GEE 4812:
Principles of Geomatics

Single Point Positioning

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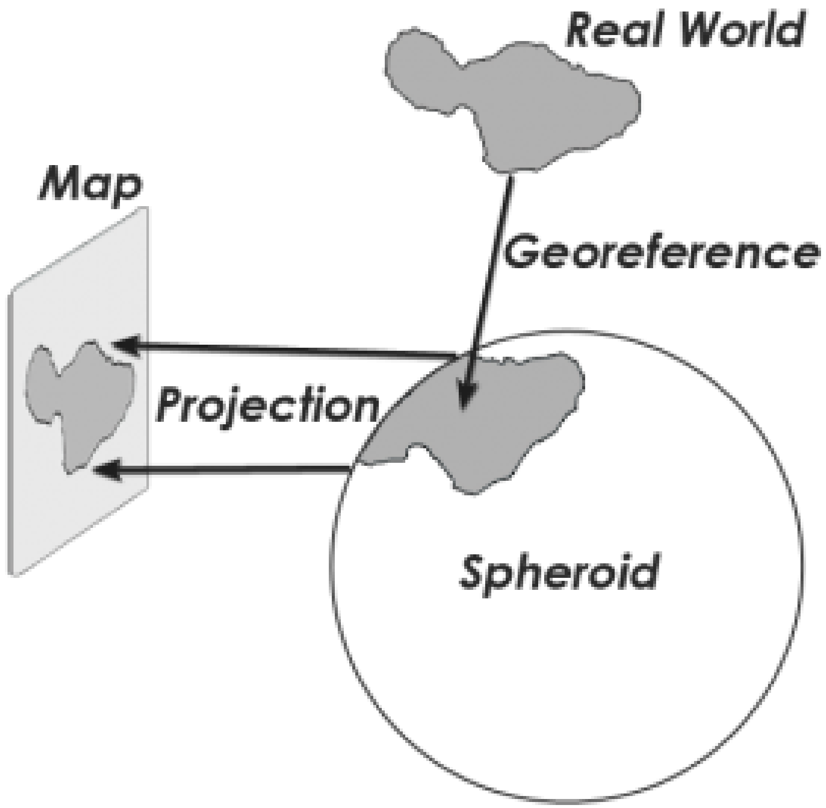
Coordinate Systems: Introduction

- The relative position of a feature or a point is defined by means of coordinates in a certain coordinate system.
- Therefore, a point on the earth may be located by geographical coordinates, i.e. longitude, latitude and altitude.
- Maps are made as plane reproduction of the earth.

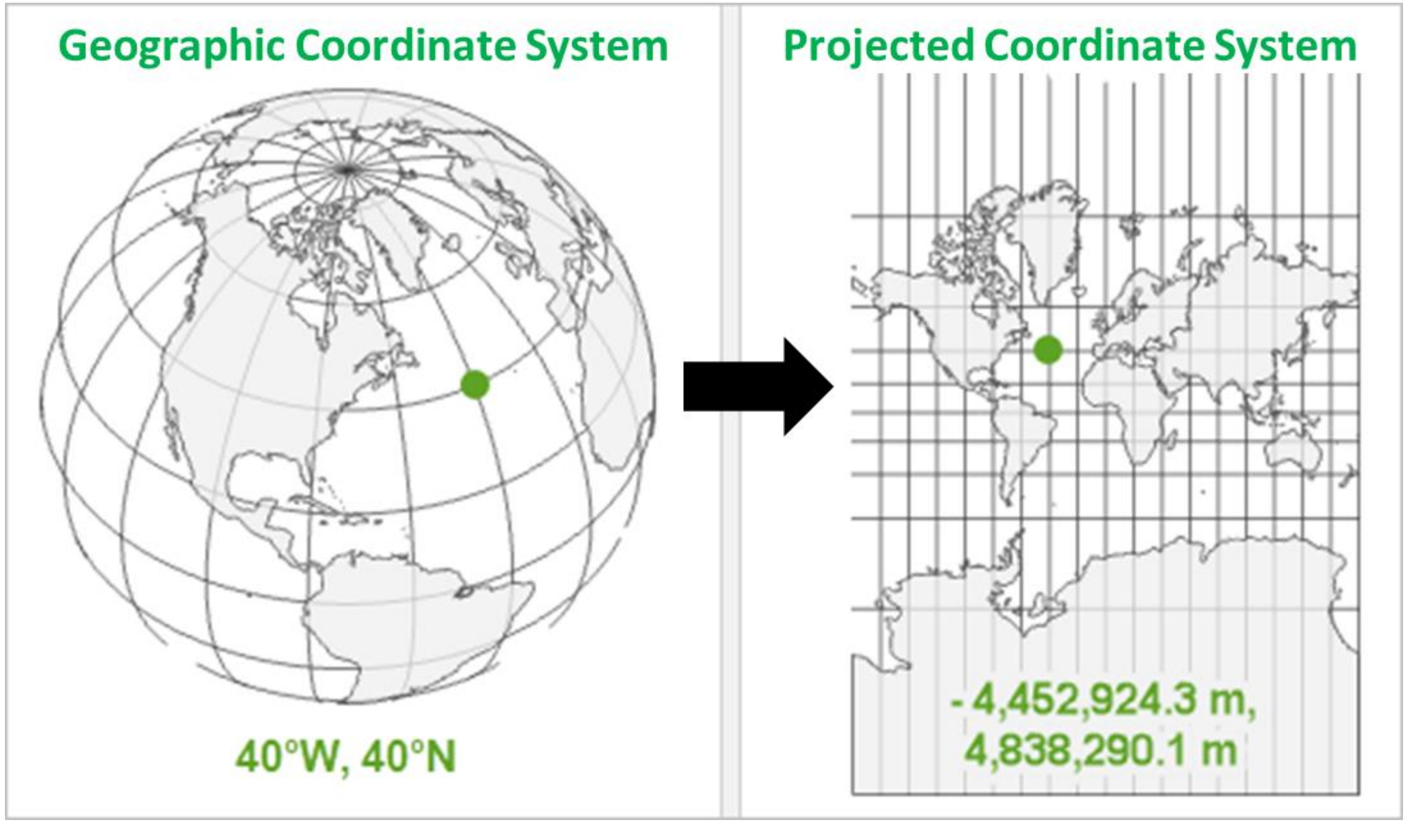
Coordinate Systems: Projection

- Since the earth is spheroidal, a projection system must be used to reproduce the earth on the plane (flat surface).
- Thus the projection system defines mathematically a link between the earth and the earth on the map.



Coordinate Systems: Projection

- Earth projected from 3D (spheroid) to 2D (flat surface).



