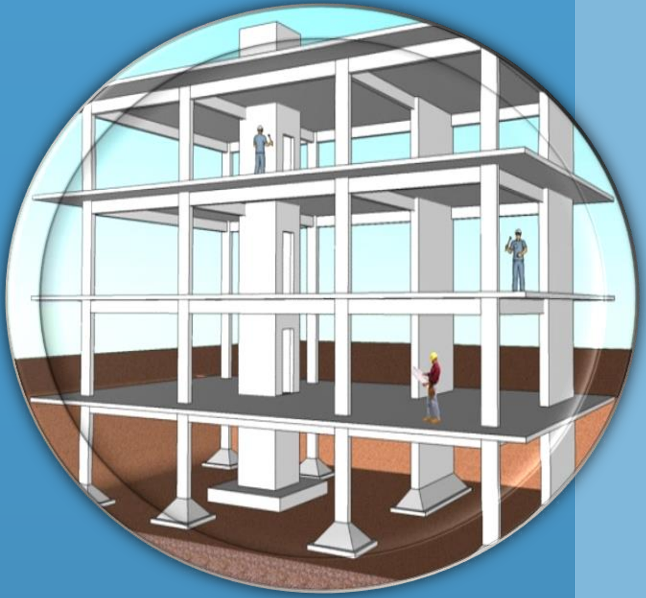
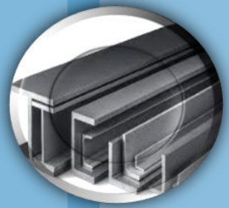
UNIVERSITY OF ZAMBIA
School of Engineering
Department of Civil & Environmental Eng.



CEE 3222: THEORY OF STRUCTURES

INTRODUCTION



Contents

Course Content

- Introduction
- Analysis of Statically Determinate Structures
  - Simple Beams and Frames, Cantilever Beams, Continuous Beams with Hinges, Three-Hinged Archs and Frames, Roof Trusses,
  - Deflections,
- Analysis of Statically Indeterminate Structures
  - Force Method
  - Moment Distribution (Cross) Method

Purpose

- This chapter provides a discussion of some of the preliminary aspects of structural analysis. The phases of activity necessary to produce a structure are presented first, followed by an introduction to the basic types of structures, their components, and supports. Finally, a brief explanation is given of the various types of loads that must be considered for an appropriate analysis and design.

Philosophy of Structures

A structure refers to a system of connected parts used to support a load. Important examples related to civil engineering include

- buildings,
- bridges and
- towers;

and in other branches of engineering,

- ship and aircraft frames,
- tanks, pressure vessels,
- mechanical systems, and
- electrical supporting structures

Such structures are composed of one or more solid elements arranged so that the whole structures as well as their components are capable of holding themselves without appreciable geometric change during loading and unloading.

Philosophy of Structures

- A structure refers to a system of connected parts used to support a load.
- When designing a structure to serve a specified function for public use, the engineer must account for its safety, esthetics, and serviceability, while taking into consideration economic and environmental constraints.
- Often this requires several independent studies of different solution before final judgment can be made as to which structural form is most appropriate.
- This design process is both creative and technical and requires a fundamental knowledge of material properties and the laws of mechanics which govern material response.

Philosophy of Structures

- Once a preliminary design of a structure is proposed, the structure must then be analyzed to ensure that it has its required stiffness and strength.
- To analyze a structure properly, certain idealizations must be made as to how the members are supported and connected together.
- The loadings are determined from codes and local specifications, and the forces in the members and their displacements are found using the theory of structural analysis.
- The results of this analysis then can be used to redesign the structure, accounting for a more accurate determination of the weight of the members and their size.

Philosophy of Structures

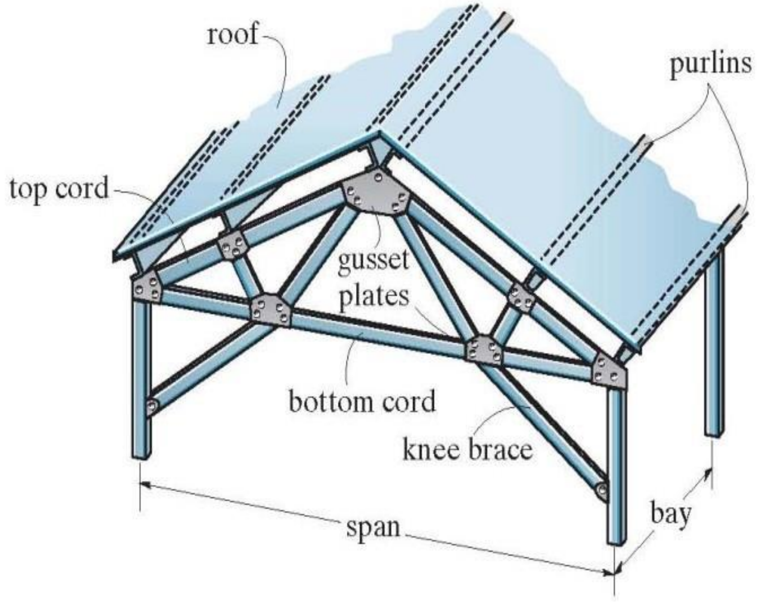
- Structural design, therefore, follows a series of successive approximations in which every cycle requires a structural analysis.
- The structural analysis is applied to civil engineering structures; however, the method of analysis described can also be used for structures related to other fields of engineering

Philosophy of Structures

- The design of a structure involves many considerations, among which are four major objectives that must be satisfied:
  - **The structure must meet the performance requirement (utility).**
  - **The structure must carry loads safely (safety).**
  - **The structure should be economical in material, construction, and cost (economy).**
  - **The structure should have a good appearance (aesthetics).**

Philosophy of Structures

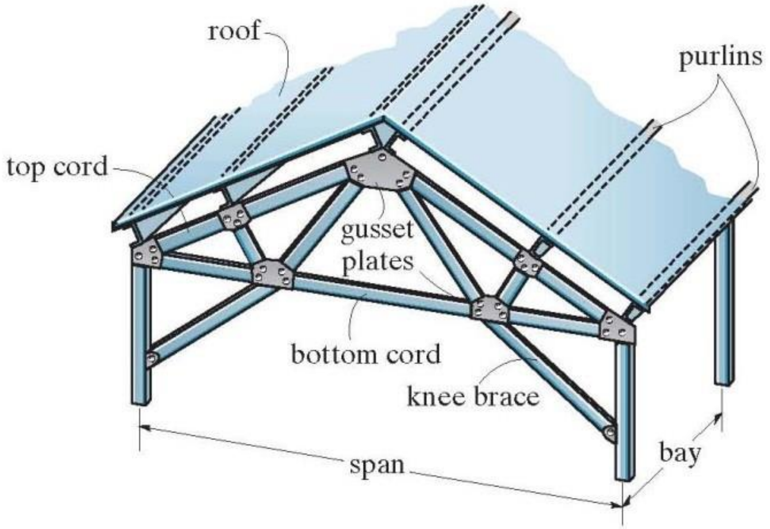
- Consider, for example, the roof truss resting on columns shown below.



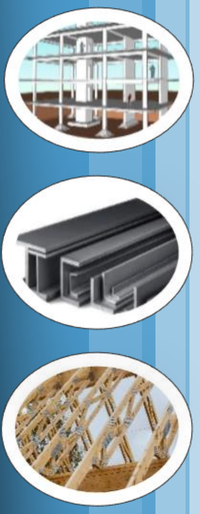
- The purposes of the roof truss and of the columns are, on the one hand, to hold in equilibrium their own weights, the load of roof covering and the wind and snow.
- Also to provide rooms for housing a family, for a manufacturing plant, or for other uses.
- During its development the design is generally optimized to achieve minimum expenditure for materials and construction uses.

Philosophy of Structures

- The complete design of a structure is outlined in the following stages:
  - **Developing a general layout**
  - **Investigating the loads**
  - **Stress analysis**
  - **Selection of elements**
  - **Drawing and detailing**



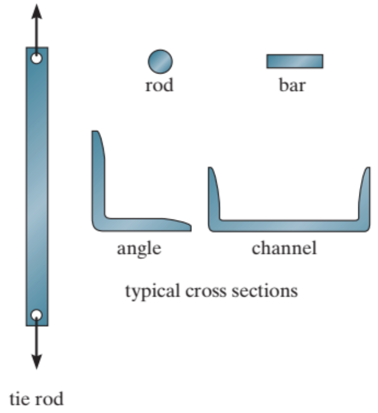
- These five stages are interrelated and may be subdivided and modified
- In many cases they must be carried out more or less simultaneously



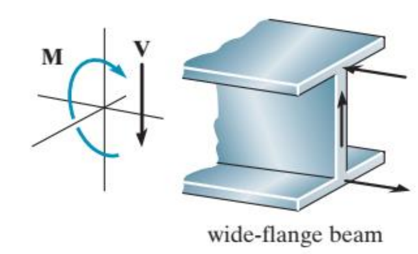
Structural Elements

Tie Rods

Structural members subjected to a tensile force are often referred to as tie rods or bracing struts. Due to the nature of this load, these members are rather slender, and are often chosen from rods, bars, angles, etc.



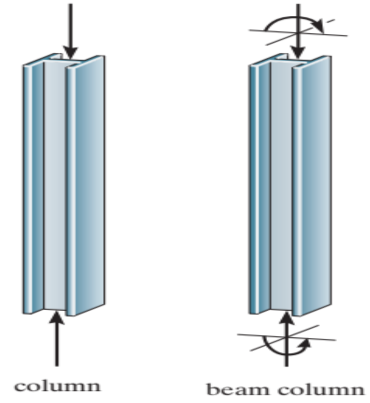
Beams



Beams are usually straight horizontal members used primarily to carry vertical loads. Quite often they are classified according to the way they are supported.

Columns

Members that are generally vertical and resist axial compressive loads are referred to as columns. Occasionally, columns are subjected to both an axial load and a bending moment and are called beam column.



It is important for a structural engineer to recognize the various types of elements composing a structure and to be able to classify structures as to their form and function.

Types of Structures

The combination of structural elements and the materials from which they are composed is referred to as a structural system.

Trusses consist of slender elements, usually arranged in triangular fashion. There are two types 1. Planar Trusses and 2. Space Trusses.

Cables are usually flexible and carry their loads in tension an are used to support bridges and building roofs. The arch achieves its strength in compression, since it has a reverse curvature to that of the cable. The arch must be rigid, however, in order to maintain its shape, and this results in secondary loadings involving shear and moment.

Trusses

Cables and Arches

Frames

Surface Structure

Frames are often used in buildings and are composed of beams and columns that are either pin or fixed connected and extend in two or three dimensions. .

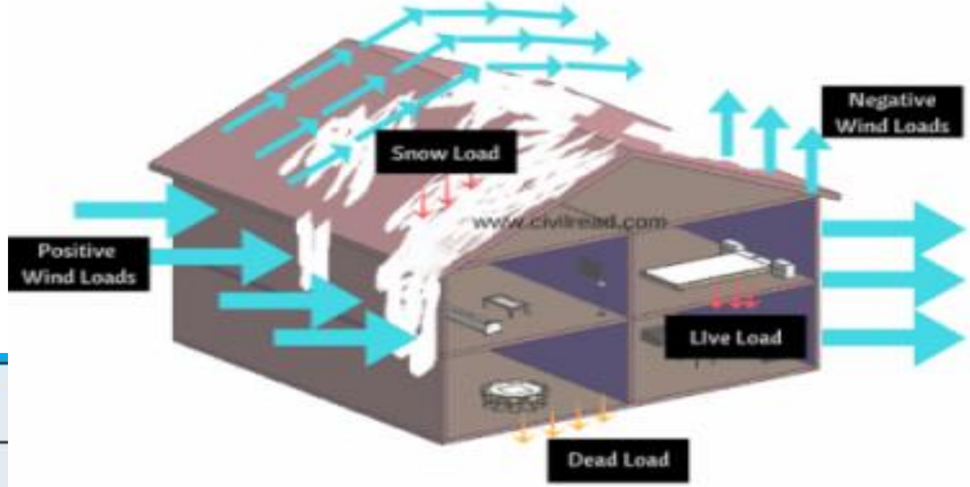
A surface structure is made from a material having a very small thickness compared to its other dimensions. Their material acts as a membrane that is subjected to pure tension.

