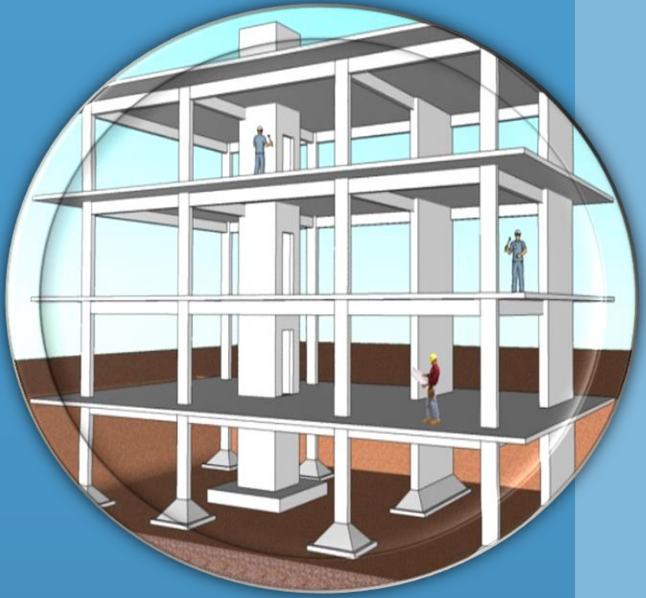
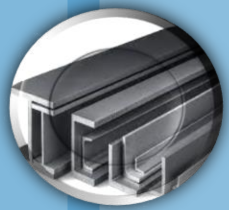
UNIVERSITY OF ZAMBIA
School of Engineering
Department of Civil & Environmental Eng.



CEE 3222: THEORY OF STRUCTURES

Lecture 2.1

INFLUENCE LINES FOR STATICALLY DETERMINATE STRUCTURES- BEAMS



Mr. MWABA MSc, B.Eng., R.Eng., PEIZ,

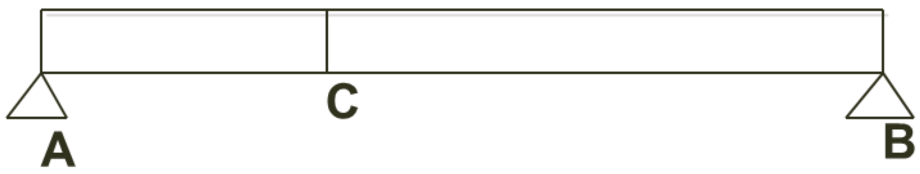
Contents

Influence Lines for Statically Determinate Structures

- Introduction - What is an influence line?
- Influence lines for beams
- Qualitative influence lines - Muller-Breslau Principle
- Influence lines for floor girders
- Influence lines for trusses
- Live loads for bridges
- Maximum influence at a point due to a series of concentrated loads
- Absolute maximum shear and moment

Introduction To Influence Lines

- Influence lines describe the variation of an analysis variable (reaction, shear force, bending moment, twisting moment, deflection, etc.) at a point (say at C in Figure 6.1)
- Why do we need the influence lines? For instance, when loads pass over a structure, say a bridge, one needs to know when the maximum values of shear/reaction/bending-moment will occur at a point so that the section may be designed.



- Notations:
  - Normal Forces - +ve forces cause +ve displacements in +ve directions
  - Shear Forces - +ve shear forces cause clockwise rotation & - ve shear force causes anti-clockwise rotation
  - Bending Moments: +ve bending moments cause “cup holding water” deformed shape

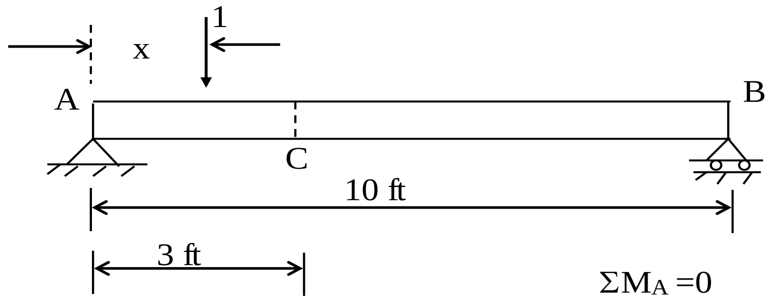
Influence Lines For Beams

- Procedure:
  (1) Allow a unit load (either 1b, 1N, 1kip, or 1 tonne) to move over beam from left to right
  (2) Find the values of shear force or bending moment, at the point under consideration, as the unit load moves over the beam from left to right
  (3) Plot the values of the shear force or bending moment, over the length of the beam, computed for the point under consideration

Influence Lines For Beams

3.3 MOVING CONCENTRATED LOAD

3.3.1 Variation of Reactions R_A and R_B as functions of load position



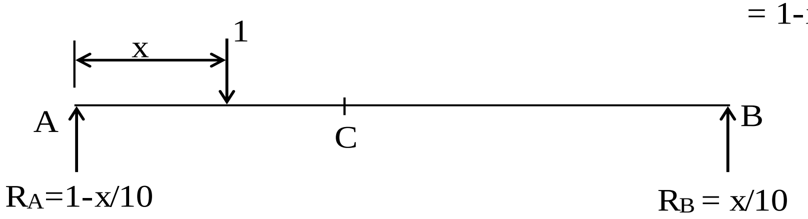
$\Sigma M_A = 0$

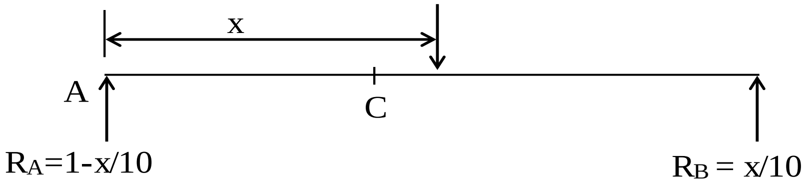
$(R_B)(10) - (1)(x) = 0$

$R_B = x/10$

$R_A = 1 - R_B$

$= 1 - x/10$



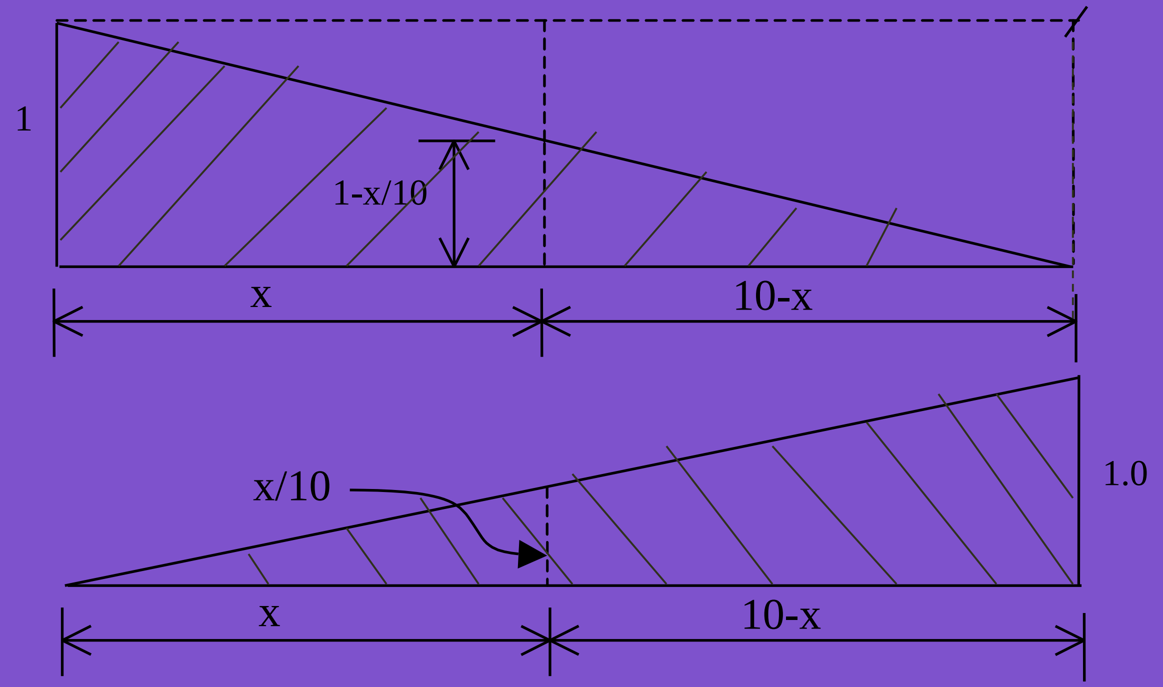


Influence Lines For Beams

R_A occurs only at A; R_B occurs only at B

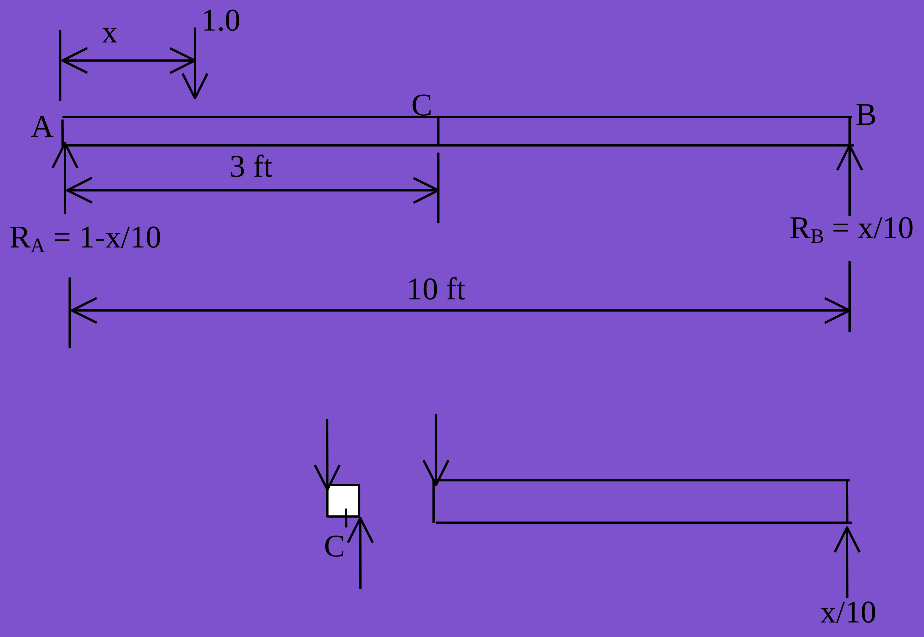
Influence line for R_A

Influence line for R_B



3.3.2 Variation of Shear Force at C as a function of load position

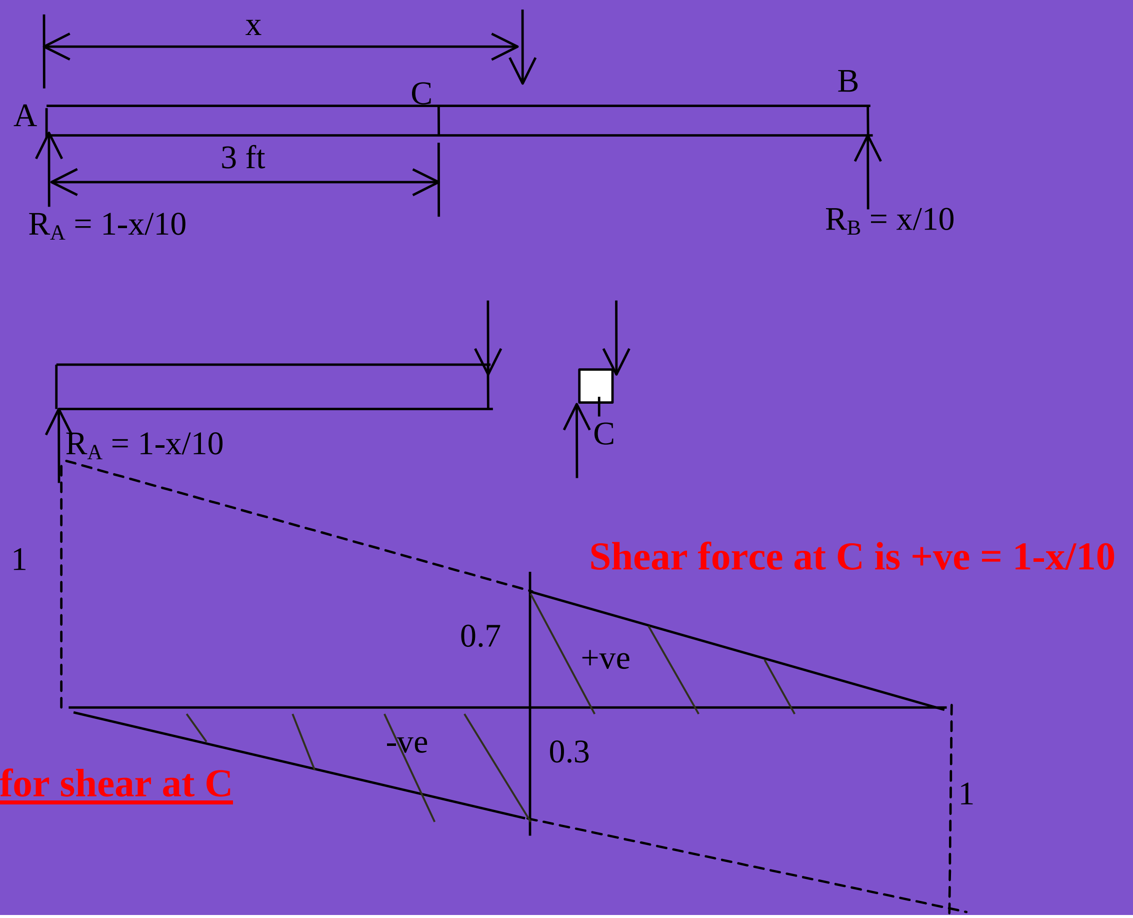
0 < x < 3 ft (unit load to the left of C)



Shear force at C is –ve, V_C =-x/10

Influence Lines For Beams

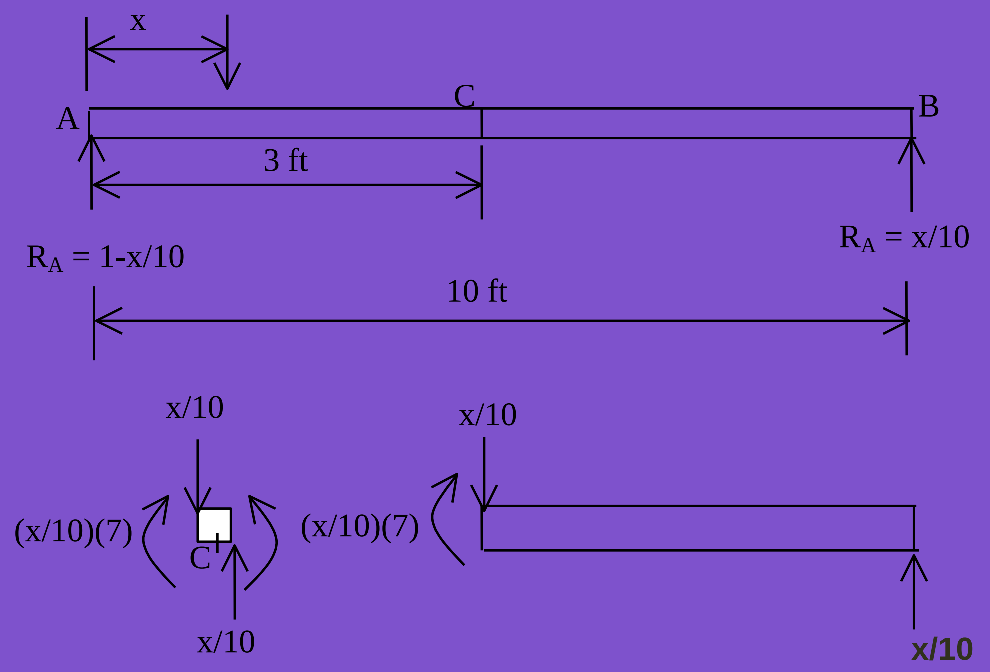
$3 < x < 10$ ft (unit load to the right of C)



Influence line for shear at C

3.3.3 Variation of Bending Moment at C as a function of load position

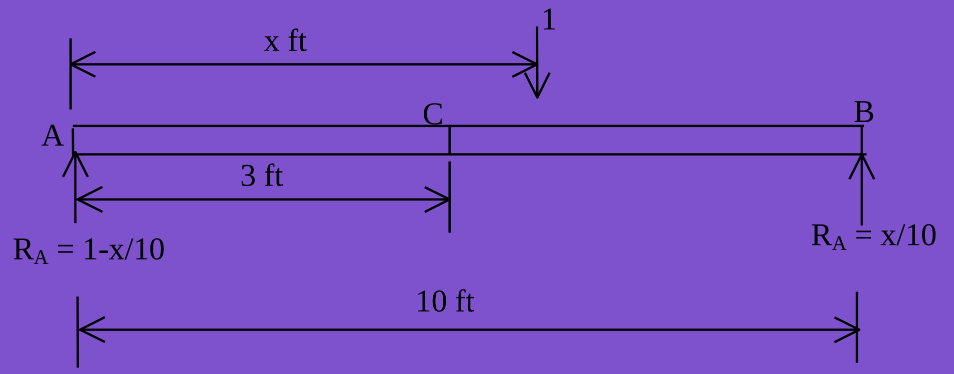
0 < x < 3.0 ft (Unit load to the left of C)

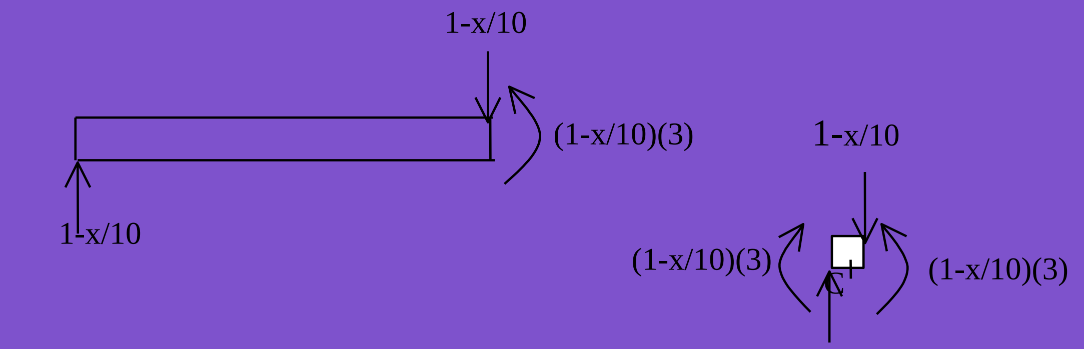


Bending moment is +ve at C

Influence Lines For Beams

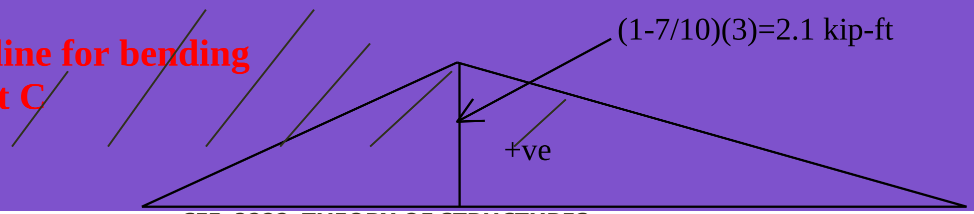
$3 < x < 10$ ft (Unit load to the right of C)



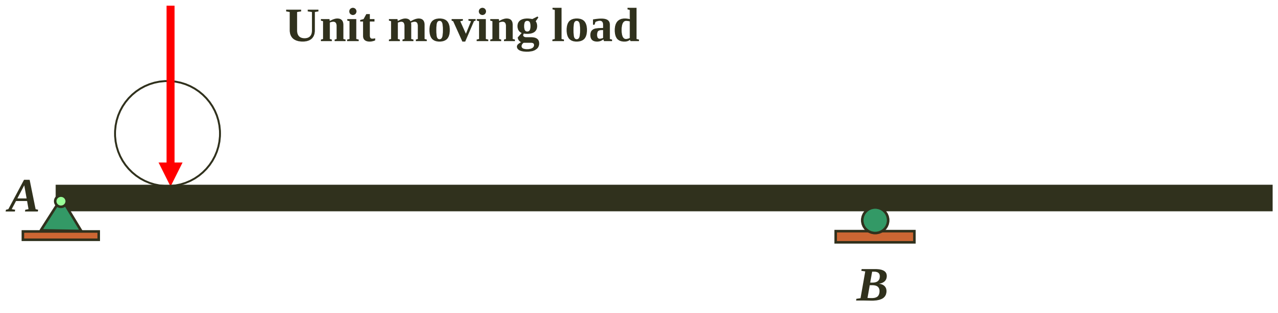


Moment at C is +ve

Influence line for bending Moment at C



Influence Lines For Beams



Procedure for Analysis

Either of the following two procedures can be used to construct the influence line at a specific point *P* in a member for any function (reaction, shear, or moment). For both of these procedures we will choose the moving force to have a *dimensionless magnitude of unity.**

Tabulate Values

- Place a unit load at various locations, *x*, along the member, and at *each* location use statics to determine the value of the function (reaction, shear, or moment) at the specified point.
- If the influence line for a vertical force *reaction* at a point on a beam is to be constructed, consider the reaction to be *positive* at the point when it acts *upward* on the beam.
- If a shear or moment influence line is to be drawn for a point, take the shear or moment at the point as positive according to the same sign convention used for drawing shear and moment diagrams. (See Fig. 4–1.)
- All statically determinate beams will have influence lines that consist of straight line segments. After some practice one should be able to minimize computations and locate the unit load *only* at points representing the *end points* of each line segment.
- To avoid errors, it is recommended that one first construct a table, listing "unit load at *x*" versus the corresponding value of the function calculated at the specific point; that is, "reaction *R*," "shear *V*," or "moment *M*." Once the load has been placed at various points along the span of the member, the tabulated values can be plotted and the influence-line segments constructed.

Influence-Line Equations

- The influence line can also be constructed by placing the unit load at a *variable* position *x* on the member and then computing the value of *R*, *V*, or *M* at the point as a function of *x*. In this manner, the equations of the various line segments composing the influence line can be determined and plotted.

*The reason for this choice will be explained in Sec. 6.2.

Influence Lines For Beams

Example 1

Construct the influence line for

a) reaction at *A* and *B*

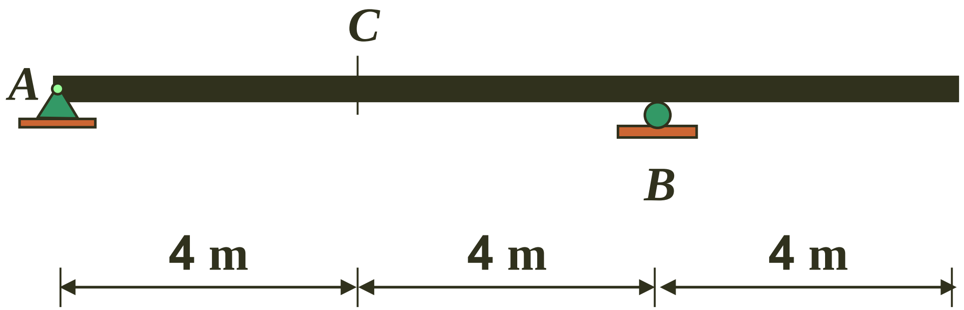
b) shear at point C

c) bending moment at point C

d) shear before and after support *B*

e) moment at point *B*

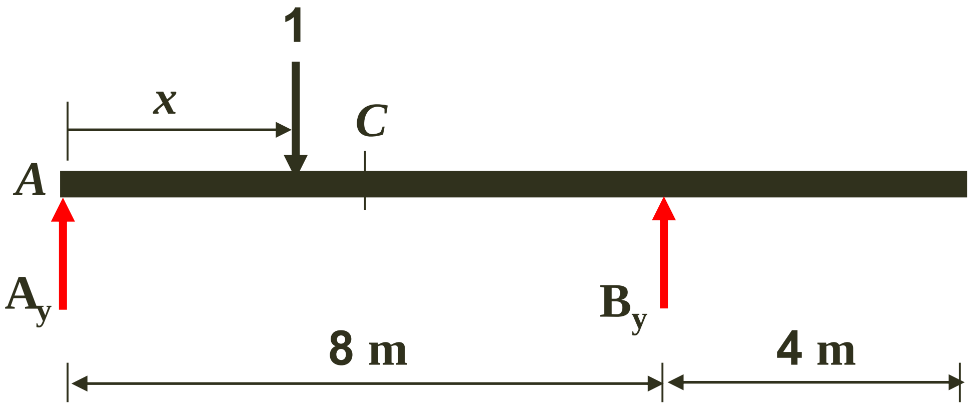
of the beam in the figure below.



