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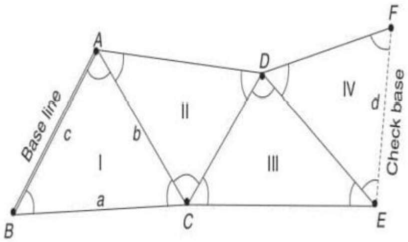
B.Eng. Geomatic Engineering
UNZA

Topic 1: Triangulation

Contents

- ✓ Principles, classification
- ✓ Steps in triangulation survey
- ✓ Base line measurements and corrections,
- ✓ base networks, Problems.



Single Chain of Triangulation

Brief Introduction – What is Triangulation?

- The object of geodetic survey is to determine the relative or absolute positions of points the surface of the earth varies precisely.
- The relative positions are determined in terms of lengths and azimuths of lines joining them.
- The absolute positions are determined in terms of latitude, longitude and elevations above the mean sea level.
- However, the distinctions between plane surveying and geodetic surveying is fundamentally of extent area rather than operations

Brief Introduction – What is Triangulation?

- The precise method of geodesy are followed in the field work of excessive plane trigonometrical survey also.
- Since the area embraced by geodetic survey for an appreciable position of the earth surface, the sphericity of the earth is taken into account while making the computations.
- The geodetic points so determined furnish the most precise control to which a more detailed survey of the intervening country may be referred

Brief Introduction – What is Triangulation?

- In triangulation survey, the system consists of a number of interconnected triangles in which the length of only one line called the base line and angles of the triangles are measured very accurately.
- Knowing the length of one side and the three angles, the length of the other two sides of the triangles can be competed.
- The vertices or apexes of the triangles are known as triangulation stations and the whole figure is called triangulation figure or triangulation system.
- The defect of triangulation method in surveying is that it tends to accumulate the errors of length and azimuth, as the length and azimuth of each line are dependent on the length azimuth of the preceding line.
- To control the accumulation or error, subsidiary bases are also selected. These stations are called **Laplace station**.

Triangulation - Purpose

Triangulation method in surveying is conducted for one or more of the following purposes:

- ✓ To establish accurate control points for plane and geodetic survey of large areas.
- ✓ To establish accurate ground control points for photogrammetric survey of large areas.
- ✓ To determine the size and shape of the earth by making observations for latitude, longitude and gravity.
- ✓ For various engineering projects it is required to build up accurate horizontal control.

Triangulation - Classification

The triangulation survey systems are classified according to the accuracy required for horizontal controls. It depends on the type of survey, extent of survey and purpose of survey. On the basis of accuracy and purpose the triangulation survey systems are classified as follows:

1. Primary or First Order Triangulation Systems:

This system of triangulation survey is of the highest grade. It provides the framework for the national control network for subsidiary triangulation stations. Primary triangulation survey is generally conducted for determination of the ***shape and size of the earth***, for ***earth's crustal movement*** studies in areas of seismic activities for engineering projects of high precision and for surveys conducted in metropolitan areas.

Triangulation - Classification

2. Secondary or Second Order Triangulation Systems:

This system of triangulation survey is of the grade slightly lower than that of the primary triangulation survey system. Generally, a secondary triangulation survey system is provided within a primary triangulation system. It provides Control points closer than those of the primary triangulation survey system. The secondary triangulation systems are connected to the primary triangulation stations at various points. ***These are used for detailed surveys in areas where great accuracy is not required***. It is used to establish control for interstate and inland subdivisions.

3. Tertiary or Third Order Triangulation Systems:

This system of triangulation survey is of the grade lower than that of the secondary triangulation system. It is used to provide control points between stations of primary and secondary triangulation systems. Tertiary triangulation systems are used to establish control for local development, topographic surveys and other such projects where lower accuracy can be accepted. It is also known as topo triangulation, as it is normally used for providing control in topographic surveys.

Triangulation – Single Point Positioning

❖ SINGLE POINT POSITIONING

There are 3 types of elementary measurements. These are *Distance*, *Bearing* and A*ngle*. Bearings can be measured either directly by the use of a compass or indirectly by a known bearing + angle. Under this section, two types of single point positioning techniques are discussed. These are:

Intersection and R*esection*.