Common types of structural systems

Types of Loads

- Design loading for a structure is often specified in codes
  - **General building codes**
  - **Design codes**

General Building Codes
Minimum Design Loads for Buildings and Other Structures, ASCE/SEI 7-10, American Society of Civil Engineers
International Building Code
Design Codes
Building Code Requirements for Reinforced Concrete, Am. Conc. Inst. (ACI)
Manual of Steel Construction, American Institute of Steel Construction (AISC)
Standard Specifications for Highway Bridges, American Association of State Highway and Transportation Officials (AASHTO)
National Design Specification for Wood Construction, American Forest and Paper Association (AFPA)
Manual for Railway Engineering, American Railway Engineering Association (AREA)



Loads
↓
Structural forms Elements
↓
carrying primary loads
↓
Various supporting members
↓
Foundation

Types of Loads – Dead loads

- Dead loads are gravity loads of constant magnitudes and fixed positions that act permanently on the structure.
- Dead loads consist of the **weights of the structural system itself** and **of all other material and equipment permanently attached to the structural system**.
- The actual weight of the structure is computed by using the member sizes and the unit weights of materials.

Material	Unit Weight kN/m^3
Aluminum	25.9
Brick	18.8
Concrete, reinforced	23.6
Structural steel	77.0
Wood	6.3

- The weights of permanent service equipment, such as heating and air-conditioning systems, are usually obtained from the manufacturer.

Types of Loads – Dead loads

TABLE 1.2 Minimum Densities for Design Loads from Materials*

	lb/ft³	kN/m³
Aluminum	170	26.7
Concrete, plain cinder	108	17.0
Concrete, plain stone	144	22.6
Concrete, reinforced cinder	111	17.4
Concrete, reinforced stone	150	23.6
Clay, dry	63	9.9
Clay, damp	110	17.3
Sand and gravel, dry, loose	100	15.7
Sand and gravel, wet	120	18.9
Masonry, lightweight solid concrete	105	16.5
Masonry, normal weight	135	21.2
Plywood	36	5.7
Steel, cold-drawn	492	77.3
Wood, Douglas Fir	34	5.3
Wood, Southern Pine	37	5.8
Wood, spruce	29	4.5

*Minimum Densities for Design Loads from Materials, Reproduced with permission from American Society of Civil Engineers *Minimum Design Loads for Buildings and Other Structures*, ASCE/SEI 7-10. Copies of this standard may be purchaed from ASCE at www.pubs.asce.org, American Society of Civil Engineers.

TABLE 1.3 Minimum Design Dead Loads*

Walls	psf	kN/m²
4-in. (102 mm) clay brick	39	1.87
8-in. (203 mm) clay brick	79	3.78
12-in. (305 mm) clay brick	115	5.51
Frame Partitions and Walls		
Exterior stud walls with brick veneer	48	2.30
Windows, glass, frame and sash	8	0.38
Wood studs 2 × 4 in. (51 × 102 mm), unplastered	4	0.19
Wood studs 2 × 4 in. (51 × 102 mm), plastered one side	12	0.57
Wood studs 2 × 4 in. (51 × 102 mm), plastered two sides	20	0.96
Floor Fill		
Cinder concrete, per inch (mm)	9	0.017
Lightweight concrete, plain, per inch (mm)	8	0.015
Stone concrete, per inch (mm)	12	0.023
Ceilings		
Acoustical fiberboard	1	0.05
Plaster on tile or concrete	5	0.24
Suspended metal lath and gypsum plaster	10	0.48
Asphalt shingles	2	0.10
Fiberboard, $\frac{1}{2}$-in. (13 mm)	0.75	0.04

*Minimum Design Dead Loads, Reproduced with permission from American Society of Civil Engineers *Minimum Design Loads for Buildings and Other Structures*, ASCE/SEI 7-10, American Society of Civil Engineers.

Types of Loads – Live loads

Occupancy or Use	Live Load kN/m²
Hospital patient rooms, residential dwellings, apartments, hotel guest rooms, school classrooms	1.92
Library reading rooms, hospital operating rooms and laboratories	2.87
Dance halls and ballrooms, restaurants, gymnasiums	4.79
Light manufacturing, light storage warehouses, wholesale stores	6.00
Heavy manufacturing, heavy storage warehouses	11.97

- Varies in magnitude & location
- Building loads
  - Depends on the purpose for which the building is designed
  - These loadings are generally tabulated in local, state or national code
  - Uniform, concentrated loads

- Live loads are loads of varying magnitudes and/or positions caused by the use of the structure.
- The magnitudes of design live loads are usually specified in building codes.
- The position of a live load may change, so each member of the structure must be designed for the position of the load that causes the maximum stress in that member

Types of Loads – Live loads

TABLE 1.4 Minimum Live Loads*

	Live Load			Live Load	
Occupancy or Use	psf	kN/m²	Occupancy or Use	psf	kN/m²
Assembly areas and theaters			Residential		
Fixed seats	60	2.87	Dwellings (one- and two-family)	40	1.92
Movable seats	100	4.79	Hotels and multifamily houses		
Garages (passenger cars only)	40	1.92	Private rooms and corridors	40	1.92
Office buildings			Public rooms and corridors	100	4.79
Lobbies	100	4.79	Schools		
Offices	50	2.40	Classrooms	40	1.92
Storage warehouse			First-floor corridors	100	4.79
Light	125	6.00	Corridors above first floor	80	3.83
Heavy	250	11.97			

*Minimum Live Loads, Reproduced with permission from American Society of Civil Engineers *Minimum Design Loads for Buildings and Other Structures*, ASCE/SEI 7-10, American Society of Civil Engineers.

$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL}A_T}}\right) \quad \text{(FPS units)}$$

$$L = L_o\left(0.25 + \frac{4.57}{\sqrt{K_{LL}A_T}}\right) \quad \text{(SI units)}$$

where

L = reduced design live load per square foot or square meter of area supported by the member.

L_o = unreduced design live load per square foot or square meter of area supported by the member (see Table 1.4).

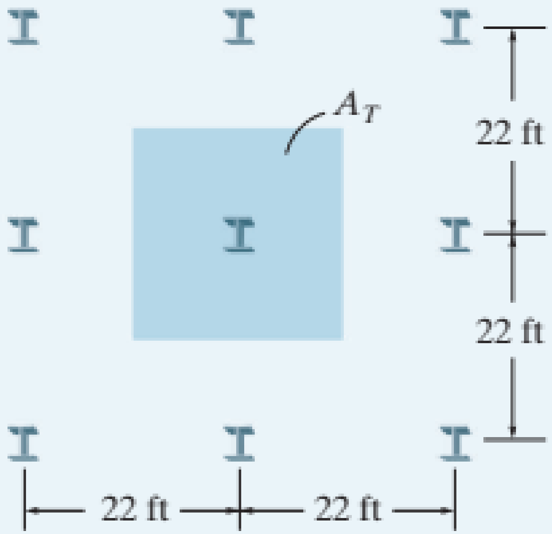
K_{LL} = live load element factor. For interior columns $K_{LL} = 4$.

A_T = tributary area in square feet or square meters.*

The reduced live load defined by Eq. 1–1 is limited to not less than 50% of L_o for members supporting one floor, or not less than 40% of L_o for members supporting more than one floor. No reduction is allowed for loads exceeding 100 lb/ft² (4.79 kN/m²), or for structures used for public assembly, garages, or roofs. Example 1.2 illustrates Eq. 1–1's application.

Example 1.1

A two-story office building shown in the photo has interior columns that are spaced 22 ft apart in two perpendicular directions. If the (flat) roof loading is 20 lb/ft^2, determine the reduced live load supported by a typical interior column located at ground level.



Solution 1.1

SOLUTION

As shown in Fig. 1–9, each interior column has a tributary area or effective loaded area of $A_T = (22 \text{ ft})(22 \text{ ft}) = 484 \text{ ft}^2$. A ground-floor column therefore supports a roof live load of

$$F_R = (20 \text{ lb/ft}^2)(484 \text{ ft}^2) = 9680 \text{ lb} = 9.68 \text{ k}$$

This load cannot be reduced, since it is not a floor load. For the second floor, the live load is taken from Table 1.4: $L_o = 50 \text{ lb/ft}^2$. Since $K_{LL} = 4$, then $4A_T = 4(484 \text{ ft}^2) = 1936 \text{ ft}^2$ and $1936 \text{ ft}^2 > 400 \text{ ft}^2$, the live load can be reduced using Eq. 1–1. Thus,

$$L = 50\left(0.25 + \frac{15}{\sqrt{1936}}\right) = 29.55 \text{ lb/ft}^2$$

The load reduction here is $(29.55/50)100\% = 59.1\% > 50\%$. O.K. Therefore,

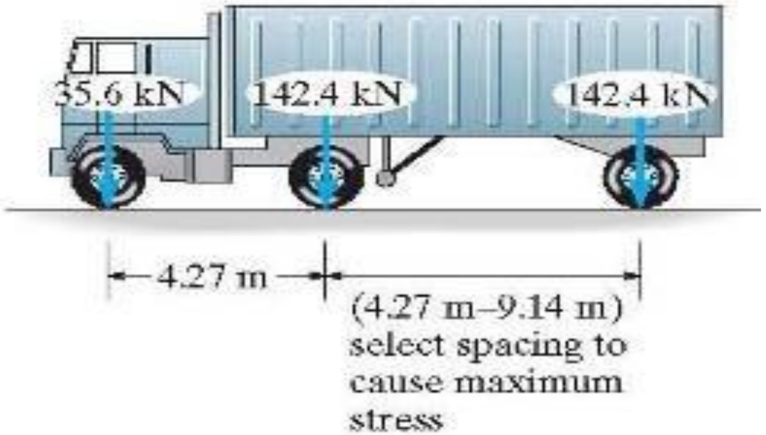
$$F_F = (29.55 \text{ lb/ft}^2)(484 \text{ ft}^2) = 14\,300 \text{ lb} = 14.3 \text{ k}$$

The total live load supported by the ground-floor column is thus

$$F = F_R + F_F = 9.68 \text{ k} + 14.3 \text{ k} = 24.0 \text{ k} \qquad \textit{Ans.}$$

Types of Loads – Live loads

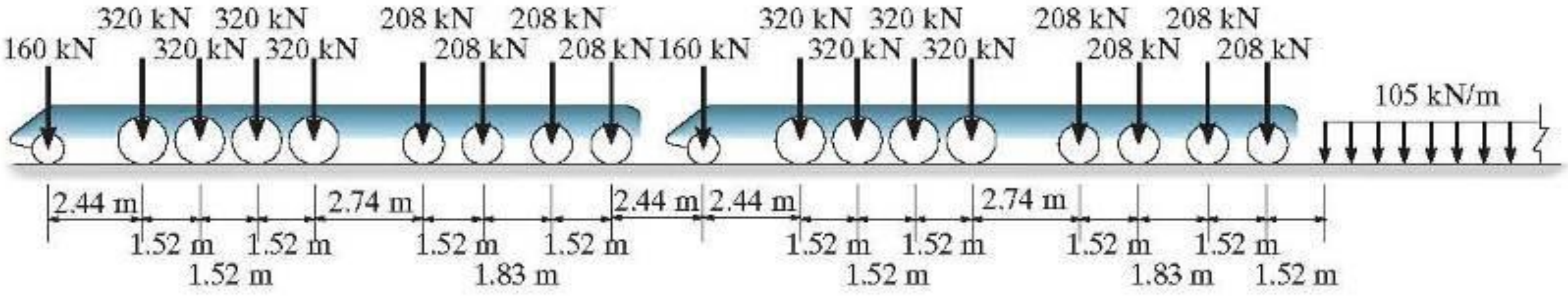
- Highway Bridge loads
  - Primary live loads are those due to traffic
  - Specifications for truck loadings are reported in AASHTO (American Association of State Highway and Transportation Officials)
  - For 2-axle truck, these loads are designated with H followed by the weight of truck in tons and another no. gives the year of the specifications that the load was reported



