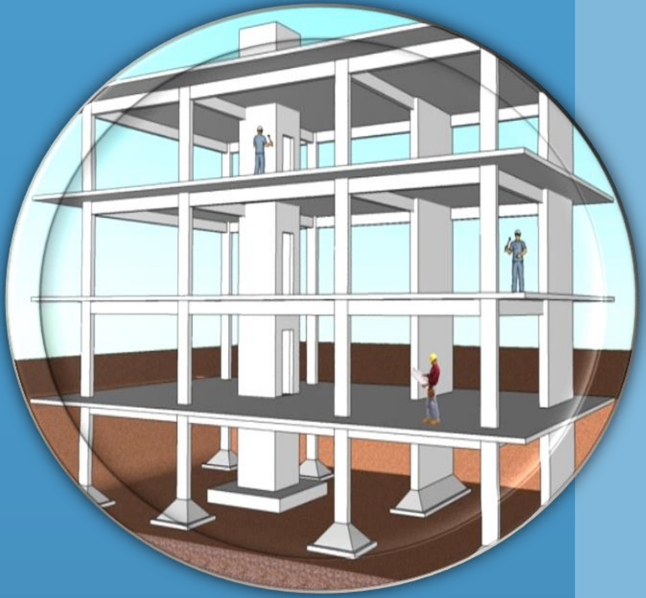
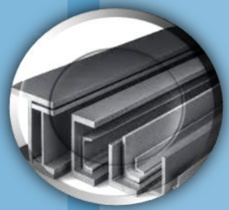
UNIVERSITY OF ZAMBIA
School of Engineering
Department of Civil & Environmental Eng.



Lecture 1.2

CEE 3222: THEORY OF STRUCTURES

REVISION OF STATICS

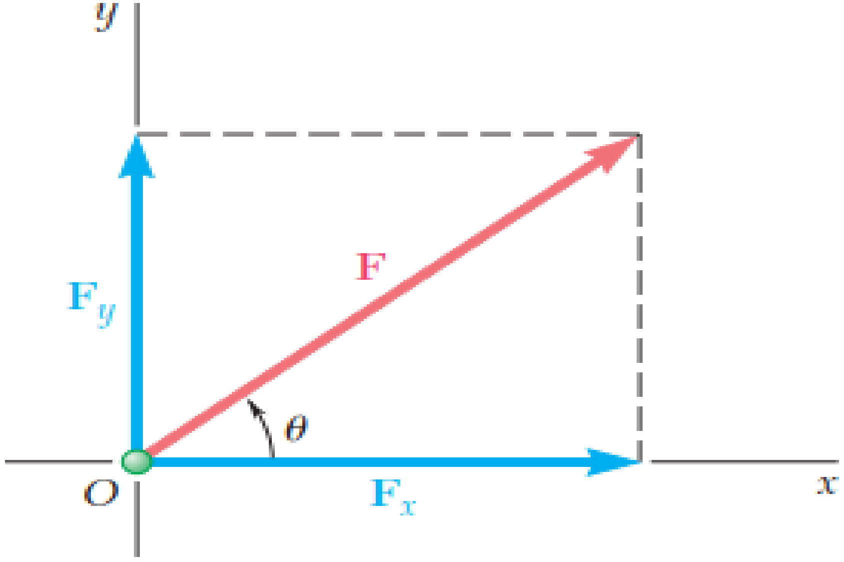


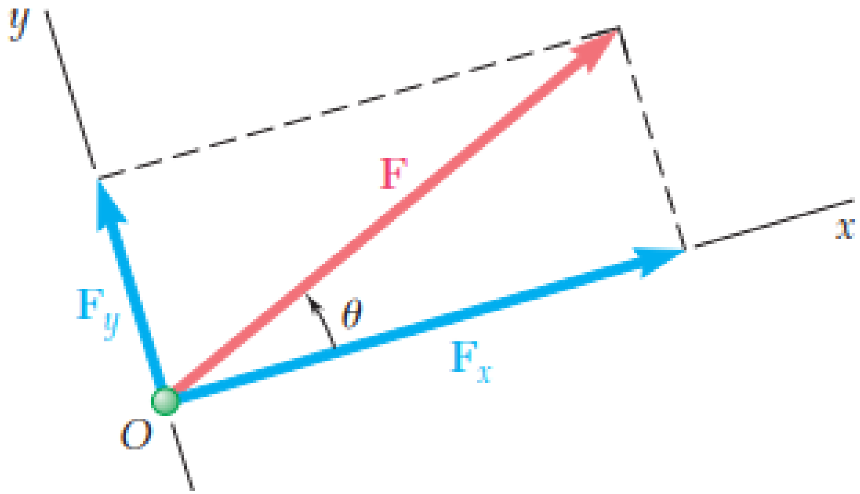
Contents

FORCE VECTORS

Addition of a System of Coplanar Forces

- *In many problems it will be found desirable to resolve a force into two components which are perpendicular to each other*
- *When a force is resolved into two components along the x and y axes, the components are then called rectangular components*
- *The parallelogram drawn to obtain the two components is a rectangle, and Fx and Fy are called rectangular components*



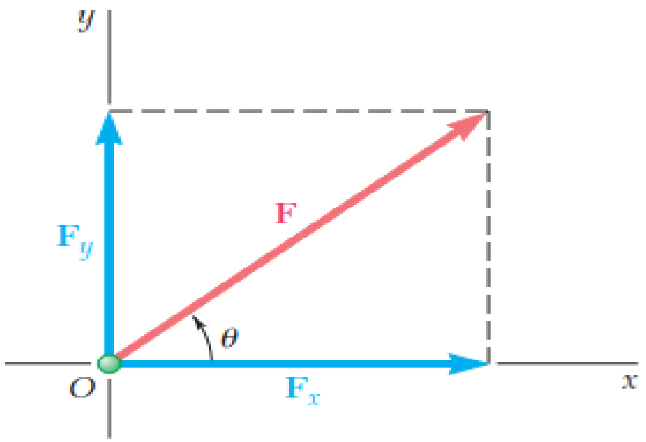


FORCE VECTORS

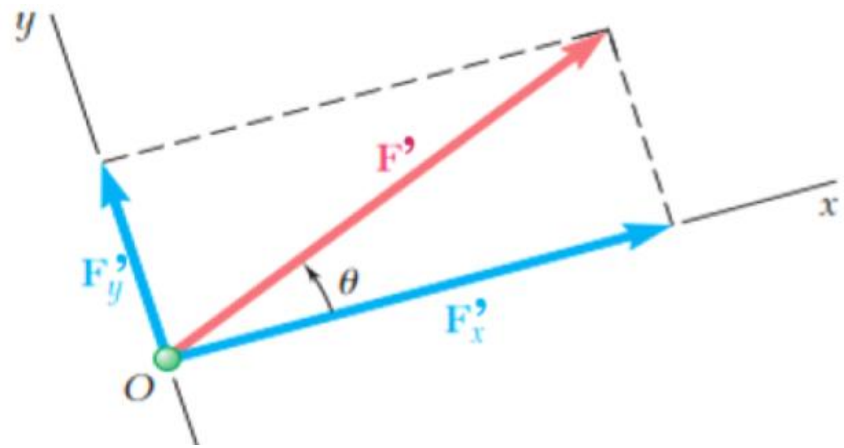
Addition of a System of Coplanar Forces

- *In this way it becomes easier to obtain resultants for more than two forces by getting the components of each force along specified axes and add them algebraically instead of using successive application of the parallelogram law*
- *Note that the axes will be horizontal and vertical or directed at any inclination as long as they remain perpendicular to one another*
- *By parallelogram law it is required that:*

F = Fx + Fy



F' = F'x + F'y



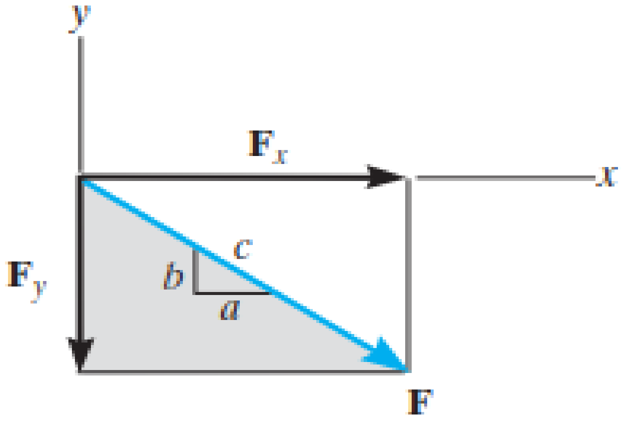
FORCE VECTORS

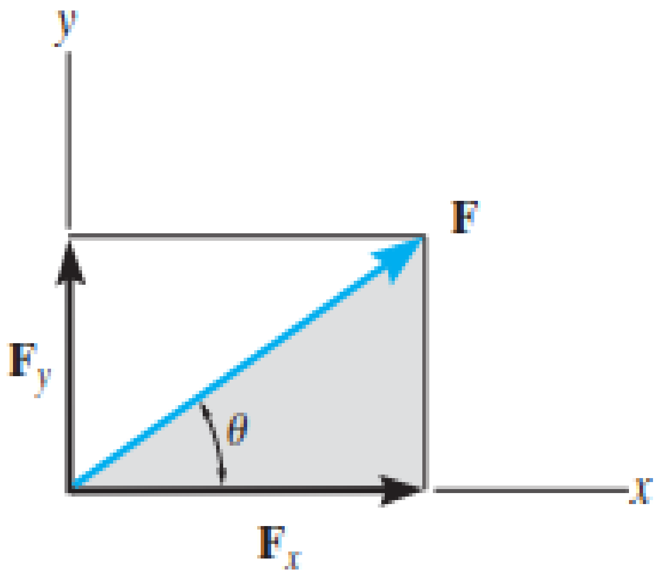
Addition of a System of Coplanar Forces

- *Figures below show that the sense of direction of each force component is represented graphically by the arrowhead*
- *For solving problems however, we need to establish a notation for representing the directional sense of the rectangular components*
- *The two ways that we will use are:*