INTERSECTION

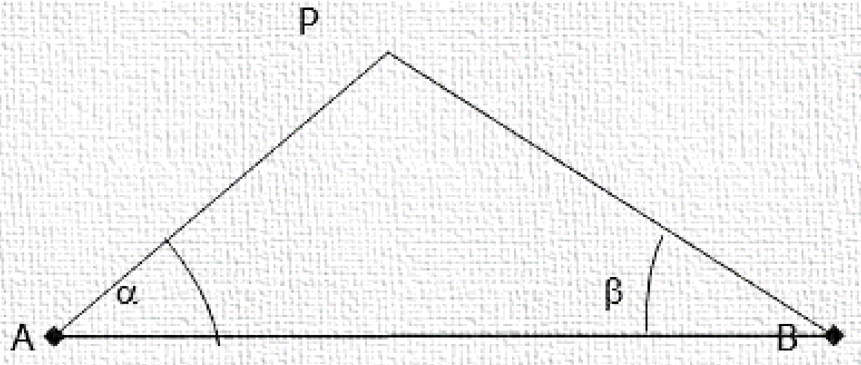
With intersection a minimum of two known points is needed to coordinate a third point, and the unknown point is just measured to. There are a number of combinations of quantities to be measured. These are:

- Distance + distance
- Distance + bearing
- Distance + angle
- Bearing + bearing
- Bearing + angle
- Angle + angle

This method of coordinating a point is suitable in cases where the point to be coordinated can not be set on. Such points as radio masts, electricity poles, aerials, etc.

Triangulation – Single Point Positioning

❖ Standard Intersection Problem



Example: Intersection by angles.

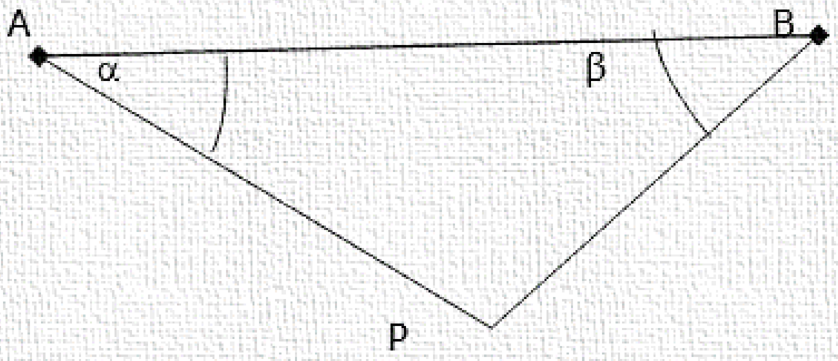


Fig. 1

Given: (Ya, Xa) and (Yb, Xb)
Measured or Derived: α and β
Wanted: Yp, Xp)

<u>Known points</u>

A 174.86 (E) 967.01(N)
B 551.49 (E) 684.54(N)

Measured angles

α	53° 06′ 42″
β	64° 17′ 20″

Triangulation – Single Point Positioning

❖ Standard Intersection Problem

Solution

- Compute a join between A and B i.e., distance and bearing
- Compute the bearing to P from A and P from B i.e., $\varphi_{AP} = \varphi_{AB} + \alpha$ and $\varphi_{BP} = \varphi_{AB} \pm 180° - \beta$
- Compute the distance AP and BP using Sine rule
- After that the coordinates of P are

$$X_P = X_A + d_{AP}\cos\varphi_{AP}$$

$$Y_P = Y_A + d_{AP}\sin\varphi_{AP} \qquad \text{from point A}$$

$$X_P = X_B + d_{BP}\cos\varphi_{BP}$$

$$Y_P = Y_B + d_{BP}\sin\varphi_{BP} \qquad \text{from point B}$$

The final set of coordinates is the average of the two.

Triangulation – Single Point Positioning

❖ Resection

This technique involves the coordination of a point by occupying the unknown point and observing *only* directions to at least three (3) known points. This is very useful if you have control well on high ground and visibility from the unknown point possible. The configuration of the points may differ from situation to situation but the formulas to use are the same. Basically, you have 3 possible configurations for a three-point resection.