HS 20–44 loading

Types of Loads – Live loads

- Railway Bridge loads
  - Loadings are specified in AREA
  - A modern train having a 320kN (72k) loading on the driving axle of the engine is designated as an E-72 loading



E-72 loading

Types of Loads – Impact loads

Impact Loads

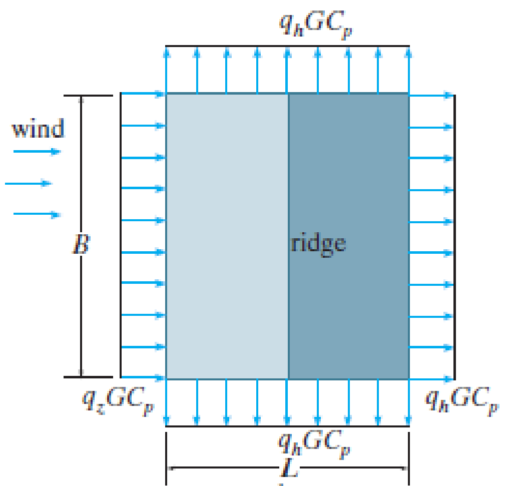
- When live loads are applied rapidly to a structure, they cause larger stresses than those that would be produced if the same loads would have been applied gradually. The dynamic effect of the load that causes this increase in stress in the structure is referred to as impact.
- The % increase of the live loads due to impact is called the impact factor, I

$$I = \frac{15.24}{L + 38.1} < 0.3$$

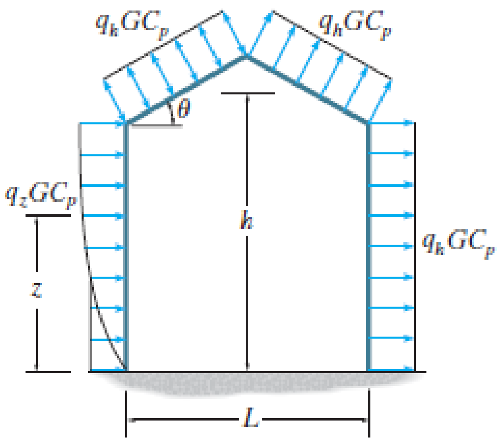
L = length of the span in m that is subjected to the live load

Types of Loads – Wind loads

- Wind loads are produced by the flow of wind around the structure. The magnitudes of wind loads that may act on a structure depend the following:
  - The geographical location of the structure
  - Obstructions in its surrounding terrain (such as nearby buildings)
  - The geometry of the structure
  - The vibrational characteristics of the structure itself.



plan



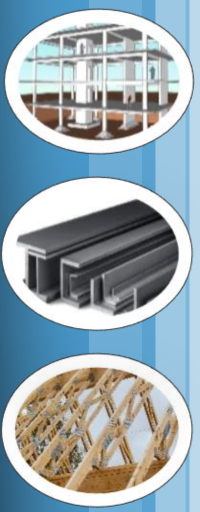
elevation

- Most of the methods to estimate wind loads are based on the basic relationship between the wind speed V and the dynamic pressure q induced on a flat surface normal to the wind flow.

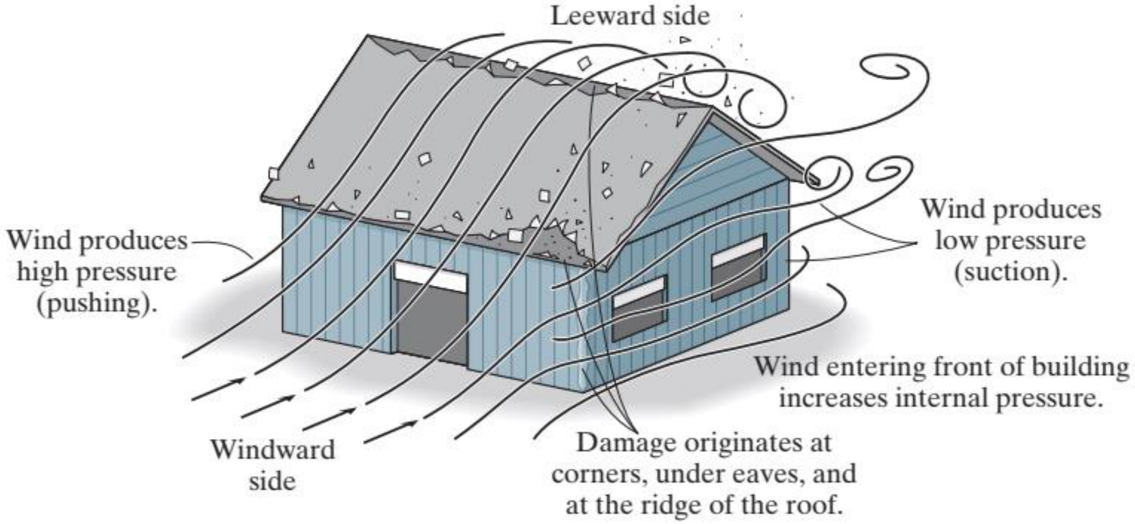
$$q = \frac{1}{2}\rho V^2$$

- ρ is the unit weight of the air.
- Using the unit weight of air of 12.02 N/m^3 for the standard atmosphere and expressing the wind speed V in meters per second (m/s)

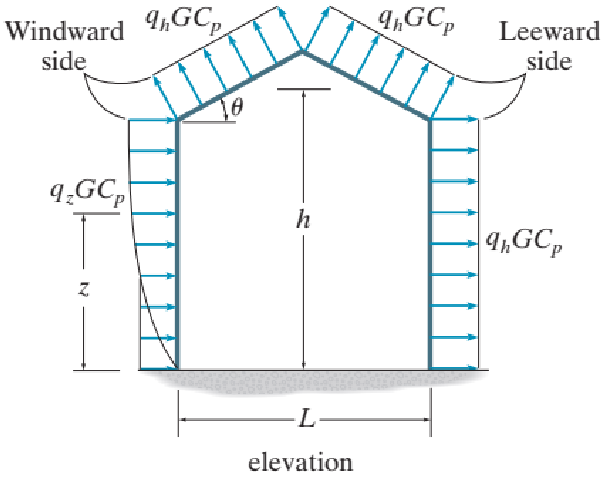
$$q = 0.613V^2$$



Types of Loads – Wind loads



- The effects of lateral loadings developed by wind, can cause racking, or leaning of a building frame. To resist this effect, engineers often use cross bracing, knee or diagonal bracing, or shear walls. Examples that show the use of these members are indicated in the photos.



Wind direction	Windward angle θ		Leeward angle
	h/L	10°	θ = 10°
Normal to ridge	≤0.25 0.5 >1.0	−0.7 −0.9 −1.3	−0.3 −0.5 −0.7

Maximum negative roof pressure coefficients, C_p, for use with q_h

Types of Loads – Snow loads

- Design loadings depend on building’s general shape & roof geometry, wind exposure, location and its importance
- Snow loads are determined from a zone map reporting 50-year recurrence interval

Types of Loads – Snow loads

- If a roof is flat, defined as having a slope of less than 5%, then the pressure loading on the roof can be obtained by modifying the ground snow loading, p_g, by the following empirical formula

$$\boxed{p_f = 0.7C_eC_tI_sp_g} \quad (1\text{–}5)$$

Here

C_e = an exposure factor which depends upon the terrain. For example, for a fully exposed roof in an unobstructed area, $C_e = 0.8$, whereas if the roof is sheltered and located in the center of a large city, then $C_e = 1.2$.

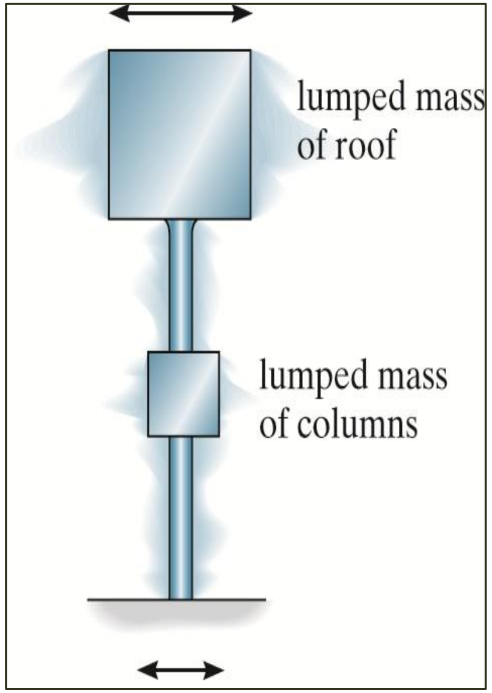
C_t = a thermal factor which refers to the average temperature within the building. For unheated structures kept below freezing $C_t = 1.2$, whereas if the roof is supporting a normally heated structure, then $C_t = 1.0$.

I_s = the importance factor as it relates to occupancy. For example, $I_s = 0.80$ for agriculture and storage facilities, and $I_s = 1.20$ for schools and hospitals.

If $p_g \leq 20\ \text{lb/ft}^2\ (0.96\ \text{kN/m}^2)$, then use the *largest value* for p_f, either computed from the above equation or from $p_f = I_sp_g$. If $p_g > 20\ \text{lb/ft}^2\ (0.96\ \text{kN/m}^2)$, then use $p_f = I_s(20\ \text{lb/ft}^2)$.

Types of Loads – Earthquake loads

- Earthquake produce loadings through its interaction with the ground & its response characteristics
- Their magnitude depends on amount & type of ground acceleration, mass & stiffness of structure
  - Top block is the lumped mass of the roof
  - Middle block is the lumped stiffness of all the building's columns
- During earthquake, the ground vibrates both horizontally & vertically.