Coordinate Systems: Types

- There are two coordinate systems used in plane surveying are:

1. Rectangular Plane Coordinates

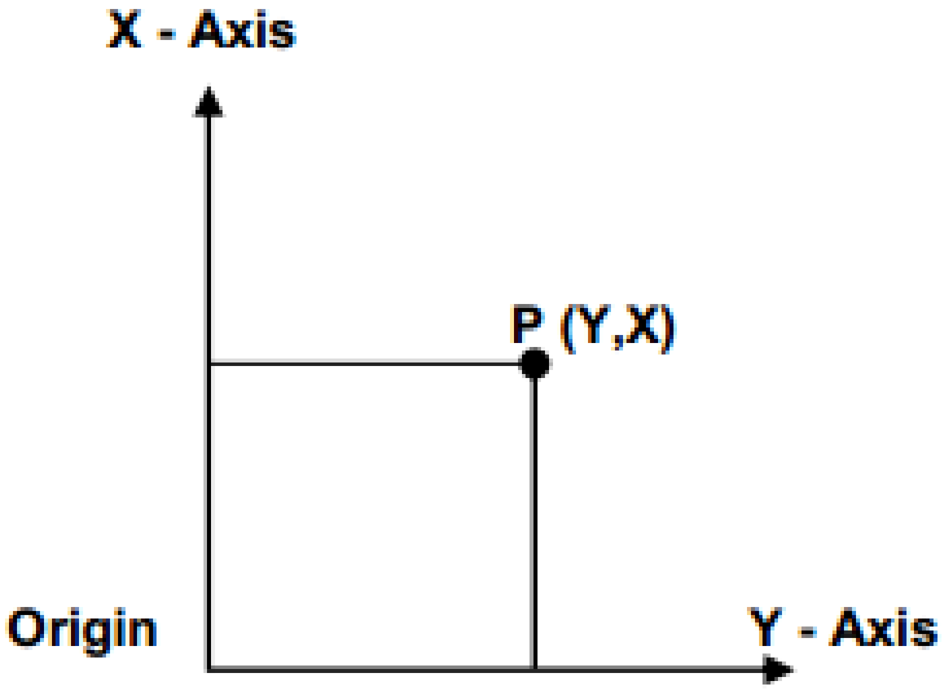
2. Polar Plane Coordinates

Rectangular Plane Coordinates

- In surveying, the coordinate systems are different from mathematical coordinate systems in that the X- axis defined along the North-South direction and the Y- axis along the East-West direction.
- For example, a point P is expressed as P (Y, X). (see figure)

Rectangular Plane Coordinates

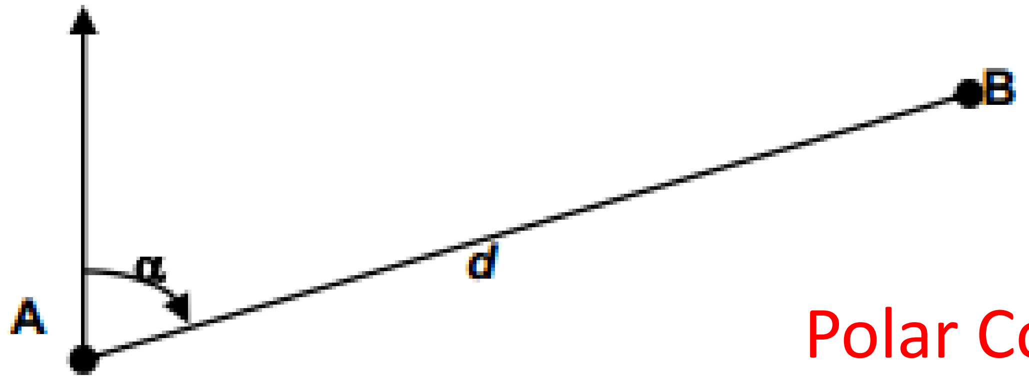
- That is the X-axis is positive to the north, which is the zero direction and the Y-axis is positive to the east.
- The calculations in both systems are exactly the same.



Polar Plane Coordinates

- The relative position of two points can also be given by distance (d) between the points and the bearing (α).

- These are referred to as Polar coordinates. (Figure below)



Polar Coordinates = (d, α)

Join and Polar

- **Join** : means the determination of the distance and bearing between two (2) points given their rectangular coordinates.
- **Polar** : means the determination of the coordinates of the second point given the coordinates of the first point, and the distance and bearing to the second point.

Bearing

- **Bearing** : this is a clockwise angle from the reference direction (say like from the North).

- This bearing can be defined either as a "Whole Circle Bearing" (WCB), that is between 0^o and 360^o from the North direction (reference) or as a Quadrant (Reduced) Bearing, that is between 0^o and 90^o with respect to the North or South and the line to the point.

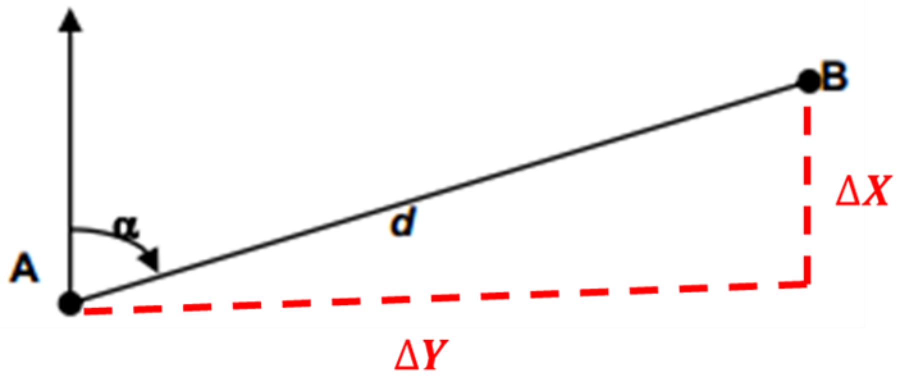
$$\text{Bearing} = \arctan\left(\Delta Y / \Delta X\right)$$