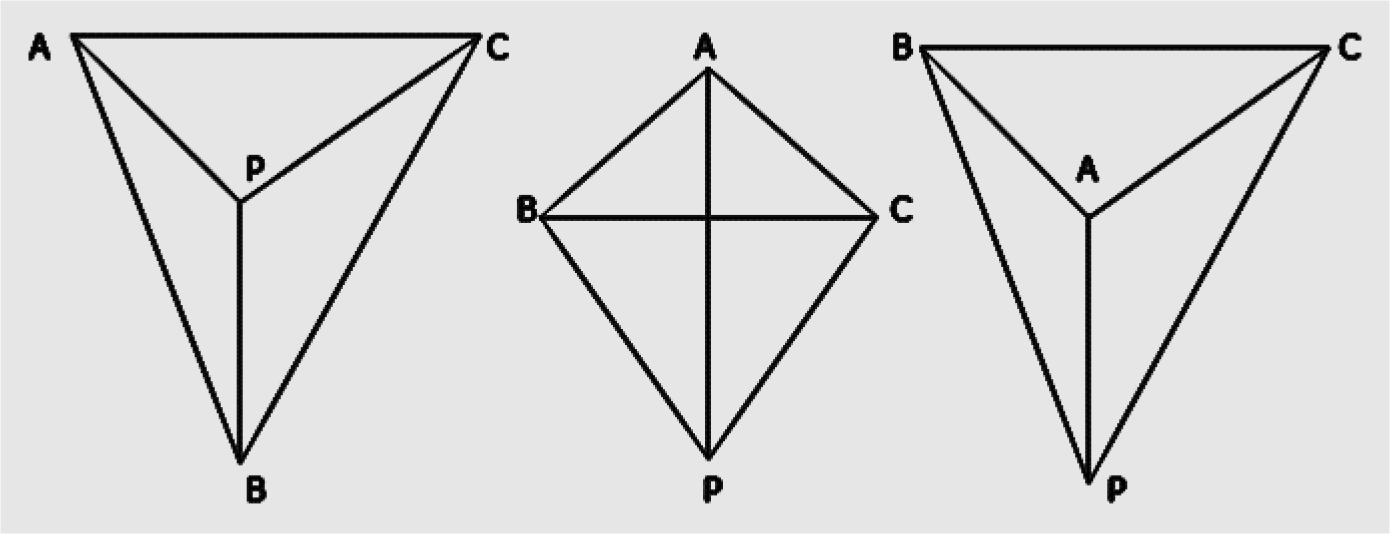


Fig. 2

Triangulation – Single Point Positioning

There are three ways of determining the coordinates with resection. Only two will be discussed under this section, namely the Tienstra and Collins methods.