Types of Loads – Earthquake loads

- The horizontal accelerations create shear forces in the column that put the block in sequential motion with the ground.
- If the column is stiff & the block has a small mass, the period of vibration of the block will be short, the block will accelerate with the same motion as the ground & undergo slight relative displacements
- If the column is very flexible & the block has a large mass, induced motion will cause small accelerations of the block & large relative displacement

Types of Loads – Other loads

- Hydrostatic & Soil Pressure
  - The pressure developed by these loadings when the structures are used to retain water or soil or granular materials
  - E.g. tanks, dams, ships, bulkheads & retaining walls

- Other natural loads
  - Effect of blast
  - Temperature changes
  - Differential settlement of foundation

Structural Design

- Material uncertainties occur due to
  - variability in material properties
  - residual stress in materials
  - intended measurements being different from fabricated sizes
  - material corrosion or decay

- Many types of loads can occur simultaneously on a structure

Structural Design

- Allowable-stress design (ASD) methods include both the material and load uncertainties into a single factor of safety. The many types of loads discussed previously can occur simultaneously on a structure, but it is very unlikely that the maximum of all these loads will occur at the same time. For example, both maximum wind and earthquake loads normally do not act simultaneously on a structure.
- For allowable-stress design the computed elastic stress in the material must not exceed the allowable stress for each of various load combinations. Load combinations specified by the ASCE 7-02 Standard

  - **Dead load**
  - **0.6 (dead load) + wind load**
  - **0.6 (dead load) + 0.7 (earthquake load)**

Structural Design

- Ultimate strength design is based on designing the ultimate strength of critical sections
- This method uses load factors to the loads or combination of loads

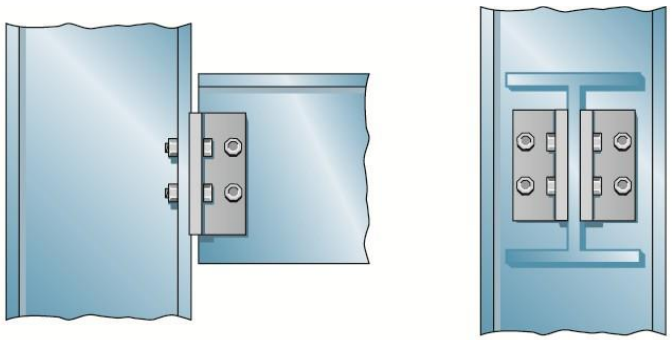
- **1.4 (Dead load)**
- **1.2 (dead load) + 1.6 (live load) + 0.5 (snow load)**
- **1.2 (dead load) + 1.5 (earthquake load) + 0.5 (live load)**

Analysis of Statically Determinate Structures

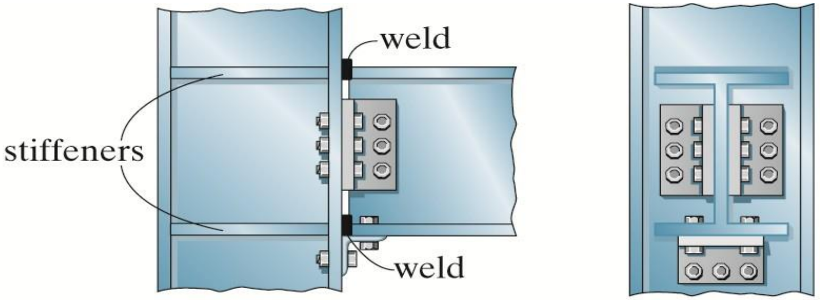
Idealized Structure

- An exact analysis of a structure can never be carried out, since estimates always have to be made of the loadings and the strength of the materials composing the structure.
- It is important to develop the ability to model or idealize a structure so that the structural engineer can perform a practical force analysis of the members
- Support Connections: Structural members are joined together in various ways depending on the intent of the designer. The three types of joints most often specified are
  - **Pin connection (allows some freedom for slight rotation)**
  - **Roller support (allows some freedom for slight rotation)**
  - **Fixed joint (allows no relative rotation)**

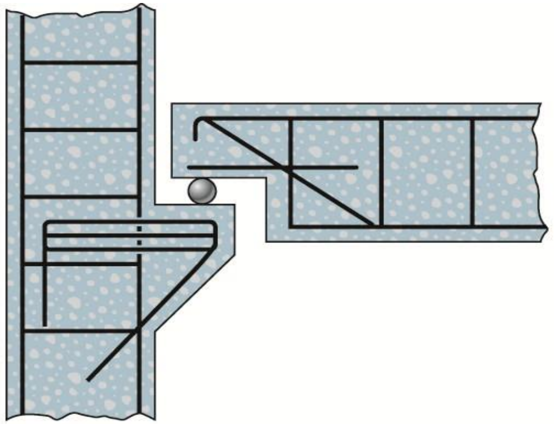
Idealized Structure



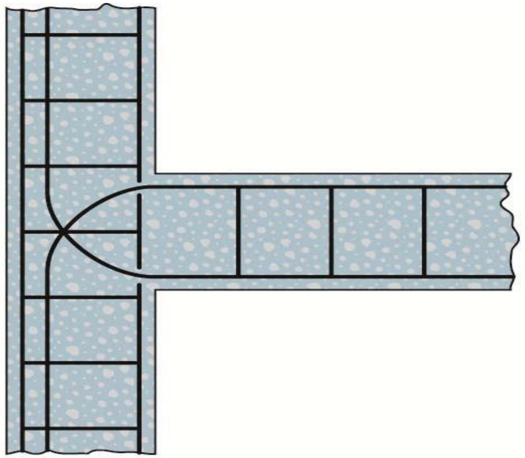
typical “pin-supported” connection (metal)



typical “fixed-supported” connection (metal)



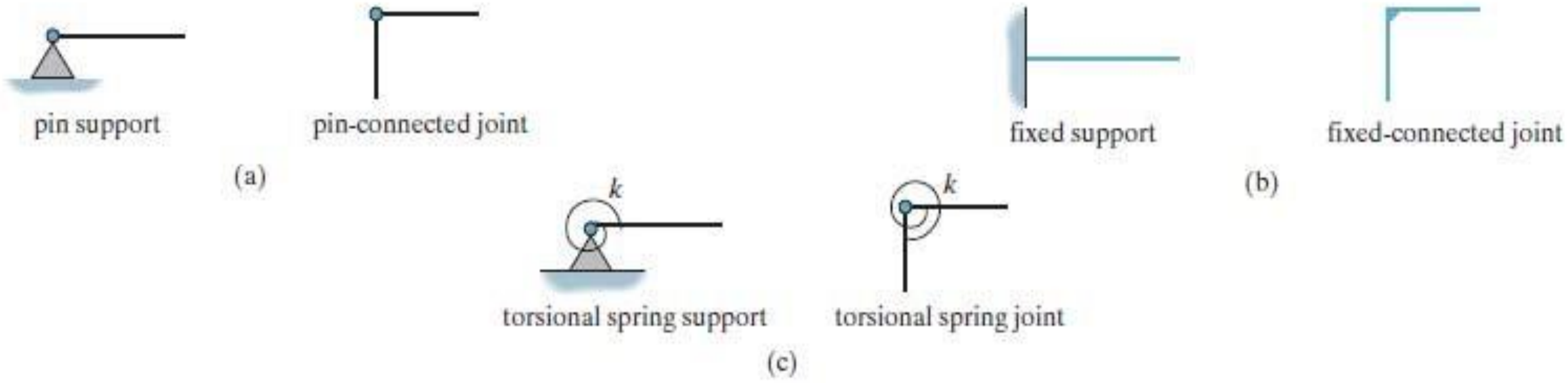
typical “roller-supported” connection (concrete)



typical “fixed-supported” connection (concrete)

Idealized Structure

- Idealized models used in structural analysis that represent various support types.



Idealized Structure

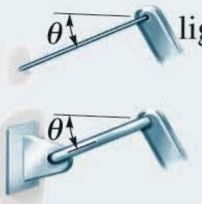
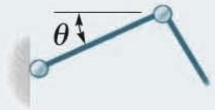
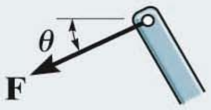
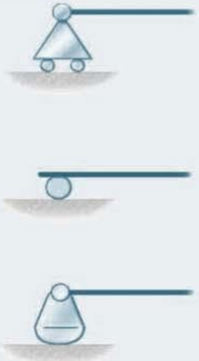
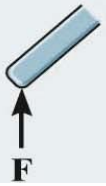
- Support Connections
  - In reality, all connections exhibit some stiffness toward joint rotations owing to friction & material behavior
  - If $k = 0$ the joint is pin and $k \to \infty$, the joint is fixed
  - When selecting the model for each support, the engineer must be aware how the assumptions will affect the actual performance
  - The analysis of the loadings should give results that closely approximate the actual loadings

Idealized Structure

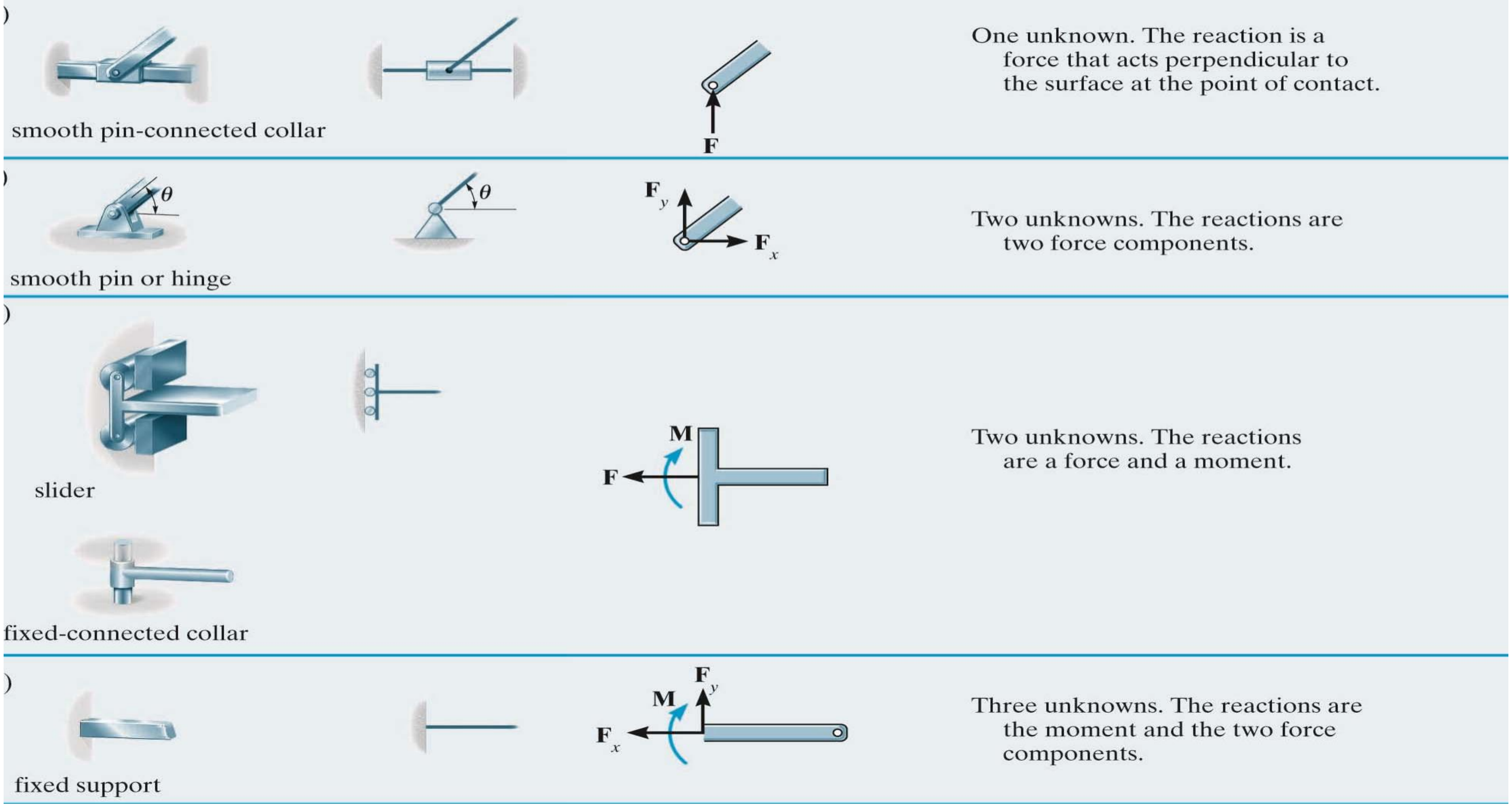
Support Connections

- In reality, all supports actually exert distributed surface loads on their contacting members. The concentrated forces and moments shown in following Table represent the resultants of these load distributions.
- This representation is, of course, an idealization; however, it is used here since the surface area over which the distributed load acts is considerably smaller than the total surface area of the connecting members.