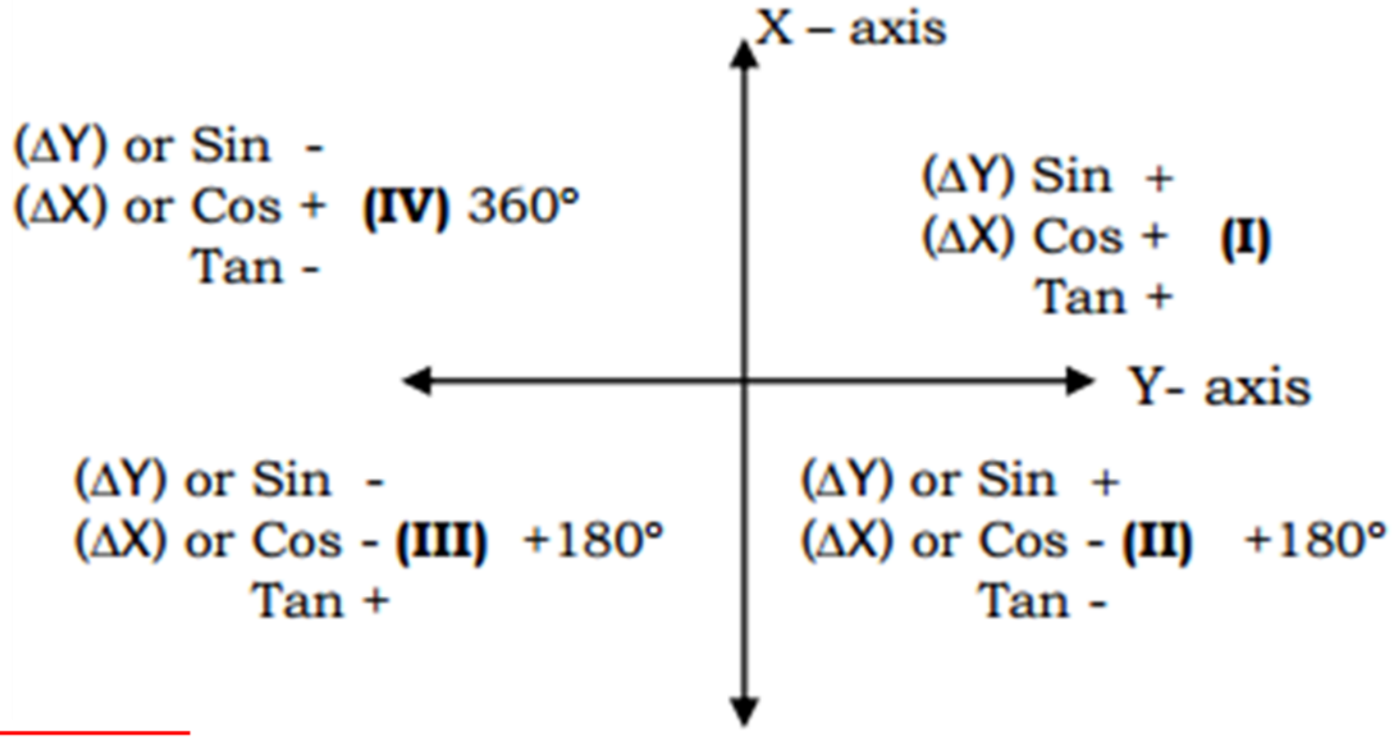
Bearing

- The computation of a bearing shown previously does not show the exact especially in the second, third and fourth quadrants.
- The quadrants are numbered clockwise starting from the upper right one.
- The quadrant where the point lies is determined by the value (+ or -) of the changes in Y and X-axes, i.e. ΔY and ΔX in the respective quadrants.
- The figure shows the values of the changes and what value has to be added.

Bearing Calculation Quadrants

Bearing = arctan (ΔY/ΔX)





$$\boxed{\begin{aligned}\Delta Y &= \sin\alpha \\ \Delta X &= \cos\alpha\end{aligned}}$$

$$d = \sqrt{\Delta Y^2 + \Delta X^2}$$

Bearing Calculation Quadrants

- 1st Quadrant : ΔY and ΔX are both positive, maintain the angle.
- 2nd Quadrant : ΔY is positive and ΔX is negative, add 180 degrees to the angle.
- 3rd Quadrant : ΔY and ΔX are both negative, add 180 degrees to the angle.
- 4th Quadrant : ΔY is negative and ΔX is positive, add 360 degrees to the angle.

Single Point Positioning

- There are 3 types of elementary measurements.
- These are Distance, Bearing and Angle.
- Bearings can be measured either directly by the use of a compass or indirectly by a known bearing + angle.
- Under this section, two types of single point positioning techniques are discussed.
- These are: **Intersection and Resection.**

Intersection

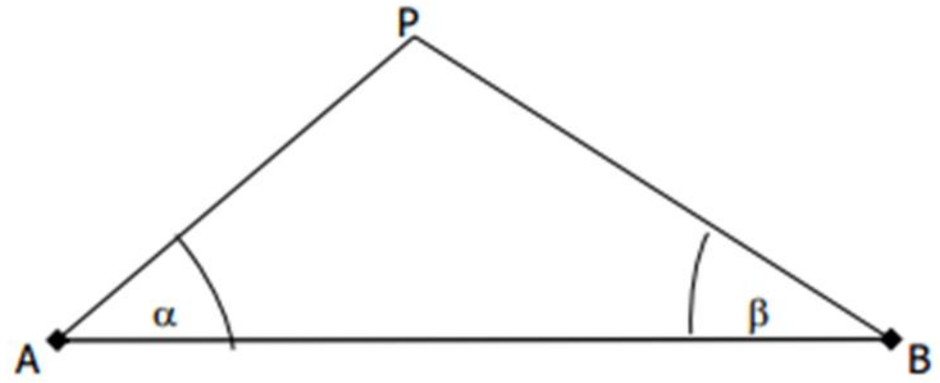
- With intersection a minimum of two known points is needed to coordinate a third point, and the unknown point is just measured to.

- There are a number of combinations of quantities to be measured. These are:

  - Distance + distance
  - Distance + bearing
  - Distance + angle
  - Bearing + bearing
  - Bearing + angle
  - Angle + angle

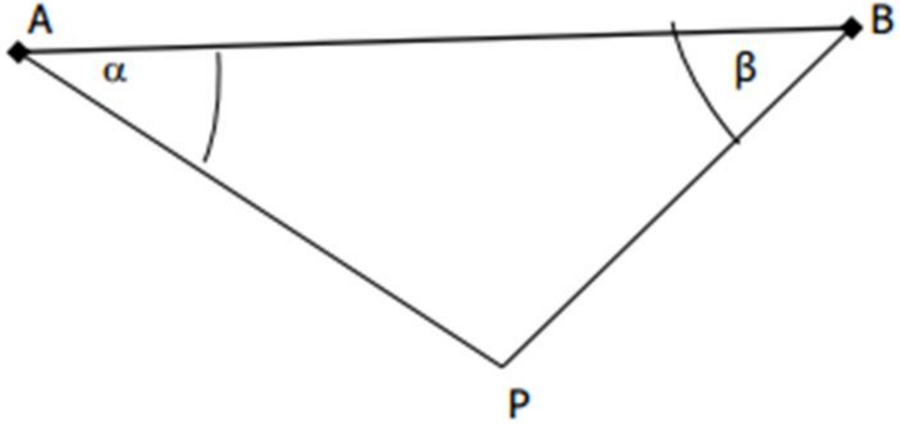
Intersection

CASE 1:



Example: Intersection by angles.