Influence Lines For Beams

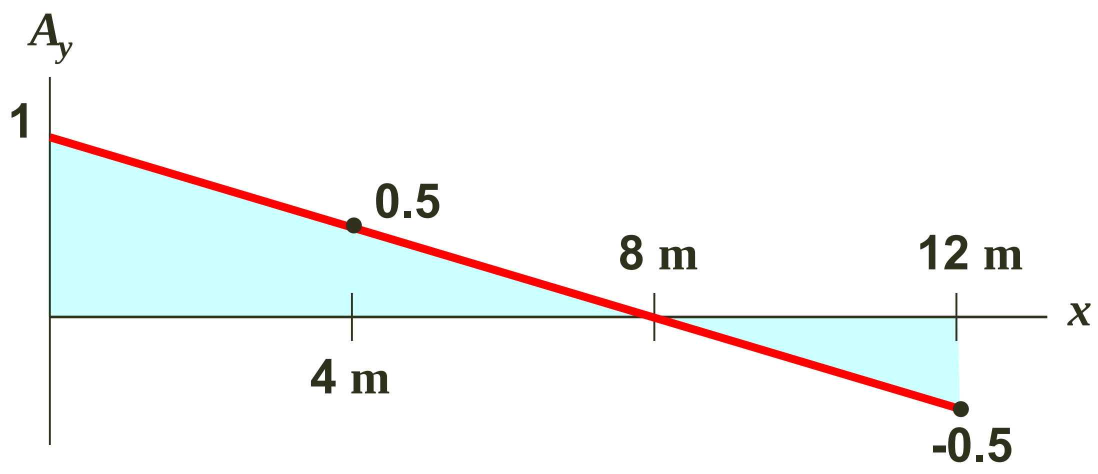
<u>SOLUTION</u>

- Reaction at *A*



x	A_y
0	1
4	0.5
8	0
12	-0.5

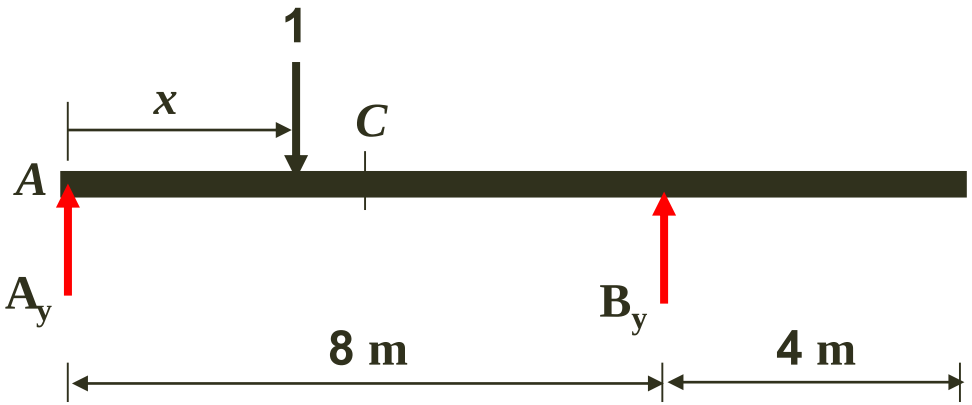
$+\circlearrowright \Sigma M_B = 0$: $\quad -A_y(8) + 1(8 - x) = 0, \quad A_y = 1 - \frac{1}{8}x$



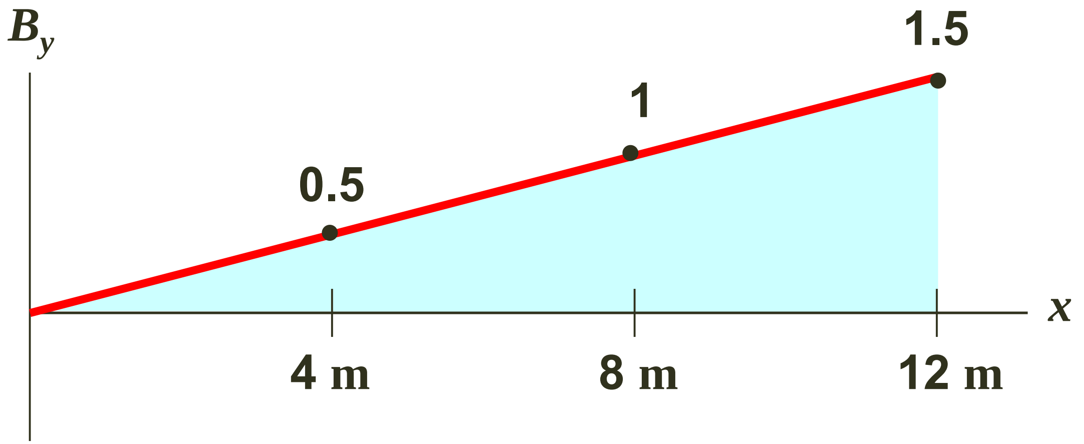
Influence Lines For Beams

- Reaction at *B*

x	B_y
0	0
4	0.5
8	1
12	1.5

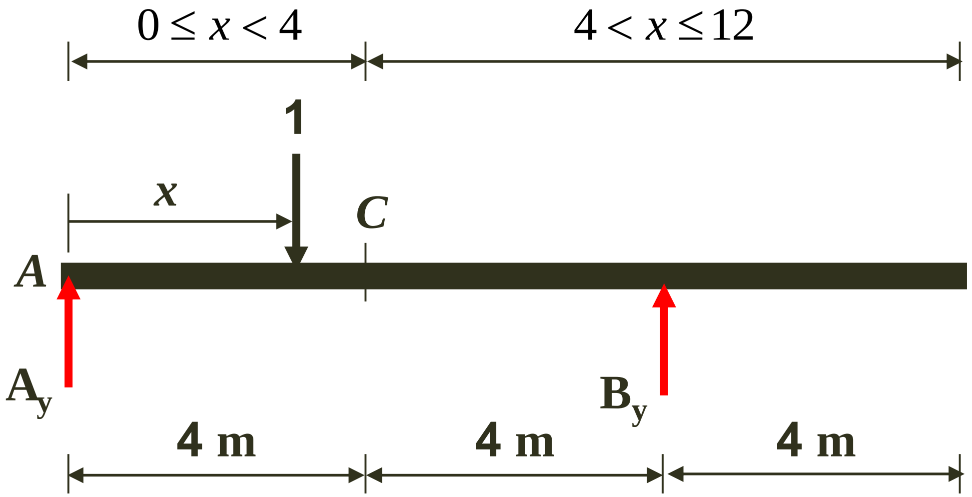


$+\circlearrowleft \Sigma M_A = 0$: $\quad B_y(8) - 1x = 0, \quad B_y = \frac{1}{8}x$

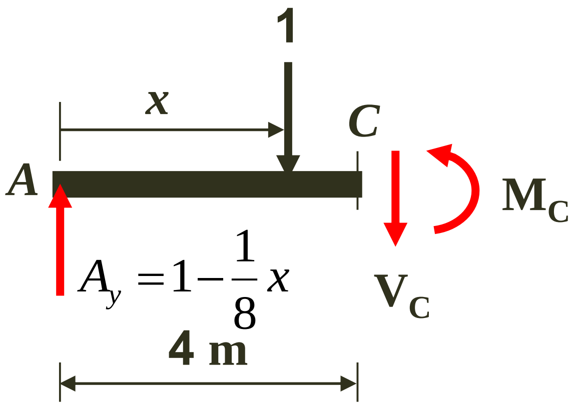


Influence Lines For Beams

- **Shear at *C***



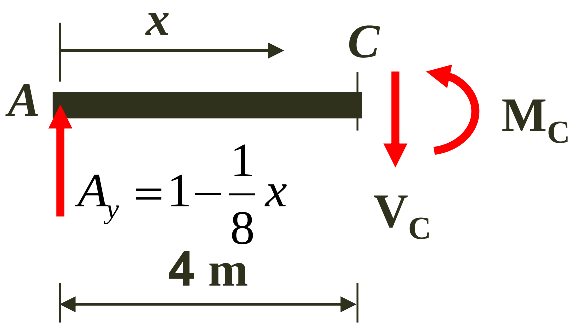
$0 \le x \le 4$



$+\uparrow \Sigma F_y = 0$: $\quad 1 - \frac{1}{8}x - 1 - V_C = 0$

$$V_C = -\frac{1}{8}x$$

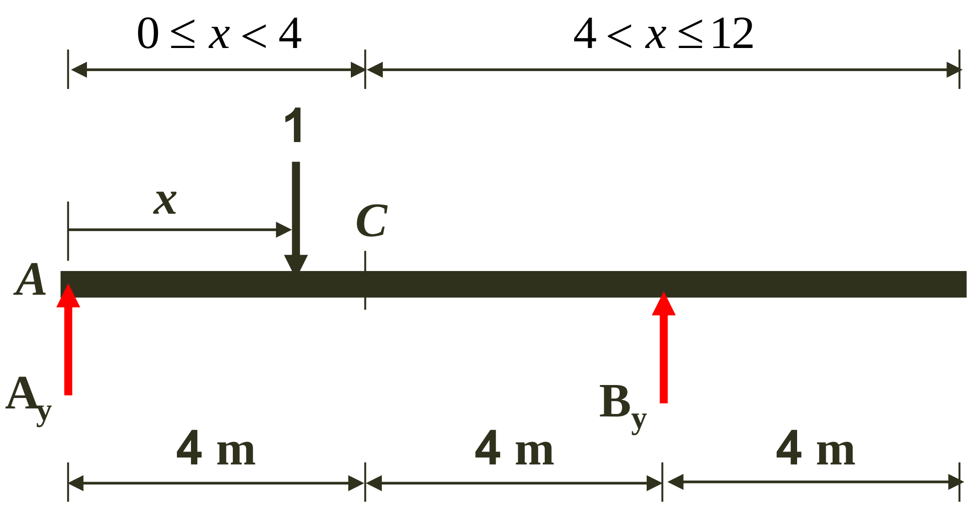
$4 < x \le 12$



$+\uparrow \Sigma F_y = 0$: $\quad 1 - \frac{1}{8}x - V_C = 0$

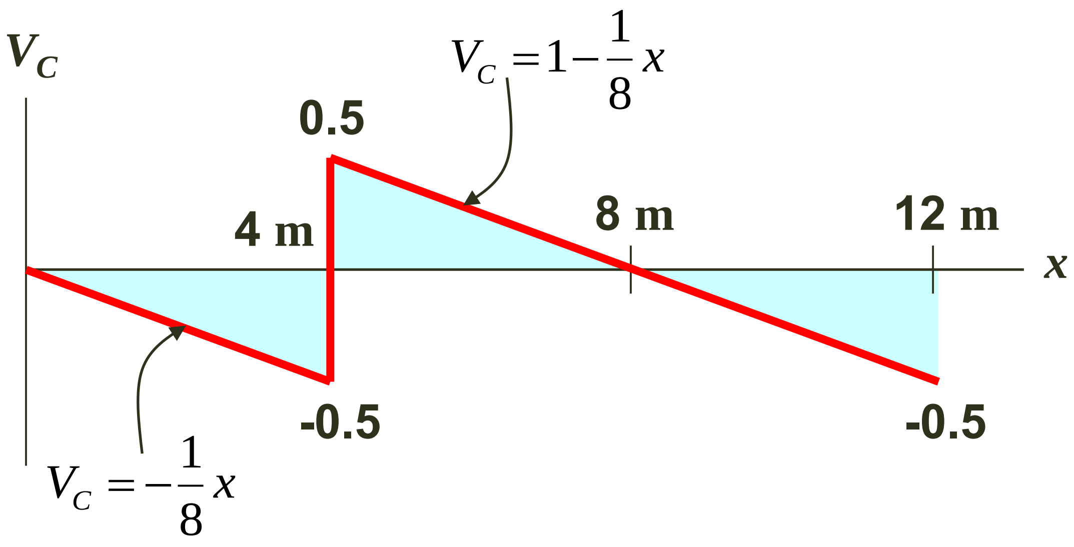
$$V_C = 1 - \frac{1}{8}x$$

Influence Lines For Beams



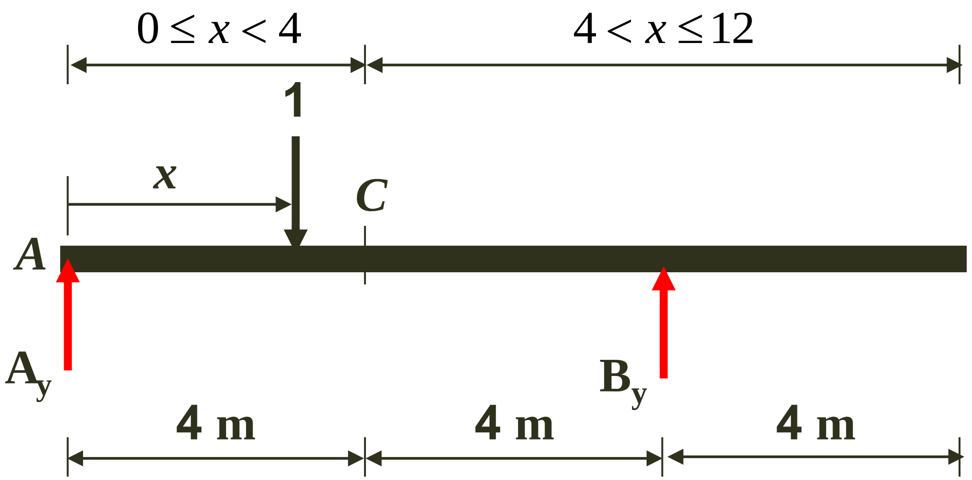
$V_C = -\frac{1}{8}x$ for x = 0 to 4^-; $V_C = 1 - \frac{1}{8}x$ for x = 4^+ to 12.

x	V_C
0	0
4^-	-0.5
4^+	0.5
8	0
12	-0.5

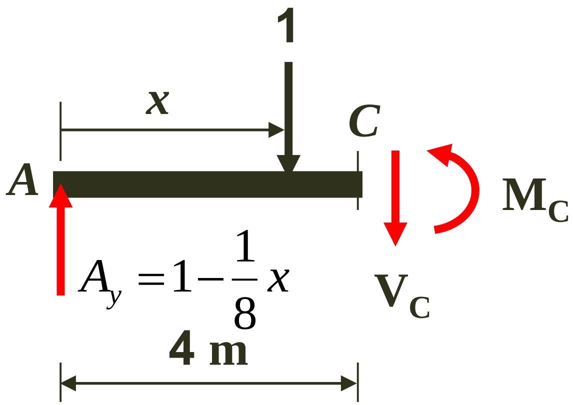


Influence Lines For Beams

- **Bending moment at *C***



$0 \le x \le 4$

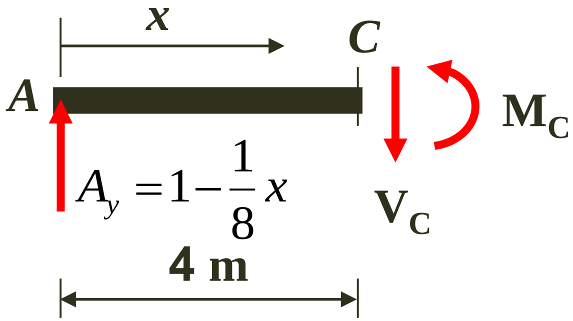


$A_y = 1 - \frac{1}{8}x$

$\circlearrowleft + \; \Sigma M_C = 0: \quad M_C + 1(4 - x) - (1 - \frac{1}{8}x)(4) = 0$

$$M_C = \frac{1}{2}x$$

$4 < x \le 12$

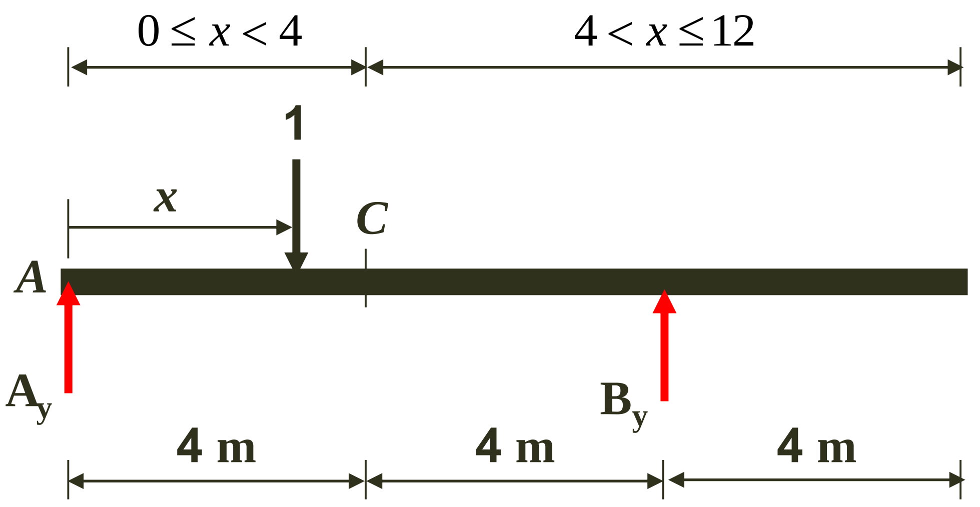


$A_y = 1 - \frac{1}{8}x$

$\circlearrowleft + \; \Sigma M_C = 0: \quad M_C - (1 - \frac{1}{8}x)(4) = 0$

$$M_C = 4 - \frac{1}{2}x$$

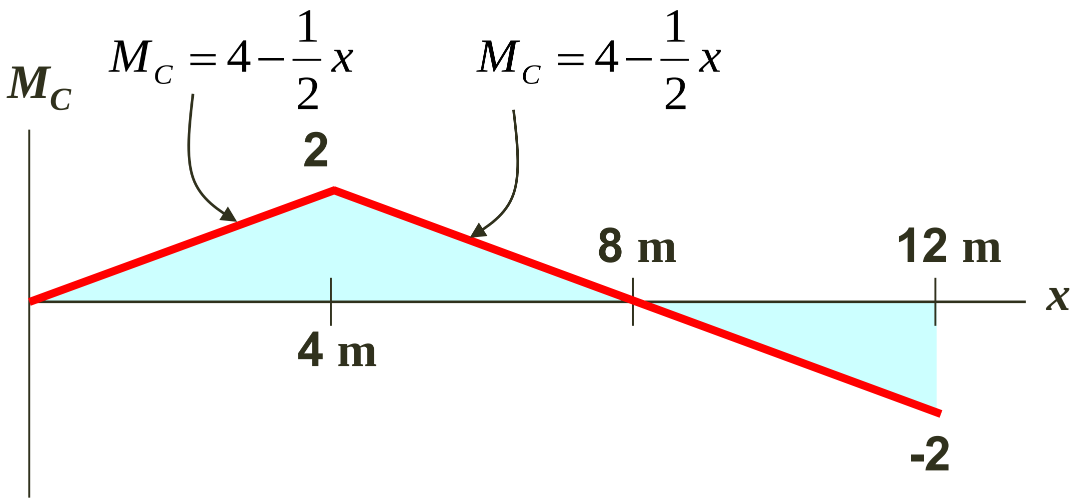
Influence Lines For Beams - Moment



x	M_C
0	0
4	2
8	0
12	-2

$M_C = \frac{1}{2}x$ (for x = 0, 4)

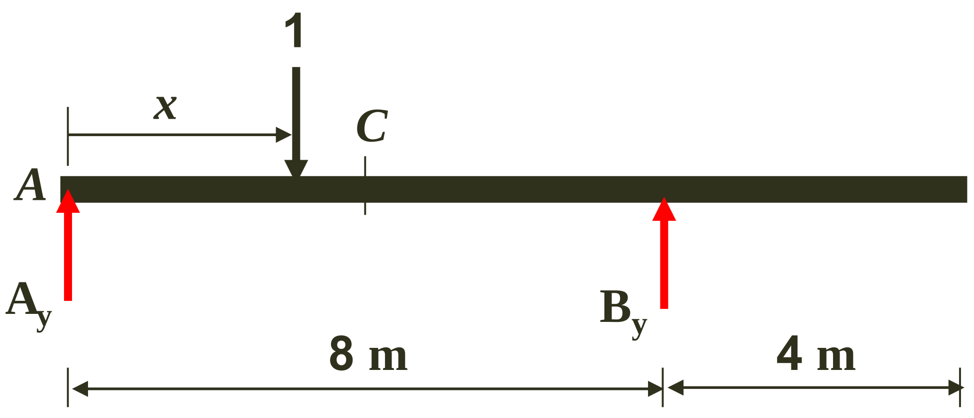
$M_C = 4 - \frac{1}{2}x$ (for x = 8, 12)



Influence Lines For Beams

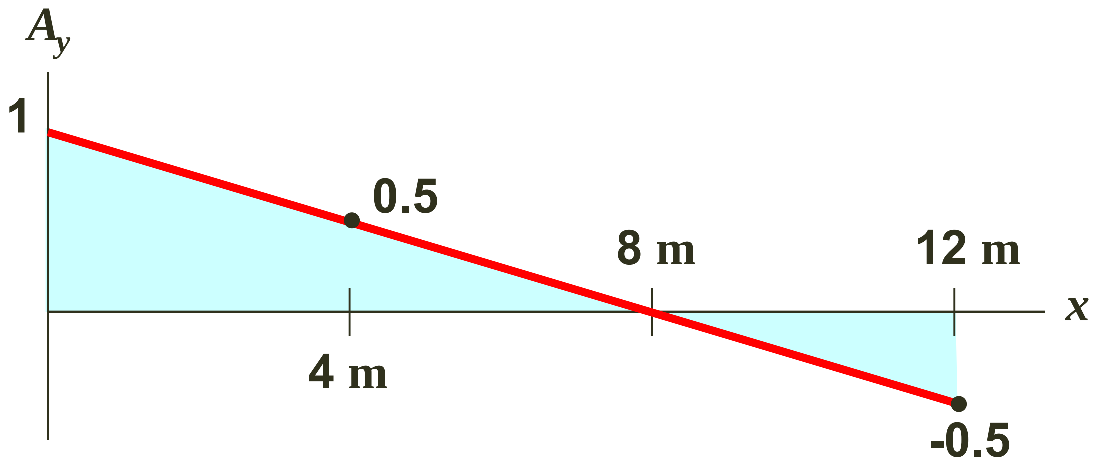
Or using equilibrium conditions:

- Reaction at *A*



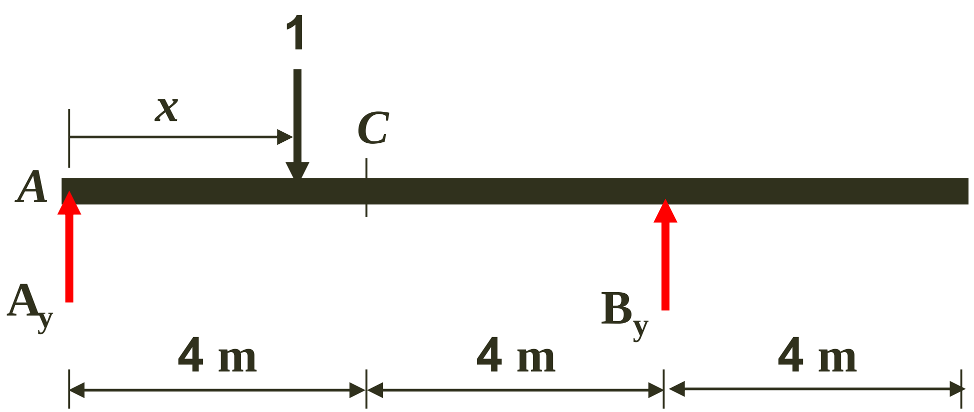
x	A_y
0	1
4	0.5
8	0
12	-0.5

$+\circlearrowleft \Sigma M_B = 0: \quad -A_y(8) + 1(8 - x) = 0, \quad A_y = 1 - \frac{1}{8}x$

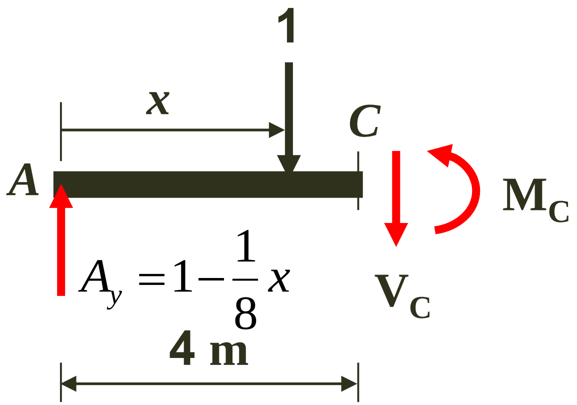


Influence Lines For Beams

- **Shear at *C***



$0 \le x \le 4$

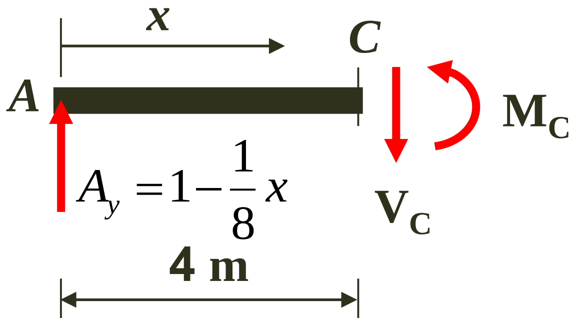


$+\uparrow \Sigma F_y = 0$:

$$A_y - 1 - V_C = 0$$

$$V_C = A_y - 1$$

$4 < x \le 12$

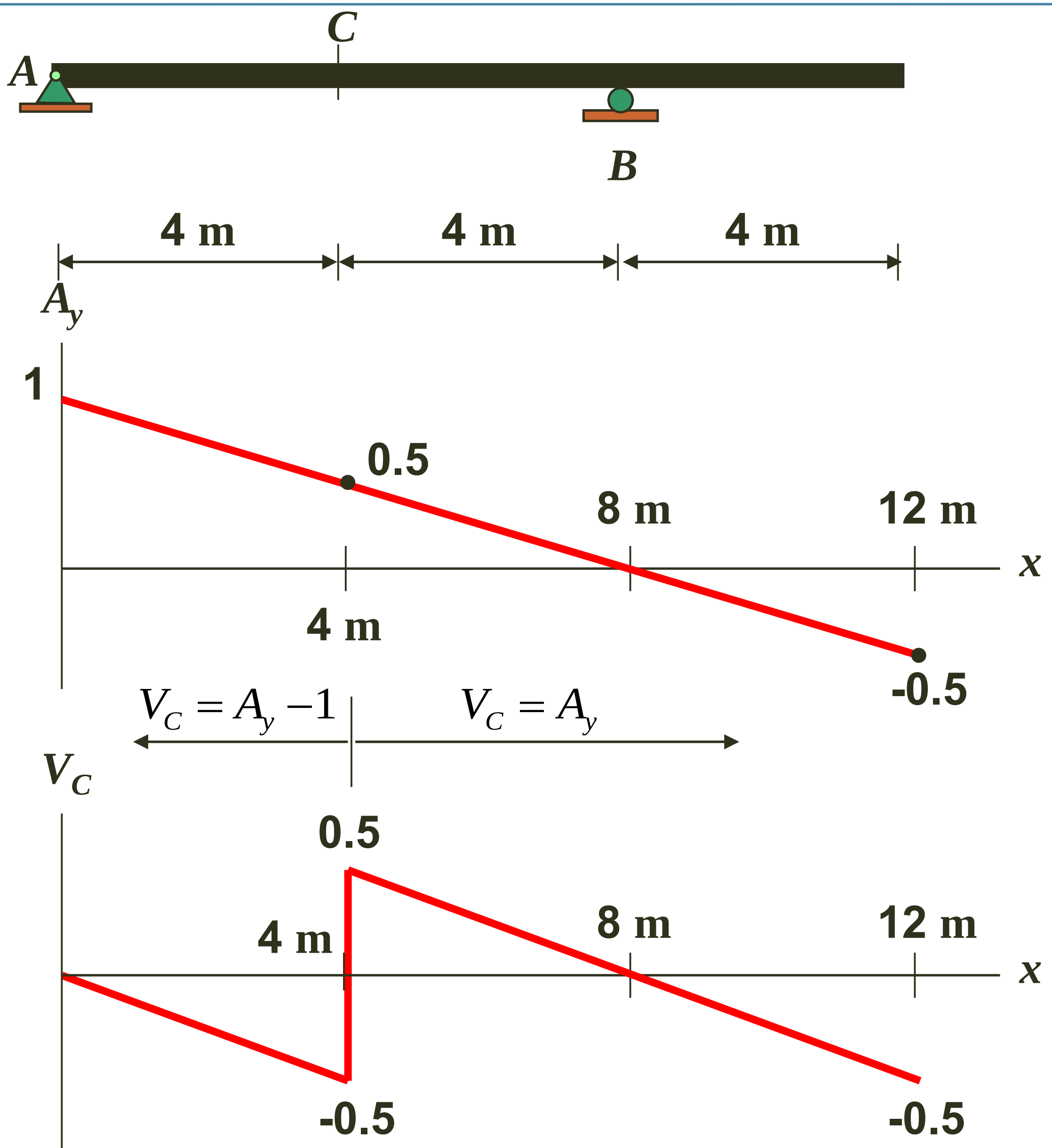


$+\uparrow \Sigma F_y = 0$:

$$A_y - V_C = 0$$

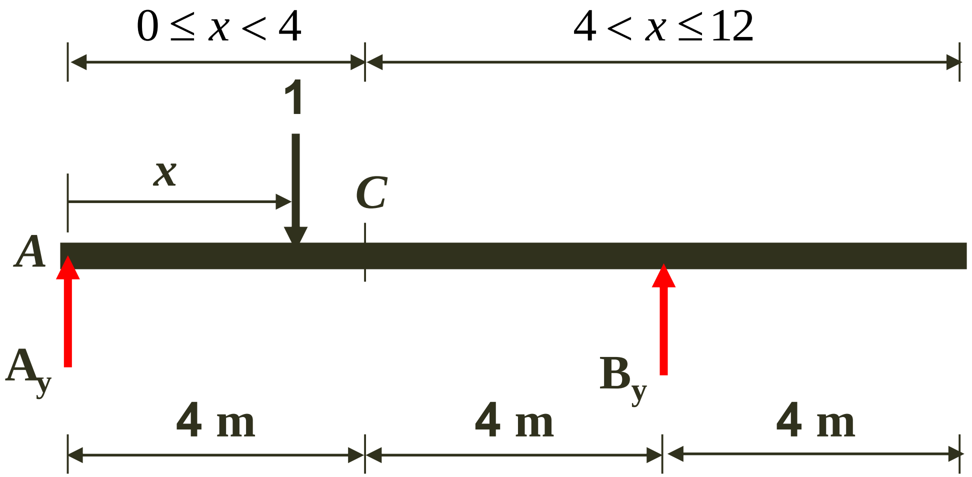
$$V_C = A_y$$

Influence Lines For Beams

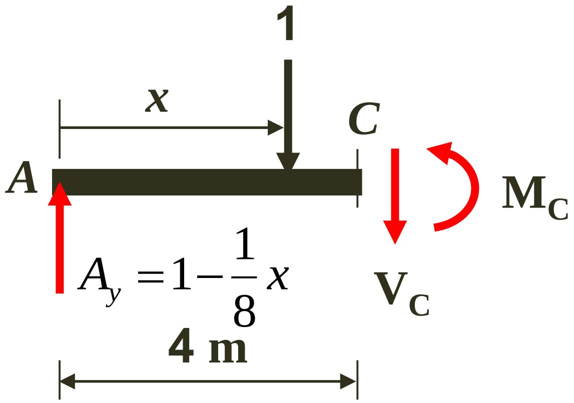


Influence Lines For Beams

- Bending moment at *C*



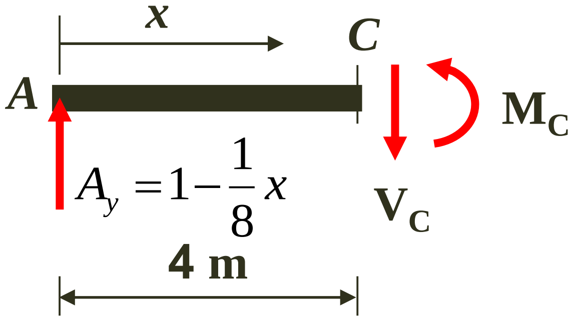
$0 \le x \le 4$



$\circlearrowleft + \; \Sigma M_C = 0$: $\quad A_y(4) + 1(4 - x) + M_C = 0$

$$M_C = 4A_y - (4 - x)$$

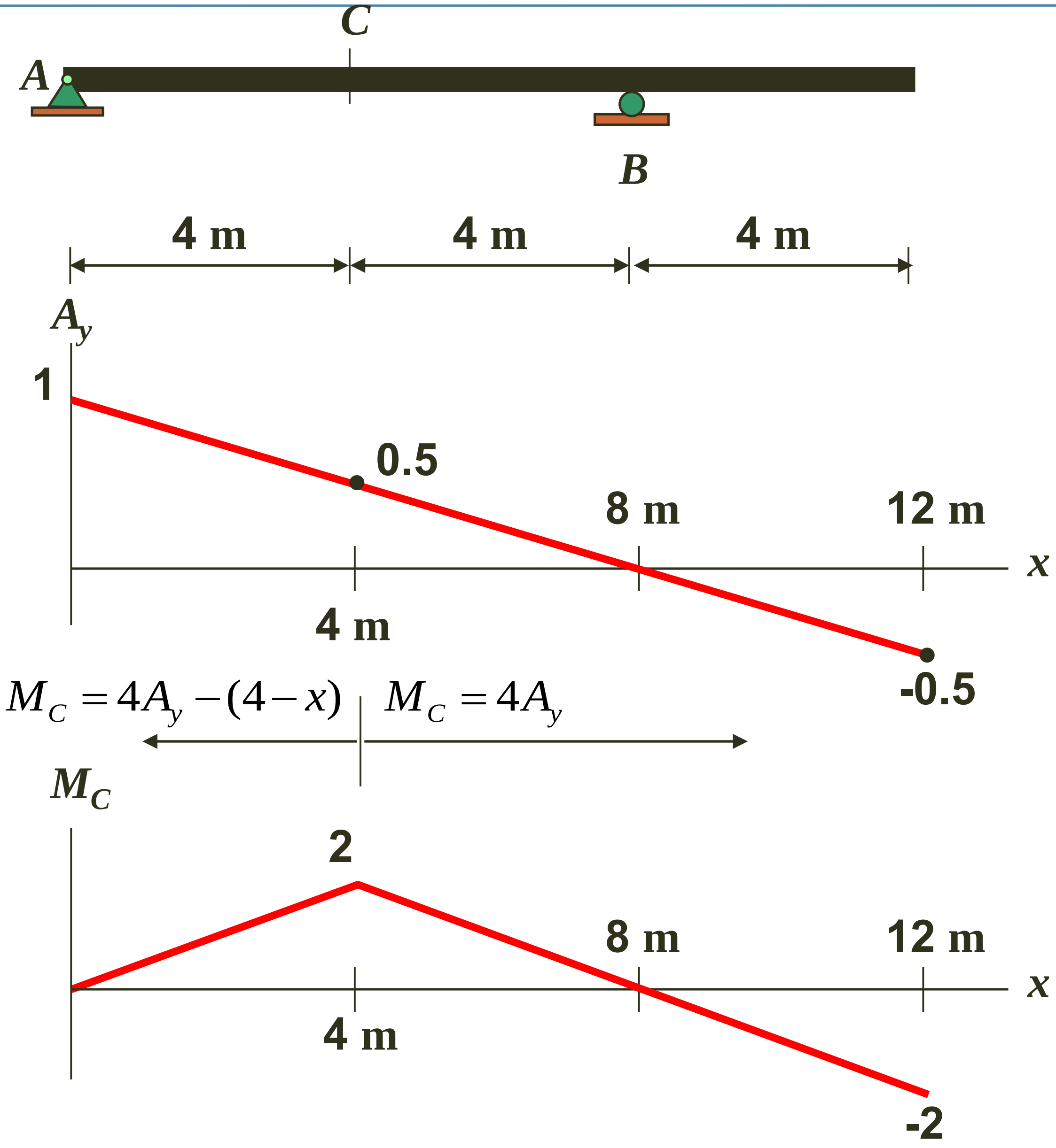
$4 < x \le 12$



$\circlearrowleft + \; \Sigma M_C = 0$: $\quad -A_y(4) + M_C = 0$

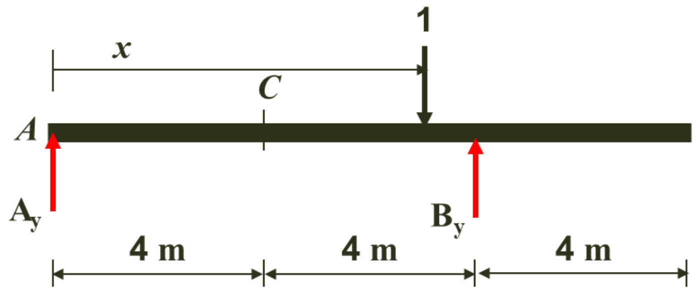
$$M_C = 4A_y$$

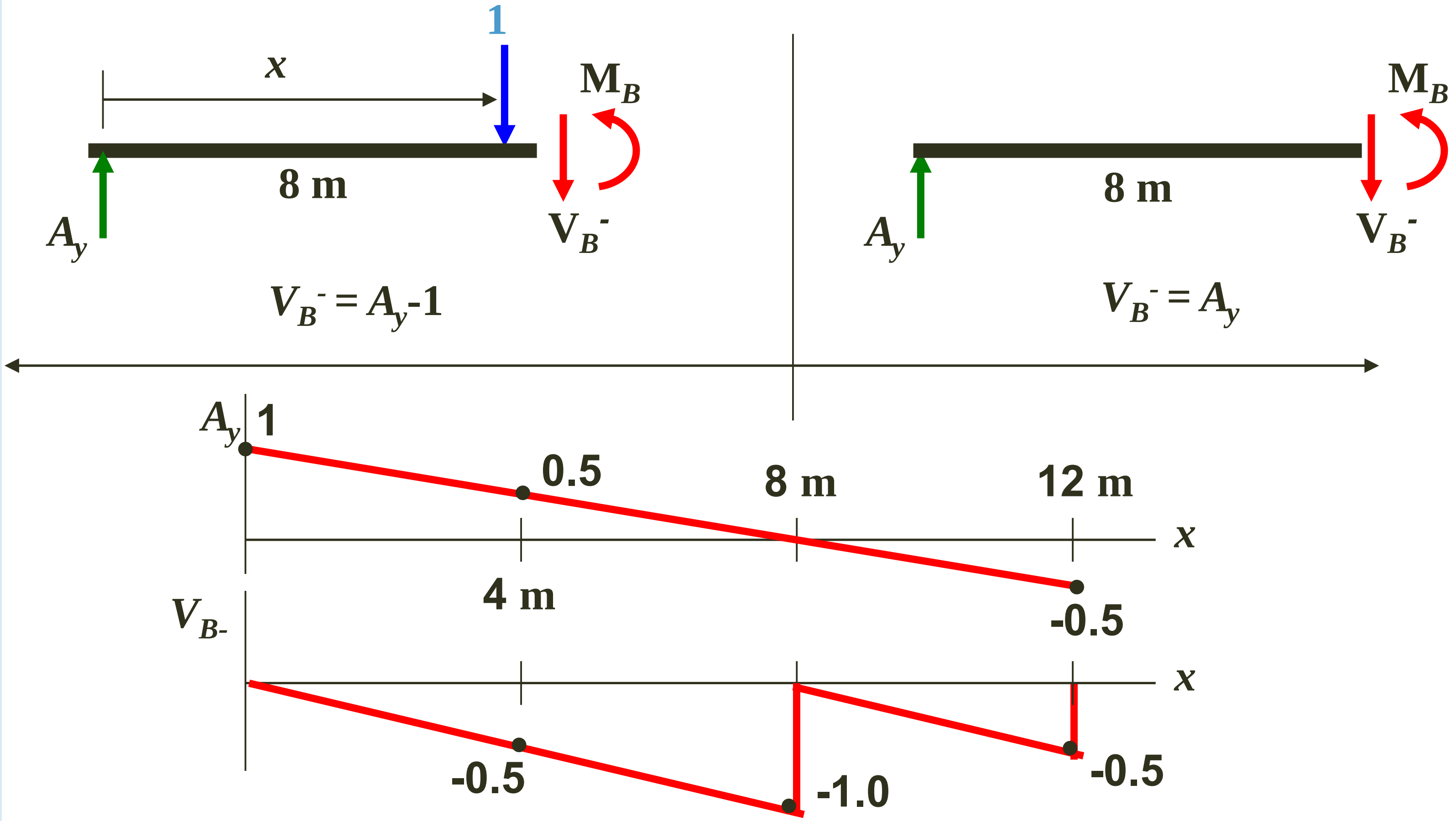
Influence Lines For Beams – Equilibrium methods



Influence Lines For Beams – Equilibrium methods

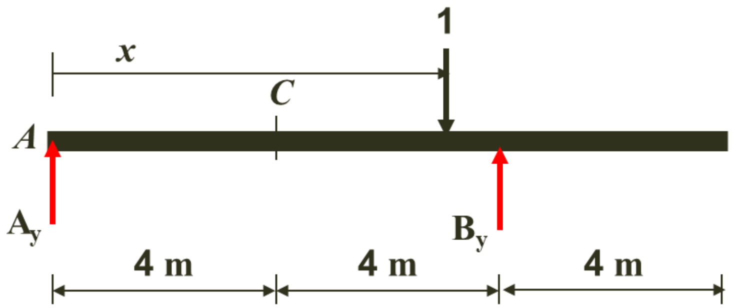
- Shear before support *B*

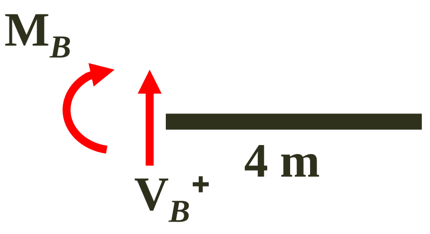


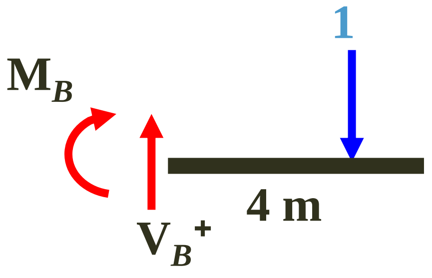


Influence Lines For Beams – Equilibrium methods

- Shear after support *B*

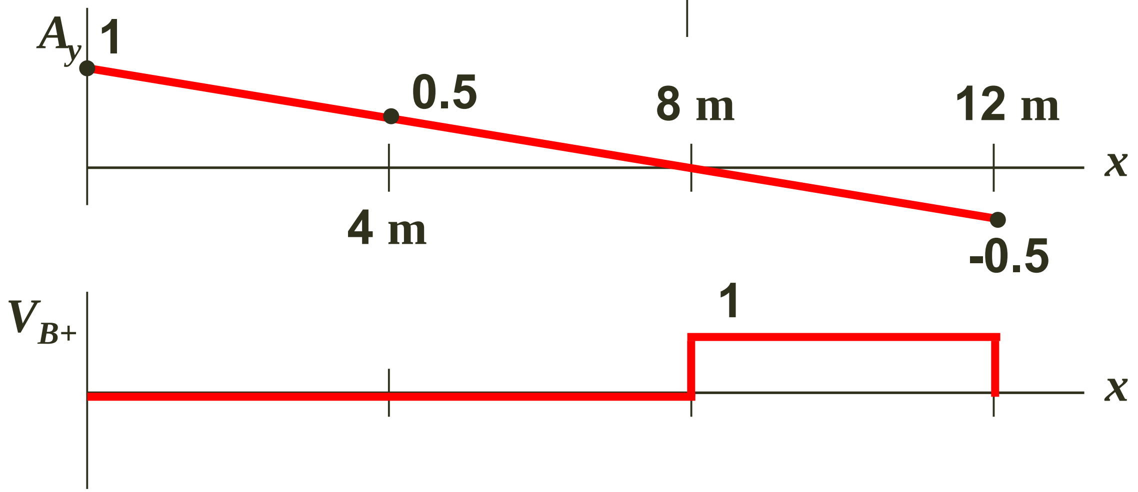






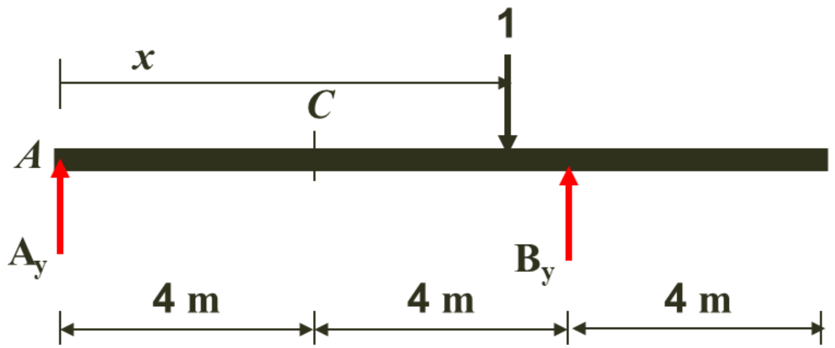
$V_B^+ = 0$

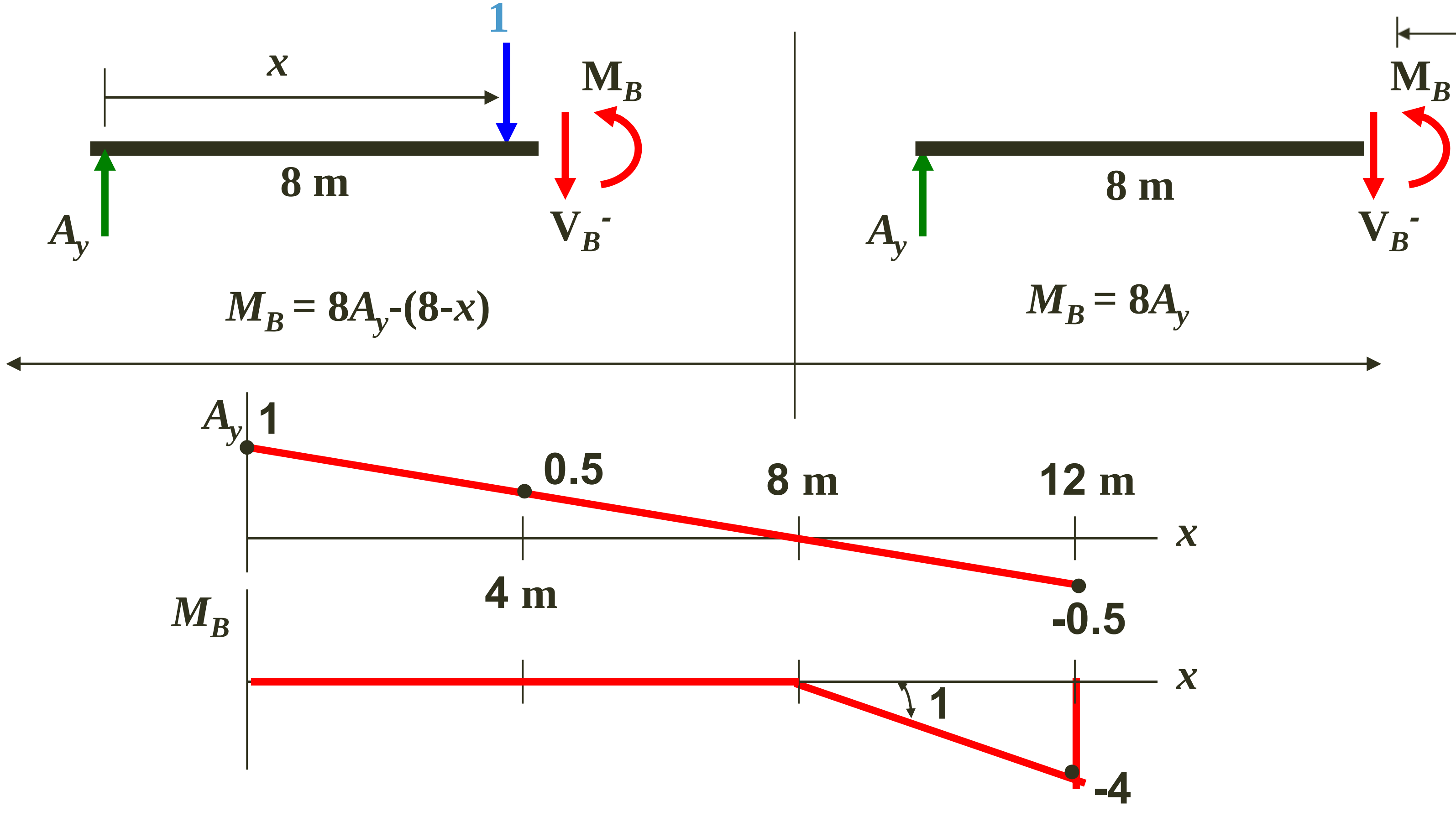
$V_B^+ = 1$



Influence Lines For Beams – Equilibrium methods

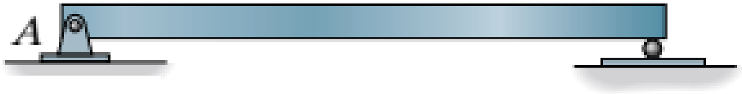
- Moment at support B

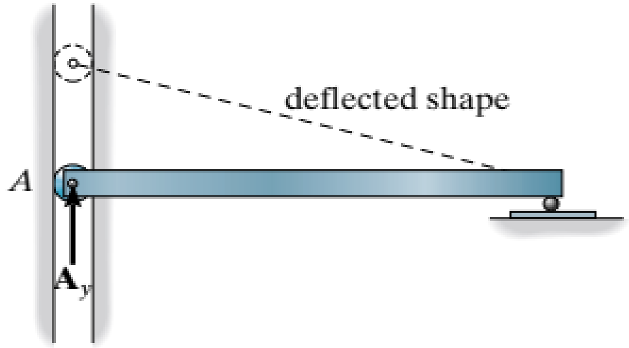


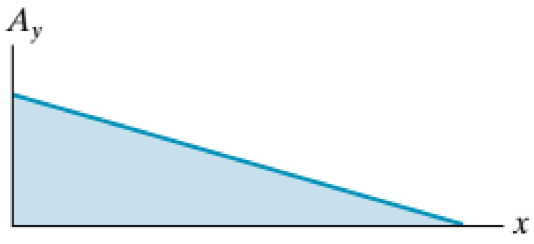


Influence Lines For Beams- Qualitative

- **Müller-Breslau principle** states that the influence line for a function (reaction, shear, or moment) is to the same scale as the deflected shape of the beam when the beam is acted upon by the function.
- In order to draw the deflected shape properly, the capacity of the beam to resist the applied function must be *removed* so the beam can deflect when the function is applied.

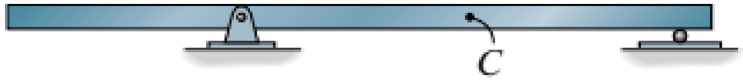




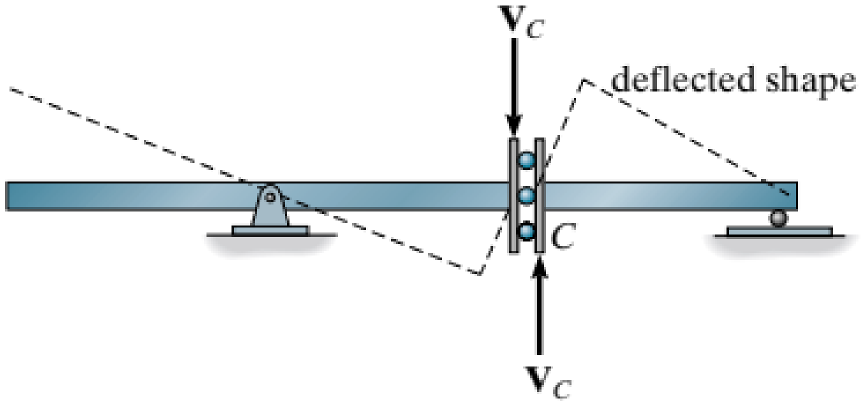


influence line for A_y

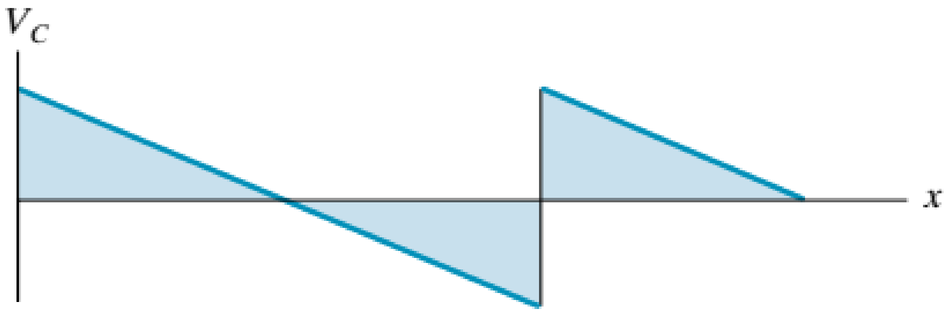
Influence Lines For Beams- Qualitative



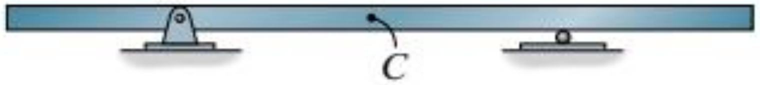
(a)



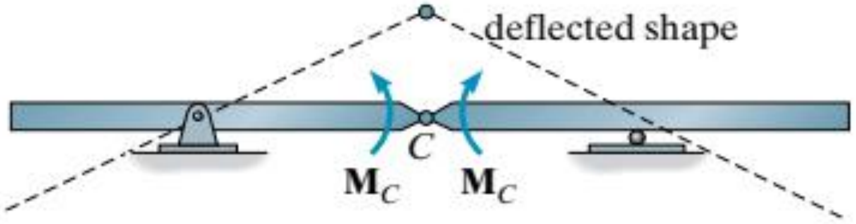
(b)



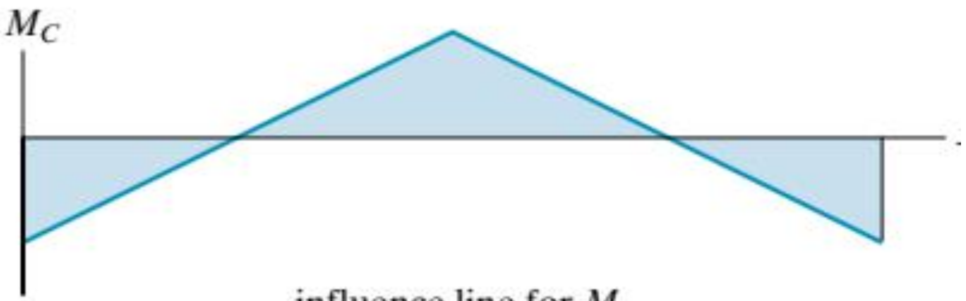
influence line for V_C



(a)



(b)

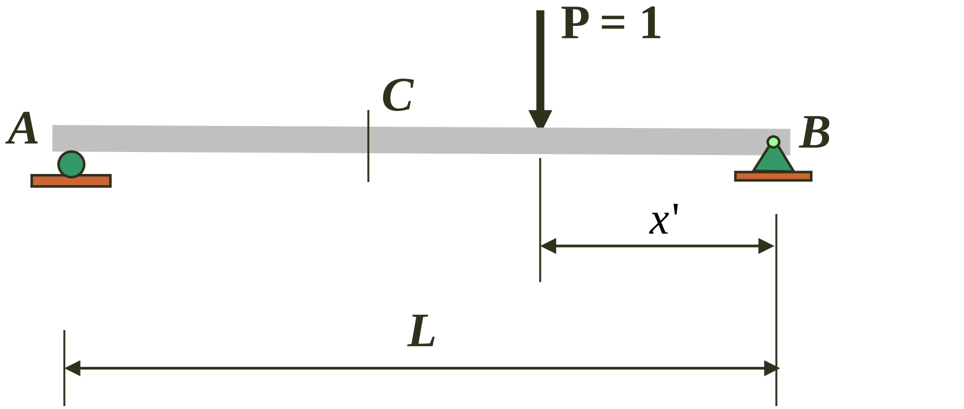


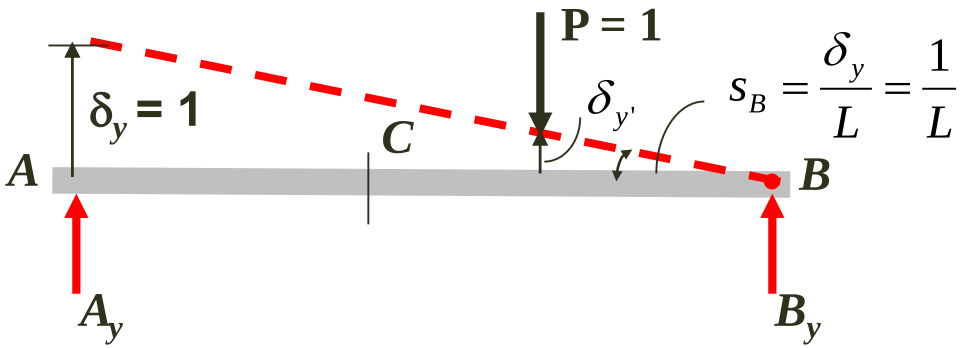
influence line for M_C

Influence Lines For Beams- Qualitative

- **Reaction**

- The proof of the Müller-Breslau principle can be established using the principle of virtual work.
- Recall that work is the product of either a linear displacement and force in the direction of the displacement or a rotational displacement and moment in the direction of the displacement.
- If the beam is given a virtual displacement δ_y at the support A, then only the support reaction A_y and the unit load do virtual work. Specifically, A_y does positive work $A_y\delta_y$ and the unit load does negative work, $-1\delta_{y'}$.