Idealized Structure

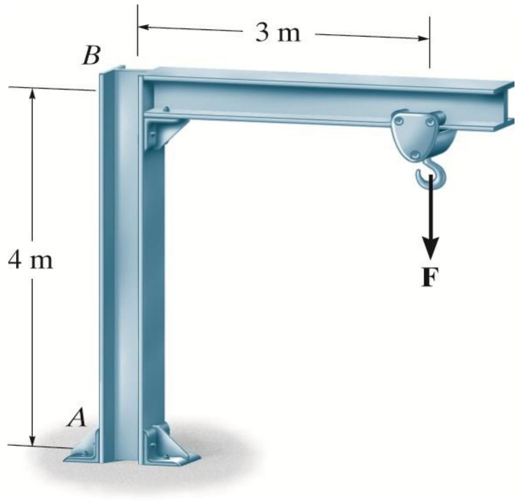
Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(1)  light cable, weightless link			One unknown. The reaction is a force that acts in the direction of the cable or link.
(2)  rollers, rocker			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(3)  smooth contacting surface			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.

Idealized Structure

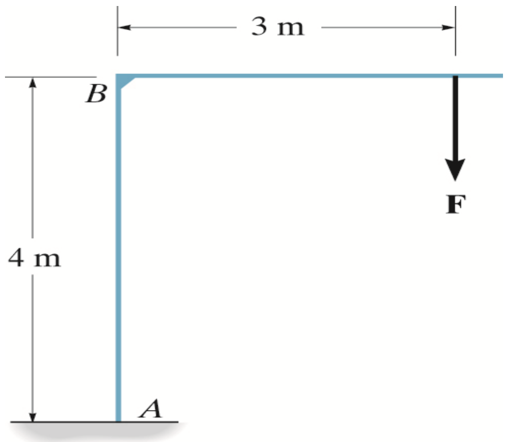


Idealized Structure

- Consider the jib crane & trolley, we neglect the thickness of the 2 main member & will assume that the joint at B is fabricated to be rigid
- The support at A can be modeled as a fixed support



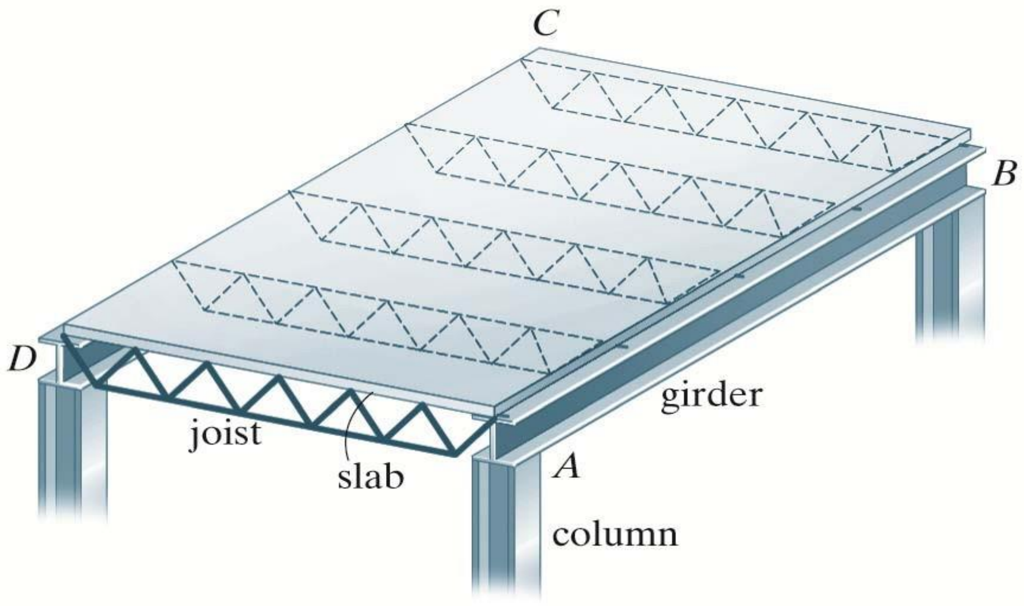
actual structure



idealized structure

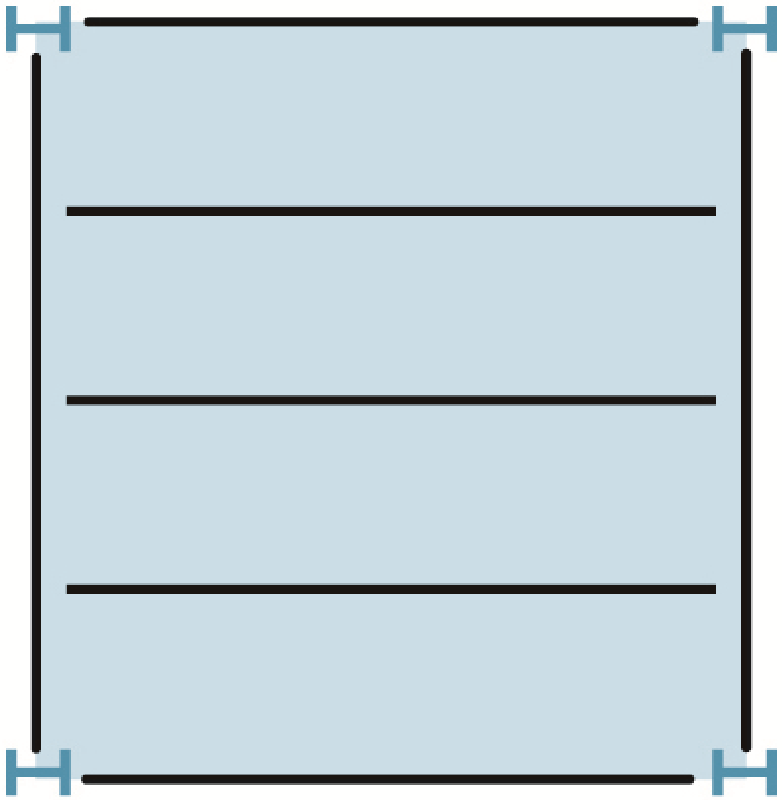
Idealized Structure

- Consider the framing used to support a typical floor slab in a building
- The slab is supported by floor joists located at even intervals
- These are in turn supported by 2 side girders AB & CD



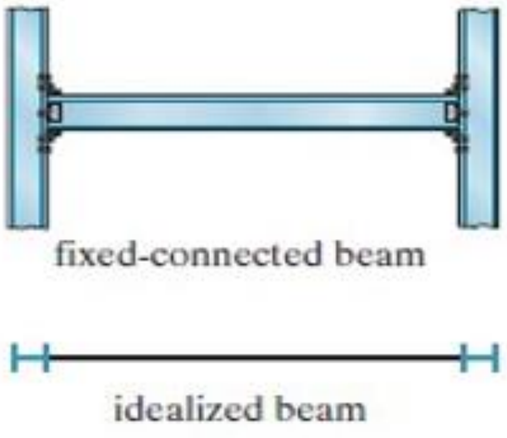
Idealized Structure

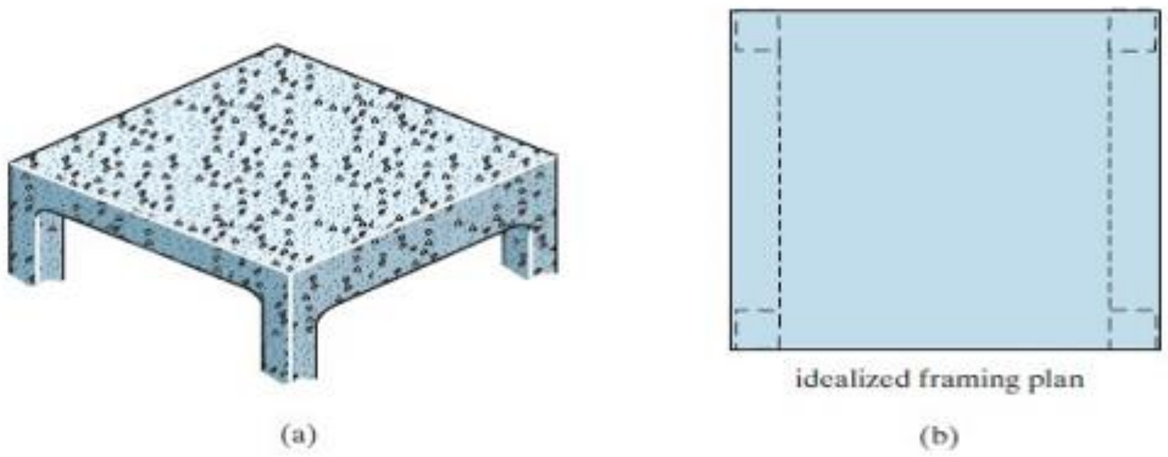
- For analysis, it is reasonable to assume that the joints are pin and/or roller connected to girders & the girders are pin and/or roller connected to columns