➢ The Scalar Notation

➢ Cartesian Vector Notation





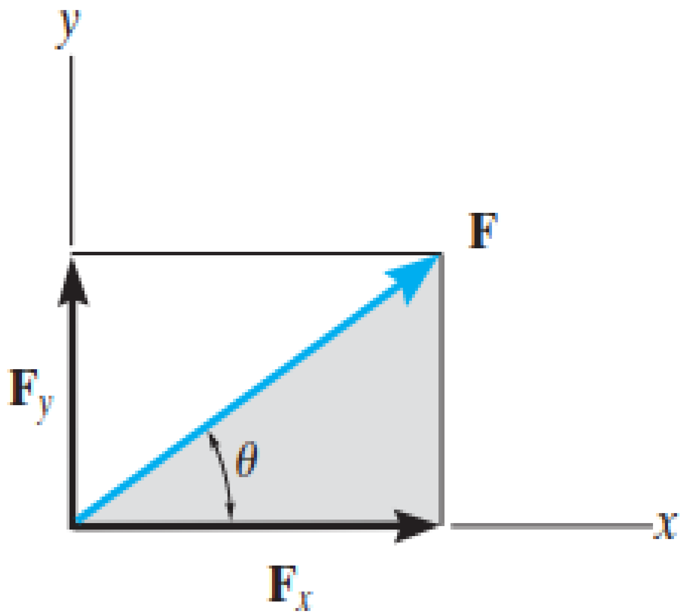
THE SCALAR NOTATION

- *The rectangular components of force F shown are found using the parallelogram law, so that **F** = Fx + Fy.*

- *Because these components form a right triangle, they can be determined from*

$$\mathbf{F}x = F\cos\theta \text{ and } \mathbf{F}y = F\sin\theta$$

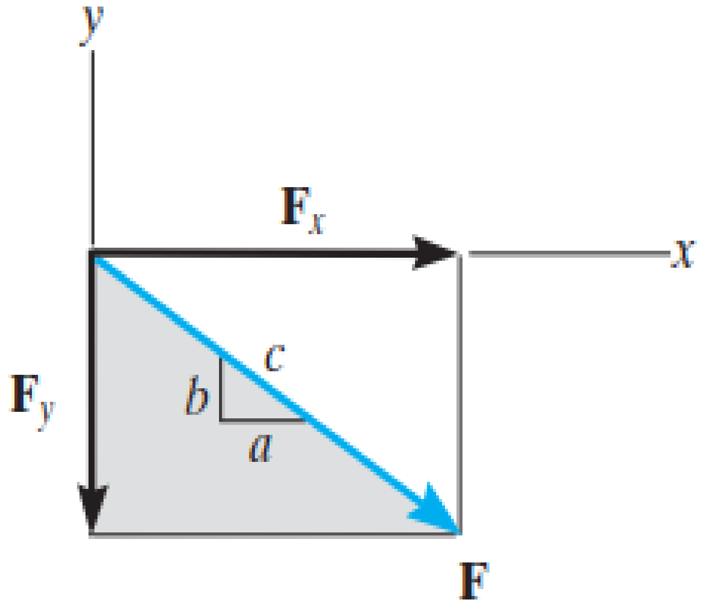
$$Tan\theta = \frac{\boldsymbol{F}y}{\boldsymbol{F}x} \quad \text{and} \quad \boldsymbol{F}x^2 + \boldsymbol{F}y^2 = \boldsymbol{F}^2$$



FORCE VECTORS

Addition of a System of Coplanar Forces

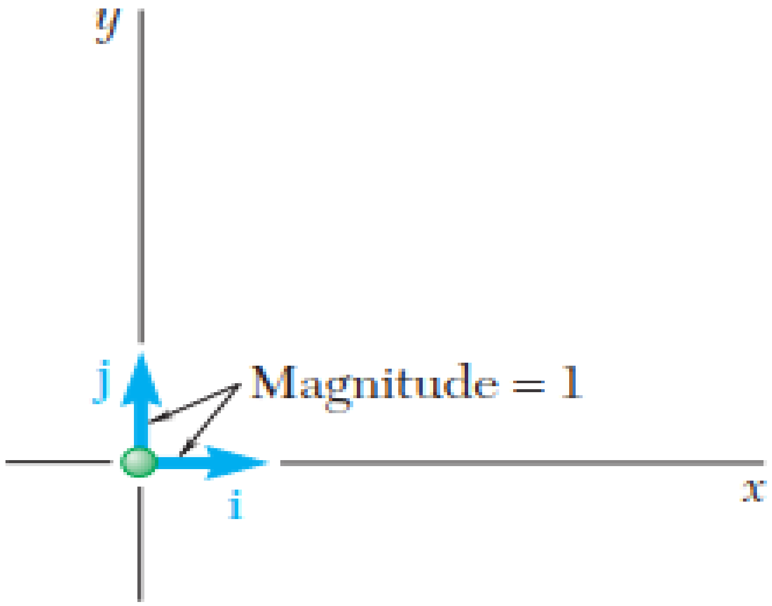
- *Instead of using the angle θ, however, th direction of F can also be defined using a sma "slope" triangle*
- *Since this triangle and the larger shaded triangl are similar, the proportional length of the side gives* $\frac{Fx}{F} = \frac{a}{c}$ and $\frac{Fy}{F} = \frac{b}{c}$
- Below shows the force notation using the scal method.



100 N ←	173 N ↑	240 N →	100 N ↓
-100 N	**+173 N**	**+240 N**	**-100 N**

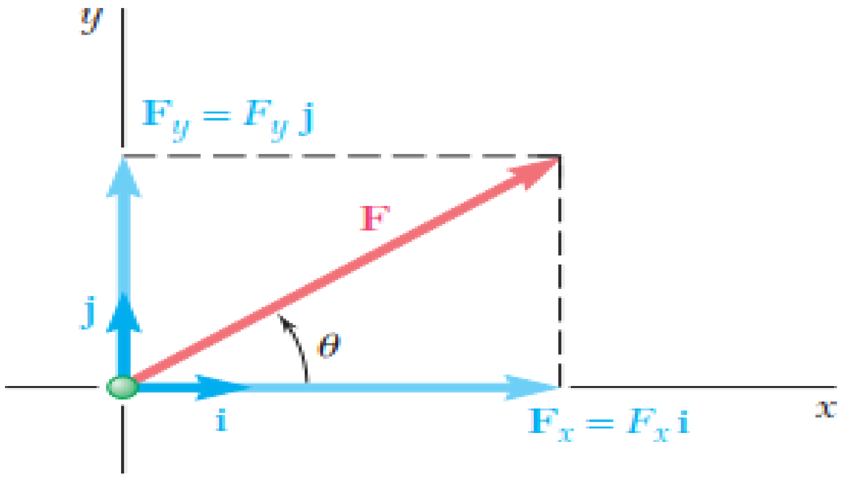
THE CARTESIAN VECTOR NOTATION

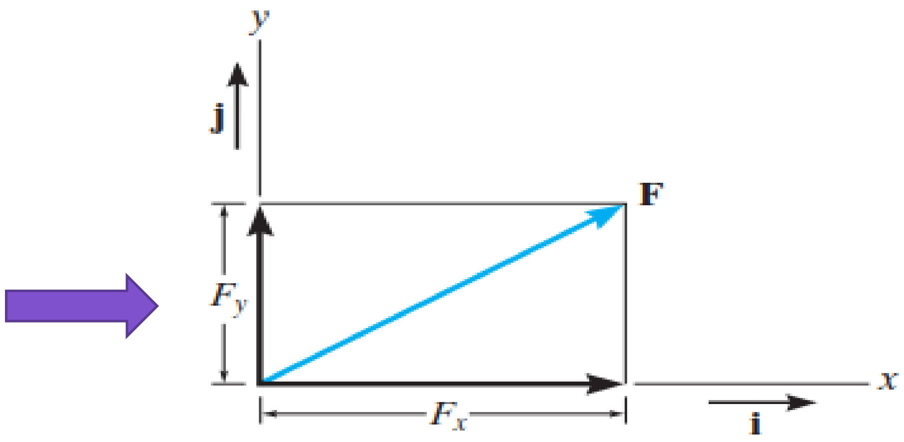
- *It is also possible to represent the x and y components of a force in terms of Cartesian unit vectors* ***i*** *and* ***j****.*
- *They are called unit vectors because they have a dimensionless magnitude of 1, and so they can be used to designate the directions of the x and y axes, respectively*
- *This methods makes vector algebra very easy and it becomes advantageous when solving problems in 3D*



THE CARTESIAN VECTOR NOTATION

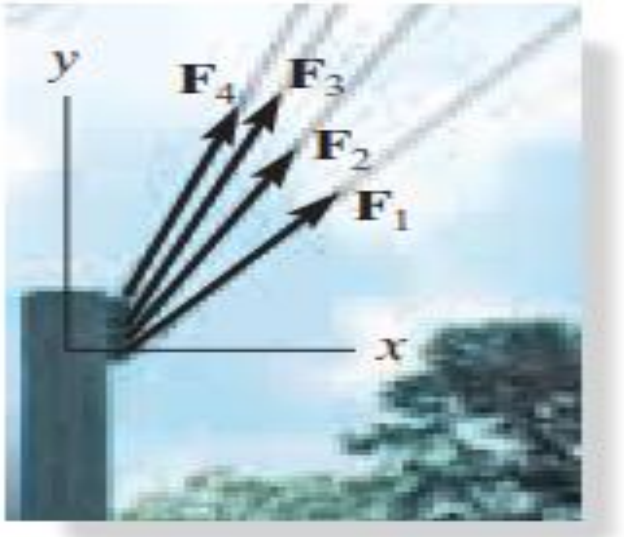
- *Recalling the definition of the product of a scalar and a vector, we note that the rectangular components Fx and Fy of a force F may be obtained by multiplying respectively the unit vectors **i** and **j** by appropriate scalars*
- *We write* $Fx = Fx\mathbf{i}$ & $Fy = Fy\mathbf{j}$ $\quad F = Fx + Fy \Rightarrow F = Fx\mathbf{i} + Fy\mathbf{j}$



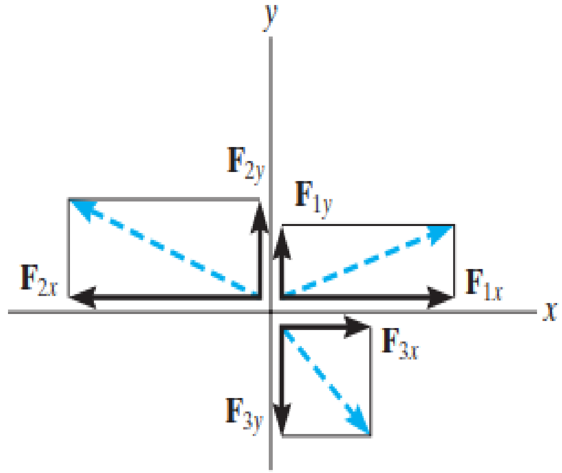


THE COPLANAR FORCE RESULTANTS

- *We can now use either of the two methods just described to determine the resultant of several coplanar forces*