Tienstra Formula

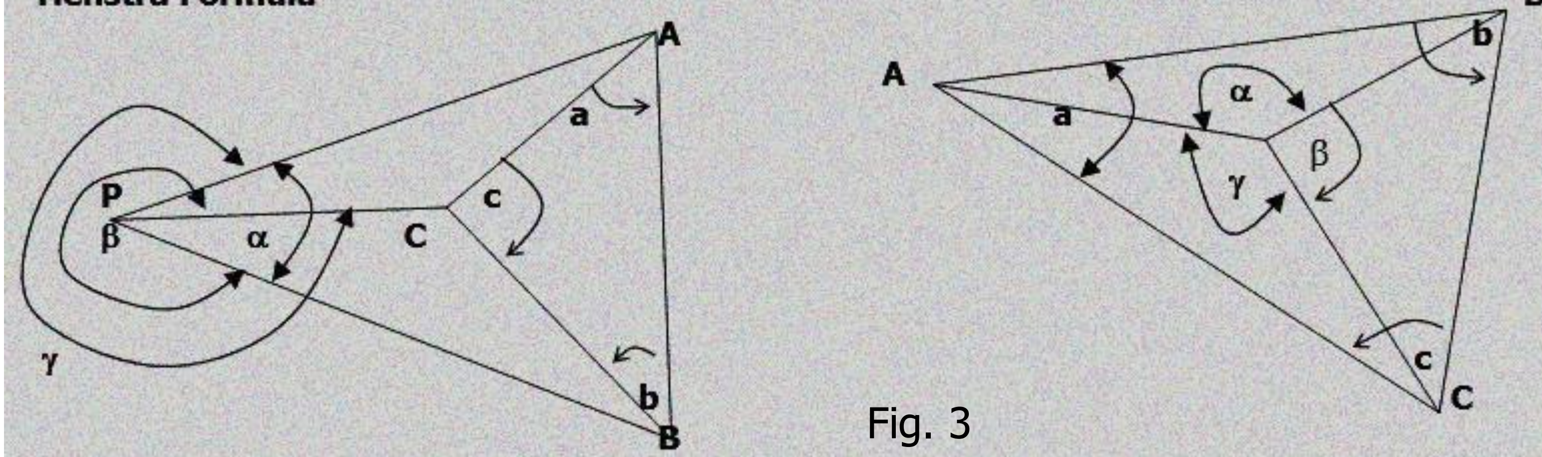


Fig. 3

$$Y_P = \frac{K_1 Y_A + K_2 Y_B + K_3 Y_C}{K_1 + K_2 + K_3} \qquad X_P = \frac{K_1 X_A + K_2 X_B + K_3 X_C}{K_1 + K_2 + K_3}$$

Where $K_1 = \frac{1}{(\cot a - \cot \beta)}$ $\quad K_2 = \frac{1}{(\cot b - \cot \gamma)}$ $\quad K_3 = \frac{1}{(\cot c - \cot \alpha)}$

Triangulation – Multiple point Positioning

Traversing is one of the many multiple point positioning methods. The positions of the control points (survey stations) are fixed by measuring the horizontal angles at each station, subtended by the adjacent stations and the horizontal distance between consecutive pairs of stations.

Traverse networks are used:

a. *As control for topographic detail surveying*
b. *As control for setting out surveys such as roads, railways, etc.*
c. *For ground control for photogrammetric mapping*
d. *In cadastral surveys*

Triangulation – Multiple point Positioning

Types of traverses

There are two categories of traverses. Depending on the way the network starts and ends, it can be open or closed.

a) Open Traverse

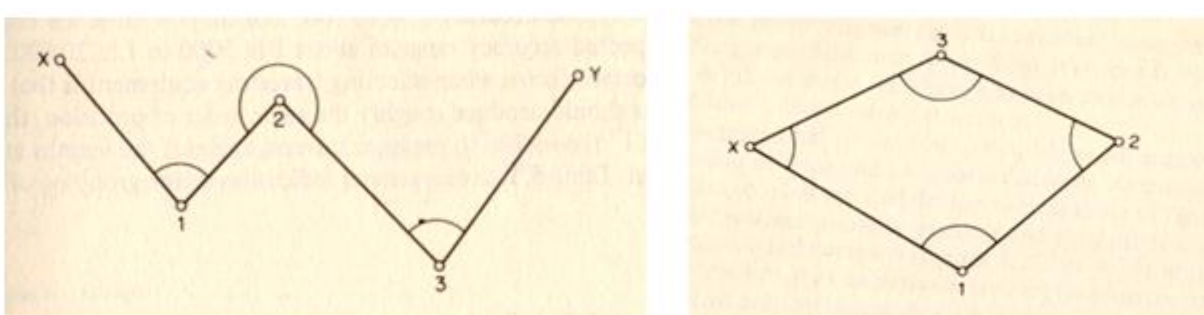
This type of traverse network starts at known points and ends on unknown point. These type of traverses are used in exceptional circumstances since there is no external check on the measurements. However, open traverses are used mainly in tunneling work where the physical situation prevents closure on known points. It is important therefore that measured angles, distances and instrument centering be very carefully checked.

Triangulation – Multiple point Positioning

a) Closed Traverse

A closed traverse network is one, which starts and ends on the known points. There are two types of closed traverses: a closed link traverse and a closed

loop traverse (see figure 1 below). A closed link traverse starts from known points, say A and B and ends at other known points C and D where as the closed loop traverse starts from known points, say A and B and ends at the same points A and B.



Link traverse

Closed loop(polygon)

Fig. 6

The advantage of both the above traverses is that there is an external check on the measurements since the traverses start and end on known points.

Triangulation – Multiple point Positioning

Traverse specifications and Accuracy

The standard of accuracy should, according to the Zambian Regulations, be

Class	Relative Accuracy	Specifications
Class A	1/12,000	Reference marks, township control
Class B	1/8,000	Township surveys
Class C	1/4, 000	Surveys not included in A and B (Farm surveys, etc.)