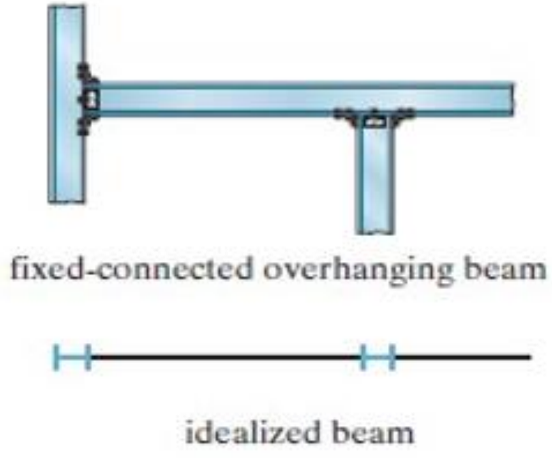


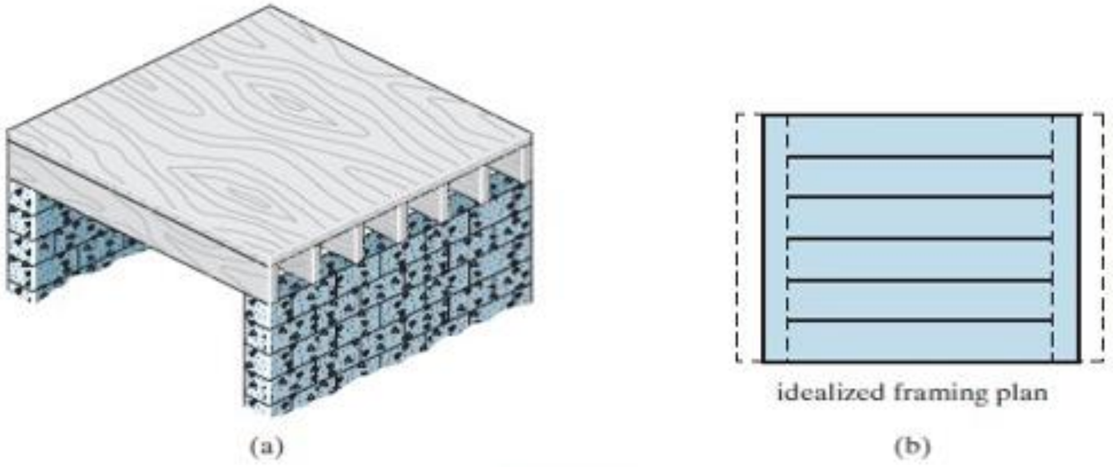
idealized framing plan

Idealized Structure







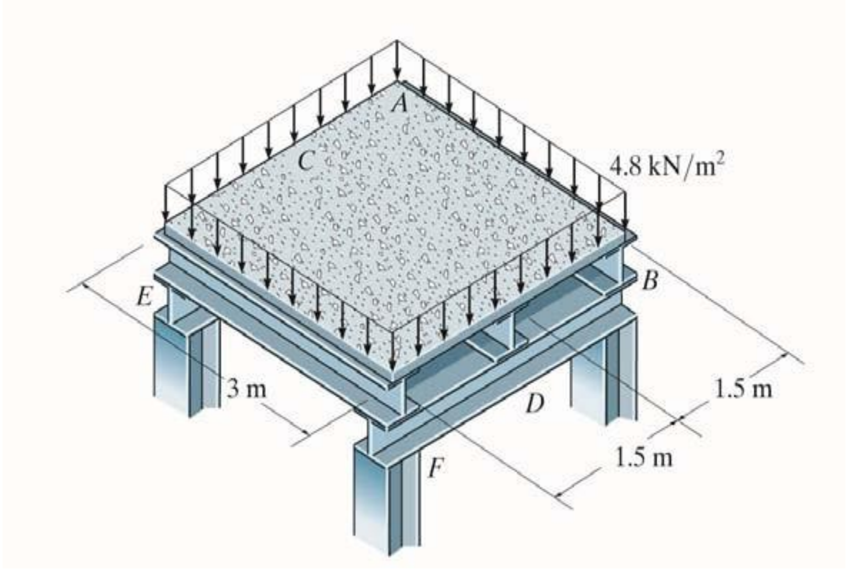


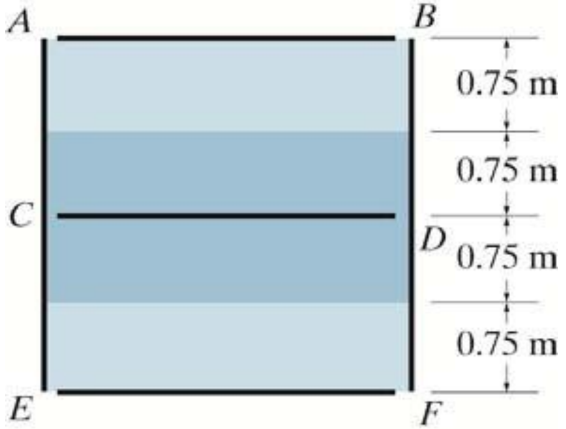
Idealized Structure

- Tributary Loadings
  - There are 2 ways in which the load on surfaces can transmit to various structural elements
    - 1-way system
    - 2-way system

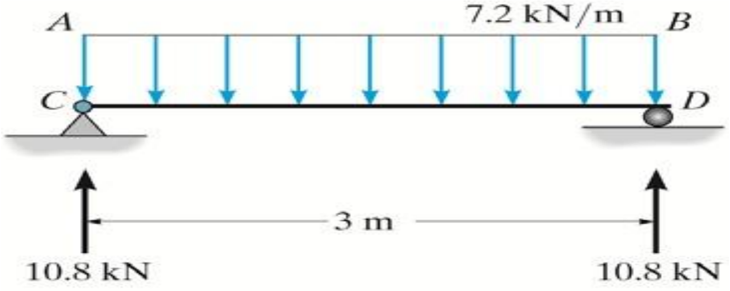
Idealized Structure

- Tributary Loadings
  - **1-way system**

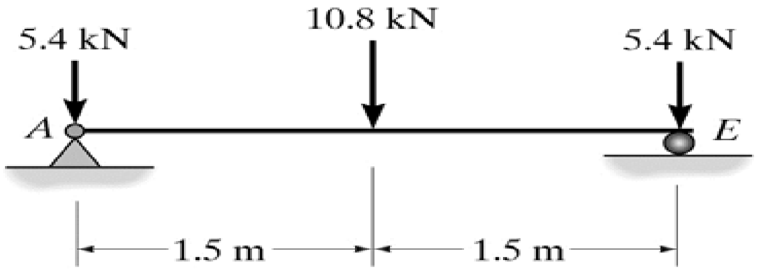




idealized framing plan



idealized beam

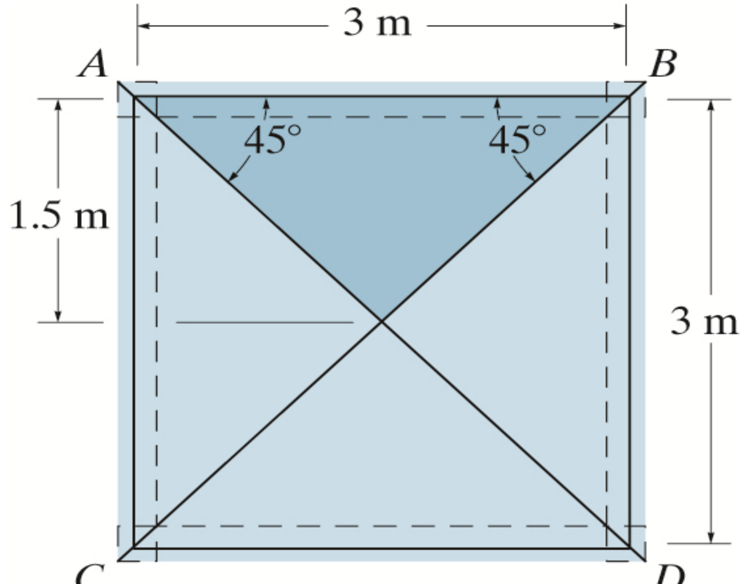


idealized girder

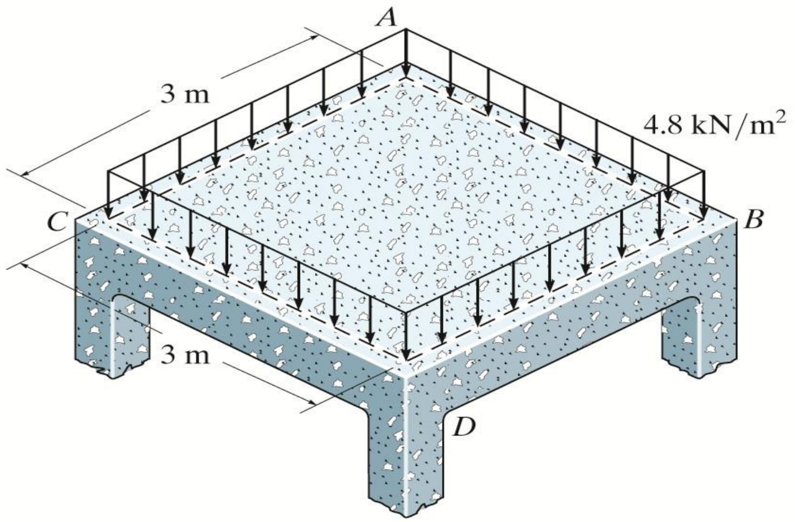
Idealized Structure

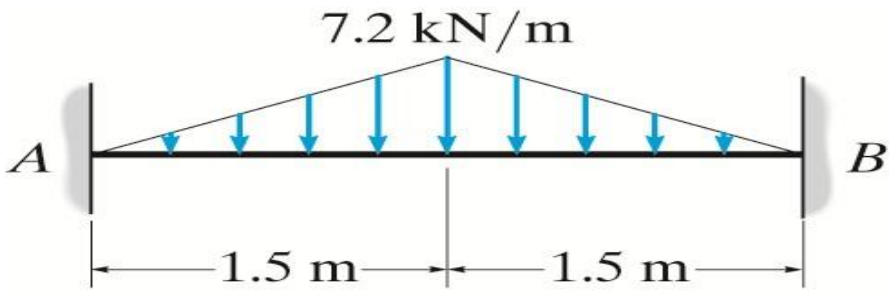
- Tributary Loadings
  - 2-way system

According to the ACI 318 concrete code, if $L_2 > L_1$ and the support ratio $(L_2/L_1) \leq 2$, then the load is assumed to be transferred to the supporting beams and girders in *two directions*.



idealized framing plan





idealized beam

Example 1.1

- The floor of a classroom is supported by the bar joists. Each joist is 4.5m long and they are spaced 0.75m on centers. The floor is made from lightweight concrete that is 100mm thick. Neglect the weight of joists & the corrugated metal deck, determine the load that acts along each joist.

Solution 1.1

Dead load, weight of concrete slab

$= (100)(0.015)$

$= 1.50 kN/m^2$

Live load $= 1.92 kN/m^2$

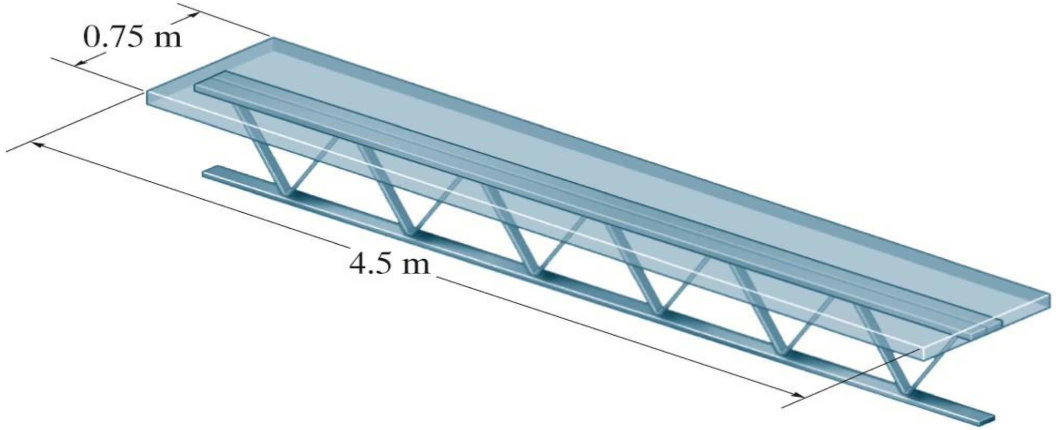
Total load $= 1.50 + 1.92 = 3.42 kN/m^2$