- *To do this, each force is first resolved into its x and y components, and then the respective components are added using scalar algebra since they are collinear*



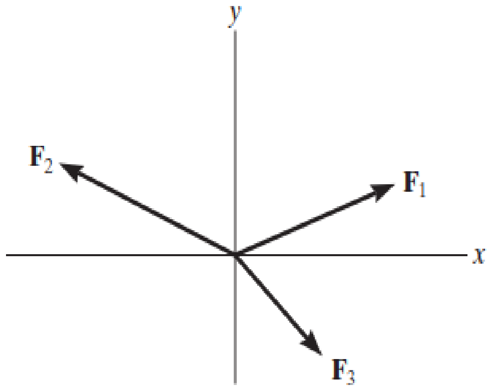
- *The resultant force is then formed by adding the resultant components using the parallelogram law*

FORCE VECTORS

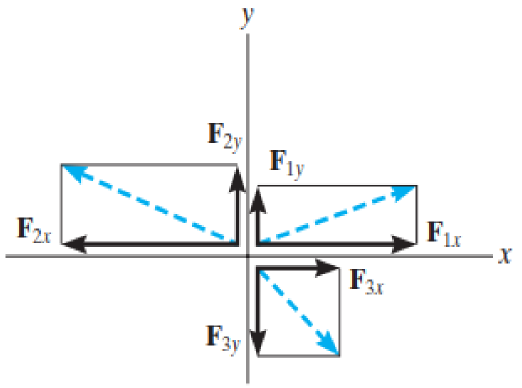
Addition of a System of Coplanar Forces

THE COPLANAR FORCE RESULTANTS

- *Consider the three concurrent forces shown which have x and y components as shown*
- *Using Cartesian vector notation (or scalar), each force is first represented as a Cartesian vector, i.e*



$\mathbf{F}1 = F1x\,\mathbf{i} + F1y\,\mathbf{j}$, $\quad \mathbf{F}2 = -\mathbf{F}2x\,\mathbf{i} + F2y\,\mathbf{j} \quad \mathbf{F}3 = F3x\,\mathbf{i} - F3y\,\mathbf{j}$



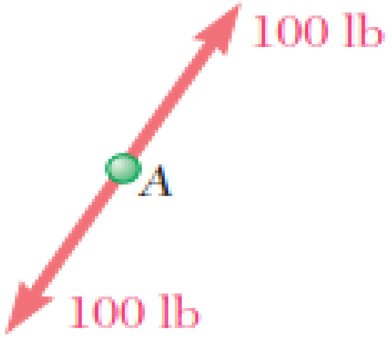
➢ *The vector resultant is therefore*

$$FR = \mathbf{F}1 + \mathbf{F}2 + \mathbf{F}3$$

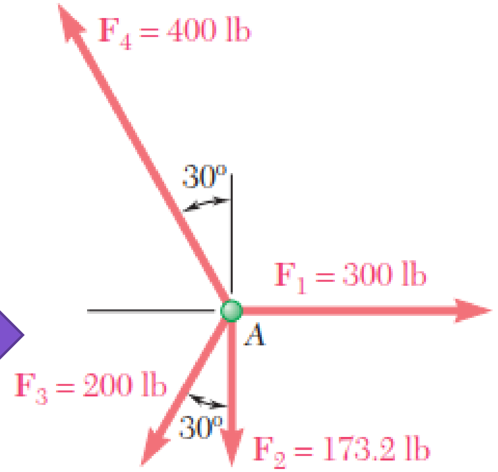
$$FR = \mathbf{F}1x\,\mathbf{i} + \mathbf{F}1y\,\mathbf{j} - F2x\,\mathbf{i} + F2y\,\mathbf{j} + F3x\,\mathbf{i} - F3y\,\mathbf{j} = (\mathbf{F}Rx)\,\mathbf{i} + (\mathbf{F}Ry)\,\mathbf{j}$$

EQUILIBRIUM OF A PARTICLE (in 2D)

- *A particle which is acted upon by two forces will be in equilibrium if the two forces have the same magnitude and the same line of action but opposite sense*
- *The resultant of the two forces is then zero*

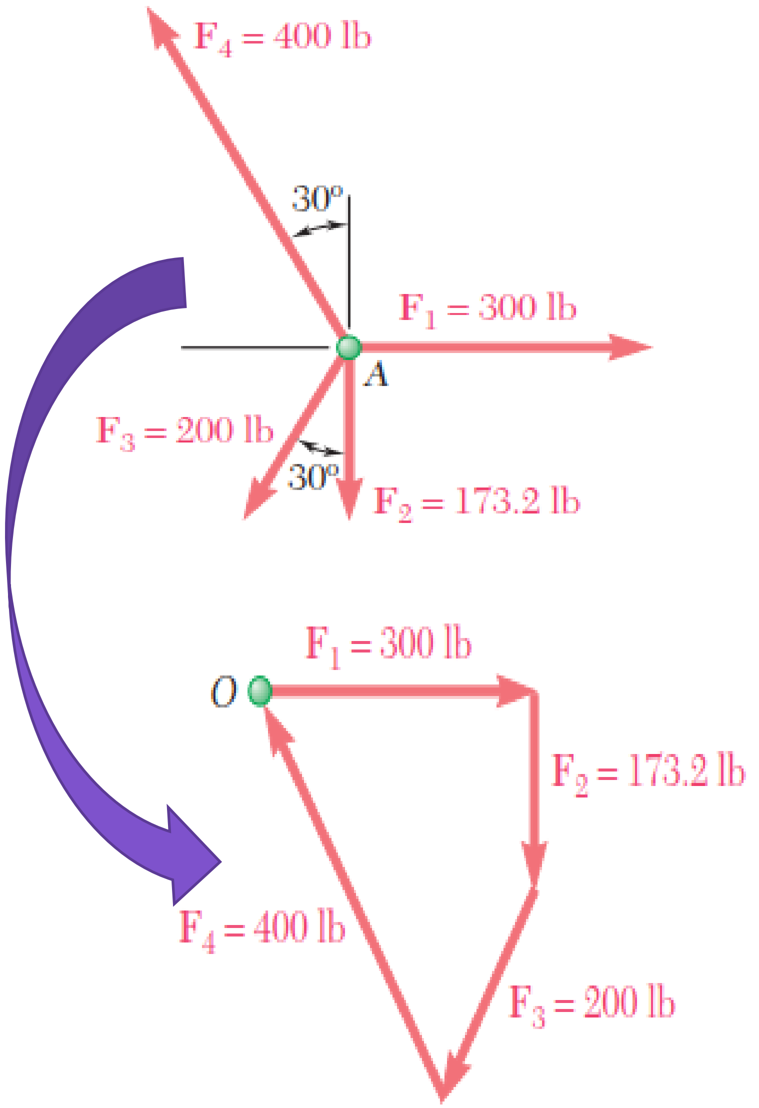


- *Another case of equilibrium of a particle is where four forces are acting on A*



EQUILIBRIUM OF A PARTICLE (in 2D)

- *In this case the resultant of the given forces is determined by the polygon rule*
- *Starting from point O with F1 and arranging the forces in tip-to-tail fashion, we find that the tip of F4 coincides with the starting point.*
- *Thus the resultant R of the given system of forces is zero, and the particle is in* ***Equilibrium***



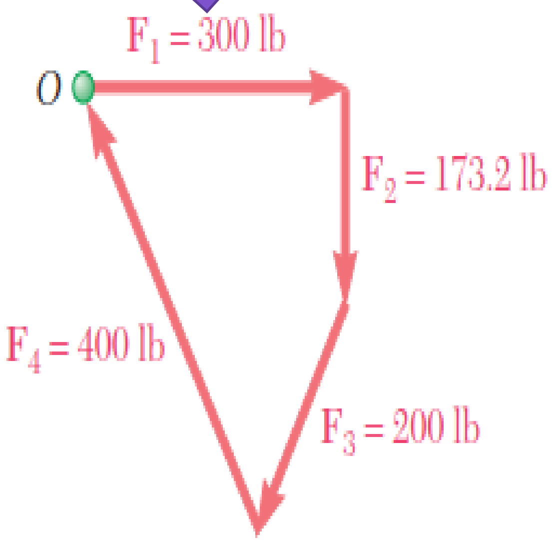
EQUILIBRIUM OF A PARTICLE (in 2D)

- *The closed polygon drawn provides a graphical expression of the equilibrium of A*
- *To express algebraically the conditions for the equilibrium of a particle, we write* $\mathbf{R} = \Sigma \mathbf{F} = 0$
- *Resolving each force F into rectangular components. we have*

$$\Sigma(F_x\mathbf{i} + F_y\mathbf{j}) = 0 \quad \text{or} \quad (\Sigma F_x)\mathbf{i} + (\Sigma F_y)\mathbf{j} = 0$$

- *We conclude that the necessary and sufficient conditions for the equilibrium of a particle are*

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0$$

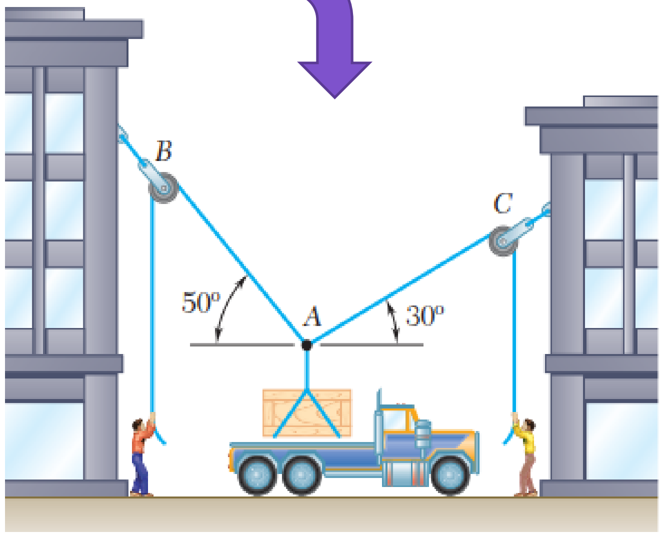


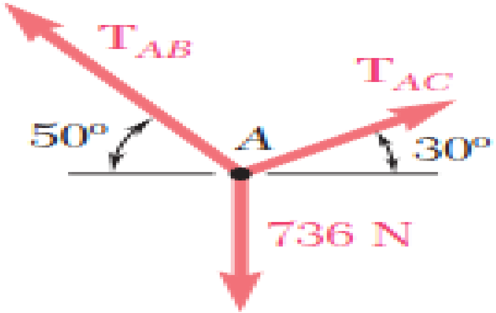
EQUILIBRIUM OF A PARTICLE - FREE-BODY DIAGRAMS (in 2D)