Generally, the accuracy of the traverse is judged on basis of the resultant closure of the traverse. This resultant closure is a function of the accuracies in the measurement of angles and distances and hence varies with the length of the traverse.

Triangulation – Multiple point Positioning

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$$\frac{\left(e_y^2 + e_x^2\right)^2}{\sum d} = \frac{e}{\sum d}$$

Where e_y is the misclosure in the Y partial between end control points.

e_x is the misclosure in the X partial between end control points.

d_i is the distances of the individual traverse legs?

Triangulation – Multiple point Positioning

In angular measurements, the allowable misclosure = **$\pm \varepsilon''(n)^{1/2}$** where ε is the estimated standard deviation of observing the angles and n is the number of instrument set ups (stations).

If the misclosure is higher than the allowable misclosure then the traverse has to be repeated.

For a closed link traverse, the measured angles or bearings are also prone to measurement errors and thus have to be checked. In the case of bearing, the forward bearing of end points (C and D, figure 6) is compared with the measured bearing of the same points (C and D, figure 6). For angle measurements, the starting bearing of the control points (A and B, figure 6) is added to the measured angles (as shown below) and the resulting end bearing is compared as explained earlier.

Triangulation – Multiple point Positioning

The distribution or adjustment of the angular misclosure is done equally since each angle is measured in the same manner and there is an equal chance of the misclosure having occurred in any of the angles.

Correction per station = - (angular misclosure/ n), n =number of stations

Triangulation –Errors in angular measurements

There are various sources of errors that are common in angular measurements:

- *Inaccurate centering of the theodolites or targets*
- *Non verticality of targets (Hint: sight the bottom part of the target if you can)*
- *Inaccurate bisection of the target*
- *Lateral refraction, wind and atmospheric effects*
- *Theodolite not level and not in adjustment when measuring angles*
- *Incorrect use of the theodolite*
- *Mistakes in reading and booking.*

Triangulation –Computation of Coordinate Differentials (Partials)

$$\Delta Y_{i,i+1} = d_{i,i+1} Sin \theta_{i,i+1}$$
$$\Delta X_{i,i+1} = d_{i,i+1} Cos \theta_{i,i+1}$$

Having adjusted and computed the bearings, then the partials for each individual legs can be computed in combination with projected horizontal distances.

Where i = 1, 2 …n, instrument station points

In an ideal situation, *the sum of the shifts (partials) in Y and X coordinates should equal to zero in the case of a closed loop traverse* since you are coming back to the same point. And in the case of a closed link traverse the sum of the *coordinate shifts in both Y and X should equal the difference in coordinates between the start and the end points*, i.e.

Triangulation –Computation of Coordinate Differentials (Partials)

For closed loop traverses

$$\sum \Delta Y = Y_C - Y_A$$

$$\sum \Delta X = X_C - X_A$$

For closed link traverses

However, due to the errors in the measured distances of the traverse and angles, there will always be a disparity on arrival at the closing point.

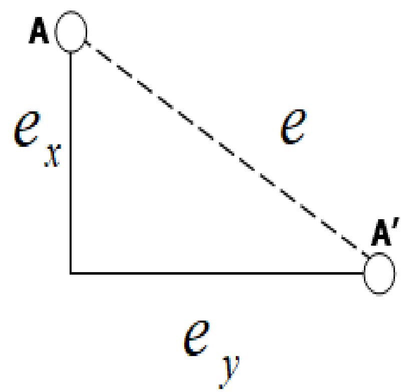

Fig. 7

The resultant displacement AA′ (e) is known as the linear misclosure and it is given by

$$e = \left(e_y^2 + e_x^2\right)^{1/2}$$

Triangulation – Error Distribution

Distribution of the linear misclosure

There are many ways of adjusting the linear misclosure of the traverse, however in this chapter we shall be restricted to two of the techniques, namely, Bowditch and the Transit methods.

(a) Bowditch Method

In this method, the values of the adjustment are directly proportional to the length of the individual traverse lines (legs) i.e.

Correction (δY_i) $$= \frac{-e_y}{\sum_{i=1}^{n} d_i} * d_i = K_y d_i$$

Correction (δX_i) $$= \frac{-e_x}{\sum_{i=1}^{n} d_i} * d_i = K_x d_i$$

Where δX_i , δX_i the coordinate (partial) correction in Y and X

e_y , e_x the partial misclosure in Y and X coordinates - constant

d_i the horizontal distance of the i[th] traverse leg

$\sum_{i=1}^{n} d_i$ sum of the distances (total length of the traverse) - constant

K_y , K_Y the resulting constants.

