with tape

t_f = field temperature

t_s = nominal temperature at which tape is operate

α = coefficient of expansion of the tape

This correction is either negative or positive. It all depends on the field temperatures.

Corrections for tape error measurements

Slope correction

When the tape is used on the ground or in catenary, the distance so obtained is slope distance. It is a well known fact that horizontal distances are the ones key to computations in surveying. These obtained distances are to be converted to horizontal values. If the distance is measured on the ground, the slope angle can be measured between the end points by way of an Abney level. If the distance is measured in catenary then the vertical angle can be measured using the theodolite.

$$C_{slope} = l_m(1-\cos\theta)$$

where

l_m = measured distance with tape

θ = the measured vertical angle between the end points

$$C_{slope} = \Delta H_{AB}^2 / 2l_m$$

where

l_m = measured distance with tape

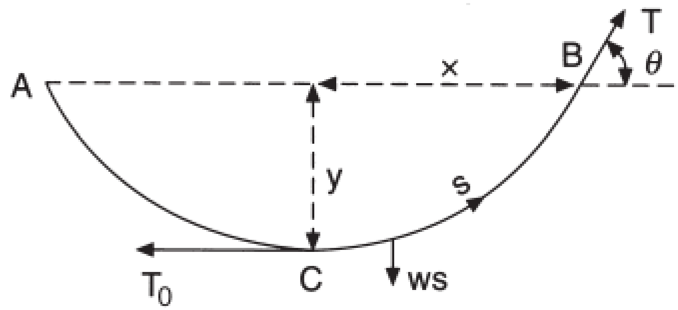
ΔH_{AB} = the difference in elevation between the end points

This correction is always negative since the slope distance is always longer than the horizontal distance.

Corrections for tape error measurements

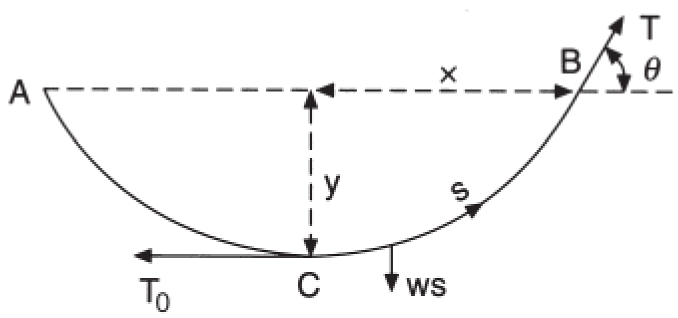
Sag Correction

When the tape is used on the ground, sag correction is not applied. When used in catenary, the distance so obtained is longer than expected due to the sagging of the tape due to its weight. This correction is always negative since the measured distance is always longer than the needed distance (see figure above)



Corrections for tape error measurements

Sag Correction



Resolving vertically $T\sin\theta = ws$

Resolving horizontally $T\cos\theta = T_0$

$$\therefore \tan\theta = \frac{ws}{T_0}$$

For a small increment of the tape

$$\frac{dx}{ds} = \cos\theta = (1+\tan^2\theta)^{-\frac{1}{2}} = \left(1 + \frac{w^2s^2}{T_0^2}\right)^{-\frac{1}{2}} = \left(1 - \frac{w^2s^2}{2T_0^2}\cdots\right)$$

$$\therefore x = \int \left(1 - \frac{w^2s^2}{2T_0^2}\right) ds$$

$$= s - \frac{w^2s^3}{6T_0^2} + K$$

When $x = 0$, $s = 0$, $\therefore K = 0$ $\quad \therefore x = s - \frac{w^2s^3}{6T_0^2}$

The sag correction for the whole span $ACB = C_s = 2(s-x) = 2\left(\frac{w^2s^3}{6T_0^2}\right)$

but $s = L/2$ $\quad \therefore C_s = \frac{w^2L^3}{24T_0^2} = \frac{w^2L^3}{24T^2}$ for small values of θ Eq. 1

i.e. $T_0 \approx T_0\cos\theta \approx T$

Corrections for tape error measurements

Sag Correction

where w = weight per unit length (N/m)
T = tension applied (N)
L = recorded length (m)
C_s = correction (m)

As $w = W/L$, where W is the *total* weight of the tape, then by substitution in *equation* Eq. 1

$$C_s = \frac{W^2 L}{24T^2}$$

Corrections for tape error measurements

Tension correction

When the tape is used on the ground or in catenary, the tension that is applied to it usually is not the standard tension. These obtained distances are to be corrected for the effects of not applying the right tension. The sign of the corrections depend the applied tension in the field.

$$C_{Tension} = l_m (T_f - T_s) / AE$$

where

l_m = measured distance with tape

A = the cross-sectional area of the tape

E = Young's Modulus of elasticity.

T_f = Tension applied to the tape.

T_s = Nominal Tension of the tape.