- *In practice, a problem in engineering mechanics is derived from an actual physical situation*
- *A sketch showing the physical conditions of the problem is known as a space diagram*
- *A large number of problems involving actual structures, however, can be reduced to problems concerning the equilibrium of a particle.*
- *This is done by choosing a significant particle and drawing a separate diagram showing this particle and all the forces acting on it.*
- *Such a diagram is called a free-body diagram (FBD)*

EQUILIBRIUM OF A PARTICLE - FREE-BODY DIAGRAMS (in 2D)

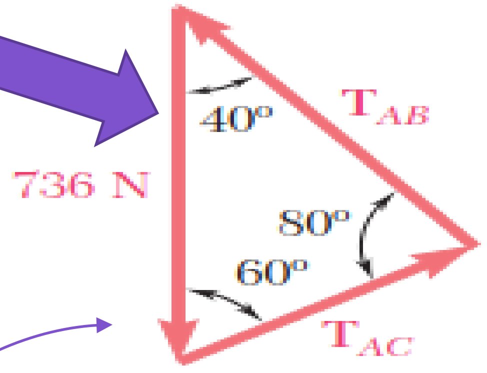
- *The next demonstration explains how to draw a useful FBD*
- *Consider the 75-kg crate shown in the space diagram*
- *This crate was lying between two buildings, and it is now being lifted onto a truck, which will remove it*
- *The crate is supported by a vertical cable, which is joined at A to two ropes which pass over pulleys attached to the buildings at B and C*
- *It is desired to determine the tension in each of the ropes AB and AC*
- *The FBD shows the forces exerted on A by the vertical cable and the two ropes.*



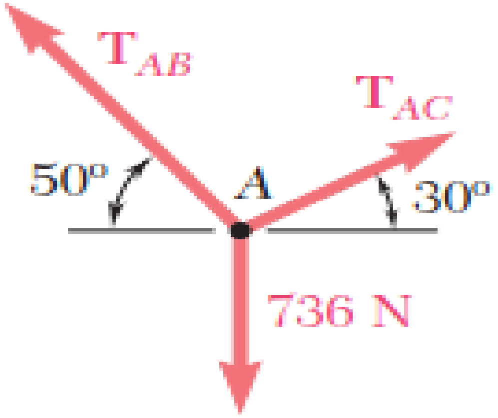


EQUILIBRIUM OF A PARTICLE - FREE-BODY DIAGRAMS (in 2D)

- *Since point A is in equilibrium, the three forces acting on it must form a closed triangle when drawn in tip-to-tail fashion*
- *The values TAB and TAC of the tension in the ropes may be found graphically if the triangle is drawn to scale, or they may be found by trigonometry*
- *If we use trigonometry method, we use the law of sines and write*



(c) Force triangle



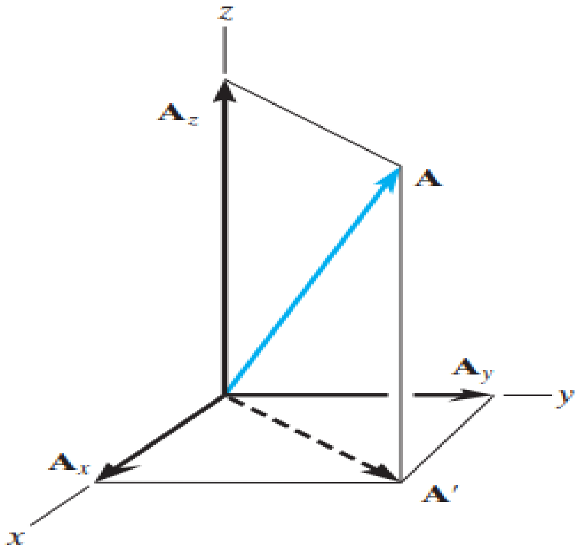
$$\frac{T_{AB}}{\sin 60°} = \frac{T_{AC}}{\sin 40°} = \frac{736 \text{ N}}{\sin 80°}$$

$$T_{AB} = 647 \text{ N} \qquad T_{AC} = 480 \text{ N}$$

FORCE VECTORS IN SPACE (3D)

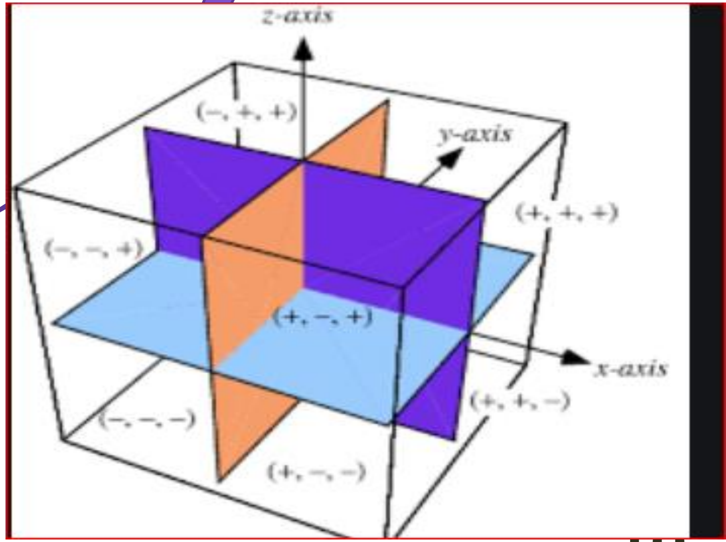
CARTESIAN VECTORS

Rectangular Components of a Vector.



- *A vector A may have one, two, or three rectangular components along the x, y, z coordinate axes, depending on how the vector is oriented relative to the axes*
- *In general, though, when A is directed within an octant of the x, y, z, then by two successive applications of the parallelogram law, we may resolve the vector into components as*
- *A = A' + Az and then A' = Ax + Ay.*

$$A = Ax + Ay + Az$$

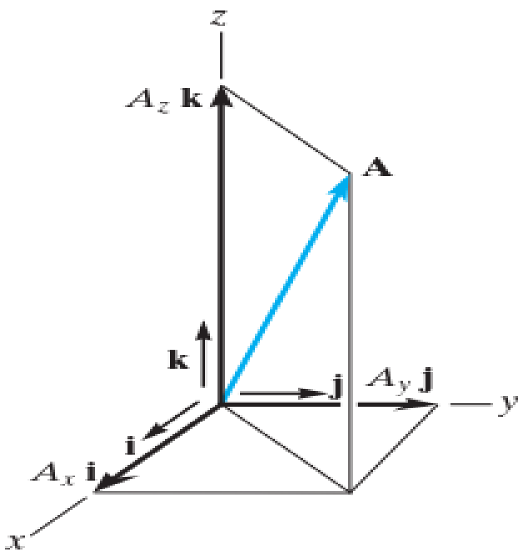


Cartesian Vectors Representation

- *Since the three components of A in Eq. A = Ax + Ay + Az act in the positive **i**, **j**, and **k** directions, we can write A in Cartesian vector form as*

$$A = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

- *There is a distinct advantage to writing vectors in this manner as separating the magnitude and direction of each component vector will simplify the operations of vector algebra, particularly in three dimensions*

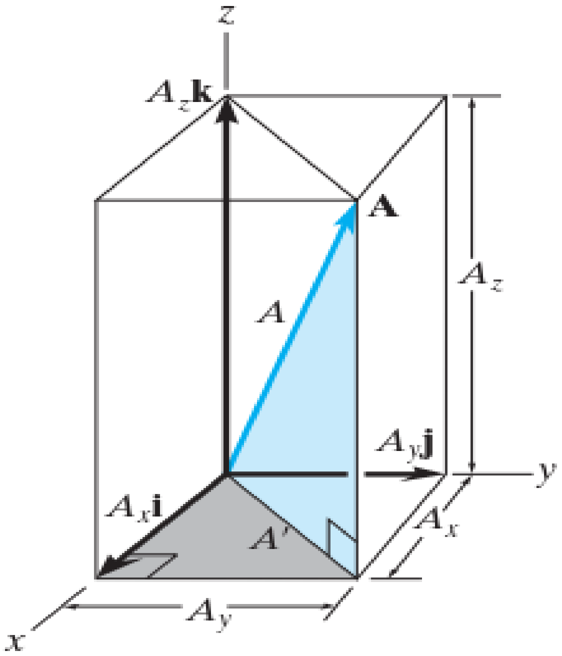


Magnitude of Cartesian Vectors

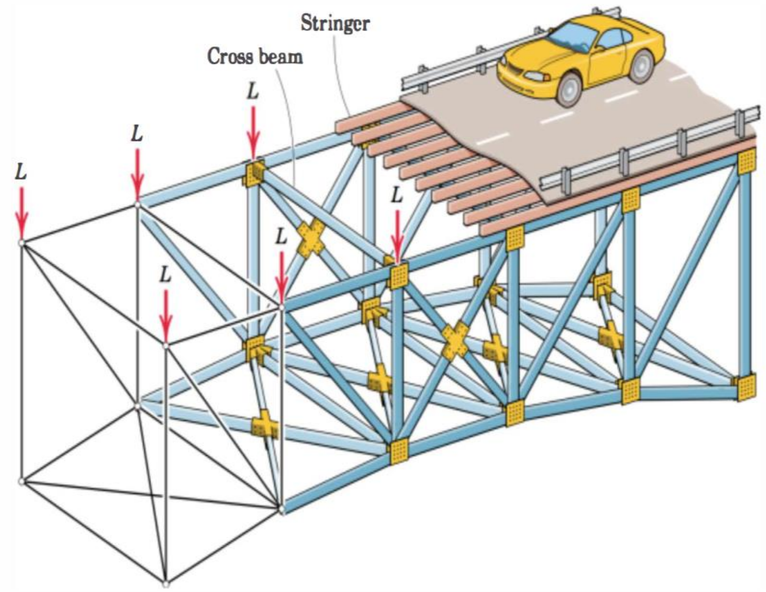
- *It is always possible to obtain the magnitude of A provided it is expressed in Cartesian vector form.*
- *As shown from the blue right triangle,* $A = \sqrt{A'^2 + Az^2}$ *and from the gray right triangle,* $A' = \sqrt{Ay^2 + Ax^2}$
- *Combining these equations to eliminate A' yields*