CASE 2:



Given: (Ya, Xa) and (Yb, Xb)
Measured or Derived: α and β
Wanted: Yp, Xp)

<u>Known points</u>

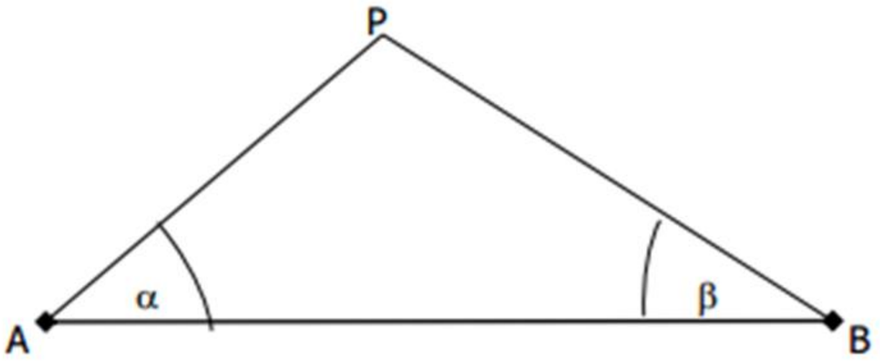
A 174.86 (E) 967.01(N)
B 551.49 (E) 684.54(N)

Measured angles

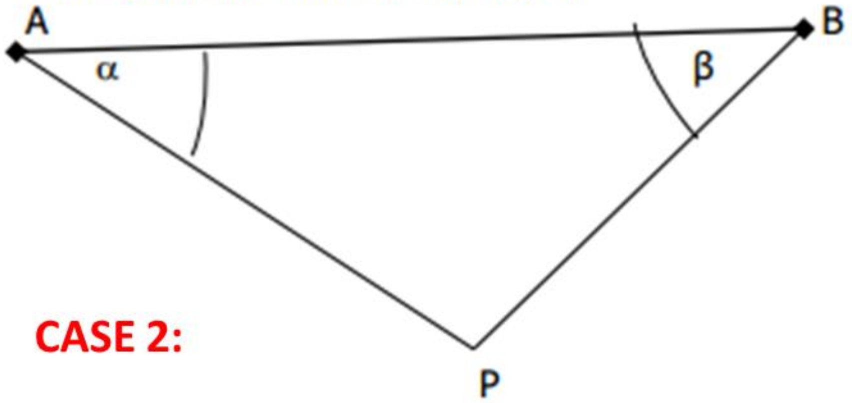
α 53° 06' 42"
B 64° 17' 20"

Intersection

CASE 1:



Example: Intersection by angles.



CASE 2:

Solution

- Compute a join between A and B i.e. distance and bearing
- Compute the bearing to P from A and P from B i.e.

$$\varphi_{AP} = \varphi_{AB} + \alpha$$ and

$$\varphi_{BP} = \varphi_{AB} \pm 180° + \beta$$

- Compute the distance AP and BP using Sine rule

- After that the coordinates of P are

$$X_P = X_A + d_{AP} \cos\varphi_{AP}$$
$$Y_P = Y_A + d_{AP} \sin\varphi_{AP}$$ from point A

$$X_P = X_B + d_{BP} \cos\varphi_{BP}$$
$$Y_P = Y_B + d_{BP} \sin\varphi_{BP}$$ from point B

The final set of coordinates is the average of the two.

Intersection

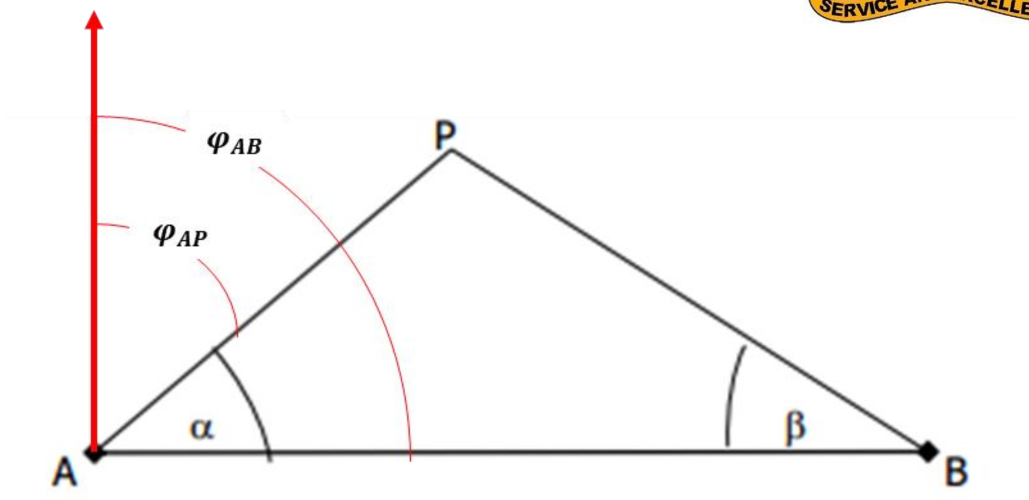
CASE 1

Finding coordinates of P from A

- $\varphi_{AB} = \tan^{-1}\left[\frac{Y_B - Y_A}{X_B - X_A}\right]$
- $\varphi_{AB} = \tan^{-1}\left[\frac{551.49 - 174.86}{684.54 - 967.01}\right]$
- $\tan^{-1}\left[\frac{376.63}{-282.47}\right] = -53^\circ 07' 49''$
- But ΔY is +ve and ΔX is –ve, so we add 180°
- $\varphi_{AB} = -53^\circ 07' 49'' + 180^\circ 00' 00''$
- $\varphi_{AB} = 126^\circ 52' 11''$
- Bearing φ_{AP}

$$\varphi_{AP} = \varphi_{AB} - \alpha$$

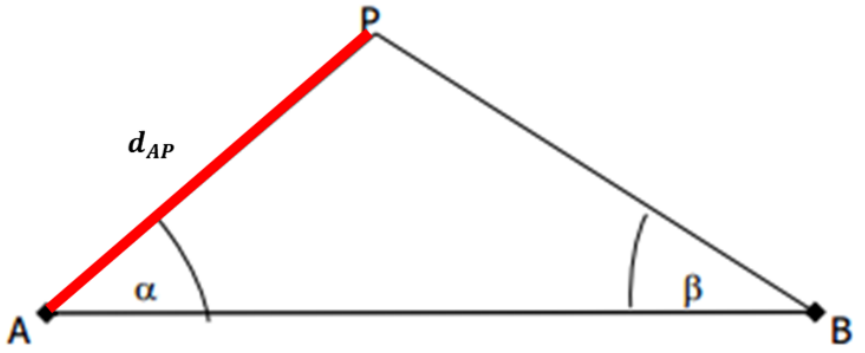
$$\varphi_{AP} = 126^\circ 52' 11'' - 53^\circ 06' 42'' = 73^\circ 45' 29''$$