Triangulation – Error Distribution

(a) Transit Method - In this method, adjustments are proportional to the values of partials (ΔY and ΔX) i.e.

Correction (δY_i) $= \dfrac{-e_y}{\left|\sum_{i=1}^{n} \Delta Y_i\right|} * \Delta Y_i$

Correction (δX_i) $= \dfrac{-e_x}{\left|\sum_{i=1}^{n} \Delta X_i\right|} * \Delta X_i$

Where δX_i (δY_i) the coordinate (partial) correction in Y and X

ΔY_i, ΔX_i the partial in Y and X for the i[th] traverse leg

$\left|\sum_{i=1}^{n} \Delta X_i\right|$, $\left|\sum_{i=1}^{n} \Delta Y_i\right|$ absolute sum of partials of the traverse.

e_y , e_x Misclosure in Y and X

Transit method, however, does not take into consideration the leg distance in determining the correction. The implication is that if a leg has a partial equal to zero i.e. ΔY and ΔX = 0, then it will have no correction, an assumption which is not practical.

Triangulation – Error Distribution

Calculation of Final (Adjusted) Coordinates

The final coordinates of the traverse points are obtained by adding or subtracting the adjusted partials, working round the traverse.

For the polygon (closed loop) traverse, the final and the initial coordinates should be the same as these represent the same point and for the link traverse, the final coordinates should equal those of the last known station in the traverse. If in both cases the expected in not achieved, it should be due to arithmetic errors or rounding off errors.

Below is an example of the traverse computation

<u>Measured angles</u>

LS498	89° 17′ 50″
TP1	90° 45′ 23″
TP2	88° 33′ 28″
TP3	93° 31′ 21″
LS497	<u>177° 52′ 12″</u>

Total **<u>540° 00′ 14″</u>**

Bearing LS498 – LS497
= 308° 49′ 03″

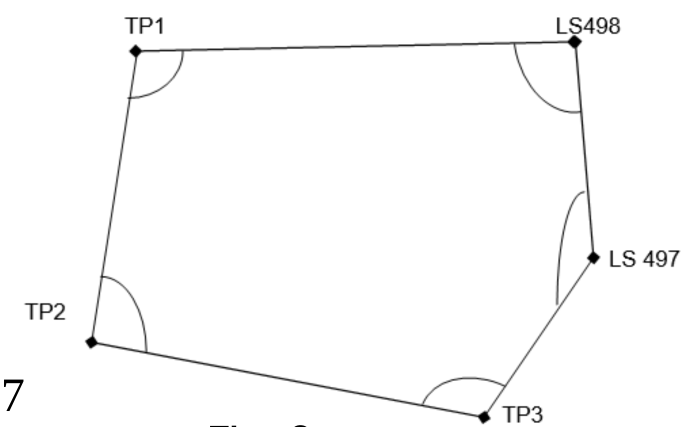

Fig. 8

Triangulation – Error Distribution

Stn	Adj. Hor. Angles	Bearing	Hor. Dists.	COORDINATE DIFFERENCES						COORDINATES	
				CALCULATED		ADJUSTMENTS		ADJUSTED			
	° ′ ″	° ′ ″	m	ΔY	ΔX	δY	δX	ΔY	ΔX	Y	X
FB1	89.17.47	038.06.50	424.573	262.058	334.048	0.002	-.009	262.060	334.039	72029.79	1701922.48
A	90.45.20	308.52.10	611.354	-475.987	383.654	0.002	-.013				
B	88.33.26	217.25.36	416.744	-253.274	-330.950						
C	93.31.18	130.56.54	227.283	171.667	-148.956						
FB2	177.52.09	128.49.03								71734.26	1702160.24
			1679.954	-295.536	237.796						

$\sum\Delta Y = -295.536$ $\sum\Delta X = 237.796$

$\underline{\Delta Y = -295.530}$ $\underline{\Delta X = 237.760}$

B - 0.006 + 0.036

Relative accuracy = $((-0.006)^2 + (0.036)^2)^{1/2} / 1679.954$

$= 1/46000$