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

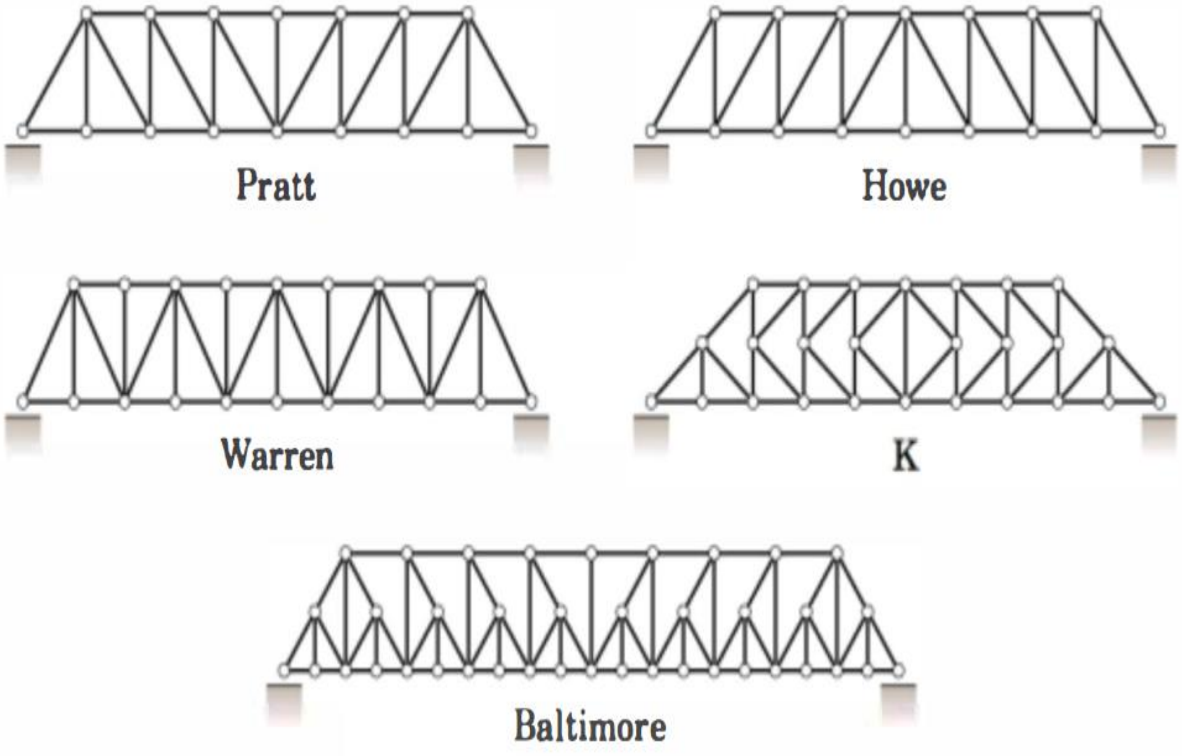
- ***Therefore the magnitude of A is equal to the positive square root of the sum of the squares of its components***



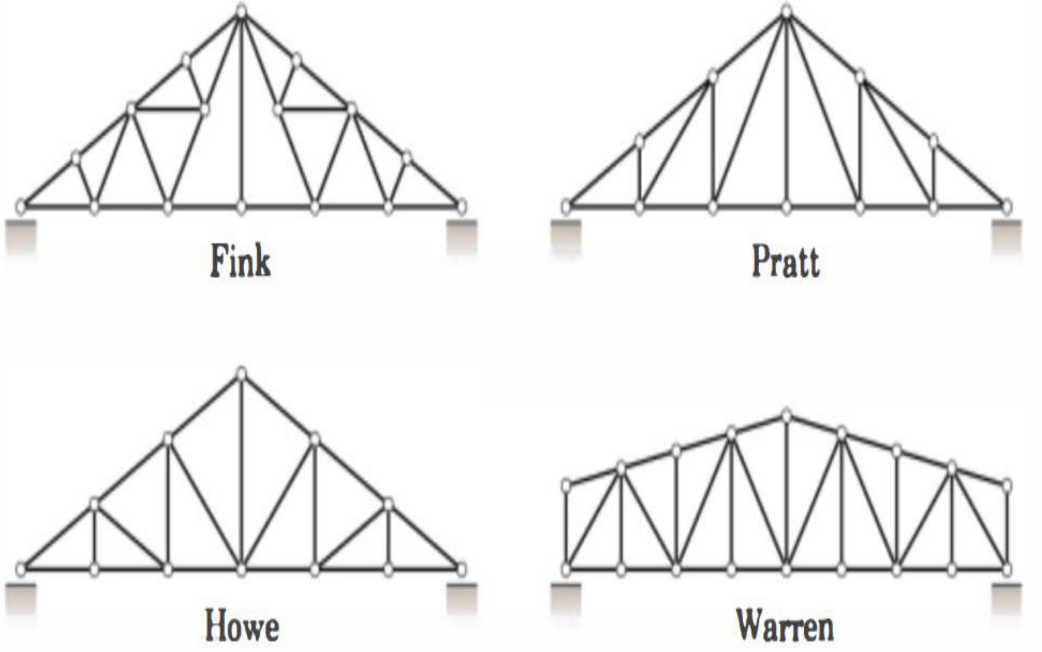
Analysis of Trusses



Analysis of Trusses

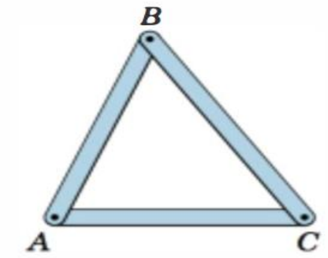


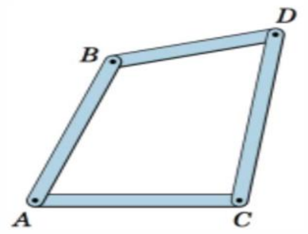
Commonly Used Bridge Trusses



Commonly Used Roof Trusses

PLANE TRUSSES



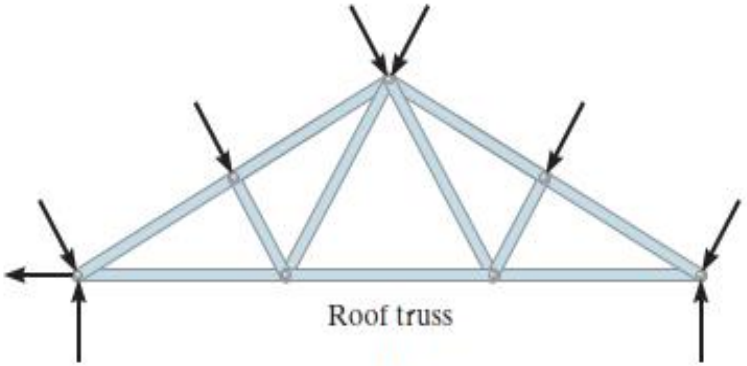


- A framework composed of members joined at their ends to form a rigid structure is called a truss. Or a truss is a structure composed of slender members joined together at their end points
- Bridges, roof supports, derricks, and other such structures common examples of trusses.
- Structural members commonly used are; beams, channels, angles, bars, and special shapes which are fastened together at their ends by welding, riveted connections, or large bolts or pins.
- When the members of the truss lie essentially in a single plane, it is called a plane truss.

PLANE TRUSSES

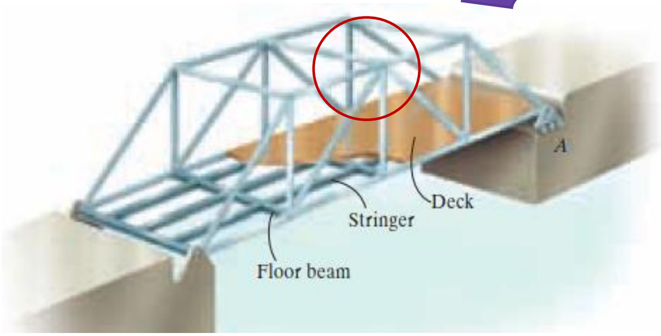
- The members commonly used in construction consist of wooden struts or metal bars.
- The truss shown in Fig. 6–1a is an example of a typical roof-supporting truss. In this figure, the roof load is transmitted to the truss at the joints by means of a series of purlins.
- Since this loading acts in the same plane as the truss, Fig. 6–1b, the analysis of the forces developed in the truss members will be two-dimensional.

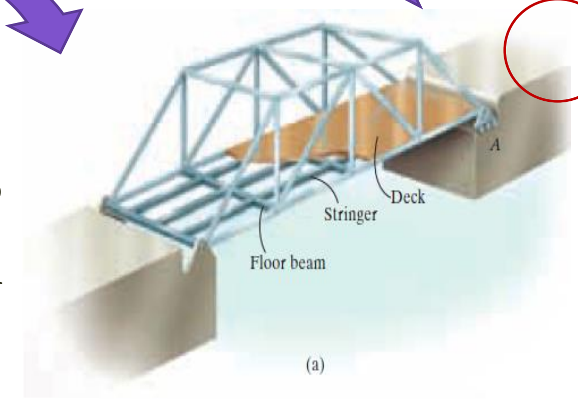
(a)



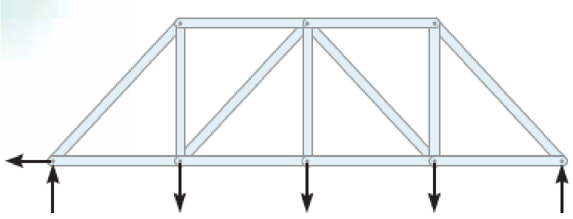
(b)

PLANE TRUSSES





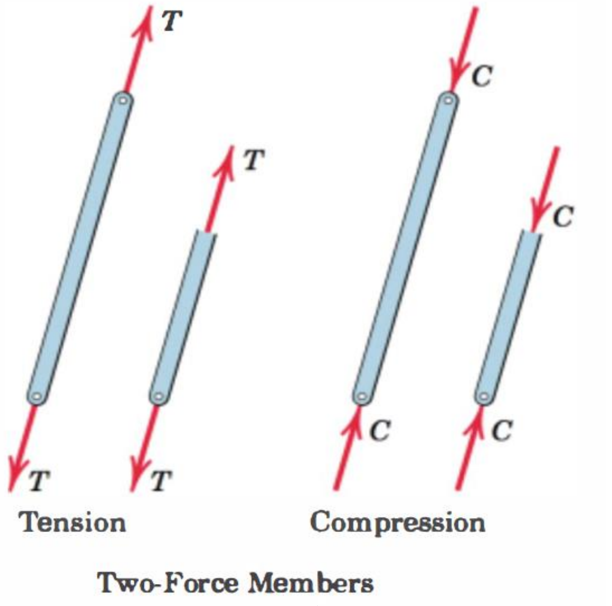
- In the case of a bridge, as shown, the load on the deck is first transmitted to stringers, then to floor beams, and finally to the joints of the two supporting side trusses. coplanar
- When bridge or roof trusses extend over large distances, a rocker or roller is commonly used for supporting one end, shown above
- This type of support allows freedom for expansion or contraction of the members due to a change in temperature or application of loads.