Intersection

CASE 1

- **Angle APB**
- $Angle\ APB = 180^{\circ}00'00'' - (\alpha + \beta)$
- $Angle\ APB = 180^{\circ}00'00'' - \left(53^{\circ}06'42'' + 64^{\circ}17'20''\right) = 62^{\circ}35'58''$
- **Find Distance AB**
- $d_{AB} = \sqrt{\Delta Y^2 + \Delta X^2} = \sqrt{(551.49 - 174.86)^2 + (684.54 - 967.01)^2} = 470.786m$
- **Calculating the Distance AP using sine rule**
- $d_{AP} = \frac{d_{AB} \sin \beta}{\sin APB} = \frac{470.786 \sin 64^{\circ}17'20''}{\sin 62^{\circ}35'58''} = 477.776m$
- **Coordinates of P from A**
- $Y_P = Y_A + d_{AP} \sin \varphi_{AP} = 174.86 + 477.776 \sin 73^{\circ}45'29'' = 633.58m$
- $X_P = X_A + d_{AP} \cos \varphi_{AP} = 967.01 + 477.776 \cos 73^{\circ}45'29'' = 1100.64m$



Intersection

CASE 1

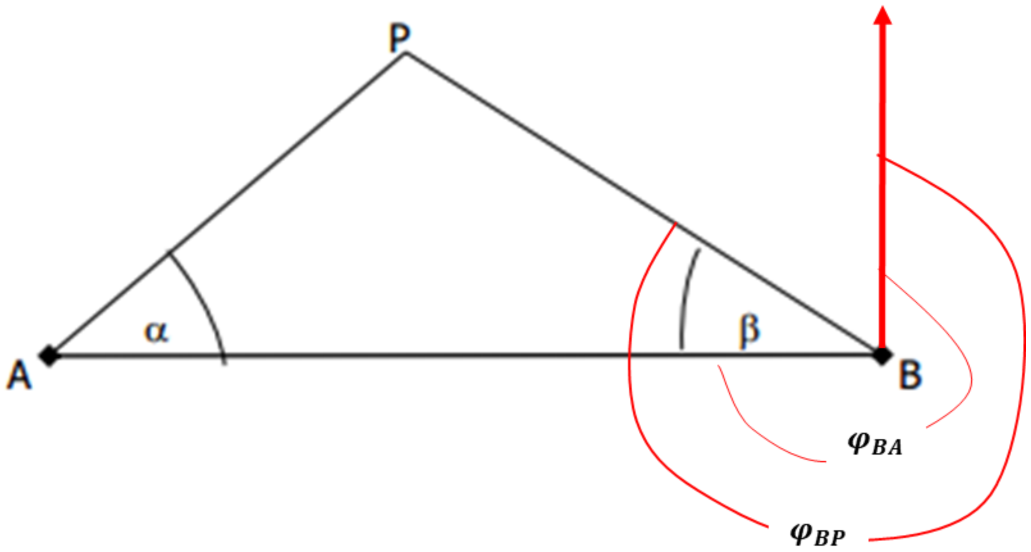
Finding coordinates of P from B

- $\varphi_{BA} = \tan^{-1}\left[\frac{Y_A - Y_B}{X_A - X_B}\right]$
- $\varphi_{BA} = \tan^{-1}\left[\frac{174.86 - 551.49}{967.01 - 684.54}\right]$
- $\tan^{-1}\left[\frac{-376.63}{282.47}\right] = -53^{\circ}07'49''$
- But ΔY is -ve and ΔX is +ve, so we add 360°
- $\varphi_{BA} = -53^{\circ}07'49'' + 360^{\circ}00'00''$
- $\varphi_{BA} = 306^{\circ}52'11''$ (note this value has a 180°difference with φ_{AB})
- **Bearing φ_{BP}**

  $\varphi_{BP} = \varphi_{BA} + \beta$

  $\varphi_{BP} = 306^{\circ}52'11'' + 64^{\circ}17'20'' = 371^{\circ}09'31''$ **(over 360°)**

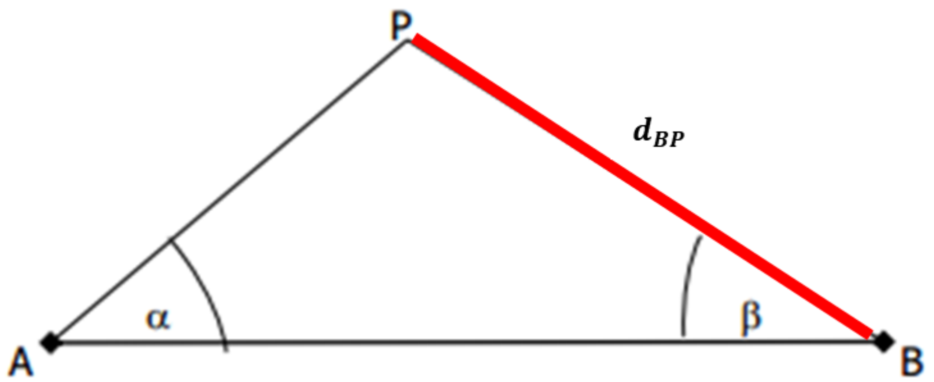
  $\varphi_{BP} = 371^{\circ}09'31'' - 360^{\circ}00'00'' = 11^{\circ}09'31''$



Intersection

CASE 1

- **Calculating the Distance BP using sine rule**
- $d_{BP} = \frac{d_{AB} \sin \alpha}{\sin APB} = \frac{470.786 \sin 53^{\circ}06'42''}{\sin 62^{\circ}35'58''} = 424.119m$
- **Coordinates of P from B**
- $Y_P = Y_A + d_{AP} \sin \varphi_{AP} = 551.49 + 424.119 \sin 11^{\circ}09'31'' = 633.57m$
- $X_P = X_A + d_{AP} \cos \varphi_{AP} = 684.54 + 424.119 \cos 11^{\circ}09'31'' = 1100.64m$
- Ideally these values should be the same with those previously calculated from A.
- In this case the X values are the same but the Y values are slightly different.
- So we take the average.
- $Y_P = \frac{633.58m + 633.57m}{2} = 633.58m$
- Therefore coordinates of P = 633.58m, 1100.64m
- Also written as **P (633.58m, 1100.64m)**