Corrections for tape error measurements

Tension correction

Generally the tape is used under standard tension, in which case there is no correction. It may, however, be necessary in certain instances to apply a tension greater than standard. From Hooke's law:

$$\text{stress} = \text{strain} \times \text{a constant}$$

This constant is the same for a given material and is called the *modulus of elasticity* (E). Since strain is a non-dimensional quantity, E has the same dimensions as stress, i.e. N/mm^2:

$$\therefore E = \frac{\text{Direct stress}}{\text{Corresponding strain}} = \frac{\Delta T}{A} \div \frac{C_T}{L}$$

$$\therefore C_T = L \times \frac{\Delta T}{AE}$$

ΔT is normally the total stress acting on the cross-section, but as the tape would be standardized under tension, ΔT in this case is the amount of stress *greater* than standard. Therefore ΔT is the difference between field and standard tension. This value may be measured in the field in kilograms and should be converted to newtons (N) for compatibility with the other units used in the formula, i.e. 1 kgf = 9.806 65 N.

Corrections for tape error measurements - Example

A base line was measured in catenary in four lengths giving 30.126, 29.973, 30.066 and 22.536 m. The differences of level were respectively 0.45, 0.60, 0.30 and 0.45 m. The temperature during observation was 10ºC and the tension applied 15kgf. The tape was standardized at 30m, at 20°C, on the flat with a tension of 5kg. The coefficient of expansion was 0.000 011 per °C, the weight of the tape 1 kg, the cross sectional area $3mm^2$ ($210\ kN/mm^2$), gravitational acceleration $g = 9.80665\ m/s^2$

a) Quote each equation used and calculate the length of the baseline.
b) What tension should have been applied to eliminate the sag error?

Corrections for tape error measurements - Example

(a) As the field tension and temperature are constant throughout, the first three corrections may be applied to the base as a whole, i.e. $L = 112.701$ m, with negligible error.

Tension

$$C_T = \frac{L\Delta_T}{AE} = \frac{112.701 \times (10 \times 9.80665)}{3 \times 210 \times 10^3} = \quad +0.0176$$

Temperature

$$C_t = LK\Delta t = 112.701 \times 0.000011 \times 10 = \quad -0.0124$$

Sag

$$C_s = \frac{LW^2}{24T^2} = \frac{112.701 \times 1^2}{24 \times 15^2} = \quad -0.0210$$

Slope

$$C_h = \frac{h^2}{2L} = \frac{1}{2 \times 30}(0.45^2 + 0.60^2 + 0.30^2) + \frac{0.45^2}{2 \times 22.536} = \quad -0.0154$$

+	−
+0.0176	
	−0.0124
	−0.0210
	−0.0154
+0.0176	−0.0488

Corrections for tape error measurements - Example

$$\text{Horizontal length of base } (D) = \text{measured length } (M) + \text{sum of corrections } (C)$$
$$= 112.701\text{ m} + (-0.031)$$
$$= 112.670\text{ m}$$

N.B. In the slope correction the first three bays have been rounded off to 30 m, the resultant second-order error being negligible.

Consider the situation where 112.670 m is the horizontal distance to be set out on site. The equivalent measured distance would be:

$$M = D - C$$
$$= 112.670 - (-0.031) = 112.701\text{ m}$$

Corrections for tape error measurements - Example

(b) To find the applied tension necessary to eliminate the sag correction, equate the two equations:

$$\frac{\Delta T}{AE} = \frac{W^2}{24T_A^2}$$

where ΔT is the difference between the applied and standard tensions, i.e. $(T_A - T_S)$.

$$\therefore \frac{T_A - T_S}{AE} = \frac{W^2}{24T_A^2}$$

$$\therefore T_A^3 - T_A^2 T_S - \frac{AEW^2}{24} = 0$$

Substituting for T_S, W, A and E, making sure to convert T_S and W to newtons gives

$$T_A^3 - 49T_A^2 - 2\,524\,653 = 0$$

Let $\quad T_A = (T + x)$

then $\quad (T + x)^3 - 49(T + x)^2 - 2\,524\,653 = 0$

$$T^3\left(1 + \frac{x}{T}\right)^3 - 49T^2\left(1 + \frac{x}{T}\right)^2 - 2\,524\,653 = 0$$

Expanding the brackets binomially gives

$$T^3\left(1 + \frac{3x}{T}\right) - 49T^2\left(1 + \frac{2x}{T}\right) - 2\,524\,653 = 0$$

$$\therefore T^3 + 3T^2x - 49T^2 - 98Tx - 2\,524\,653 = 0$$

$$\therefore x = \frac{2\,524\,653 - T^3 + 49T^2}{3T^2 - 98T}$$

assuming $T = 15$ kgf $= 147$ N, then $x = 75$ N

$\therefore$ at the first approximation $T_A = (T + x) = 222$ N