Bridge truss

Assumptions for Design

- To design both the members and the connections of a truss, it is necessary first to determine the force developed in each member when the truss is subjected to a given loading.
- To do this we will make two important assumptions;
  - ✓ The members are assumed to be connected only by frictionless pins.
  - ✓ The loads must be applied at the joints
- Because of these two assumptions, each truss member will act as a two-force member, and therefore the force acting at each end of the member will be directed along the axis of the member.



Simple Trusses

- Two methods for the force analysis of simple trusses will be discussed.
- The free-body diagram of the truss as a whole is shown in Fig. 4/6b
- *The external reactions are usually determined first, by applying the equilibrium equations to the truss as a whole*
- Then the force analysis of the remainder of the truss is performed

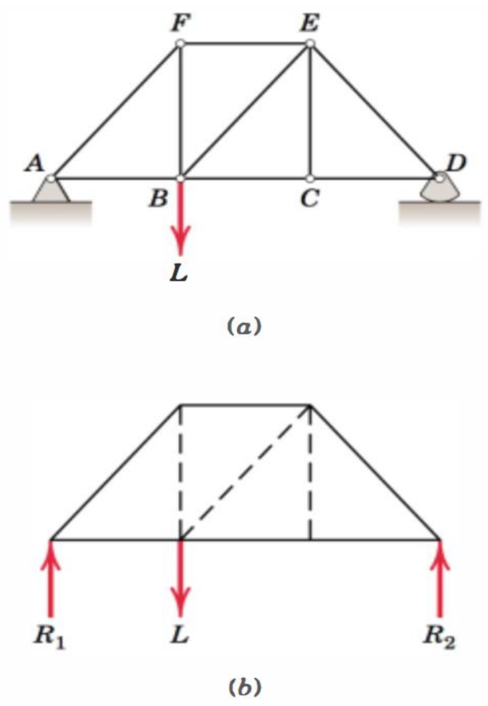
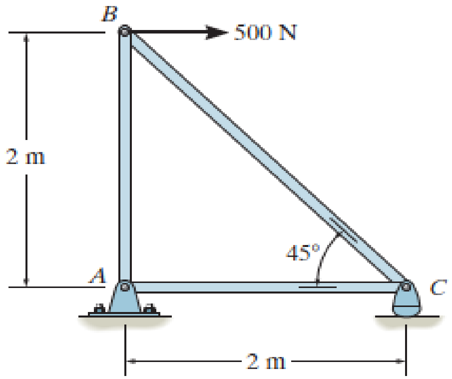
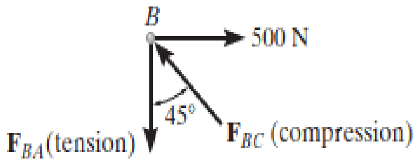


Figure 4/6

SIMPLE TRUSSES
Method of Joints

- In order to analyze or design a truss, it is necessary to determine the force in each of its members by using the method of joints.
- This method is based on the fact that if the entire truss is in equilibrium, then each of its joints is also in equilibrium.
- Hence if the FBD of each joint is drawn, the force equilibrium equations can then be used to obtain the member forces acting on each joint.
- Since the members of a plane truss are members lying in a single plane, each joint is subjected to a force system that is coplanar and concurrent.
- As a result, only $\sum F_x = 0$ and $\sum F_y = 0$ need to be satisfied for equilibrium.





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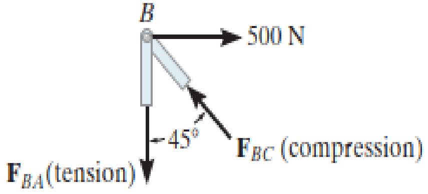
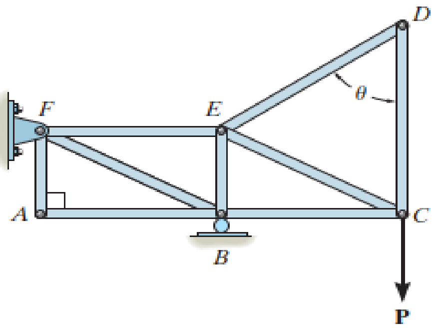


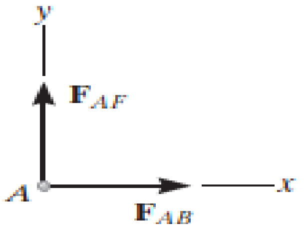
(c)

SIMPLE TRUSSES
Zero-Force Members

- Truss analysis using the method of joints is greatly simplified if we can first identify those members which support **no loading**.
- These zero-force members are used to increase the stability of the truss during construction and to provide added support if the loading is changed.
- The zero-force members of a truss can generally be found by inspection of each of the joints.
- For example, consider the truss shown in Fig. 6–11a. If a FBD of the pin at joint A is drawn, Fig. 6–11b, it is seen that members AB and AF are zero-force members.



(a)

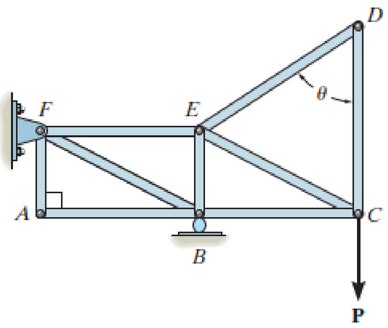


$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AB} = 0$

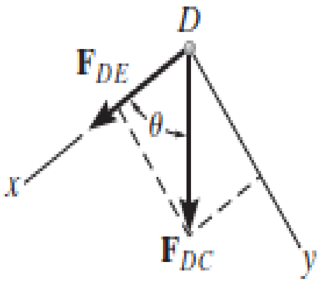
$+\uparrow \Sigma F_y = 0; \quad F_{AF} = 0$

(b)

Zero-Force Members



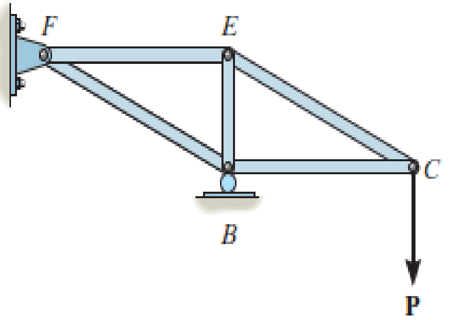
(a)



$$+\searrow \Sigma F_y = 0;\ F_{DC}\sin\theta = 0;\quad F_{DC} = 0 \text{ since } \sin\theta \neq 0$$
$$+\swarrow \Sigma F_x = 0;\ F_{DE} + 0 = 0;\quad F_{DE} = 0$$

(c)

- Note that we could not have come to this conclusion if we had considered the FBDs of joints F or B simply because there are five unknowns at each of these joints.)
- In a similar manner, consider the FBD of joint D, Fig. 6–11c. Here again it is seen that DC and DE are zero-force members.
- From these observations, we can conclude that if only two non-collinear members form a truss joint and no external load or support reaction is applied to the joint, the two members must be zero-force members.
- The load on the truss in Fig. 6–11a is therefore supported by only five members as shown in Fig. 6–11d.



(d)

SIMPLE TRUSSES

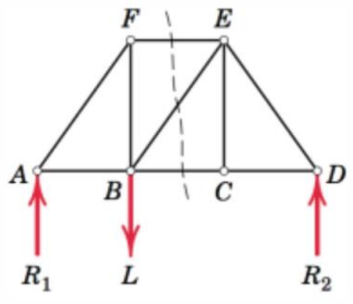
Method of Section

- When we need to find the force in only a few members of a truss, we can analyze the truss using the *method of sections.*
- It is based on the principle that if the truss is in equilibrium then any segment of the truss is also in equilibrium
- This method of sections has the basic advantage that the force in almost any desired member may be found directly from an analysis of a section which has cut that member
- Thus, it is not necessary to proceed with the calculation from joint to joint until the member in question has been reached.
- In choosing a section of the truss, we note that, in general, *not more than three members whose forces are unknown should be cut*, since there are only three available independent equilibrium relations.

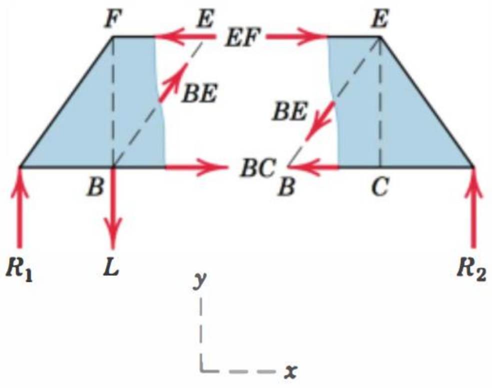
SIMPLE TRUSSES

Method of Section

- The Fig. shown will be used to illustrate how the section method works.
- The external reactions are first computed as with the method of joints, by considering the truss as a whole.
- Let us determine the force in the member BE, for example.
- An imaginary section, indicated by the dashed line, is passed through the truss, cutting it into two parts
- This section has cut three members whose forces are initially unknown.
- In order for the portion of the truss on each side of the section to remain in equilibrium, it is necessary to apply to each cut member the force which was exerted on it by the member cut away.