Intersection

CASE 2

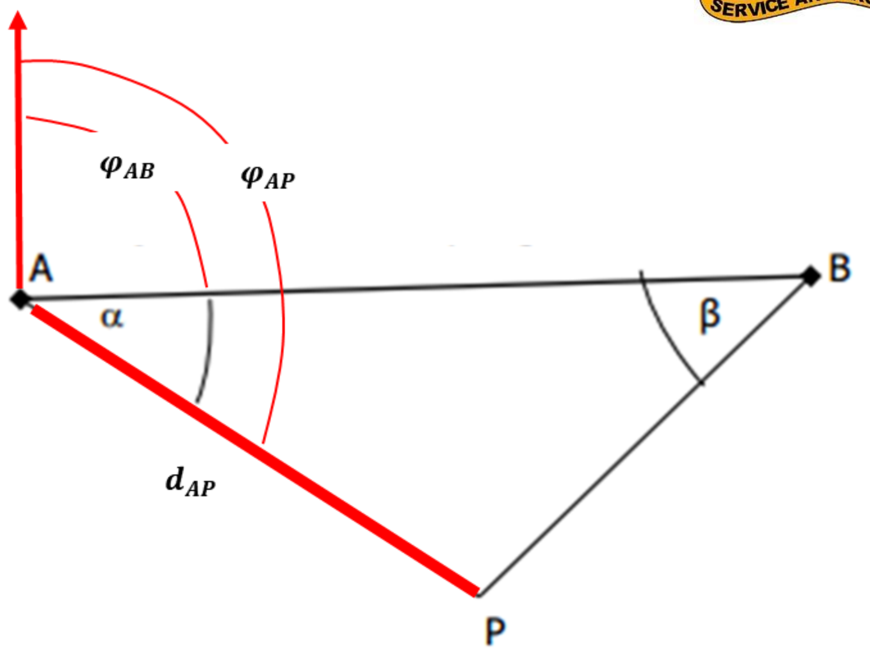
Finding coordinates of P from A

- $\varphi_{AB} = 126^\circ 52'11'', \ d_{AB} = 470.786m$ and $d_{AP} = 477.776m$
- **Bearing φ_{AP}**

  $\varphi_{AP} = \varphi_{AB} + \alpha$

  $\varphi_{AP} = 126^\circ 52'11'' + 53^\circ 06'42'' = 179^\circ 58'53''$

- **Coordinates of P from A**
- $Y_P = Y_A + d_{AP} \sin \varphi_{AP} = 174.86 + 477.776 \sin 179^\circ 58'53'' = 175.02m$
- $X_P = X_A + d_{AP} \cos \varphi_{AP} = 967.01 + 477.776 \cos 179^\circ 58'53'' = 489.23m$



Intersection

CASE 2

Finding coordinates of P from B

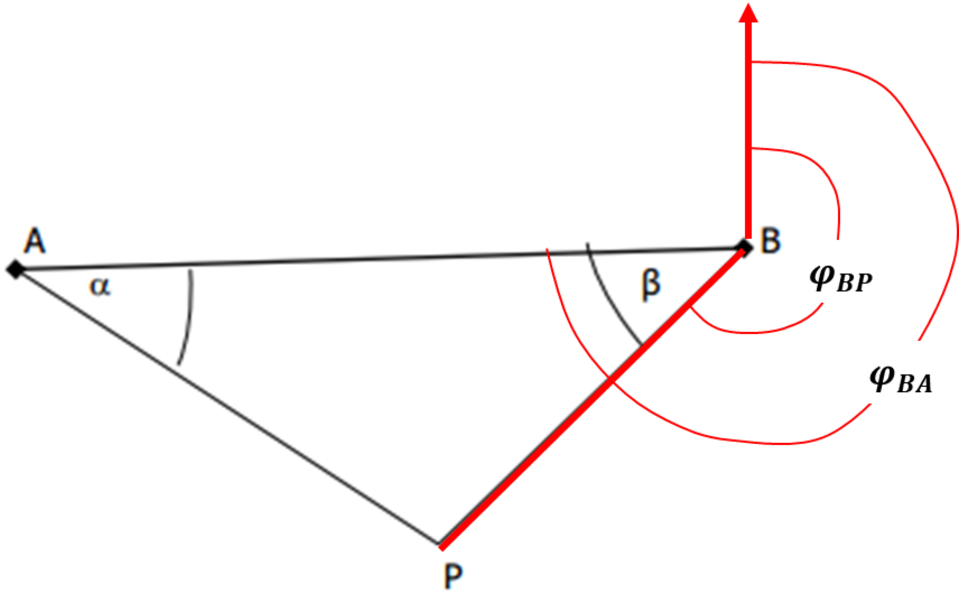
- $\varphi_{BA} = 306^\circ 52' 11'',\ d_{BA} = 470.786m$ and $d_{BP} = 424.119m$
- **Bearing φ_{BP}**

  $\varphi_{BP} = \varphi_{BA} - \beta$

  $\varphi_{BP} = 306^\circ 52' 11'' - 64^\circ 17' 20'' = 242^\circ 34' 51''$

- **Coordinates of P from A**
- $Y_P = Y_B + d_{BP} \sin \varphi_{BP} = 551.49 + 424.119 \sin 242^\circ 34' 51'' = 175.02m$
- $X_P = X_B + d_{BP} \cos \varphi_{BP} = 684.54 + 424.119 \cos 242^\circ 34' 51'' = 489.23m$

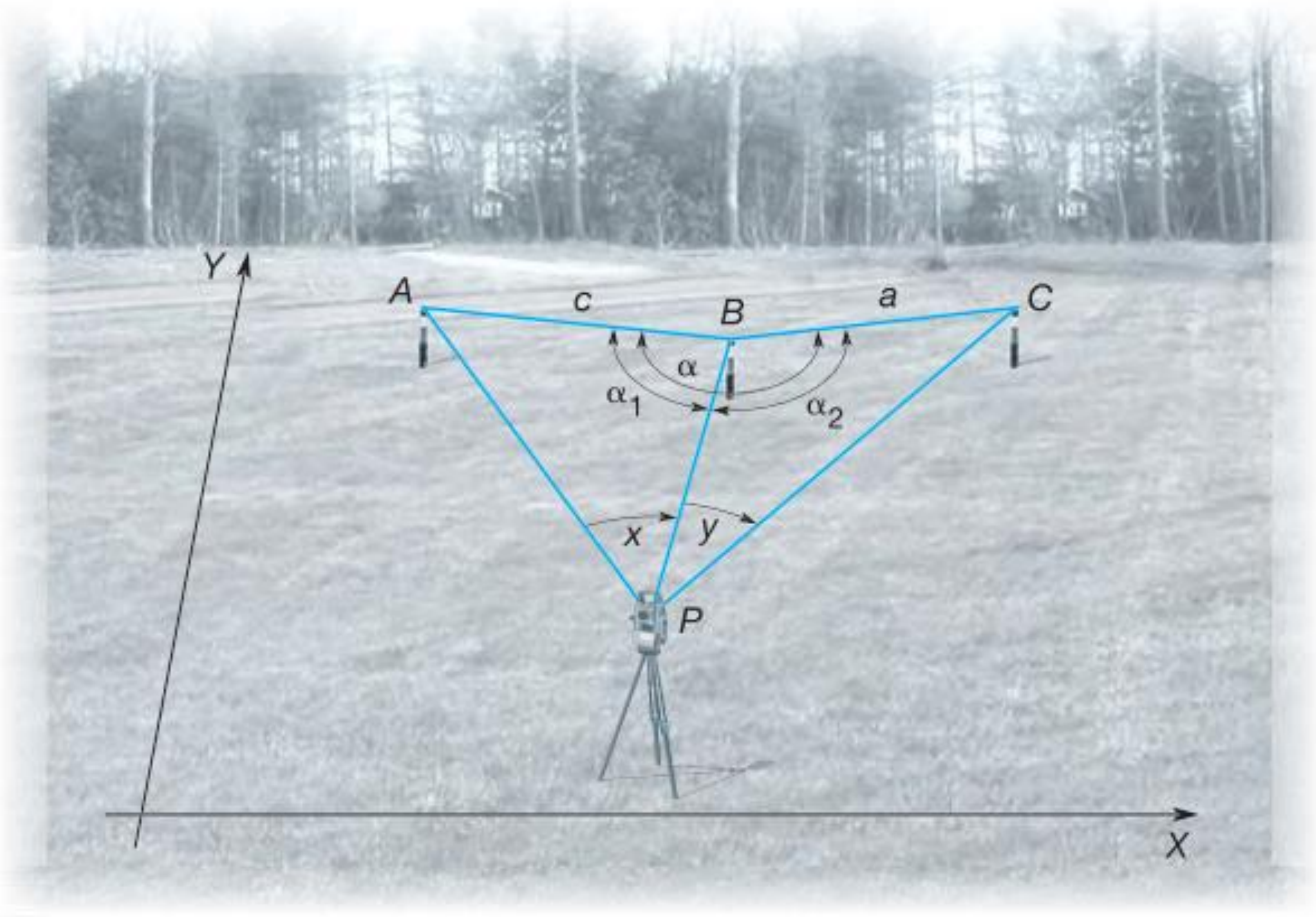
- In this case the X and Y values are the same as those calculated from A.