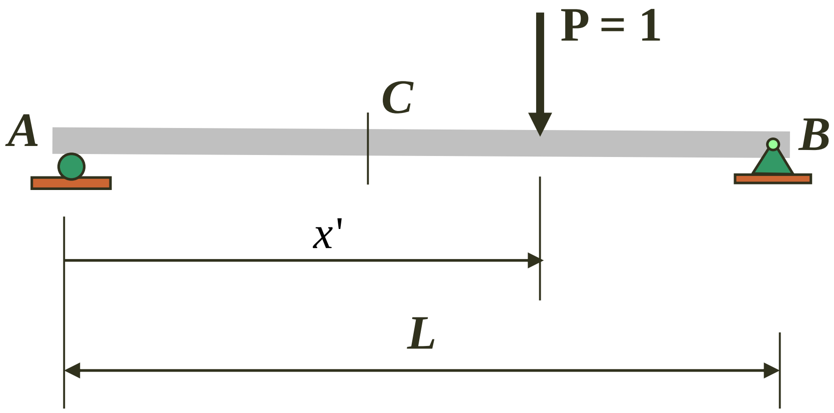


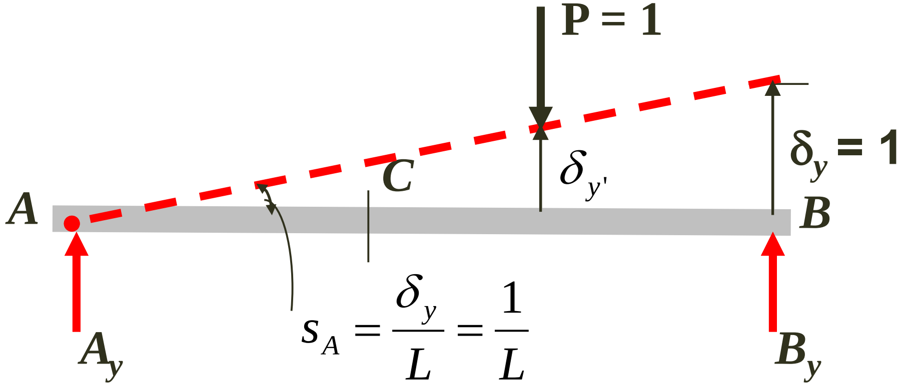


$$A_y(1) - 1(\delta_{y'}) + B_y(0) = 0$$

$$A_y = \delta_y'$$

Influence Lines For Beams- Virtual Work

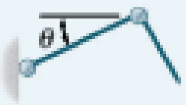
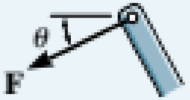
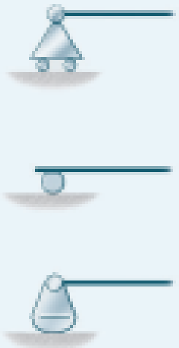
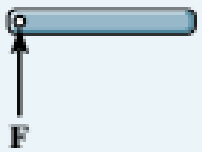
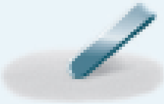
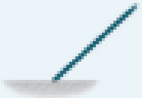


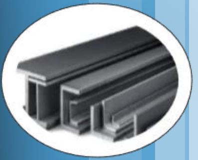


$$A_y(0) - 1(\delta_{y'}) + B_y(1) = 0$$

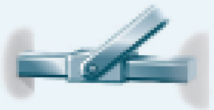
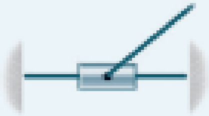
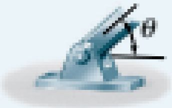
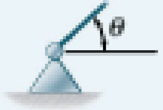
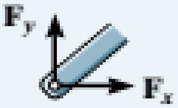
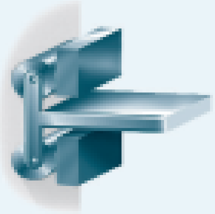
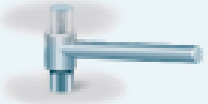
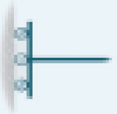
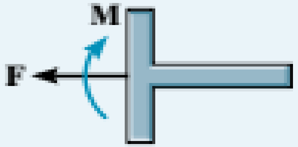
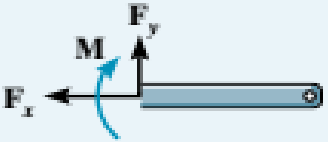
$$B_y = \delta_y'$$

Influence Lines For Beams- Qualitative

Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(1)  light cable weightless link			One unknown. The reaction is a force that acts in the direction of the cable or link.
(2)  rollers rocker			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(3)  smooth contacting surface			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.

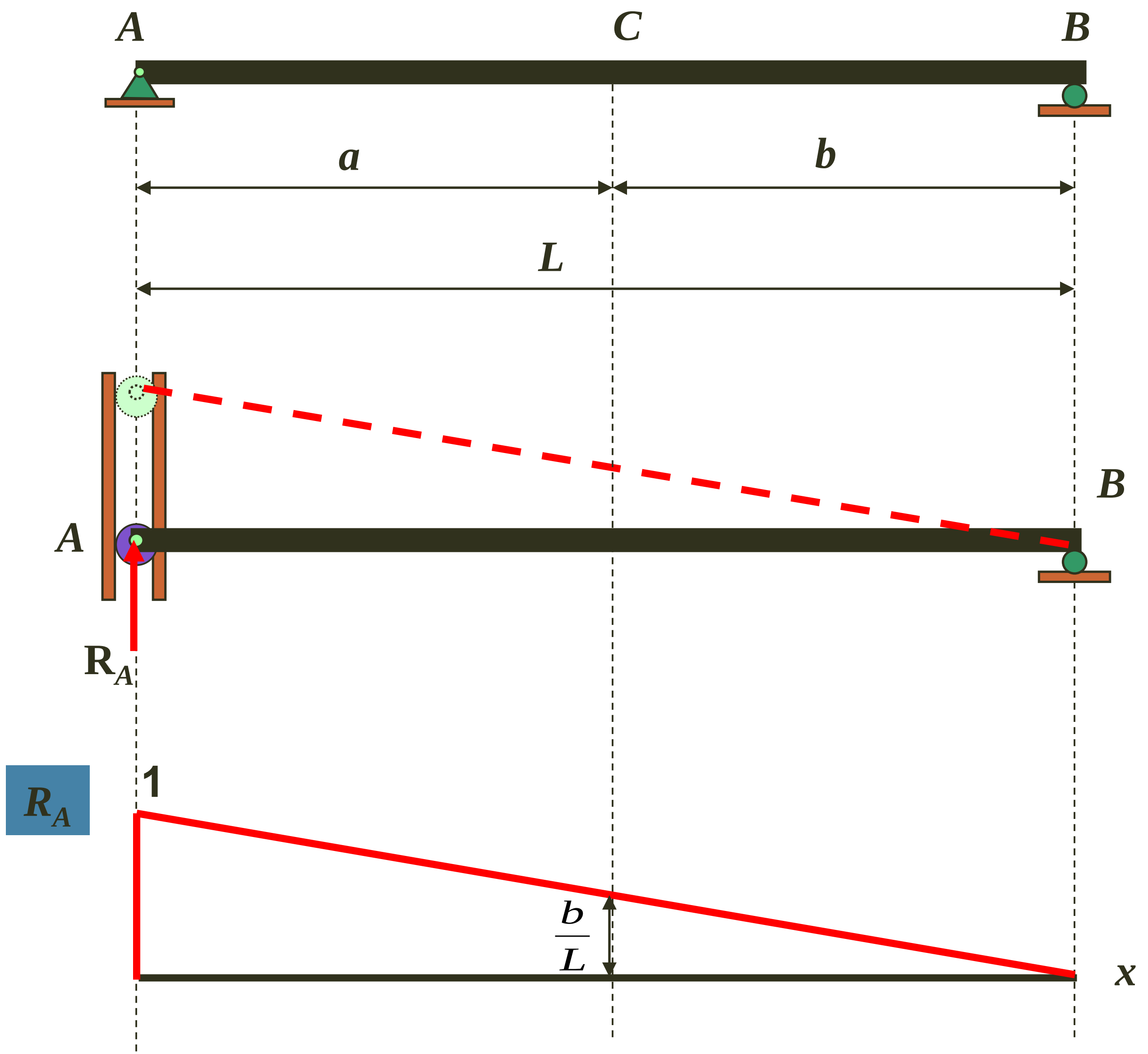




Influence Lines For Beams- Qualitative

(4)  smooth pin-connected collar			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(5)  smooth pin or hinge			Two unknowns. The reactions are two force components.
(6)  slider  fixed-connected collar			Two unknowns. The reactions are a force and a moment.
(7)  fixed support			Three unknowns. The reactions are the moment and the two force components.

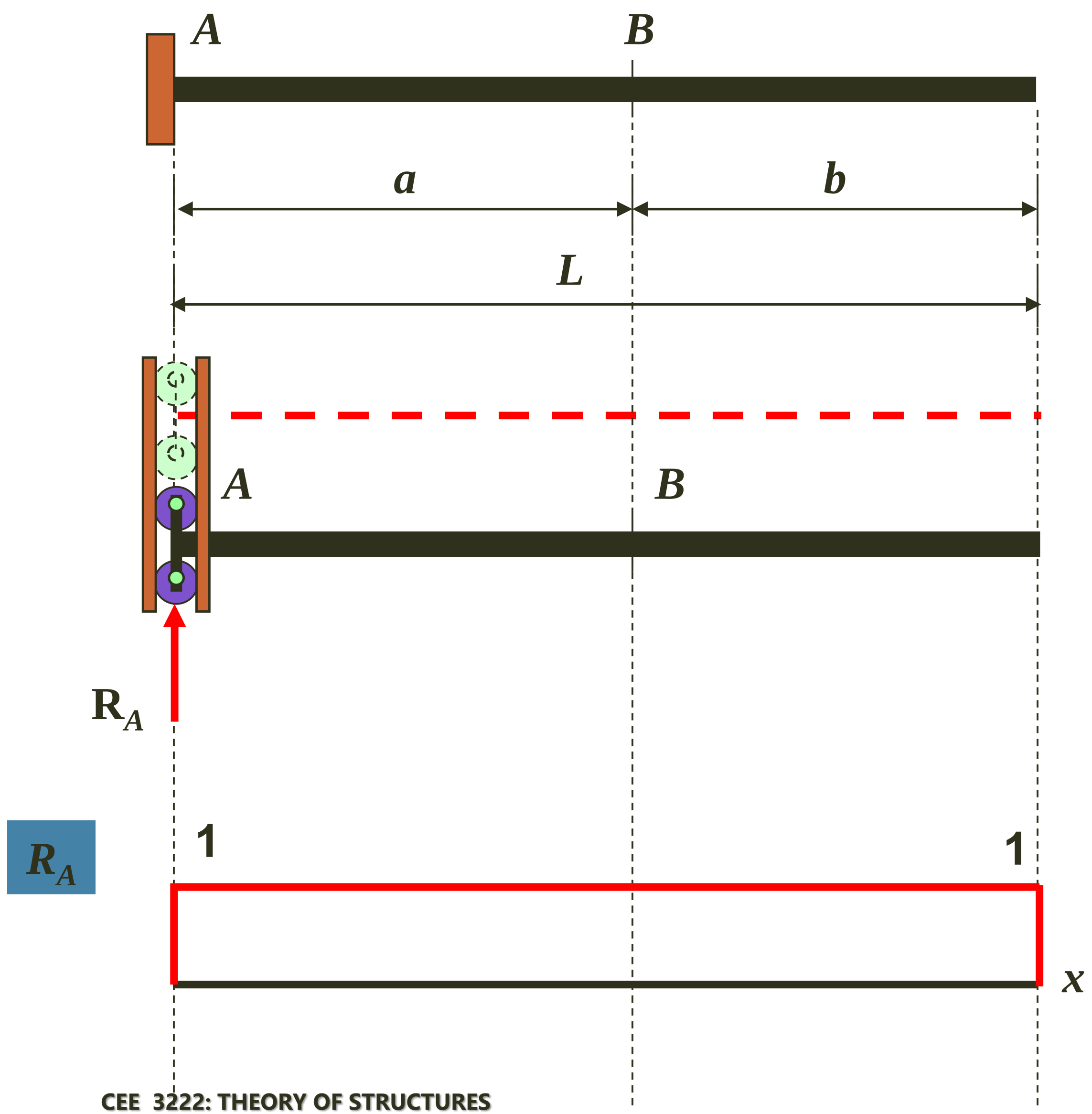
Influence Lines For Beams- Qualitative

- Pinned Support



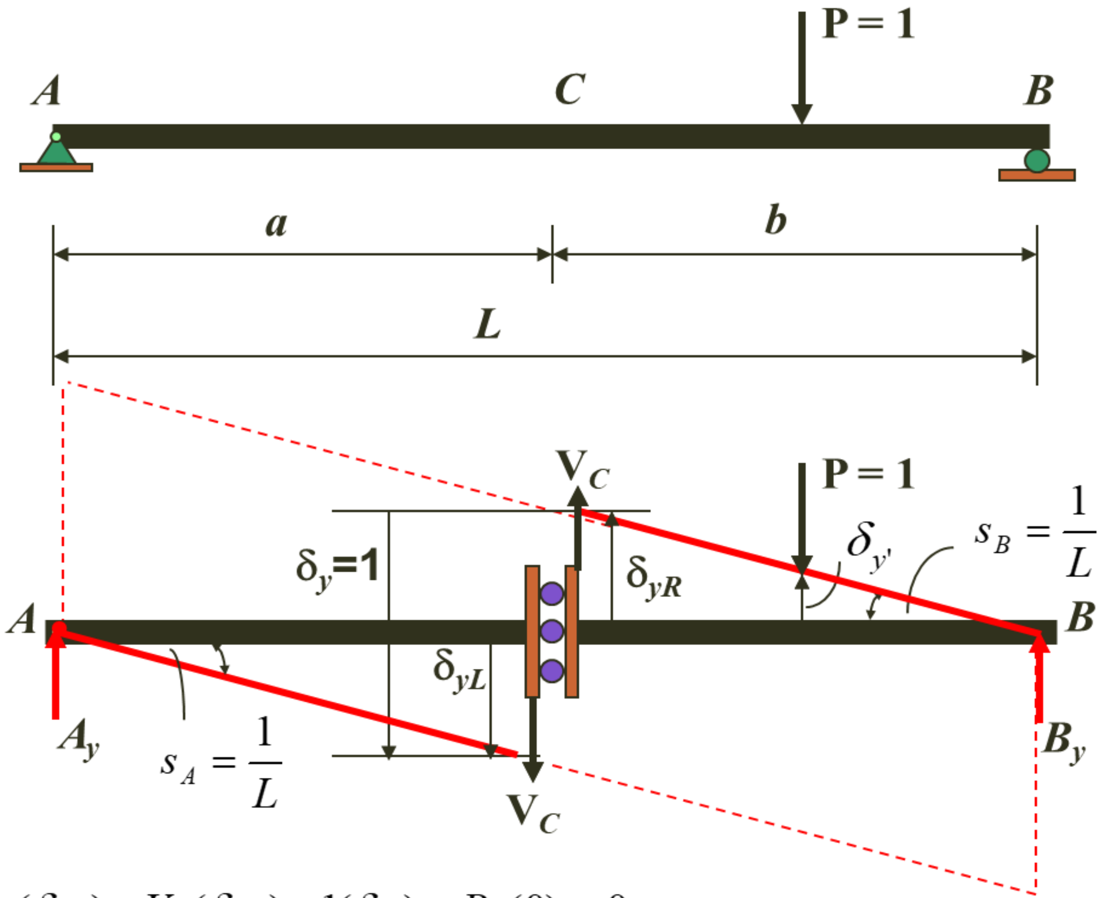
Influence Lines For Beams- Qualitative

- Fixed Support



Influence Lines For Beams- Qualitative

- **Shear**



$$A_y(0)+V_C(\delta_{yL})+V_C(\delta_{yR})-1(\delta_{y'})+B_y(0)=0$$

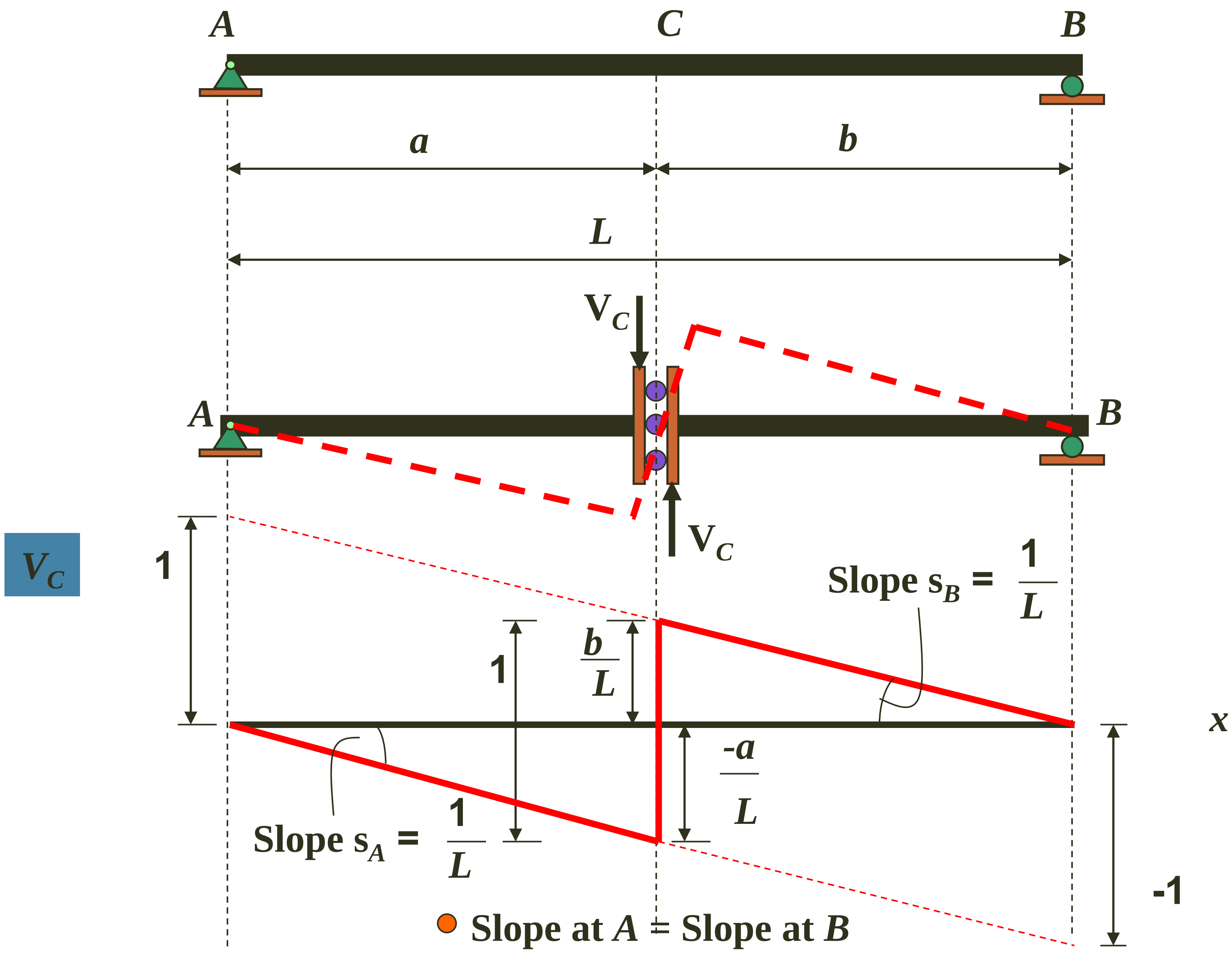
$$V_C\underbrace{(\delta_{yL}+\delta_{yR})}_{\delta_y=1}=\delta_{y'}$$

$$V_C=\delta_{y'}$$

$$slopes:|s_A|=|s_B|$$

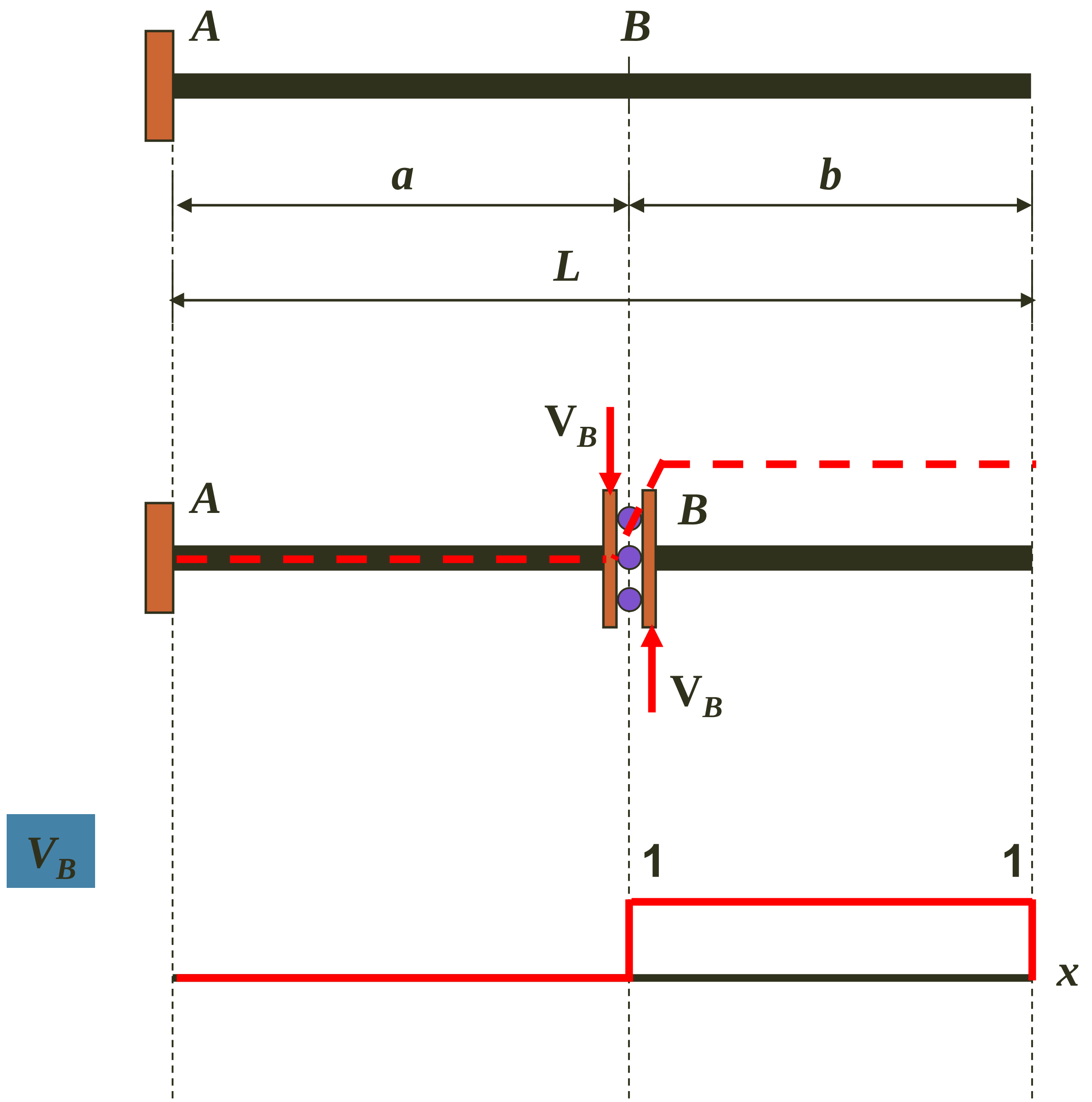
Influence Lines For Beams- Qualitative

- Pinned Support



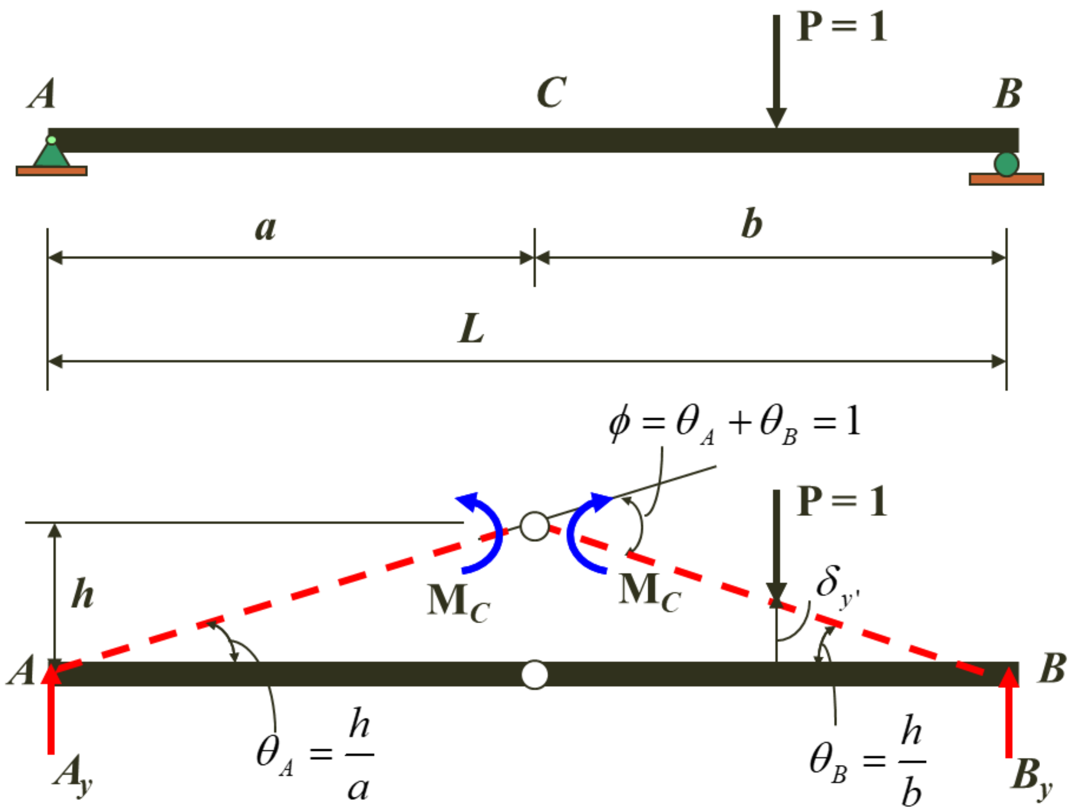
Influence Lines For Beams- Qualitative

- Fixed Support



Influence Lines For Beams- Qualitative

- **Bending Moment**



$$A_y(0) + M_C\theta_A + M_C\theta_B + 1(\delta_{y'}) + B_y(0) = 0$$

$$M_C(\theta_A + \theta_B) = \delta_{y'}$$

1

$$M_C = \delta_{y'}$$

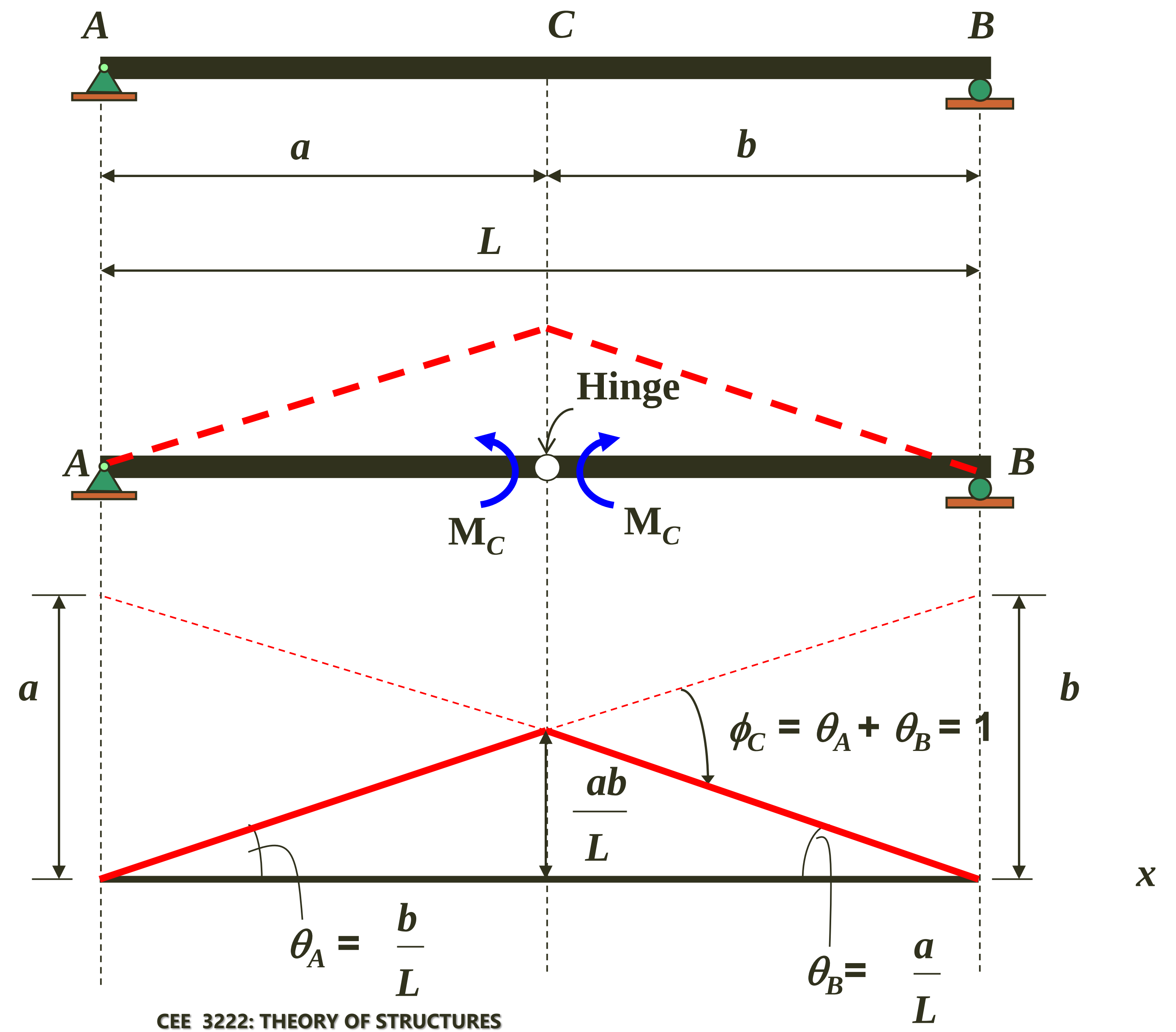
$$\left(\frac{h}{a} + \frac{h}{b}\right) = 1$$