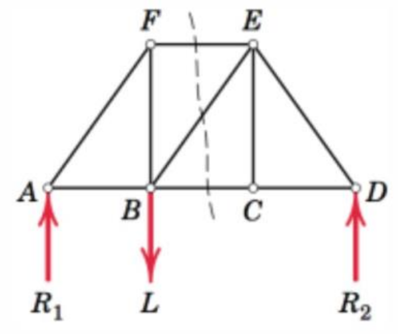
(a)



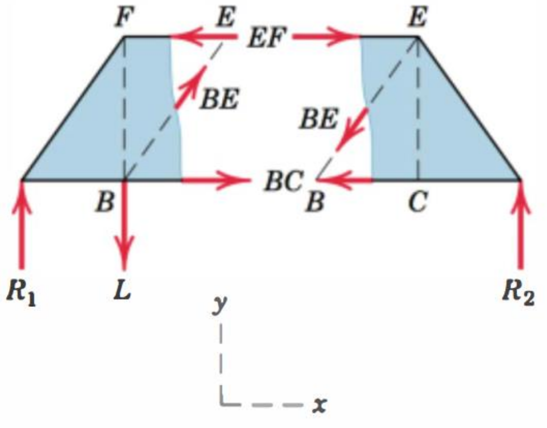
(b)

Method of Section

- For simple trusses composed of straight two-force members, these forces, either tensile (T) or compressive (C), will always be in the directions of the respective members.
- The left-hand section is in equilibrium under the action of the applied load L, the end reaction R1 and the three forces exerted on the cut members by the right-hand section which has been removed.
- We can usually draw the forces with their proper senses by a visual approximation of the equilibrium requirements. Thus, in balancing the
- moments about point B for the left-hand section, the force EF is clearly to the left, which makes it compressive, because it acts toward the cut section of member EF.



(a)



(b)

SIMPLE TRUSSES

Internal and External Redundancy

- If a plane truss has more external supports than are necessary to ensure a stable equilibrium configuration, the truss as a whole is statically indeterminate, and the extra supports constitute external redundancy.
- If a truss has more internal members than are necessary to prevent collapse when the truss is removed from its supports, then the extra members constitute internal redundancy and the truss is again statically indeterminate.
- For a truss which is statically determinate externally, there is a definite relation between the number of its members and the number of its joints necessary for internal stability without redundancy.

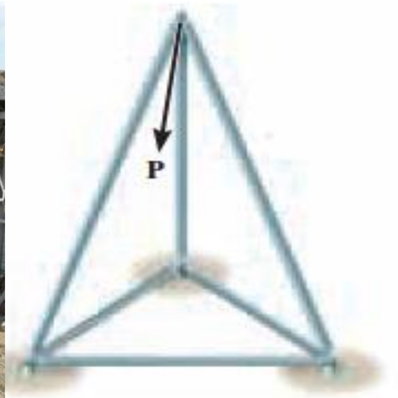
SIMPLE TRUSSES

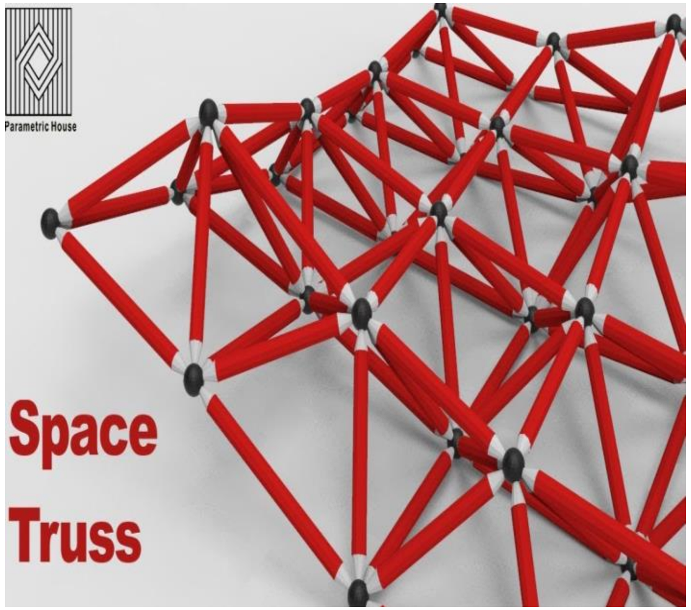
Internal and External Redundancy

- For the entire truss composed of m two-force members and having the maximum of three unknown support reactions, there are in all $m + 3$ unknowns (m tension or compression forces and three reactions).
- Thus, for any plane truss, the equation $m + 3 = 2j$ will be satisfied if the truss is statically determinate internally.
- If $m + 3 > 2j$, there are more members than independent equations, and the truss is statically indeterminate internally with redundant members present.
- If $m + 3 < 2j$, there is a deficiency of internal members, and the truss is unstable and will collapse under load.

SPACE (3D) TRUSSES

- A space truss consists of members joined together at their ends to form a stable three-dimensional structure.
- The simplest form of a space truss is a tetrahedron, formed by connecting six members together, as shown in Fig. 6–19.
- Any additional members added to this basic element would be redundant in supporting the force P.
- A simple space truss can be built from this basic tetrahedral element by adding three additional members and a joint, and continuing in this manner to form a system of multiconnected tetrahedrons.





SPACE (3D) TRUSSES

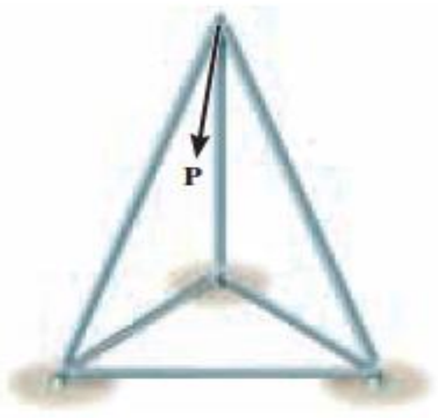
Statically Determinate Space Trusses

- When a space truss is supported externally so that it is statically determinate as an entire unit, a relationship exists between the number of its joints and the number of its member necessary for internal stability without redundancy.
- Because the equilibrium of each joint is specified by three scalar force equations, there are in all 3j such equations for a space truss with **j** joints.
- For the entire truss composed of m members there are **m** unknowns (the tensile or compressive forces in the members) plus six unknown support reactions in the general case of a statically determinate space structure.
- For any space truss, the equation $m + 6 = 3j$ will be satisfied if the truss is statically determinate internally and stable.
- A simple space truss satisfies this relation automatically

SPACE (3D) TRUSSES

Statically Determinate Space Trusses

- Starting with the initial tetrahedron, for which the equation holds, the structure is extended by adding three members and one joint at a time.
- If $m + 6 > 3j$, there are more members than there are independent equations, and the truss is statically indeterminate internally with redundant members present.
- If $m + 6 < 3j$, there is a deficiency of internal members, and the truss is unstable and subject to collapse under load.
- This relationship between the number of joints and the number of members is very helpful in the preliminary design of a stable space truss, since the configuration is not as obvious as with a plane truss, where the geometry for statically determinacy is generally quite apparent.



SPACE (3D) TRUSSES

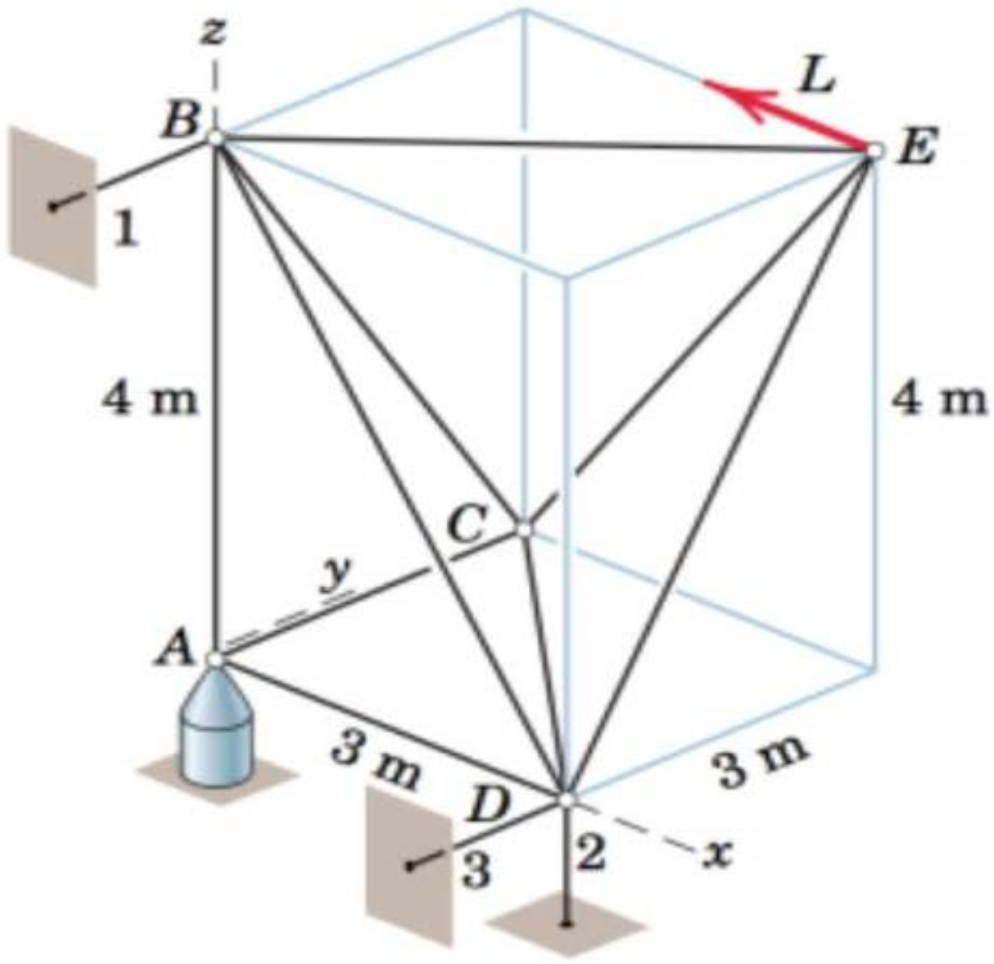
- **<u>Method of Joints</u>** may also be extended directly to space trusses by satisfying the complete vector equation $\sum F = 0$ for each joint.
- We normally begin the analysis at a joint where at least one *<u>known</u>* force acts and not <u>more than three unknown forces</u> are present.
- Adjacent joints on which not more than three unknown forces act may then be analyzed in turn.
- **<u>Method of Sections for Space Trusses</u>** needs to satisfy $\sum F = 0$ & $\sum M = 0$ for any section of the truss.
- Because the two vector equations are equivalent to six scalar equations, we conclude that, a section should not be passed through more than six members whose forces are unknown.