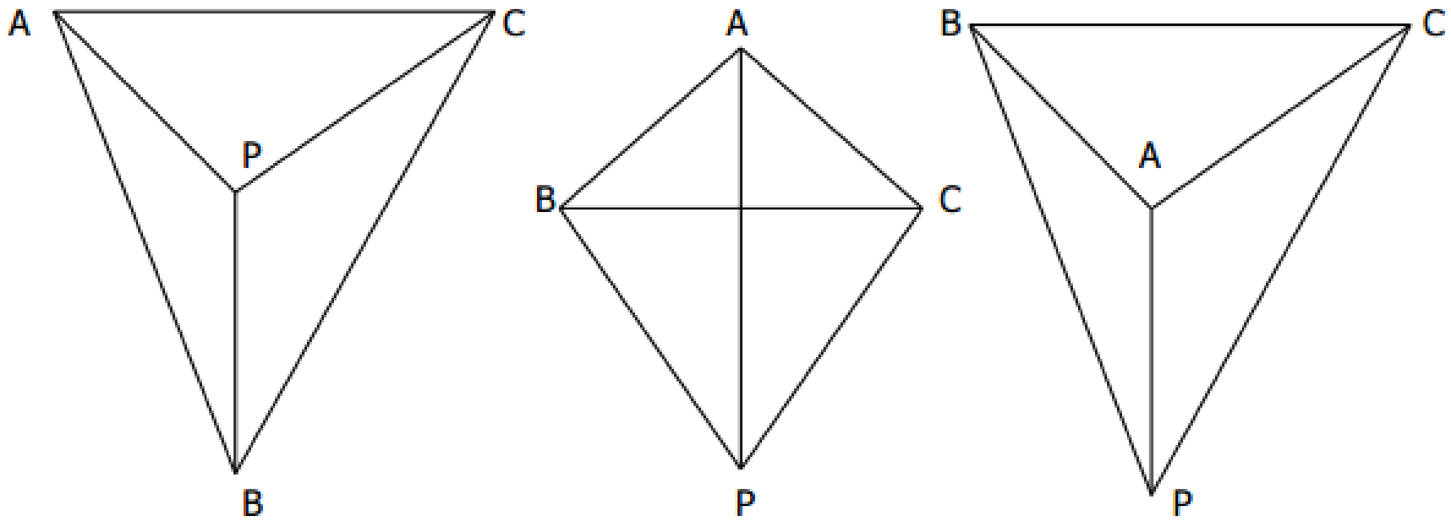
Resection

- This technique involves the coordination of a point by occupying the unknown point and observing only directions to at least three (3) known points.
- This is very useful if you have control well on high ground and visibility from the unknown point possible.
- The configuration of the points may differ from situation to situation but the formulas to use are the same.
- Basically you have 3 possible configuration for a three-point resection.

Resection

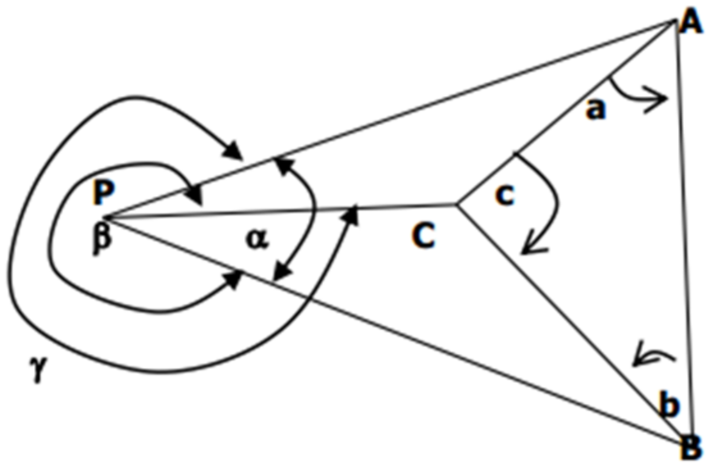


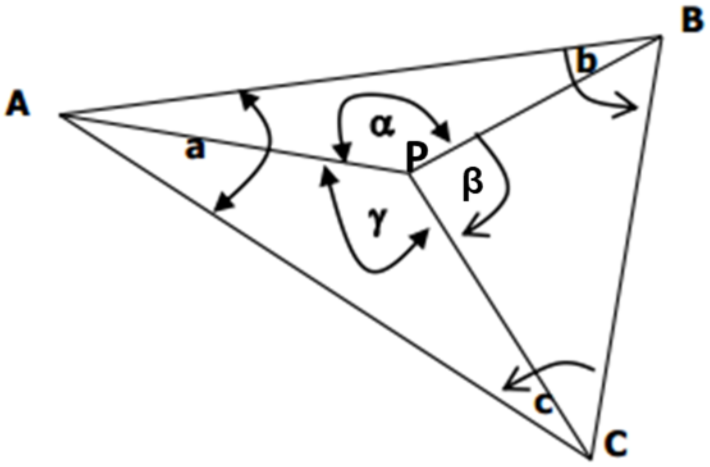
Resection



- There are a number of ways of determining the coordinates with resection.
- Only one will be discussed in this section i.e Tienstra method.