$$\frac{h(a+b)}{ab} = 1, \qquad h = \frac{ab}{(a+b)} = \frac{ab}{L}$$

Influence Lines For Beams- Qualitative

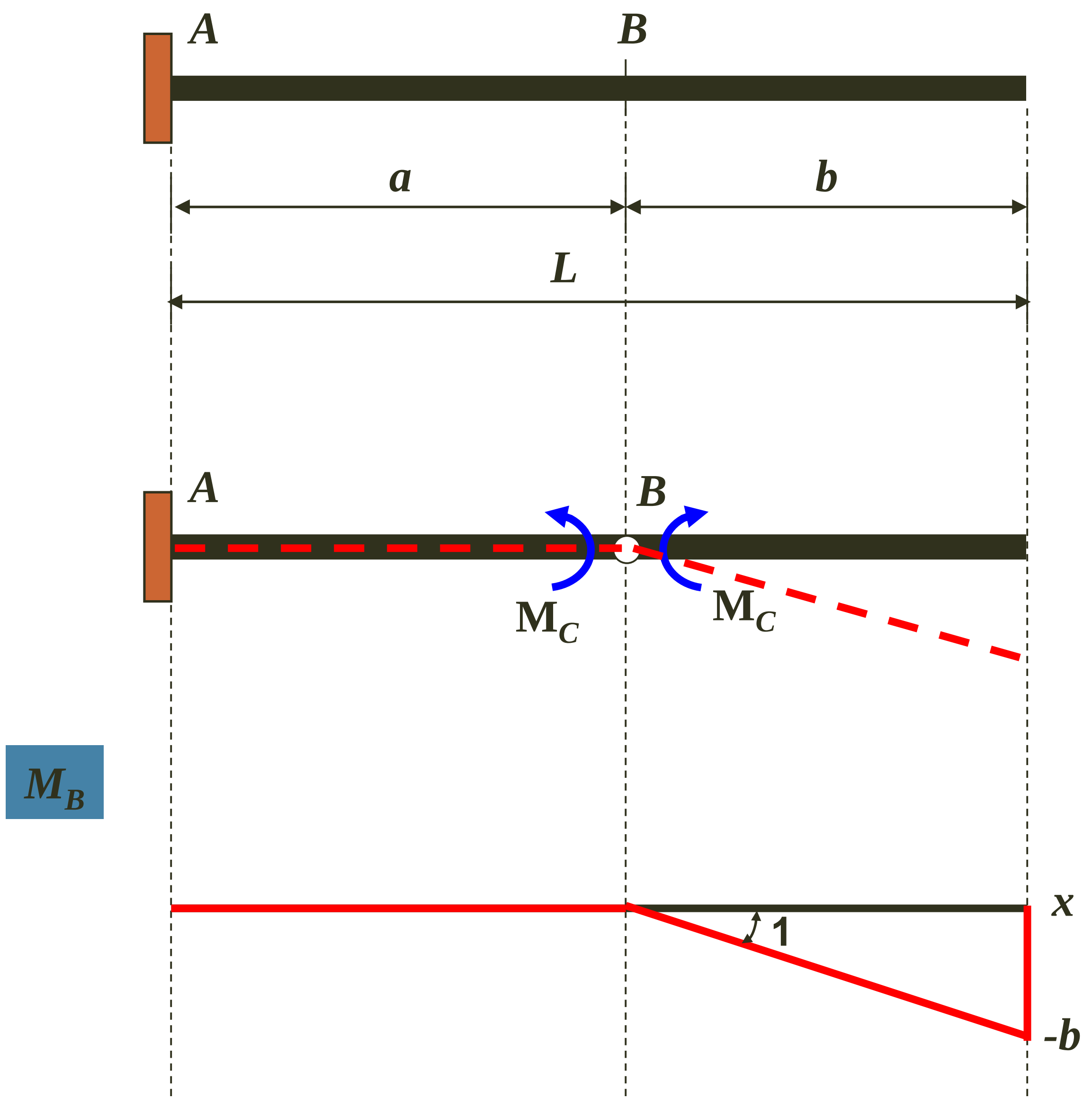
- Pinned Support

M_C



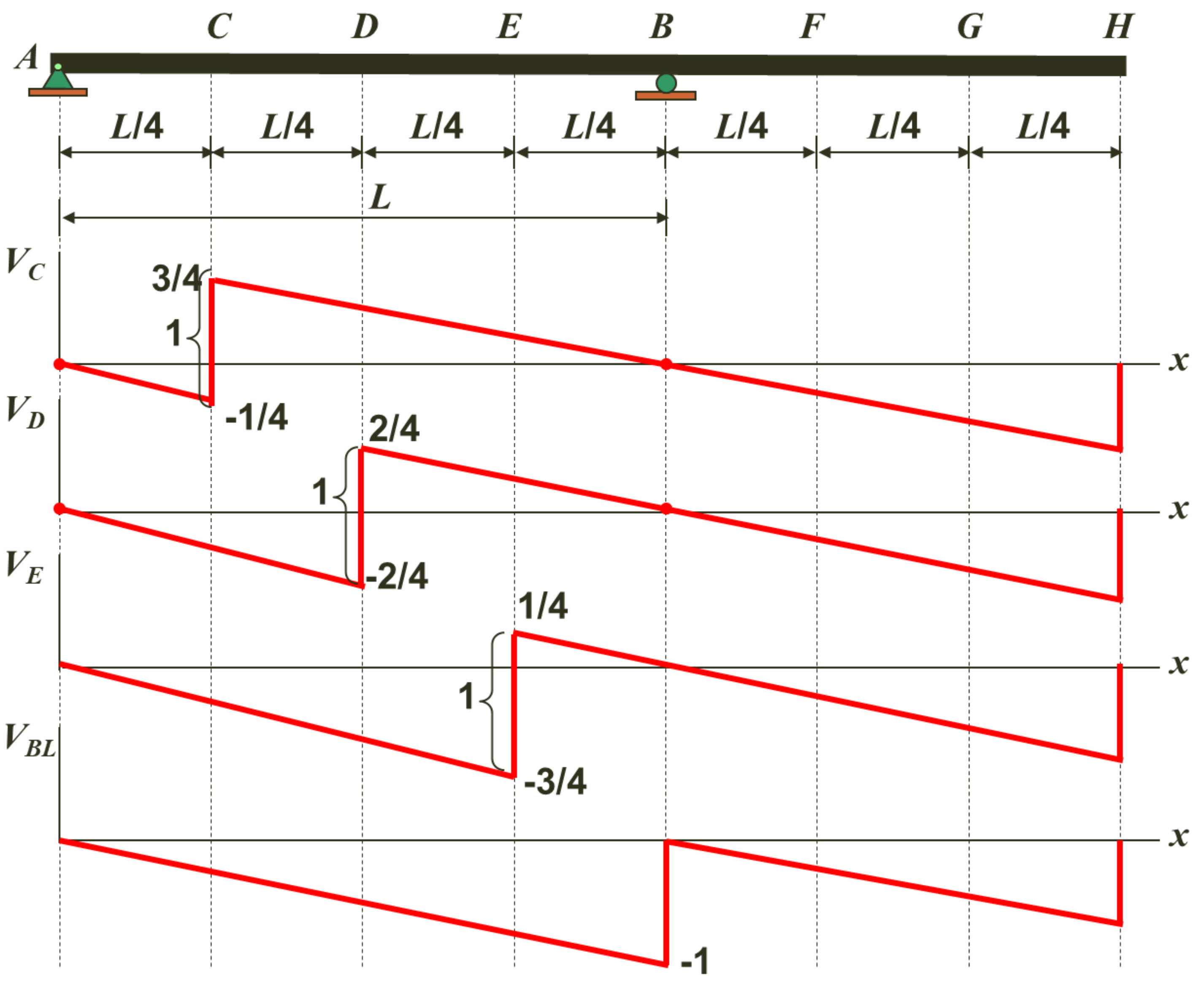
Influence Lines For Beams- Qualitative

- Fixed Support



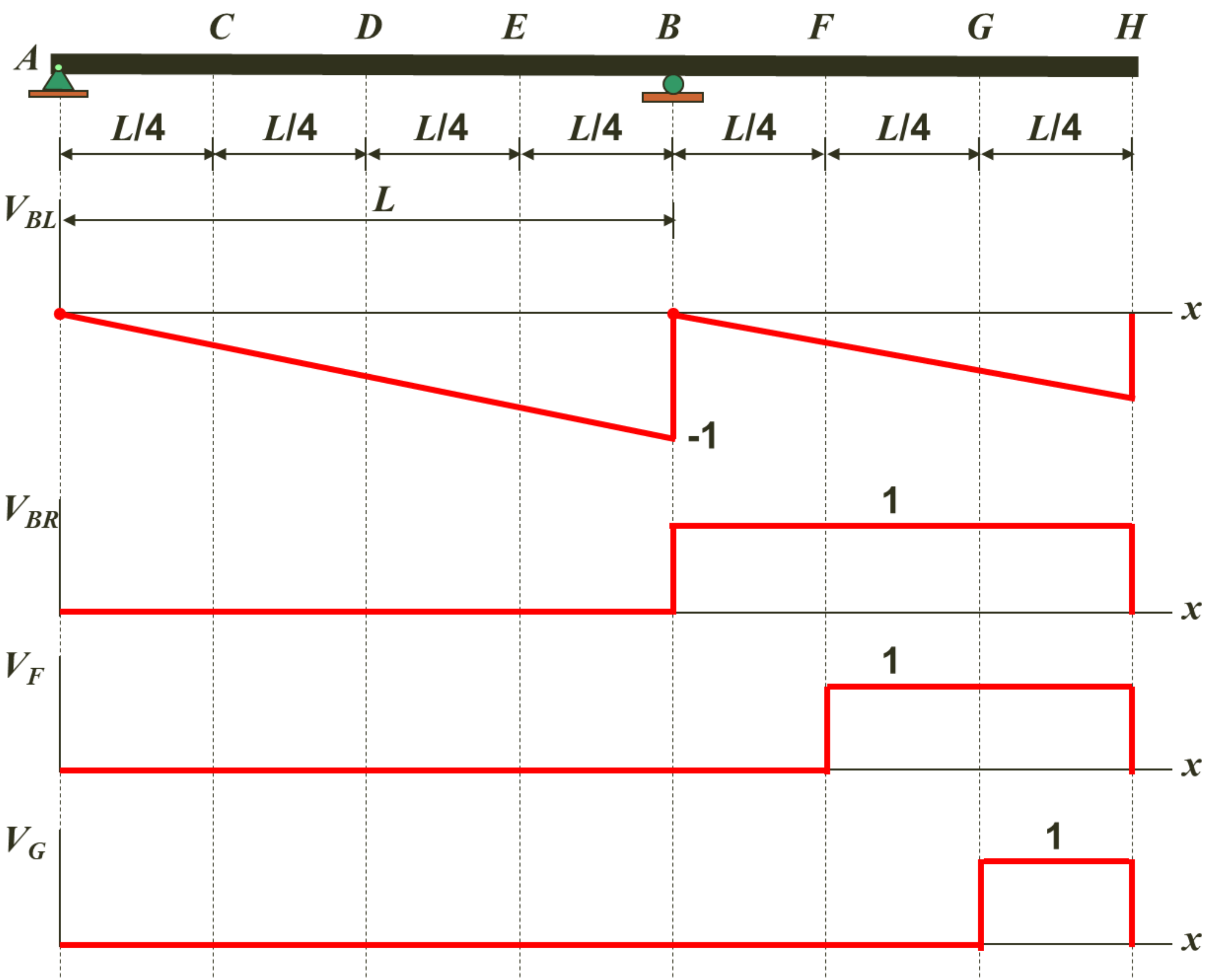
Influence Lines For Beams- General

- General Shear



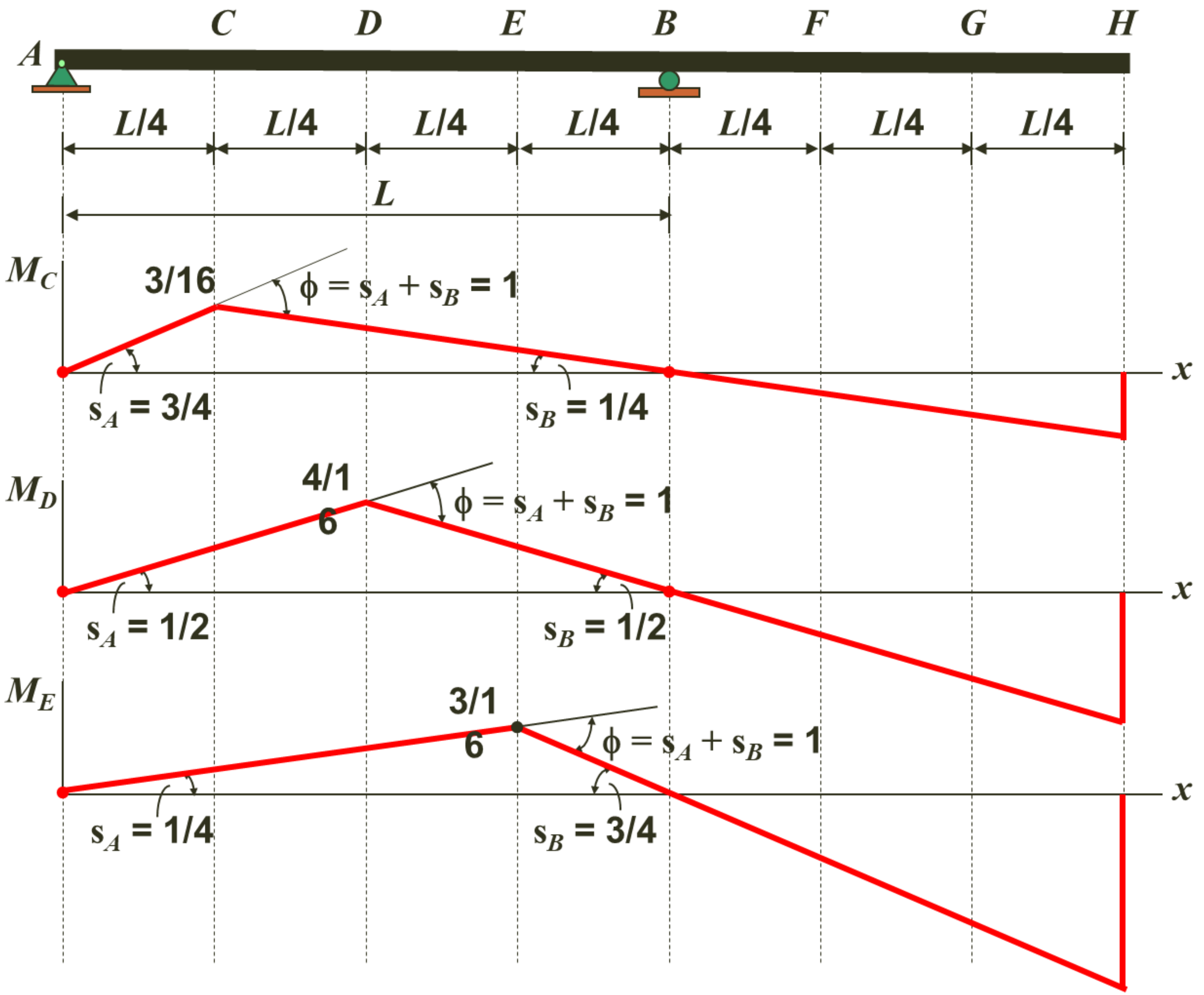
Influence Lines For Beams- General

- General Shear

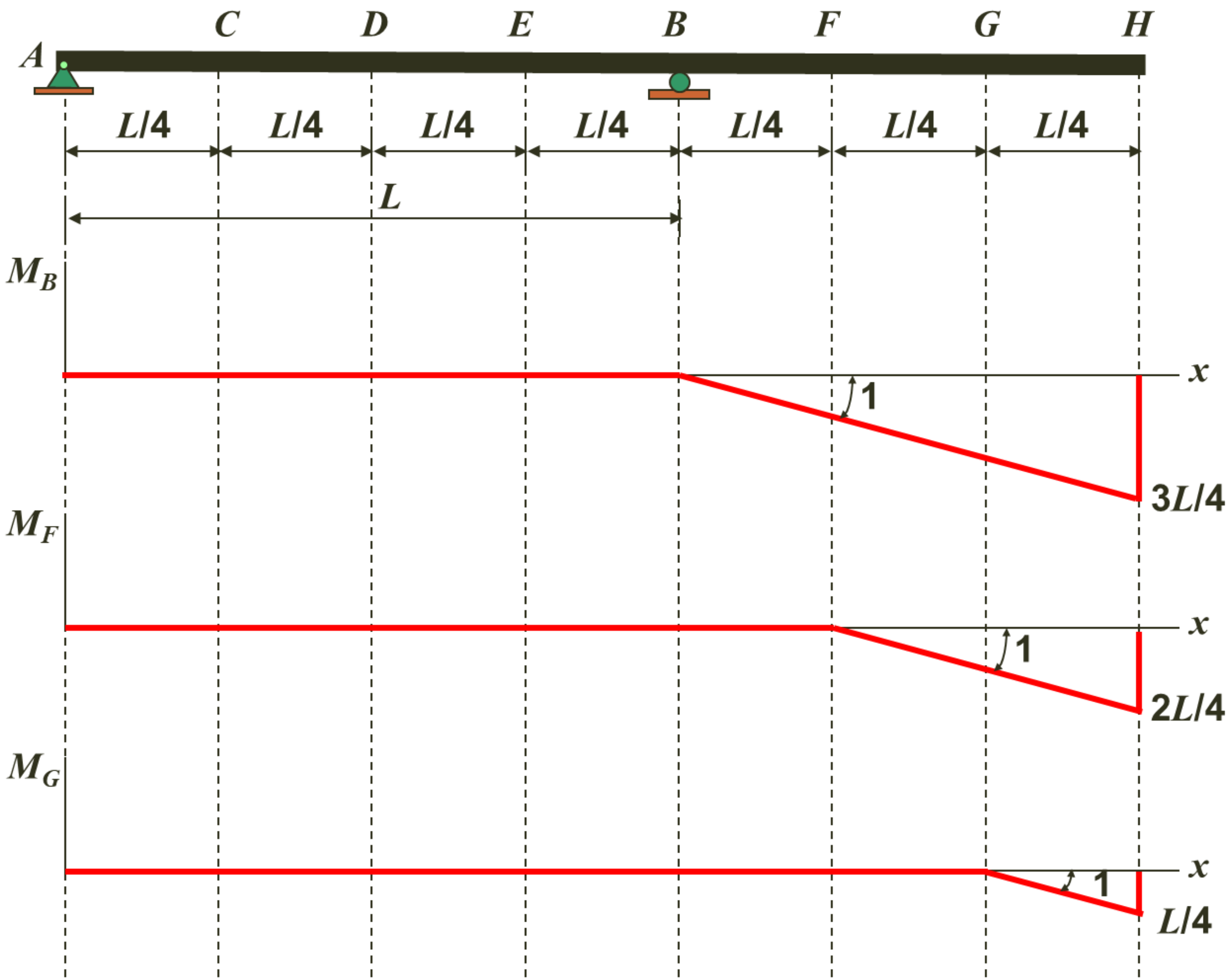


Influence Lines For Beams- General

- **General Bending Moment**



Influence Lines For Beams- General

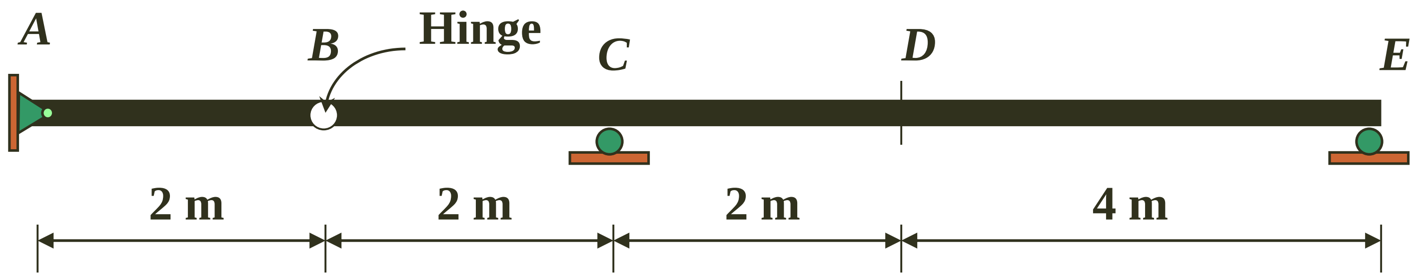


Influence Lines For Beams- Qualitative

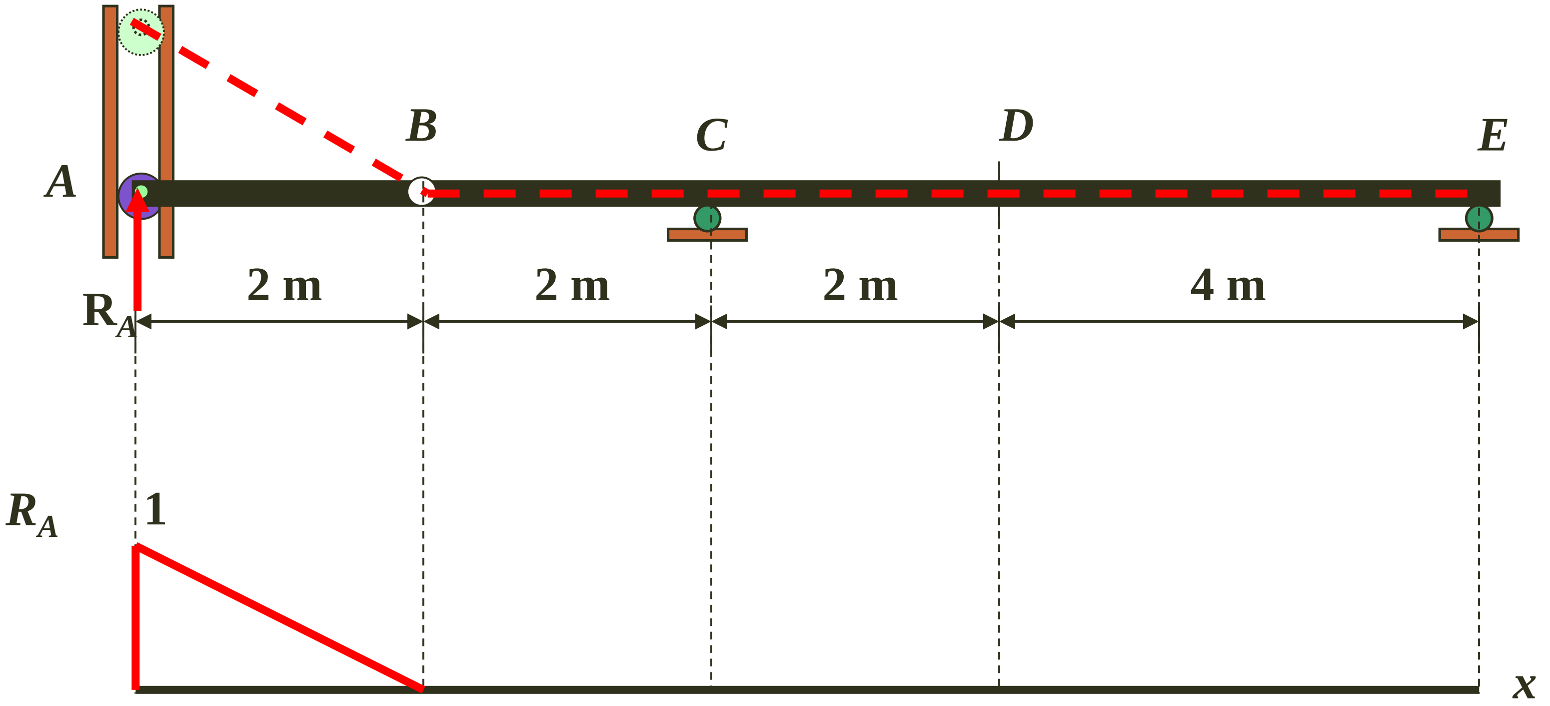
Example 6-2

Construct the influence line for

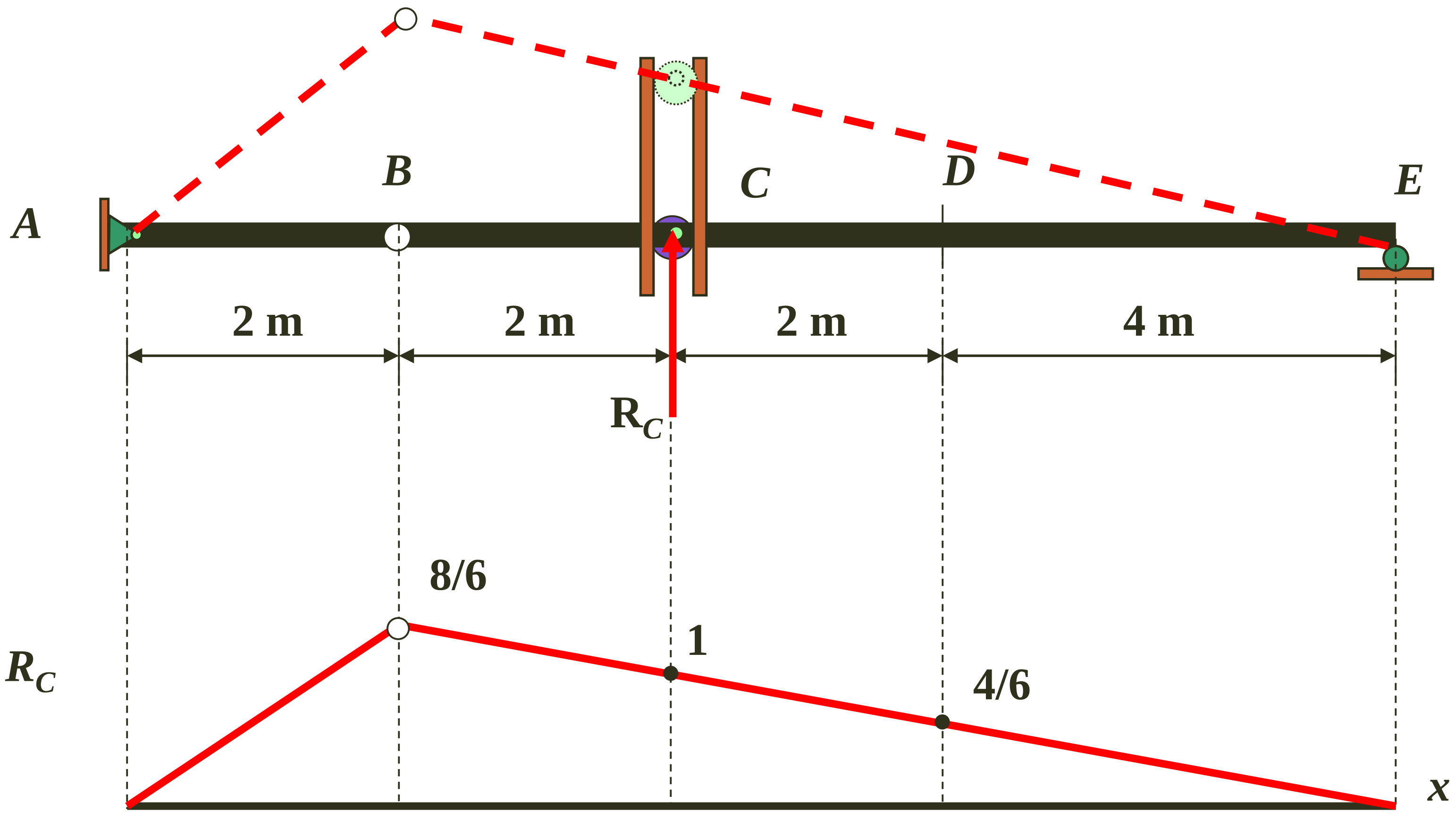
- the reaction at *A*, *C* and *E*
- the shear at *D*
- the moment at *D*
- shear before and after support *C*
- moment at point *C*