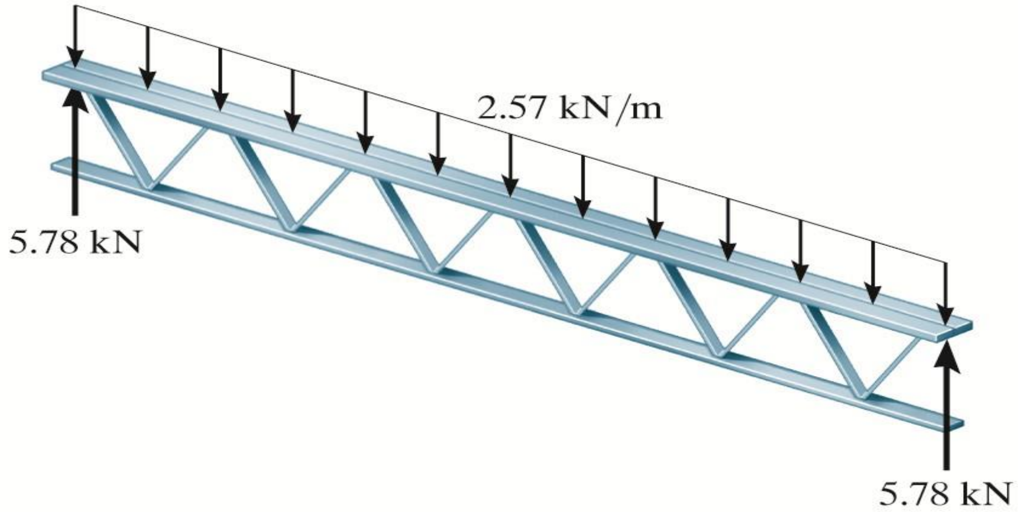
$L_1 = 0.75m$, $L_2 = 4.5m$

$L_1 / L_2 > 2 \Rightarrow$ 1- way slab

Uniform load along its length, w

$= 3.42 kN/m^2 (0.75m) = 2.57 kN/m$





Example 1.3

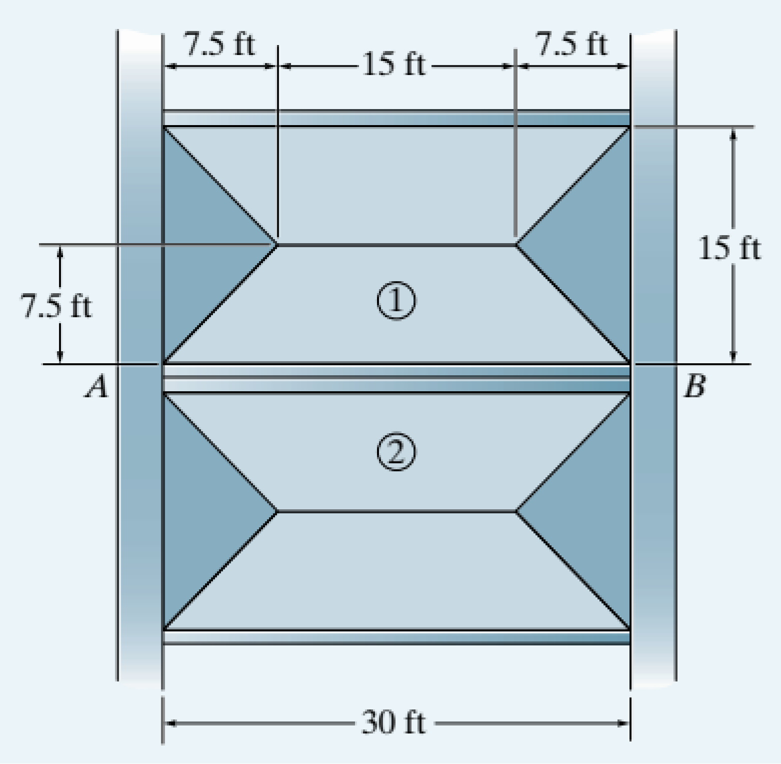
- The concrete girders shown in the photo of the passenger car parking garage span 30 ft and are 15 ft on center. If the floor slab is 5 in. thick and made of reinforced stone concrete, and the specified live load is 50 lb/ft^2 (see slide 18), determine the distributed load the floor system transmits to each interior girder

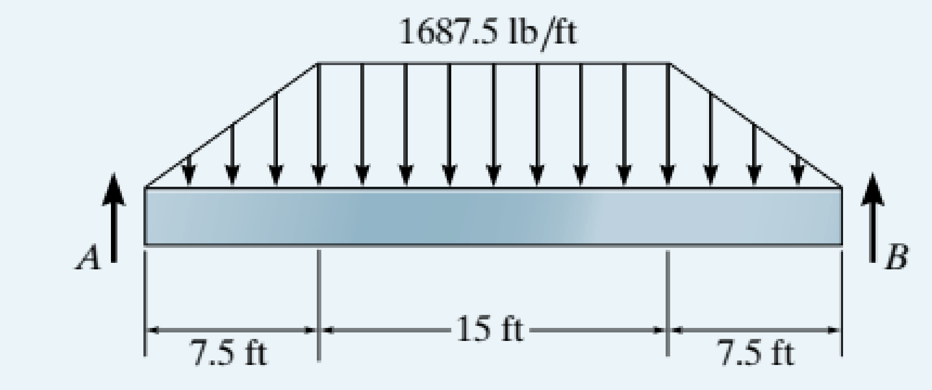
Solution 1.3

Here, $L_2 = 30$ ft and $L_1 = 15$ ft, so that $L_2/L_1 = 2$. We have a two-way slab. From Table 1.2, for reinforced stone concrete, the specific weight of the concrete is 150 lb/ft^3. Thus the design floor loading is

$$p = 150\ \text{lb/ft}^3\left(\frac{5}{12}\ \text{ft}\right) + 50\ \text{lb/ft}^2 = 112.5\ \text{lb/ft}^2$$

A trapezoidal distributed loading is transmitted to each interior girder AB from each of its two sides ① and ②. The maximum intensity of each of these distributed loadings is $(112.5\ \text{lb/ft}^2)(7.5\ \text{ft}) = 843.75$ lb/ft, so that on the girder this intensity becomes $2(843.75\ \text{lb/ft}) = 1687.5$ lb/ft, Fig. 2–17*b*. *Note:* For design, consideration should also be given to the weight of the girder.





Principle of Superposition

- The principle of superposition forms the basis for much of the theory of structural analysis. It may be stated as follows
- The total displacement or internal loadings (stress) at a point in a structure subjected to several external loadings can be determined by adding together the displacements or internal loadings (stress) caused by each of the external loads acting separately.
- For this statement to be valid it is necessary that a linear relationship exist among the loads, stresses, and displacements.

Principle of Superposition

- 2 requirements for the principle to apply:
  - The material must behave in a linear-elastic manner, so that Hooke's law is valid, and therefore the load will be proportional to displacement.
  - The geometry of the structure must not undergo significant change when the loads are applied, i.e., small displacement theory applies. Large displacements will significantly change the position and orientation of the loads.

Equilibrium Equations

- For equilibrium, the equations in 3D are as shown:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$
$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

- The principal load-carrying portions of most structures, however, lie in a single plane, and since the loads are also coplanar, the above requirements for equilibrium reduce to

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum M_o = 0$$

Determinacy

- Equilibrium equations provide both the necessary and sufficient conditions for equilibrium
- All forces can be determined strictly from these equations
- No. of unknown forces > equilibrium equations means statically indeterminate
- This can be determined using a free body diagram

Determinacy

- For a coplanar structure

$$r = 3n, \quad \text{statically determinate}$$
$$r > 3n, \quad \text{statically indeterminate}$$

- r = number force and moment reaction components, n = number of parts
- The additional equations needed to solve for the unknown equations are referred to as compatibility equations

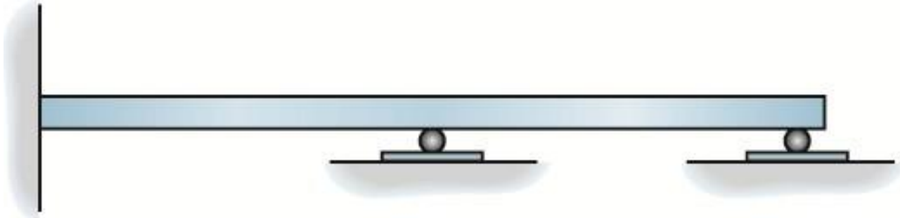
Example 1.2

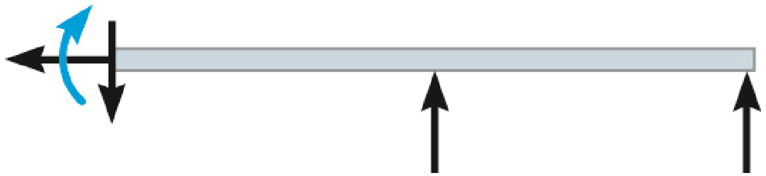
- Classify each of the beams as statically determinate or statically indeterminate. If statically indeterminate, report the no. of degree of indeterminacy. The beams are subjected to external loadings that are assumed to be known & can act anywhere on the beams.

Solution 1.2

$$r = 3,\ n = 1,\ 3 = 3(1)$$

- Statically determinate

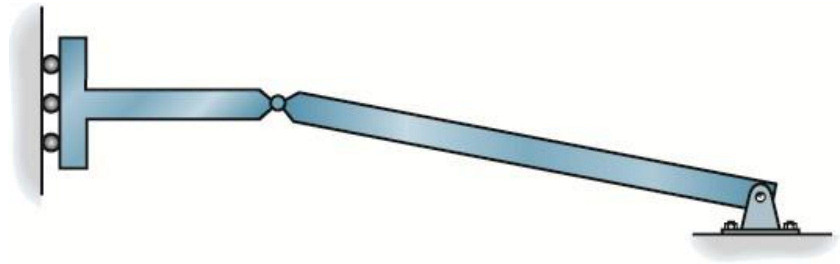


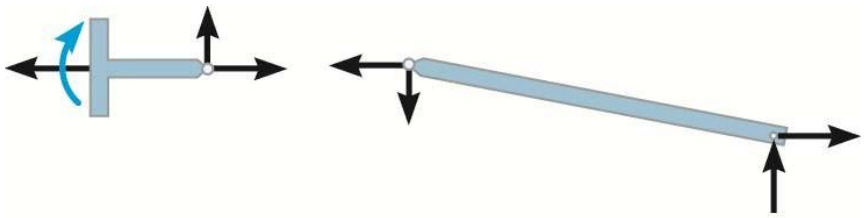


$$r = 5,\ n = 1,\ 5 > 3(1)$$

- Statically indeterminate to the second degree

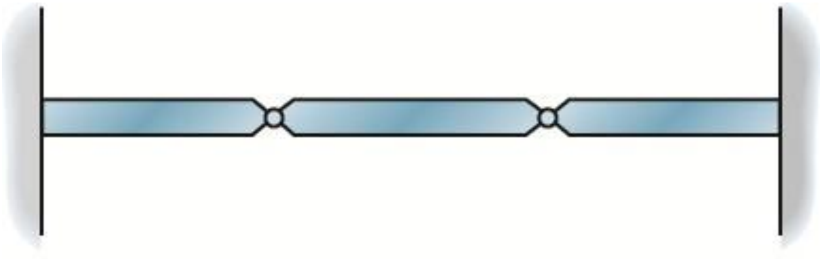
Solution





$r = 6,\ n = 2,\ 6 = 3(2)$

Statically determinate

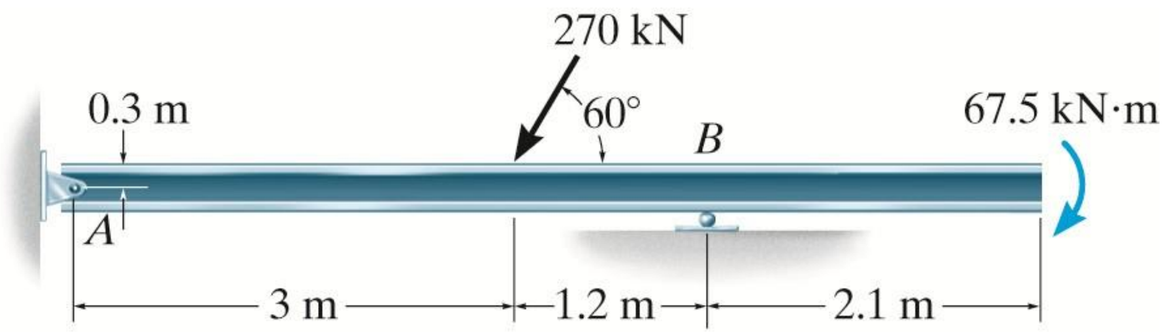


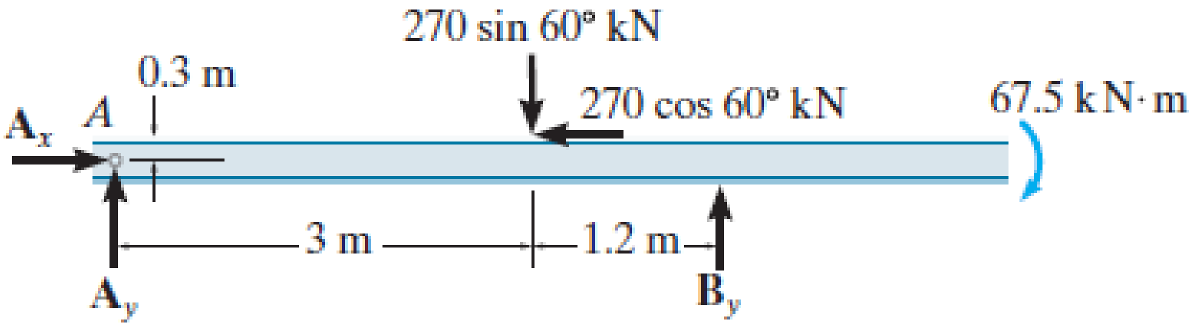
$r = 10,\ n = 3,\ 10 > 3(3)$

Statically indeterminate to the first degree

Example

Application of the Equations of Equilibrium. Determine the reactions on the beam as shown.