SPACE (3D) TRUSSES

Example

Question

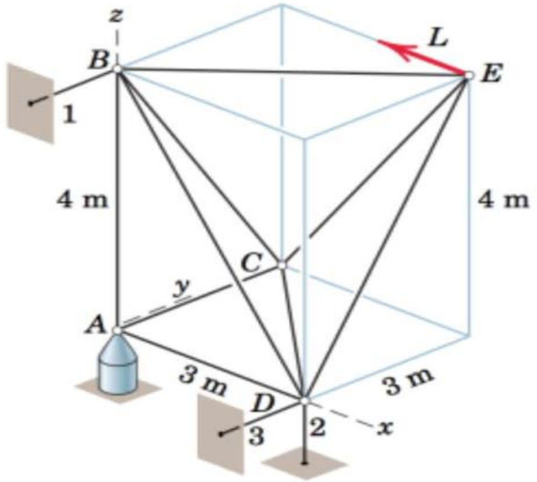
- The space truss consists of the rigid tetrahedron ABCD anchored by a ball-and-socket connection at A and prevented from any rotation about the x-, y-, or z-axes by the respective links 1, 2 , and 3. The load L is applied to joint E, which is rigidly fixed to the tetrahedron by the three additional links.
- Solve for the forces in the members at joint E and indicate the procedure for the determination of the forces in the remaining members of the truss.

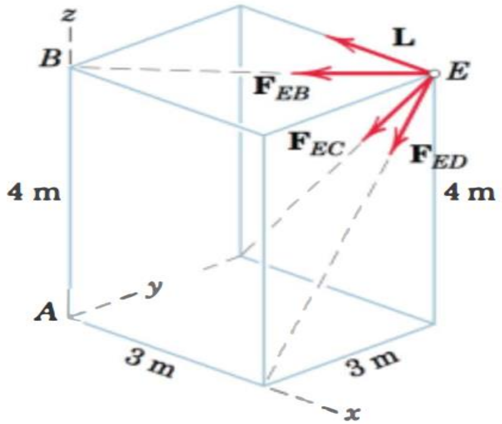


Example

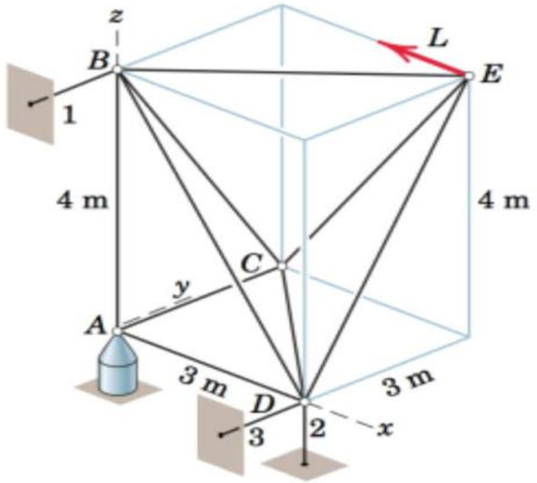
Solution

- We note first that with m = 9 members and j = 5 joints, the condition m + 6 = 3j (9+6 = 15 & 3*5 = 15) for a sufficiency of members to provide a noncollapsible structure is satisfied.
- The external reactions at A, B, and D can be calculated easily as a first step, although their values will be determined from the solution of all forces on each of the joints in succession.



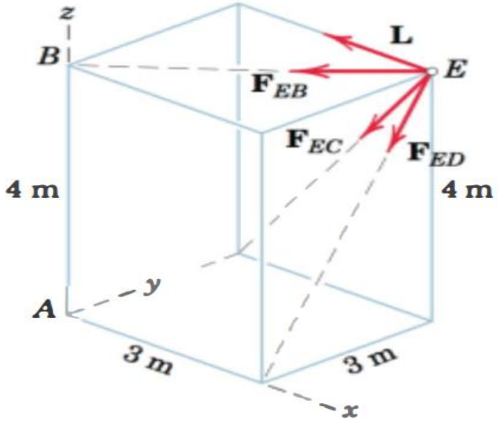


SPACE (3D) TRUSSES
Example



Solution

- We start with a joint on which at least one known force and not more than three unknown forces act, which in this case is joint E.
- The FBD of joint E is shown with all force vectors arbitrarily assumed in their positive tension directions (away from the joint). The vector expressions for the three unknown forces are



$$\mathbf{F}_{EB} = \frac{F_{EB}}{\sqrt{2}}(-\mathbf{i} - \mathbf{j}), \qquad \mathbf{F}_{EC} = \frac{F_{EC}}{5}(-3\mathbf{i} - 4\mathbf{k}), \qquad \mathbf{F}_{ED} = \frac{F_{ED}}{5}(-3\mathbf{j} - 4\mathbf{k})$$

SPACE (3D) TRUSSES

Example

Solution

Equilibrium of joint E requires

$$[\Sigma \mathbf{F} = 0] \qquad \mathbf{L} + \mathbf{F}_{EB} + \mathbf{F}_{EC} + \mathbf{F}_{ED} = \mathbf{0} \qquad \text{or}$$

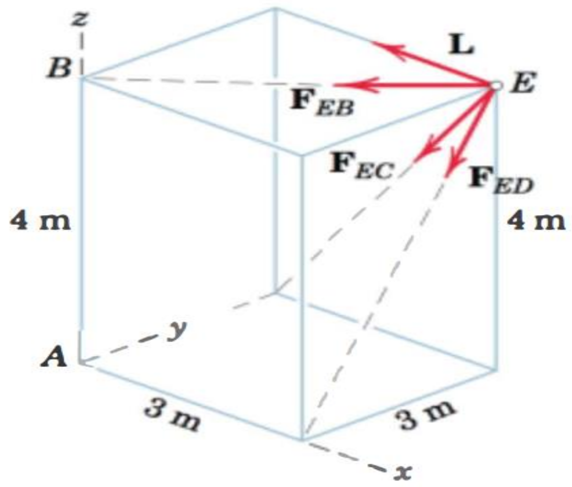
$$-L\mathbf{i} + \frac{F_{EB}}{\sqrt{2}}(-\mathbf{i} - \mathbf{j}) + \frac{F_{EC}}{5}(-3\mathbf{i} - 4\mathbf{k}) + \frac{F_{ED}}{5}(-3\mathbf{j} - 4\mathbf{k}) = \mathbf{0}$$

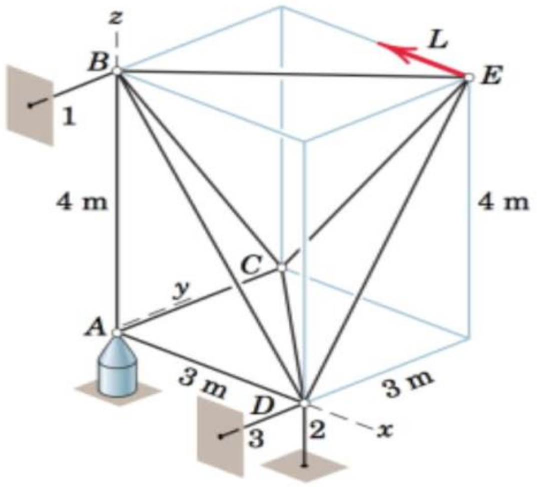
Rearranging terms gives

$$\left(-L - \frac{F_{EB}}{\sqrt{2}} - \frac{3F_{EC}}{5}\right)\mathbf{i} + \left(-\frac{F_{EB}}{\sqrt{2}} - \frac{3F_{ED}}{5}\right)\mathbf{j} + \left(-\frac{4F_{EC}}{5} - \frac{4F_{ED}}{5}\right)\mathbf{k} = \mathbf{0}$$

Equating the coefficients of the **i**-, **j**-, and **k**-unit vectors to zero gives the three equations

$$\frac{F_{EB}}{\sqrt{2}} + \frac{3F_{EC}}{5} = -L \qquad \frac{F_{EB}}{\sqrt{2}} + \frac{3F_{ED}}{5} = 0 \qquad F_{EC} + F_{ED} = 0$$



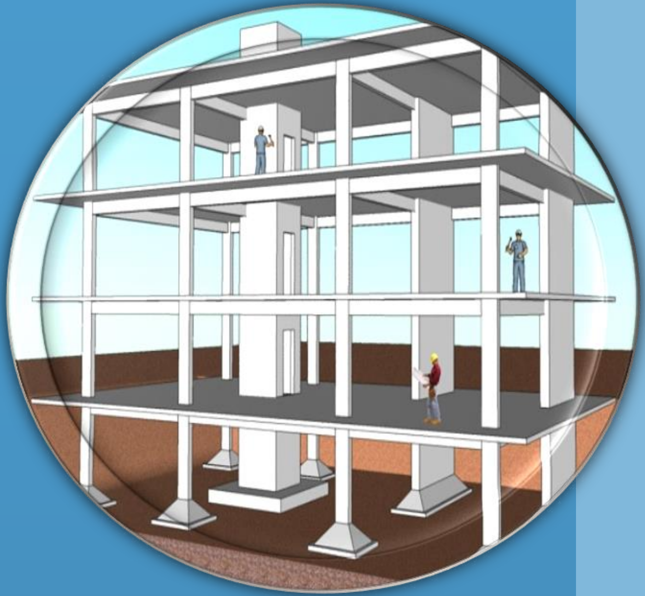


Solving the equations gives us

$$F_{EB} = -L/\sqrt{2} \qquad F_{EC} = -5L/6 \qquad F_{ED} = 5L/6 \qquad \textit{Ans.}$$

Thus, we conclude that F_{EB} and F_{EC} are compressive forces and F_{ED} is tension.

Unless we have computed the external reactions first, we must next analyze joint C with the known value of F_{EC} and the three unknowns F_{CB}, F_{CA}, and F_{CD}. The procedure is identical to that used for joint E. Joints B, D, and A are then analyzed in the same way and in that order, which limits the scalar unknowns to three for each joint. The external reactions computed from these analyses must, of course, agree with the values which can be determined initially from an analysis of the truss as a whole.



Thank You!