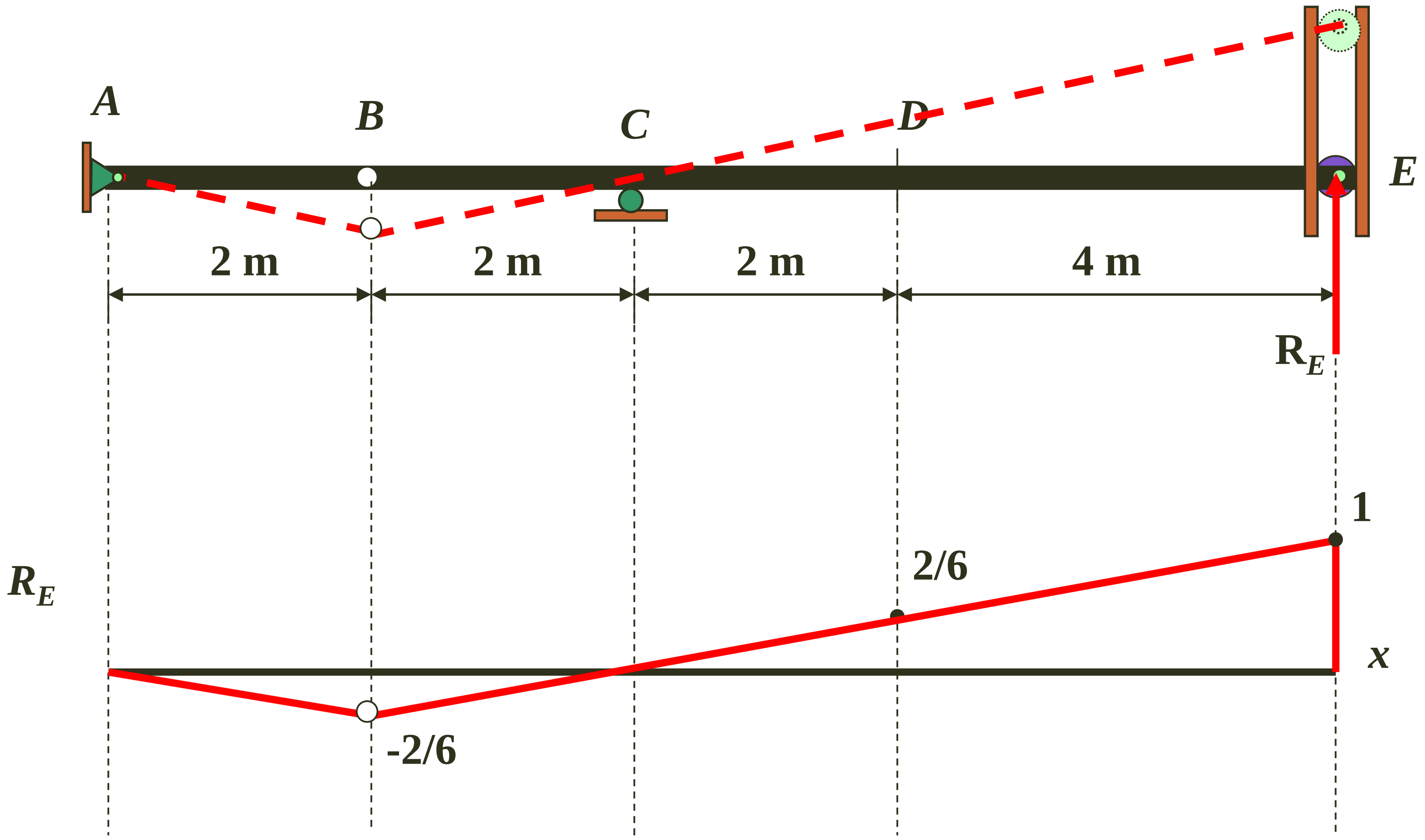
SOLUTION



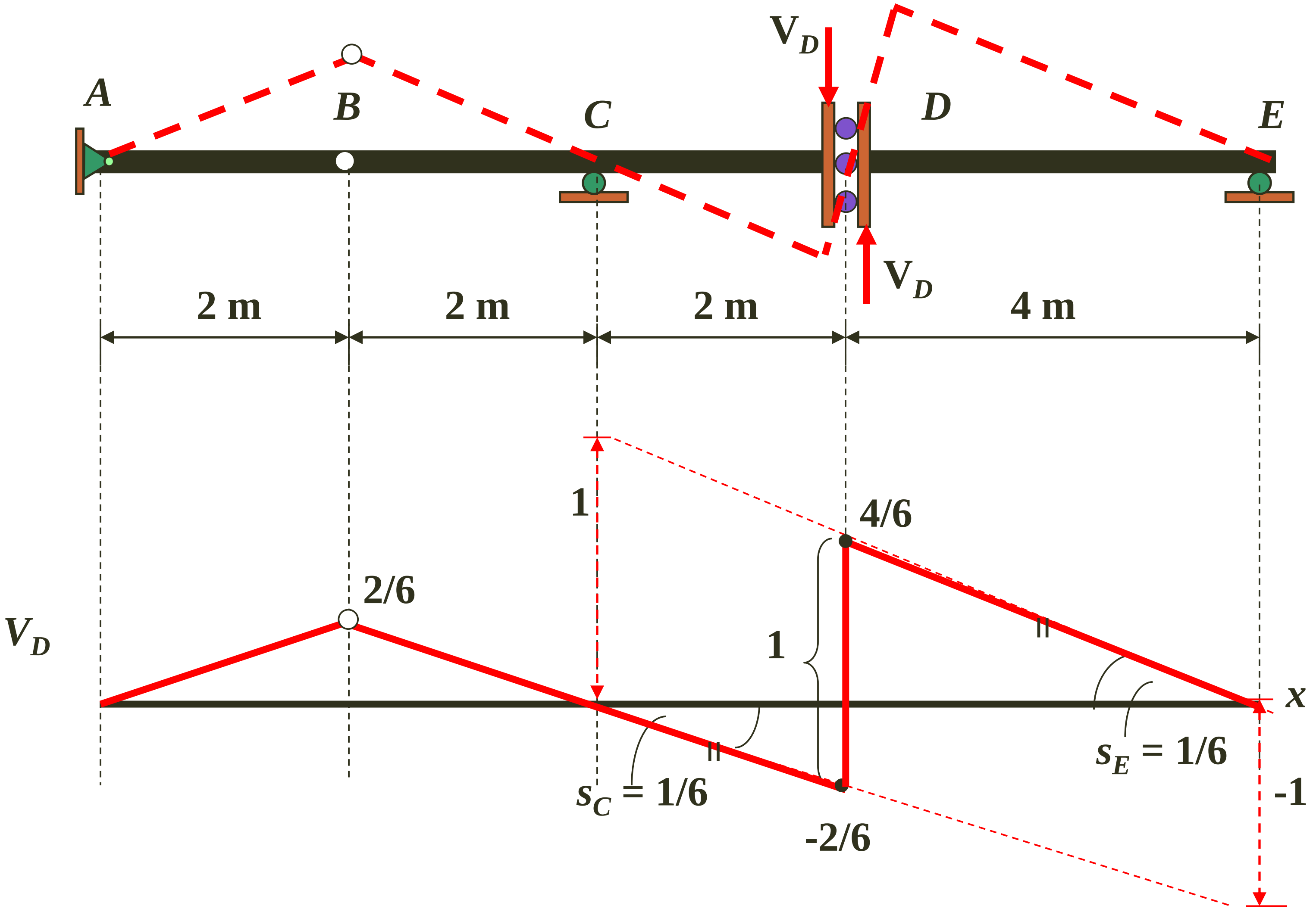
Influence Lines For Beams- Qualitative



Influence Lines For Beams- Qualitative



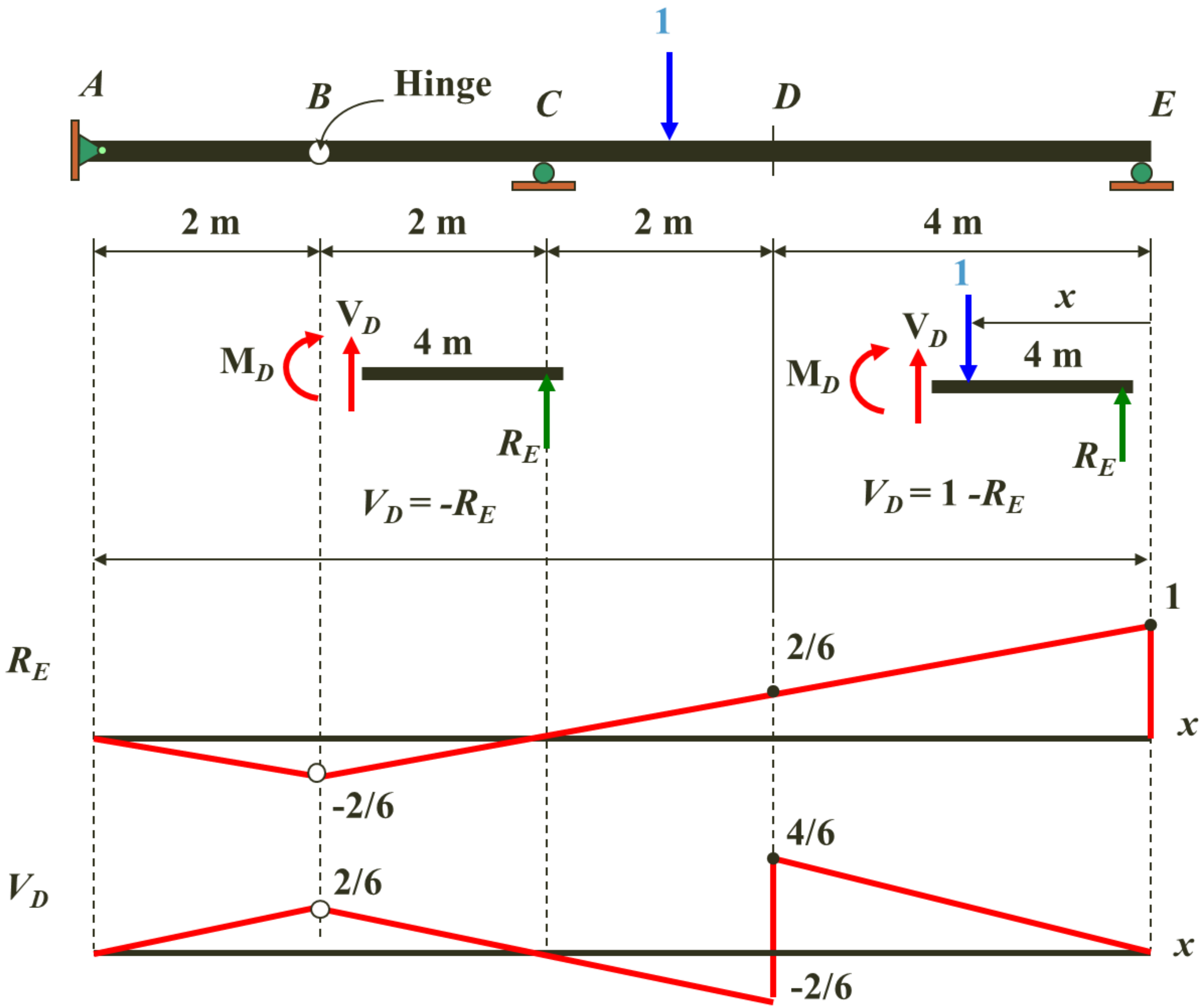
Influence Lines For Beams- Qualitative



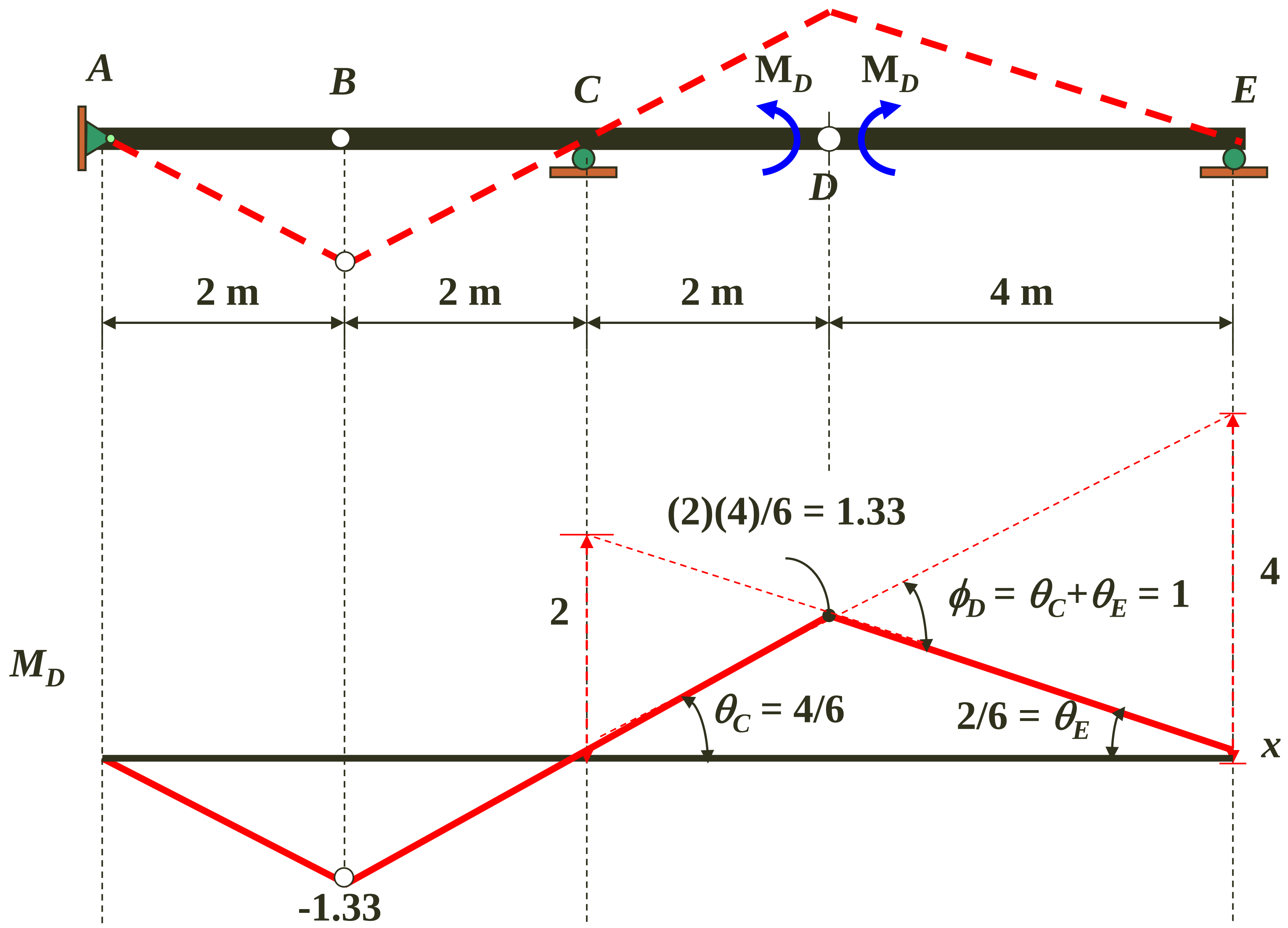
- $s_E = s_C$

Influence Lines For Beams- Equilibrium

Or using equilibrium conditions:

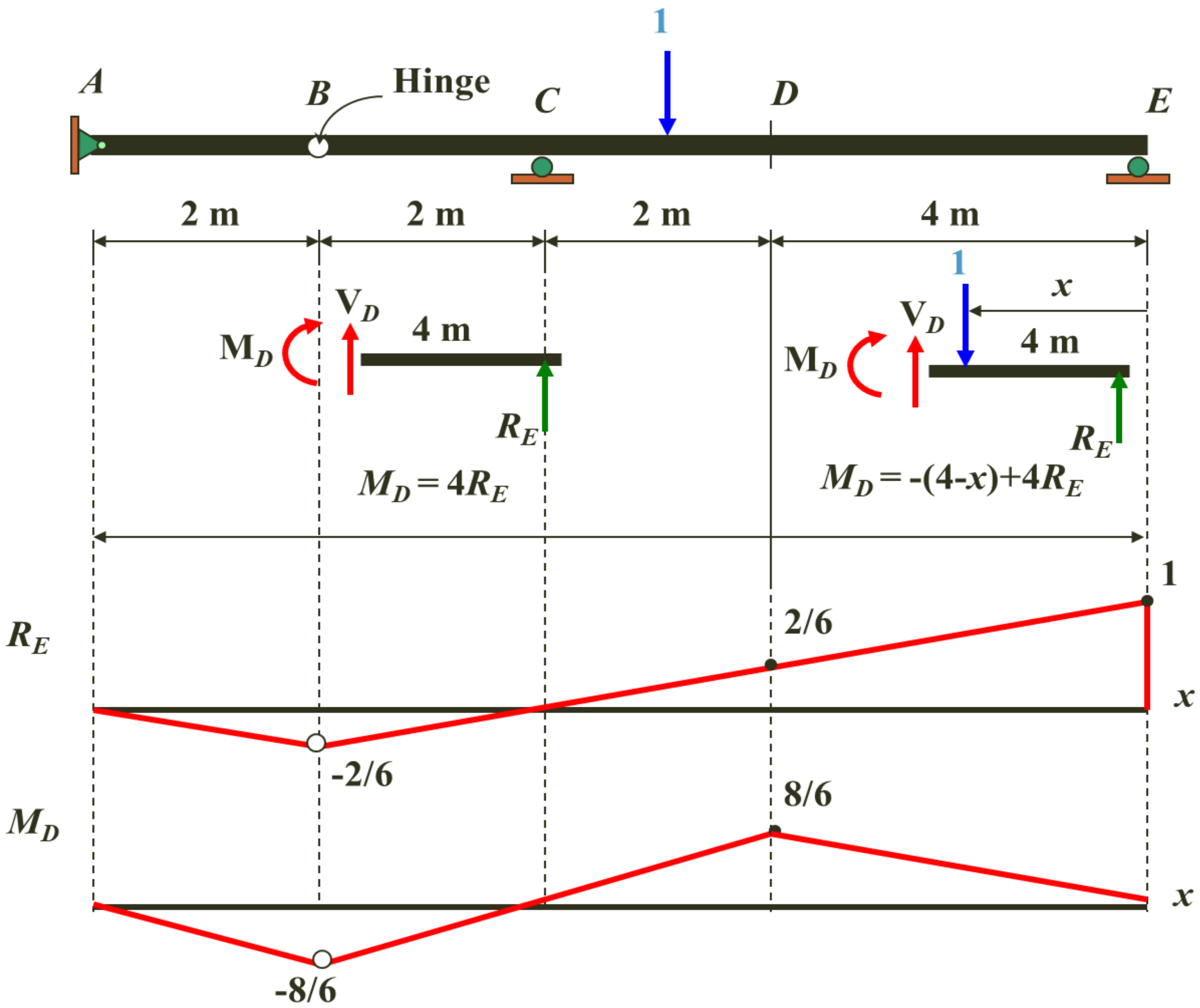


Influence Lines For Beams- Virtual Work

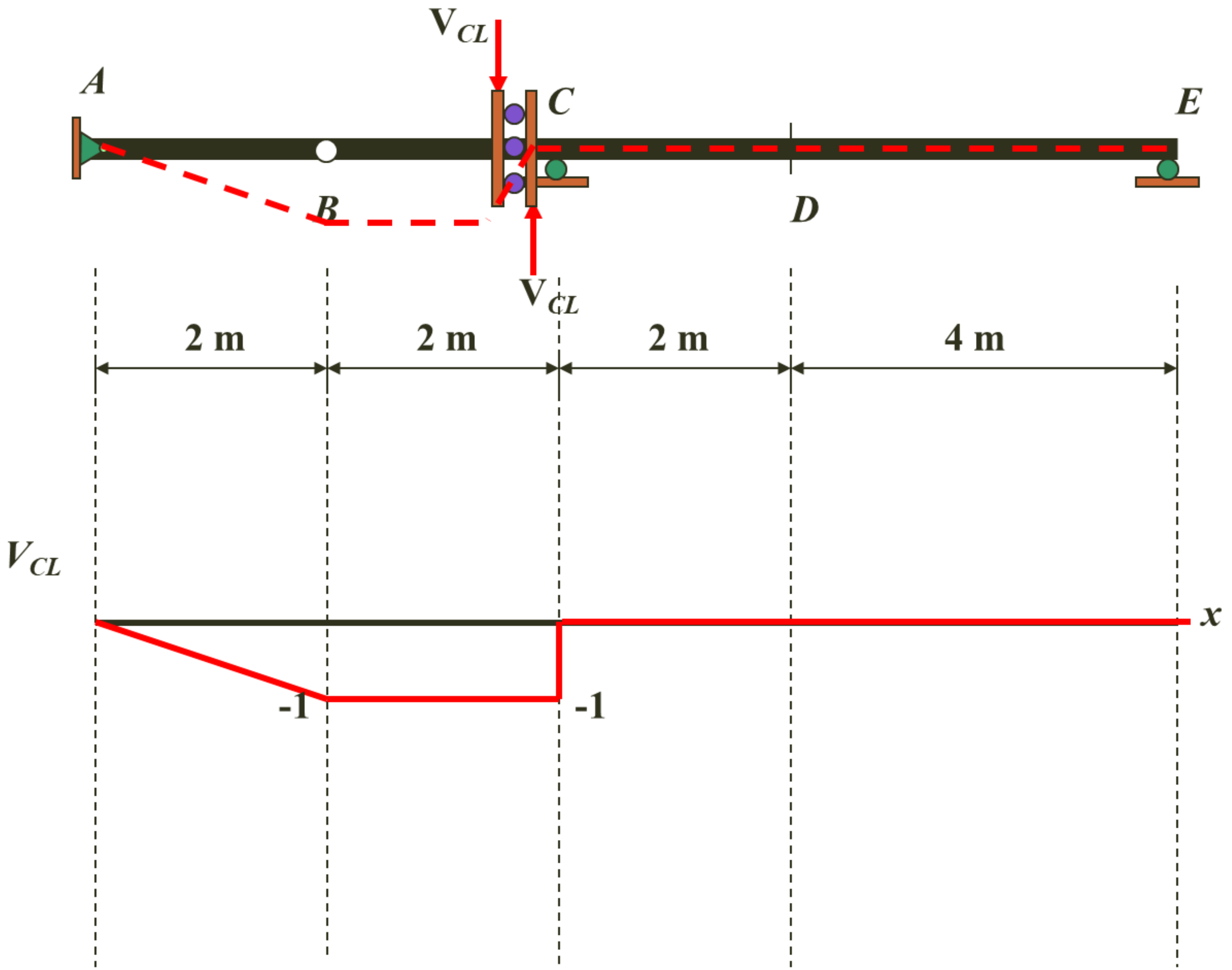


Influence Lines For Beams

Or using equilibrium conditions:

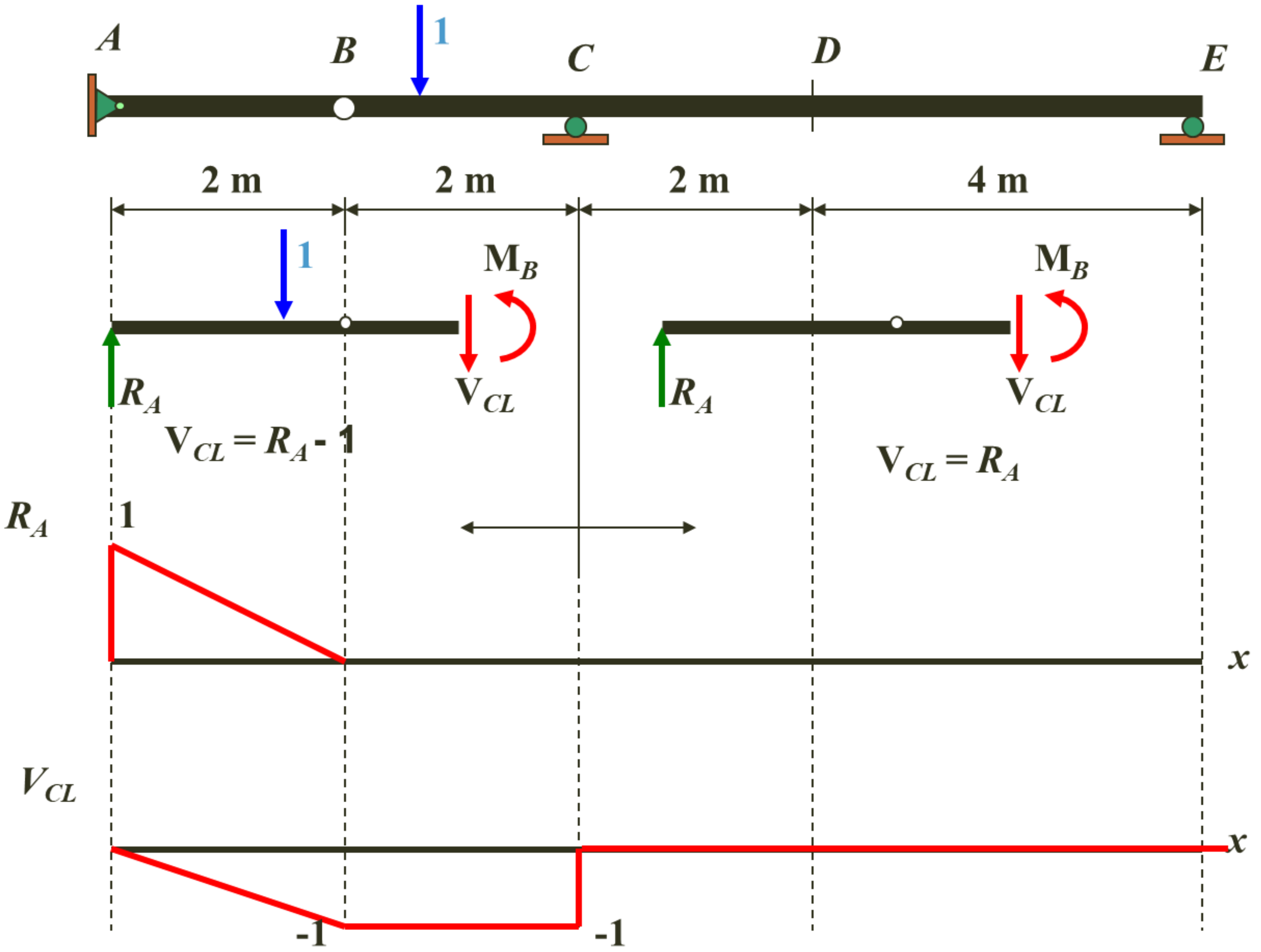


Influence Lines For Beams- Virtual Work

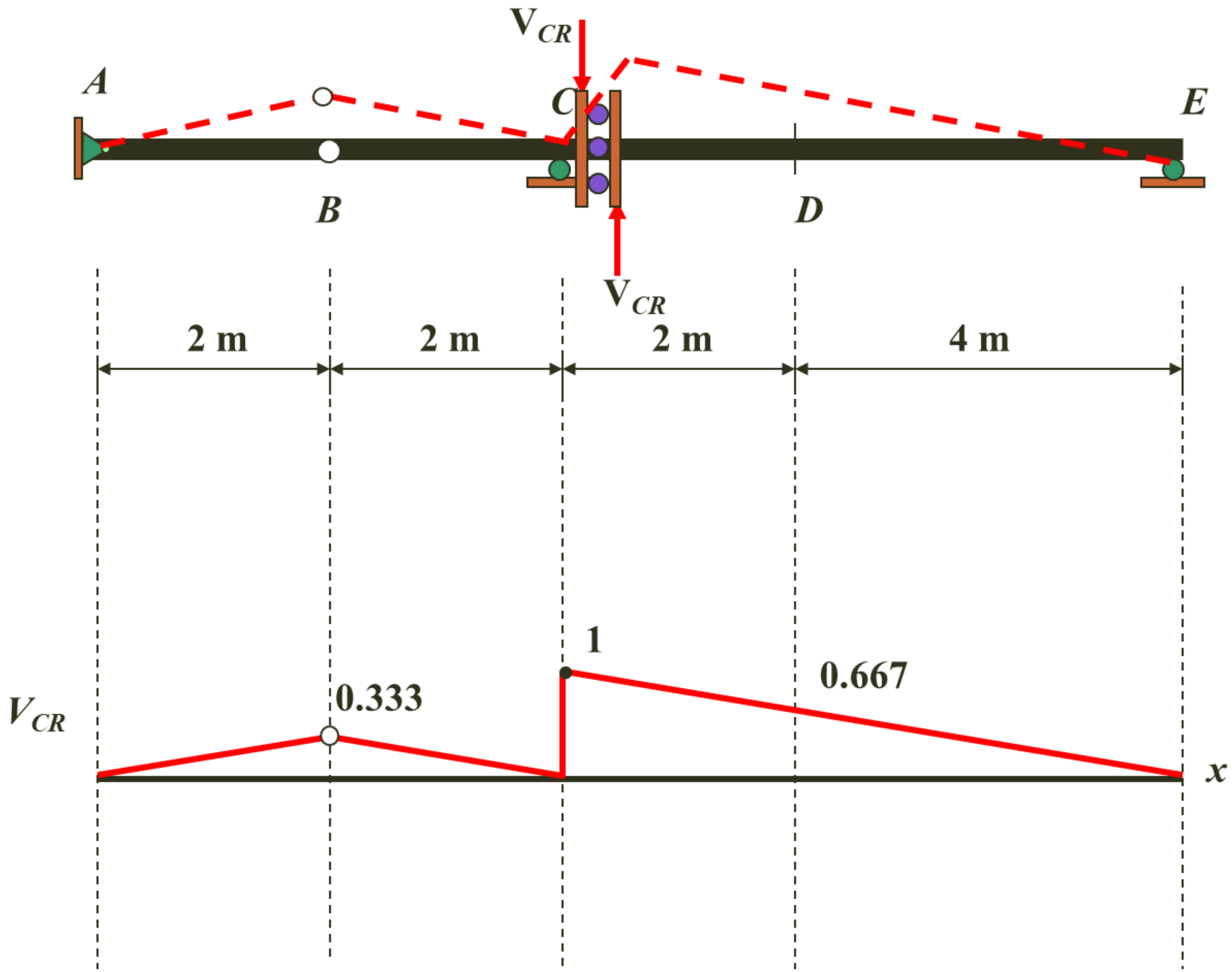


Influence Lines For Beams

Or using equilibrium conditions:

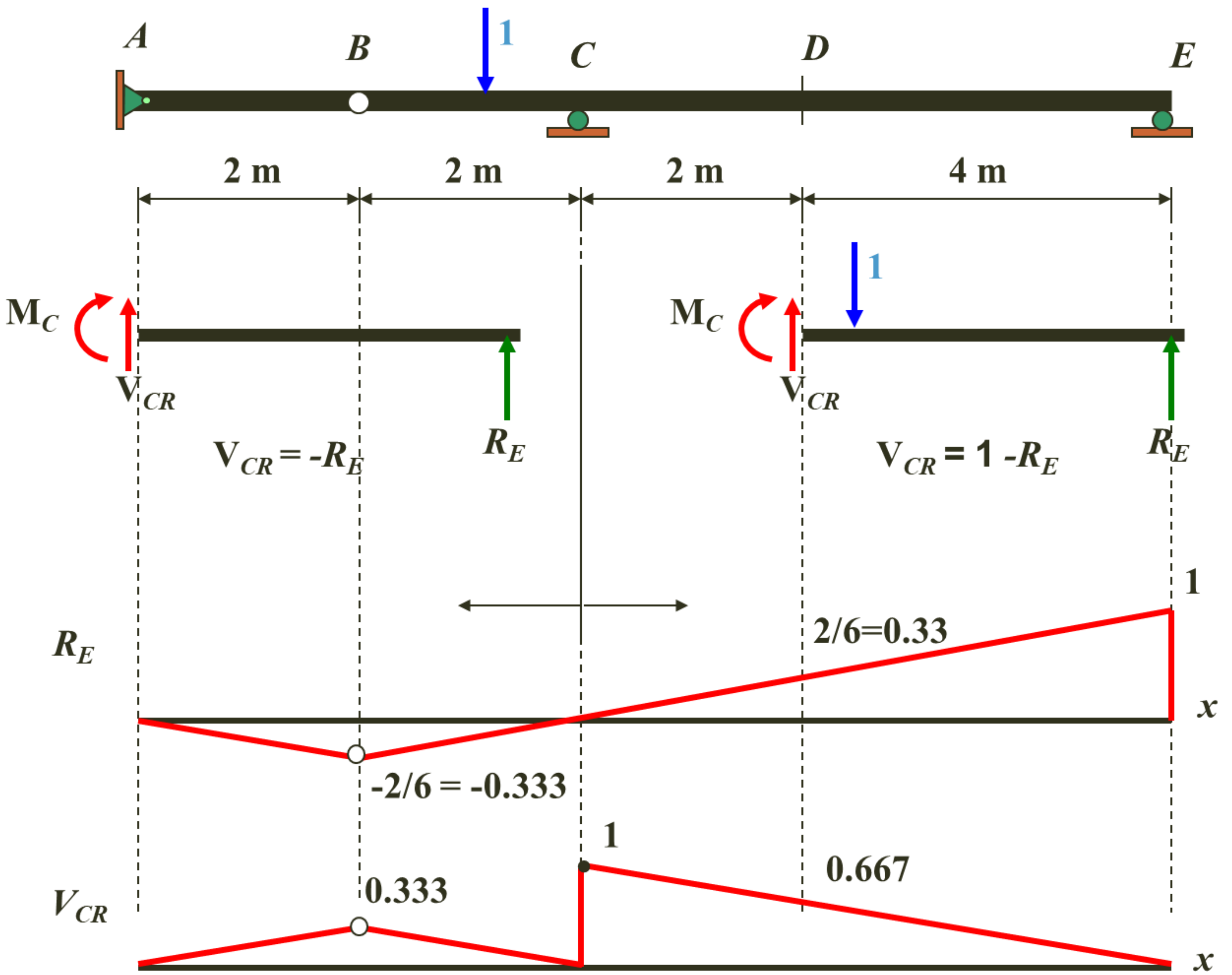


Influence Lines For Beams- Virtual Work

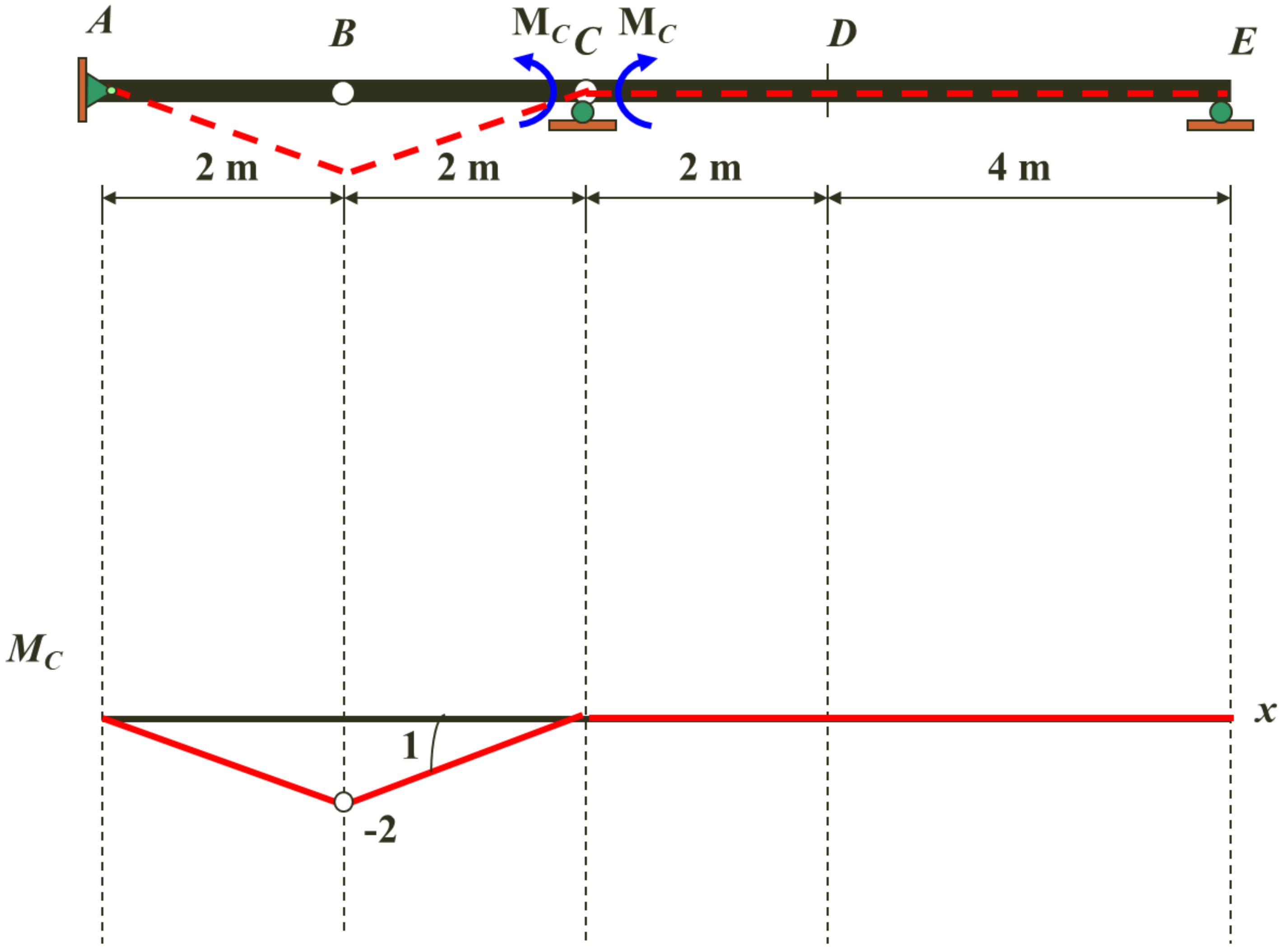


Influence Lines For Beams

Or using equilibrium conditions:

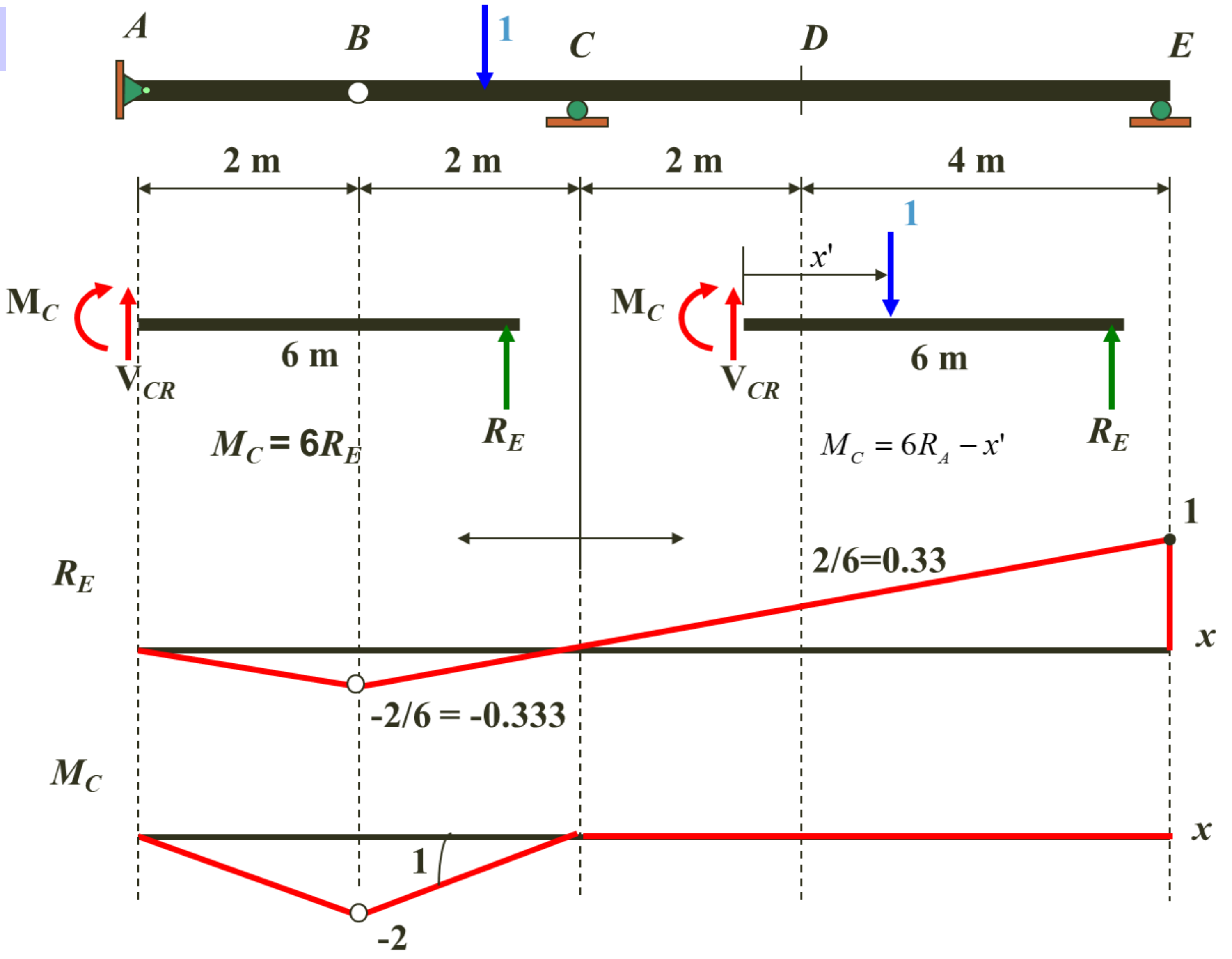


Influence Lines For Beams- Virtual Work



Influence Lines For Beams

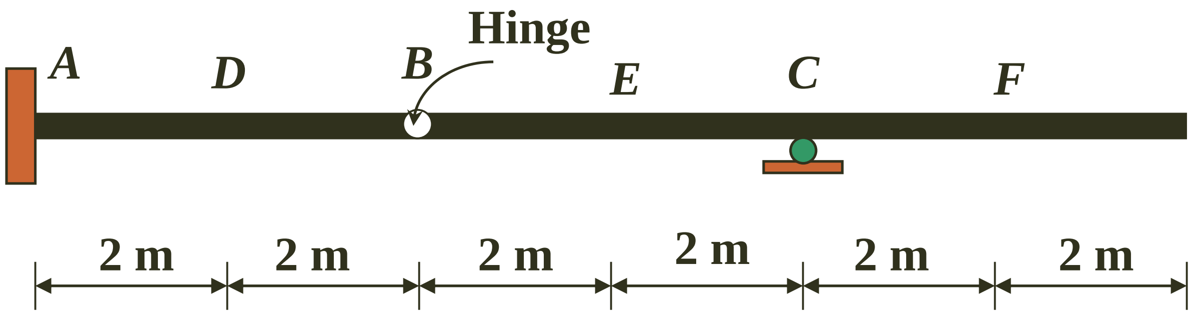
Or using equilibrium conditions:



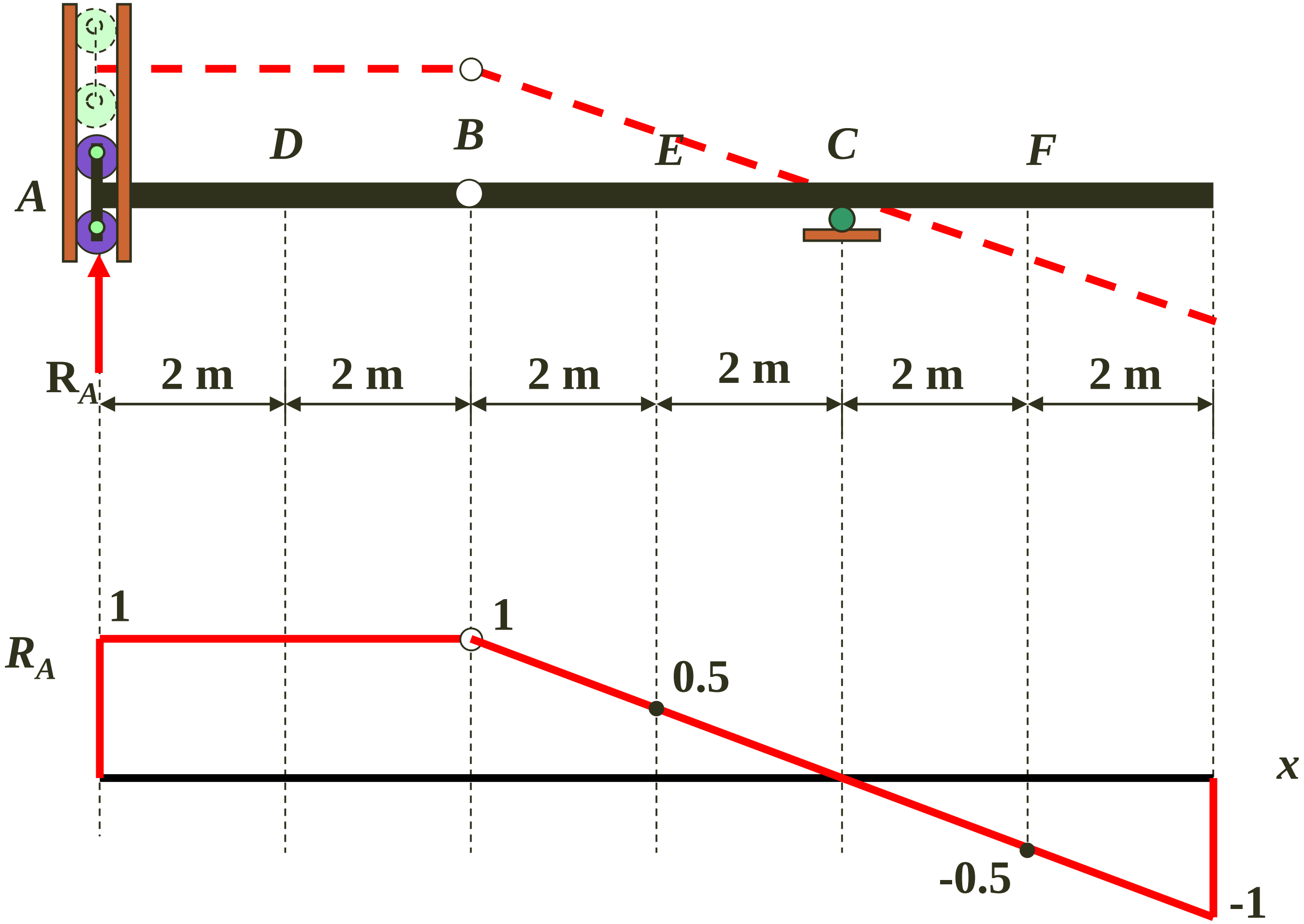
Example 6-3

Construct the influence line for

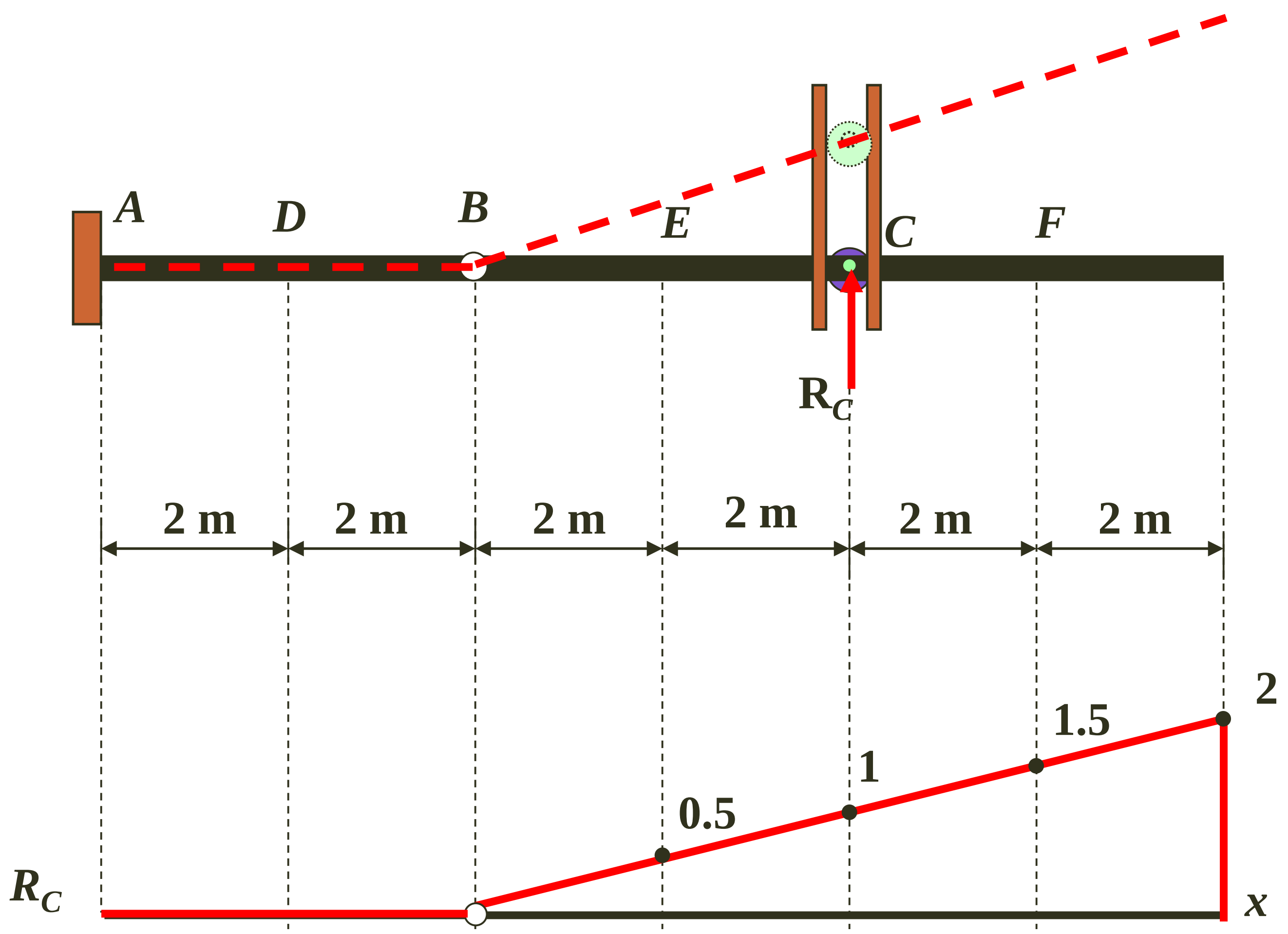
- the reaction at *A* and *C*
- shear at *D*, *E* and *F*
- the moment at *D*, *E* and *F*