Solution

$$\overset{+}{\rightarrow}\sum F_x = 0; \quad A_x - 270\cos 60^0 = 0$$

$$A_x = 135\text{kN}$$

With anti-clockwise moments in the + direction,

$$\sum M_A = 0; \quad -270\sin 60^0(3) + 270\cos 60^0(0.3) + B_y(4.2) - 67.5 = 0$$

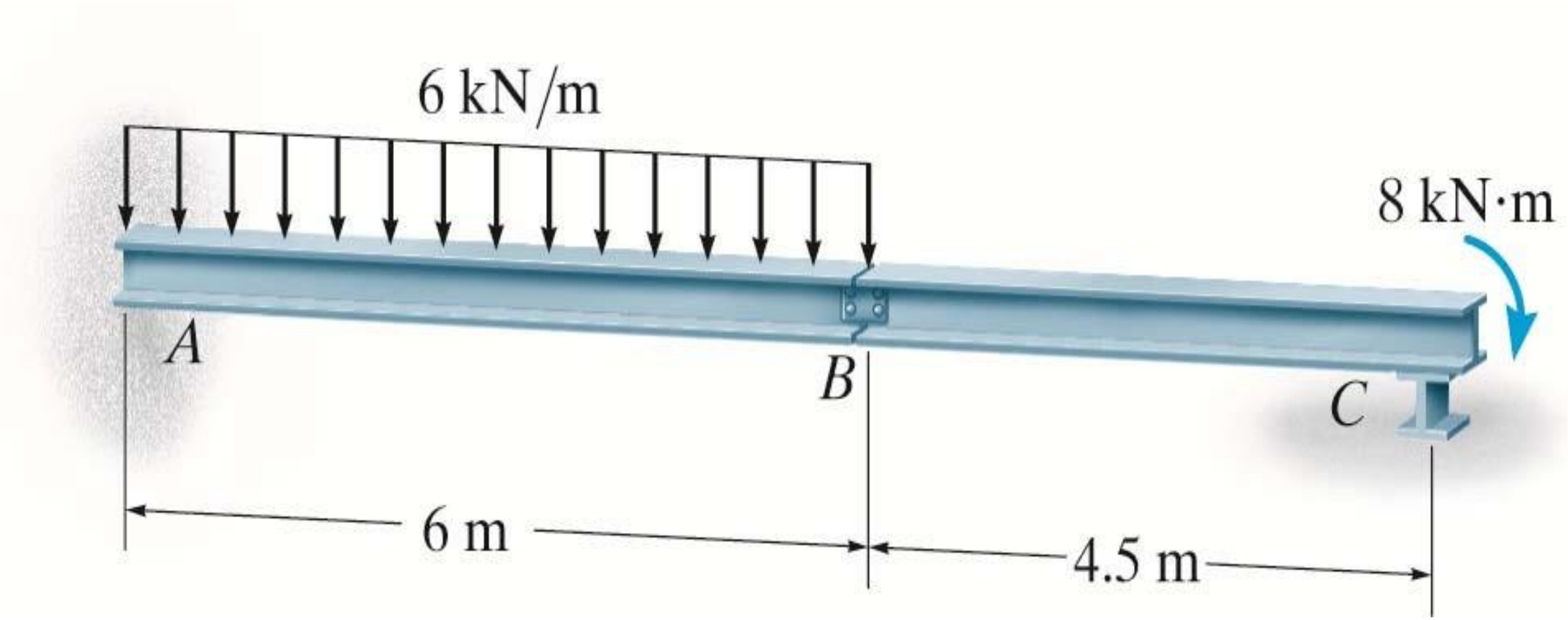
$$B_y = 173.4\text{kN}$$

$$+\uparrow \sum F_y = 0; \quad -270\sin 60^0 + 173.4 + A_y = 0$$

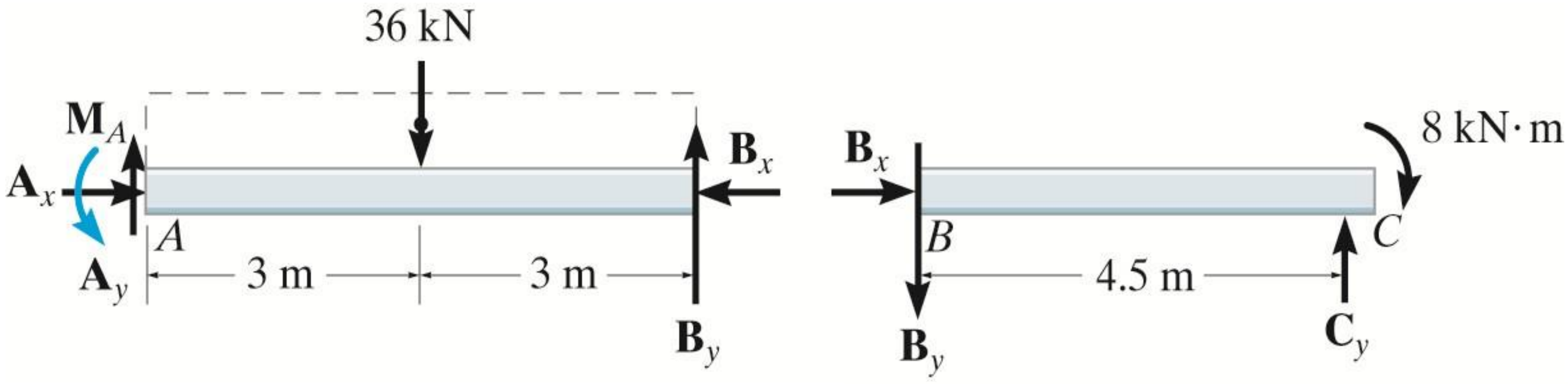
$$A_y = 60.4\text{kN}$$

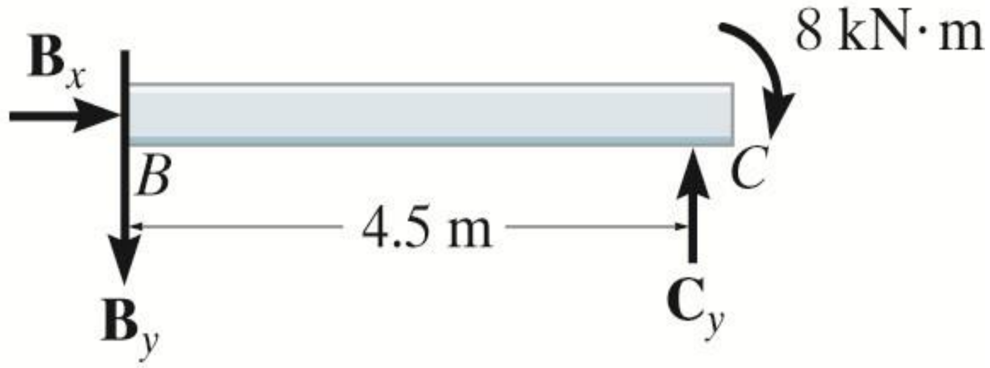
Example

The compound beam shown is fixed at A. Determine the reactions at A, B, and C. Assume that the connection at B is a pin and C is a roller.



Solution





Solution

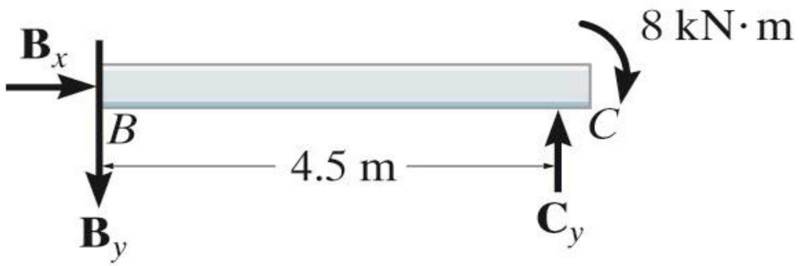
Segment *BC*:

With anti-clockwise moments in the + direction,

$$\sum M_c = 0; \quad -8 + B_y(4.5) = 0 \quad \Rightarrow B_y = 1.78\,\text{kN}$$

$$+\uparrow \sum F_y = 0; \quad -1.78 + C_y = 0 \quad \Rightarrow C_y = 1.78\,\text{kN}$$

$$\underrightarrow{+} \sum F_x = 0; \quad B_x = 0$$



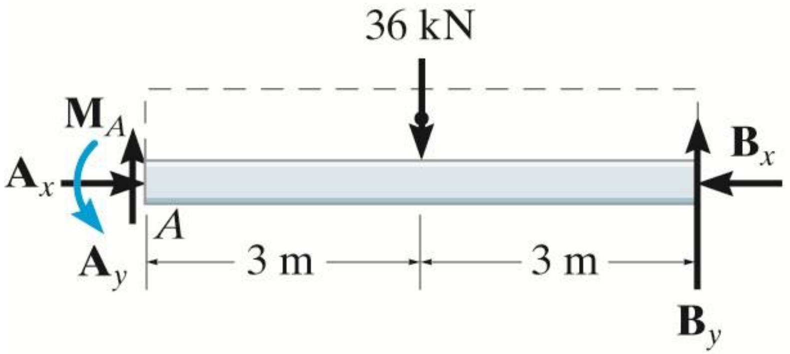
Segment *AB*:

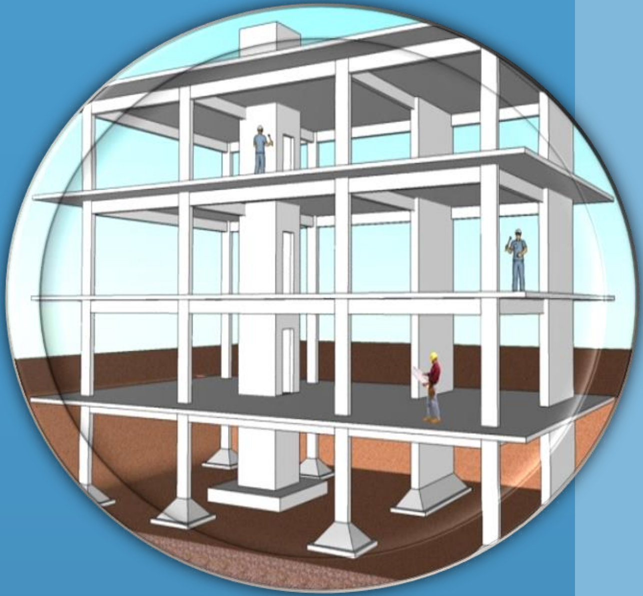
With anti-clockwise moments in the + direction,

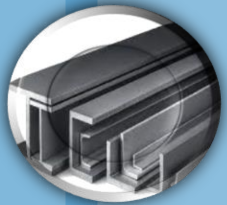
$$\sum M_A = 0; \quad M_A - 36(3) + (1.78)(6) = 0 \Rightarrow M_A = 97.3\,\text{kN}\bullet\text{m}$$

$$+\uparrow \sum F_y = 0; \quad A_y - 36 + 1.78 = 0 \Rightarrow A_y = 34.2\,\text{kN}$$

$$\underrightarrow{+} \sum F_x = 0; \quad A_x = 0$$







Thank You!