Tienstra Method





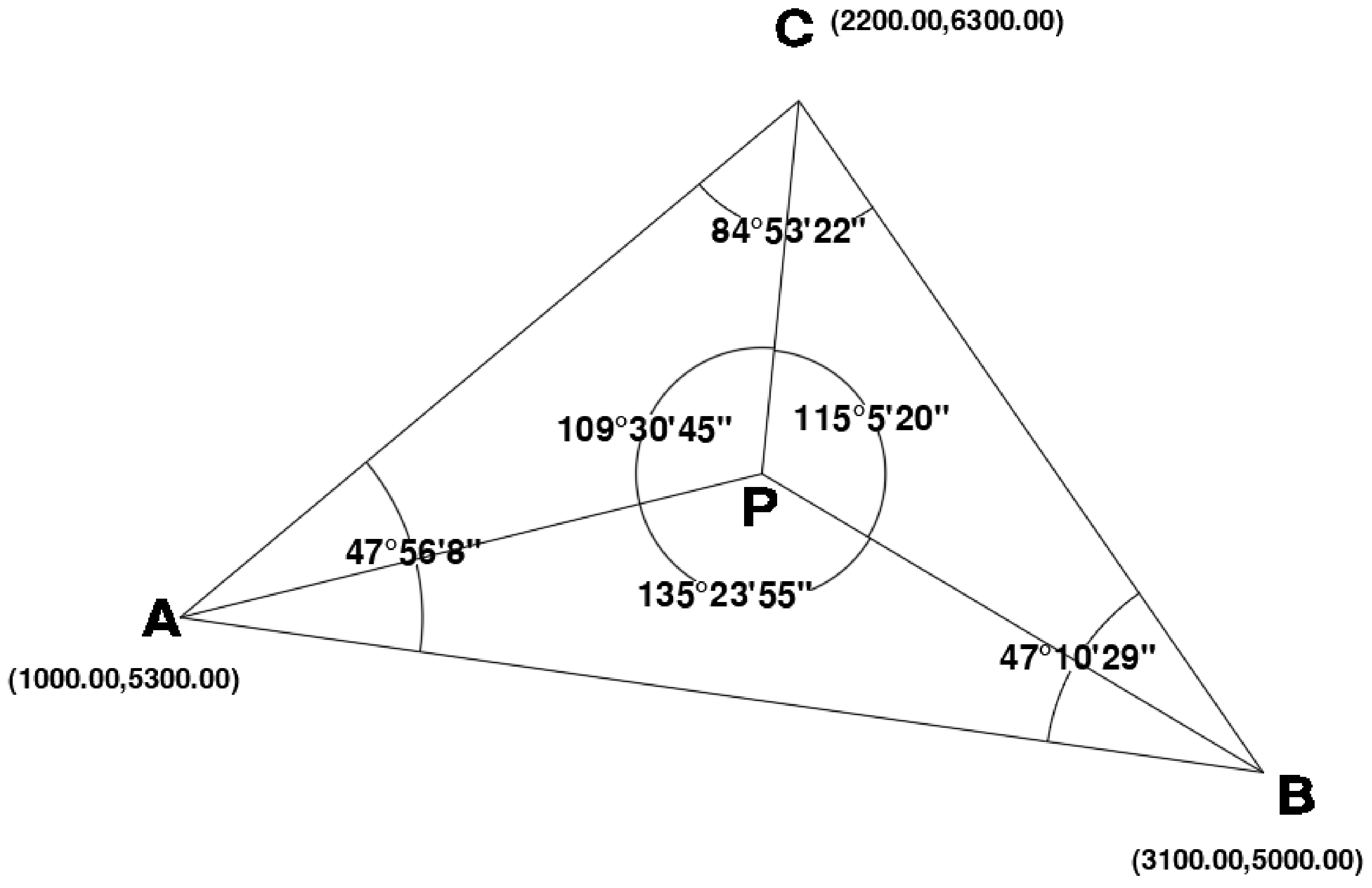
$$Y_P = \frac{K_1 Y_A + K_2 Y_B + K_3 Y_C}{K_1 + K_2 + K_3} \qquad X_P = \frac{K_1 X_A + K_2 X_B + K_3 X_C}{K_1 + K_2 + K_3}$$

Note that $\cot\theta = \dfrac{1}{\tan\theta}$

Where $K_1 = \dfrac{1}{(\cot a - \cot\beta)} \qquad K_2 = \dfrac{1}{(\cot b - \cot\gamma)} \qquad K_3 = \dfrac{1}{(\cot c - \cot\alpha)}$

Tienstra Method

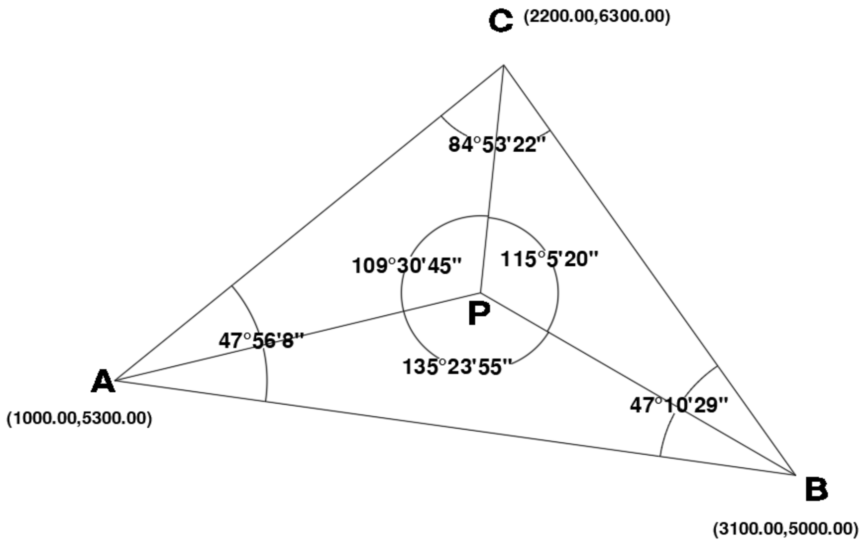
Determine the coordinates of P



Tienstra Method

- $K_1 = \dfrac{1}{(\cot 47^\circ 56' 08'' - \cot 115^\circ 05' 20'')} = 0.729585896$
- $K_2 = \dfrac{1}{(\cot 47^\circ 10' 29'' - \cot 109^\circ 30' 45'')} = 0.780521861$
- $K_3 = \dfrac{1}{(\cot 84^\circ 53' 22'' - \cot 135^\circ 23' 55'')} = 0.90625254$

- $Y_P = \dfrac{(0.729585896)(1000.00)+(0.780521861)(3100.00)+(0.90625254\ )(2200.00)}{(0.729585896)+(0.78052186)+(0.90625254)} = 2128.39m$
- $X_P = \dfrac{(0.729585896)(5300.00)+(0.780521861)(5000.00)+(0.90625254\ )(6300.00)}{(0.729585896)+(0.78052186)+(0.90625254)} = 5578.81m$



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