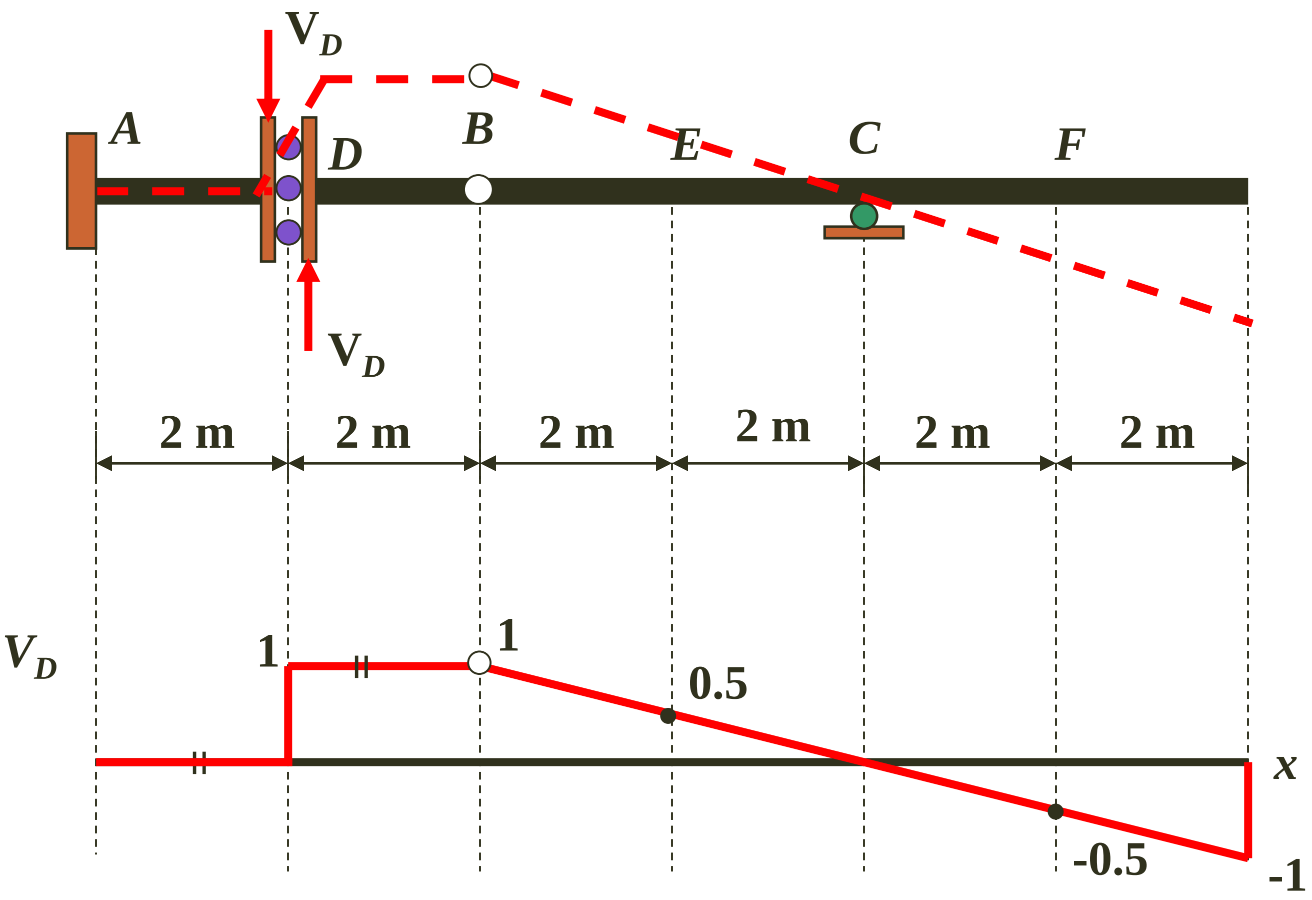
SOLUTION



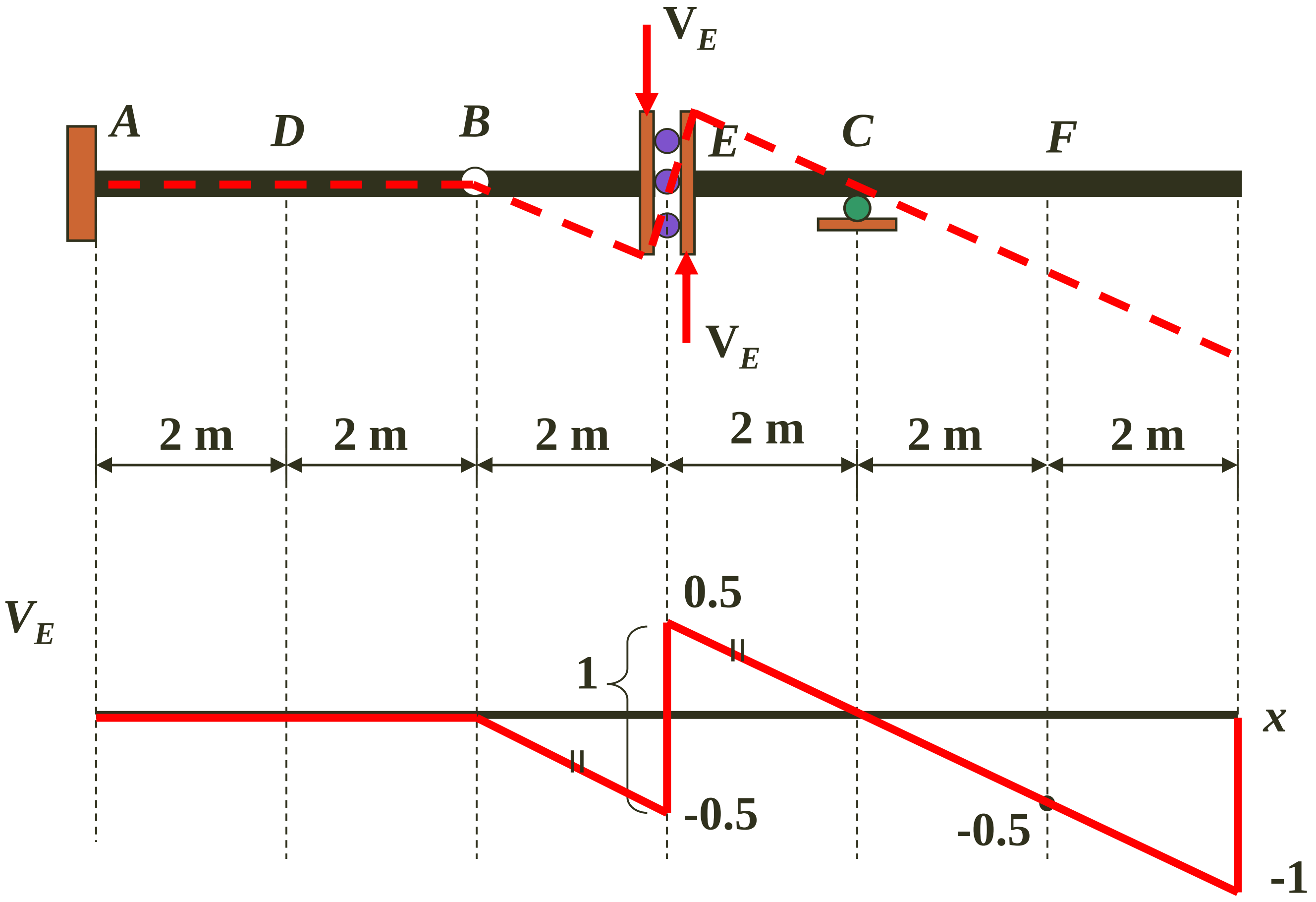
Influence Lines For Beams- Virtual Work



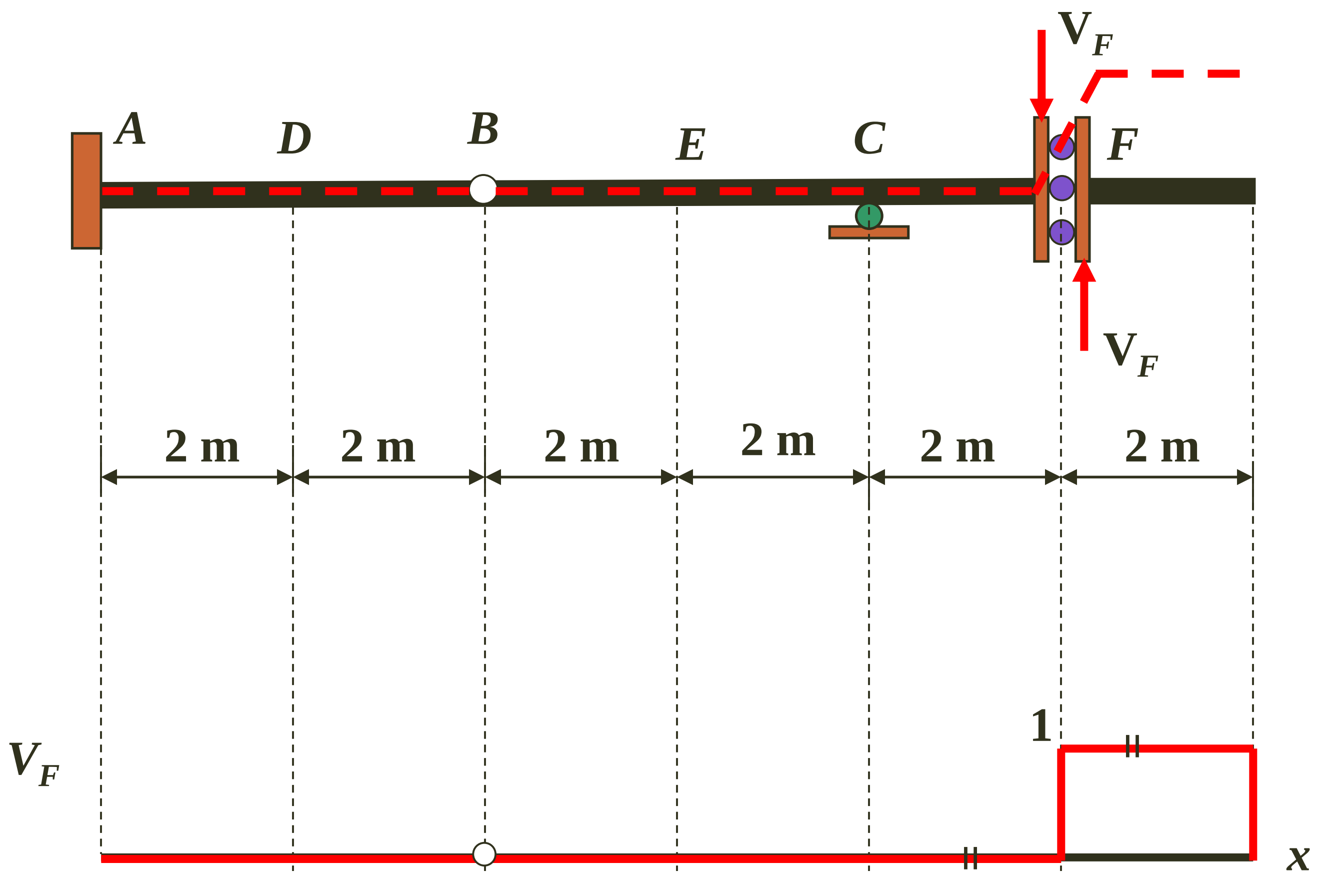
Influence Lines For Beams- Virtual Work



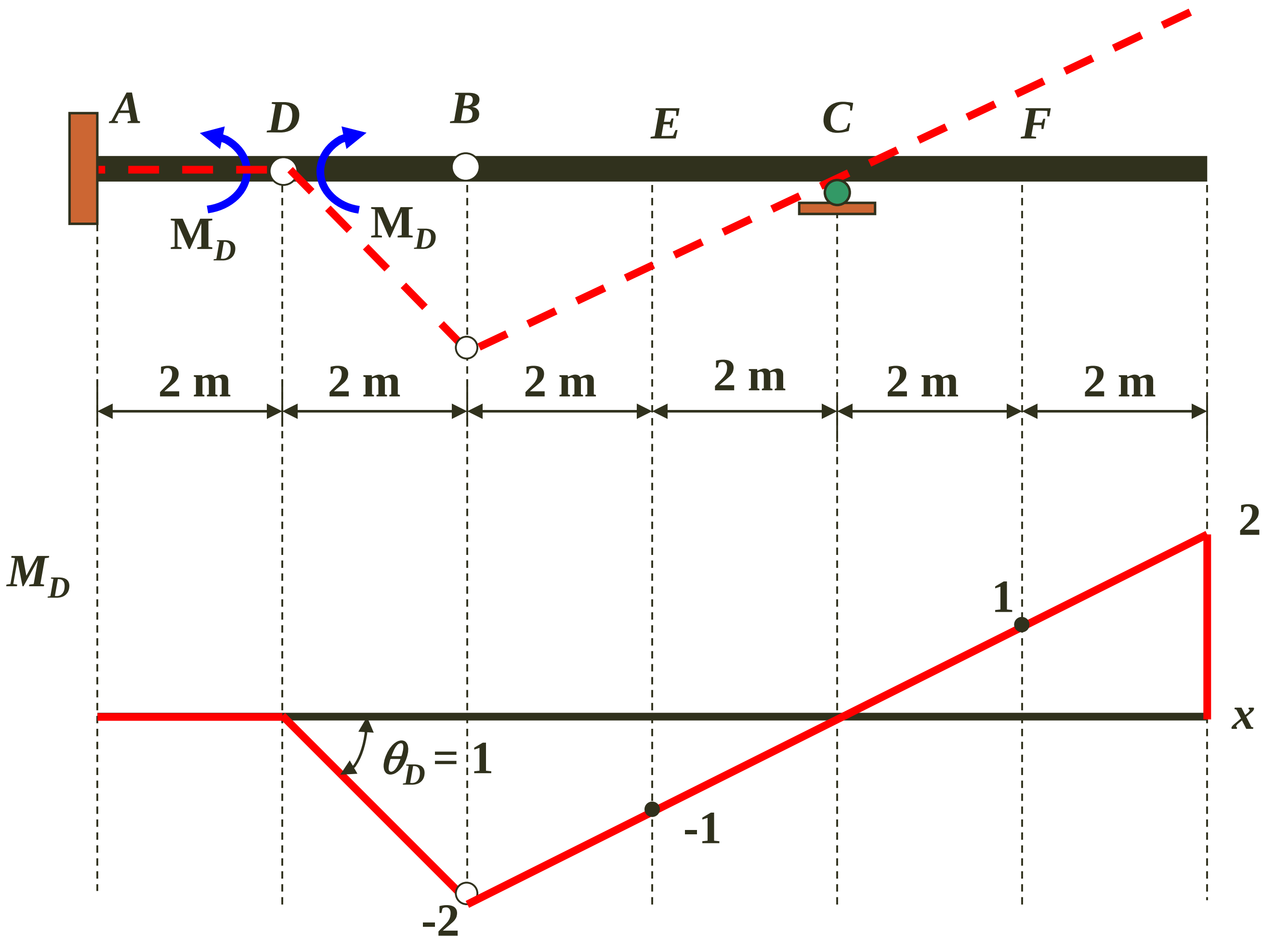
Influence Lines For Beams- Virtual Work



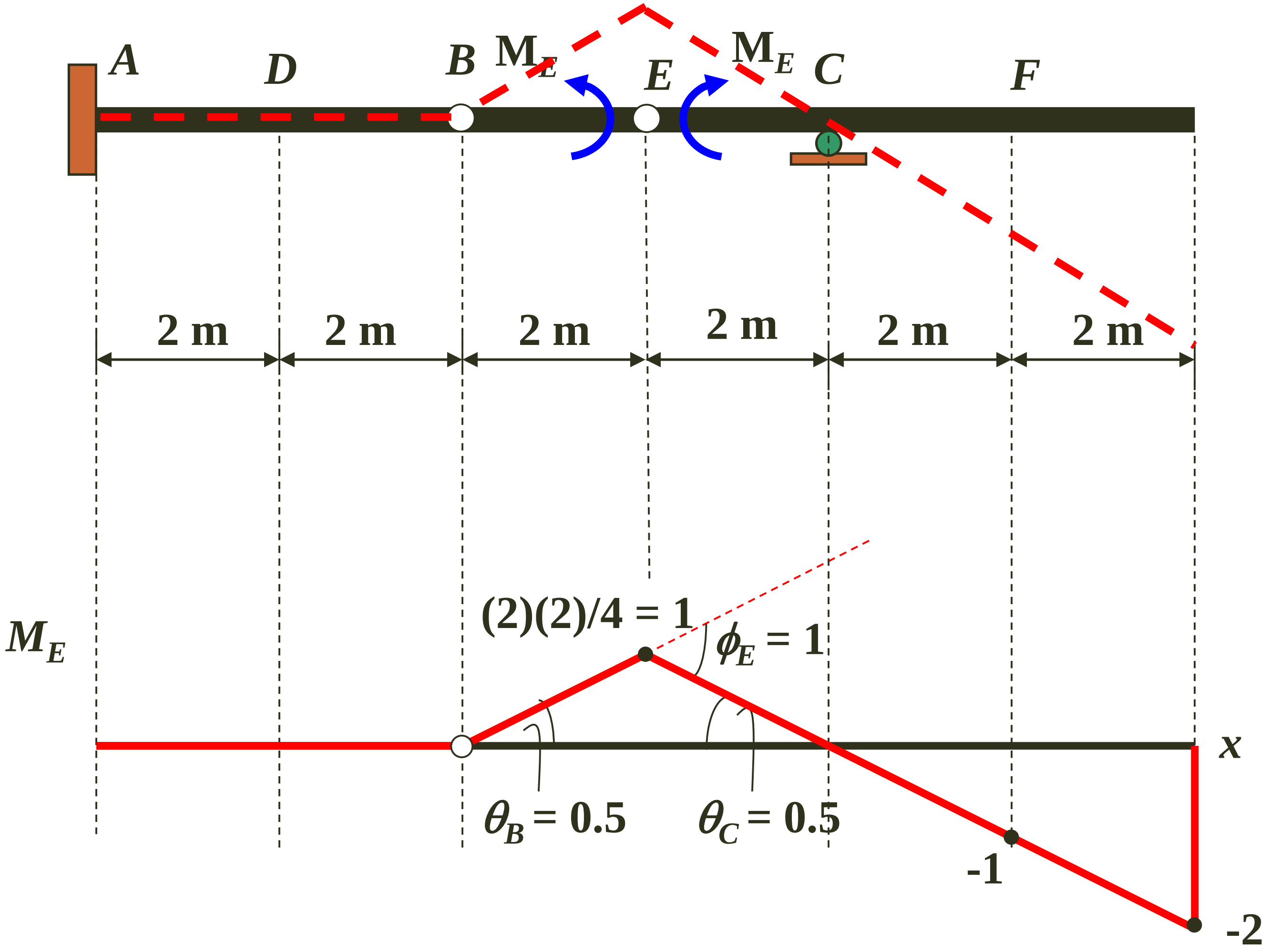
Influence Lines For Beams- Virtual Work

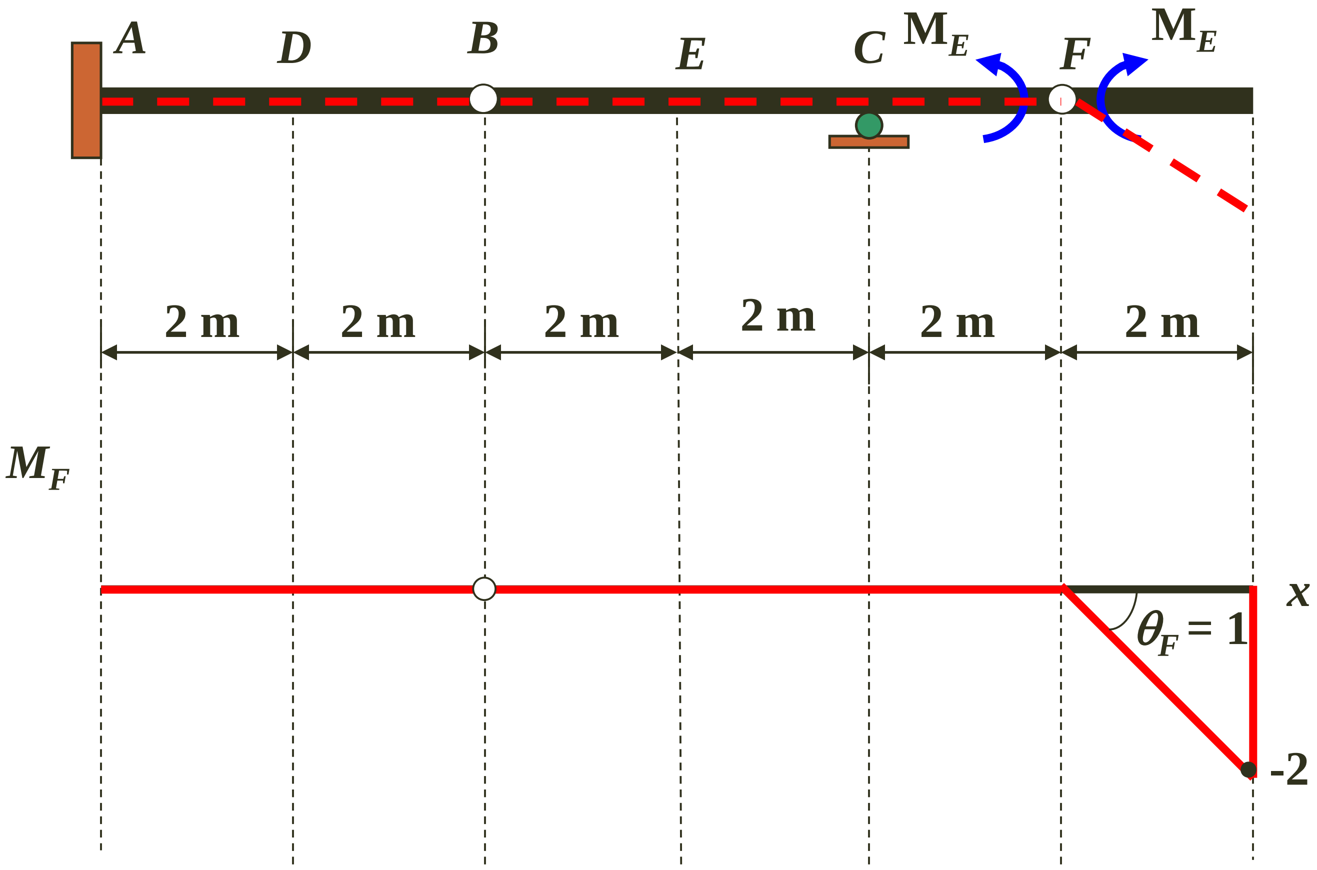


Influence Lines For Beams- Virtual Work



Influence Lines For Beams- Virtual Work



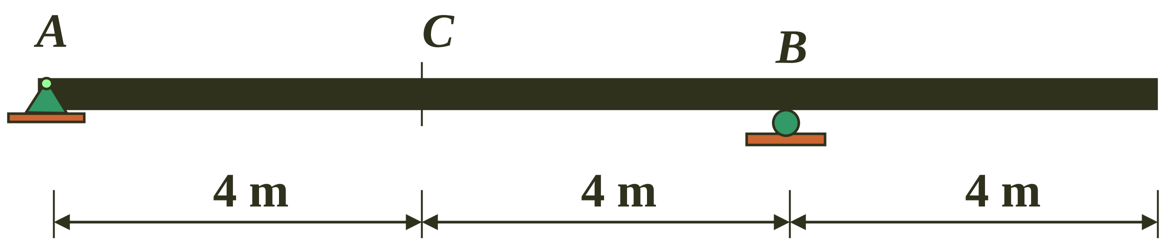

A
D
B
E
C
M_E
F
M_E
2 m
2 m
2 m
2 m
2 m
2 m
M_F
x
$\theta_F = 1$
-2

Influence Lines For Beams- The Use of

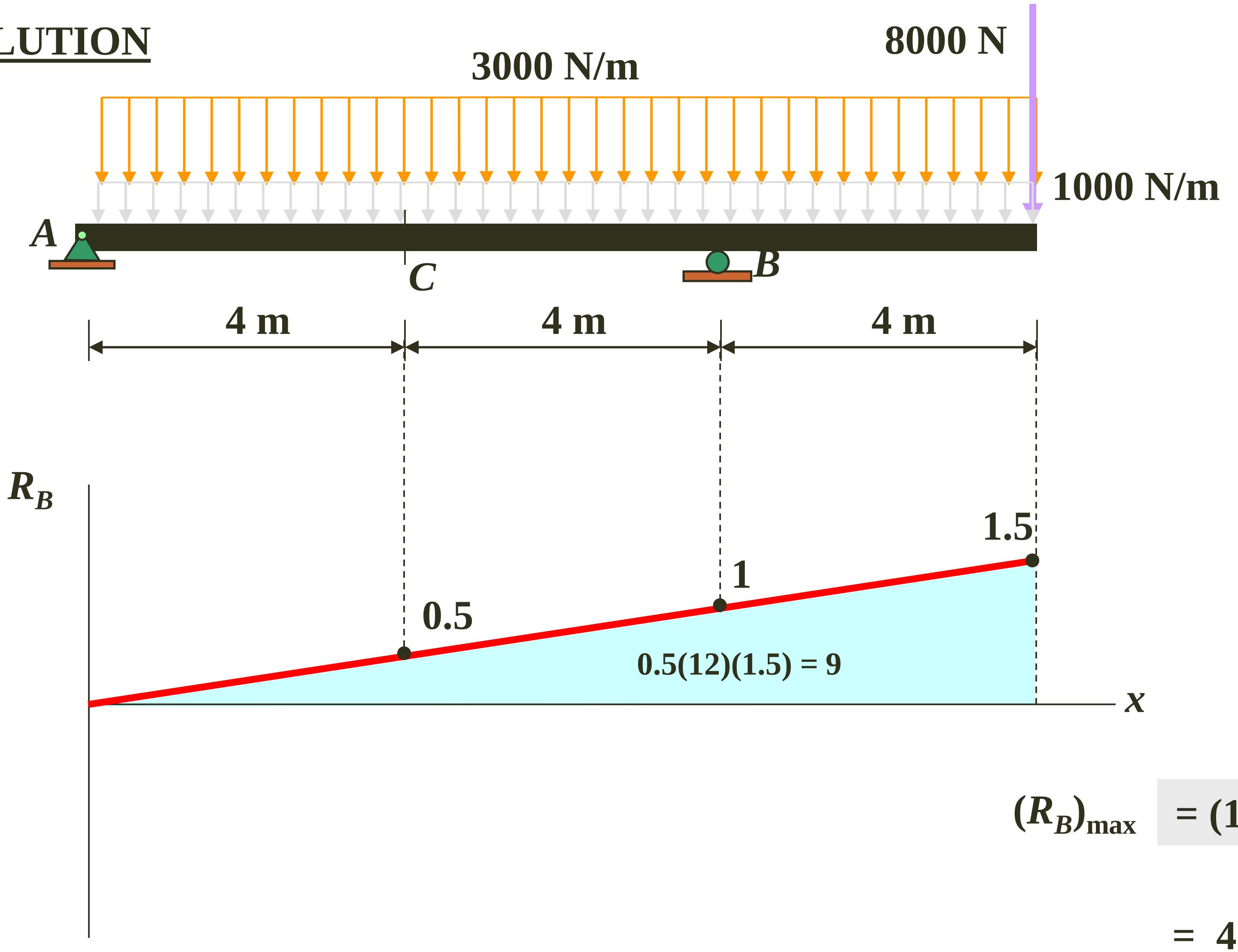
Example 6-4

Determine the maximum reaction at support *B*, the maximum shear at point *C* and the maximum positive moment that can be developed at point *C* on the beam shown due to

- a single concentrate live load of 8000 N
- a uniform live load of 3000 N/m
- a beam weight (dead load) of 1000 N/m



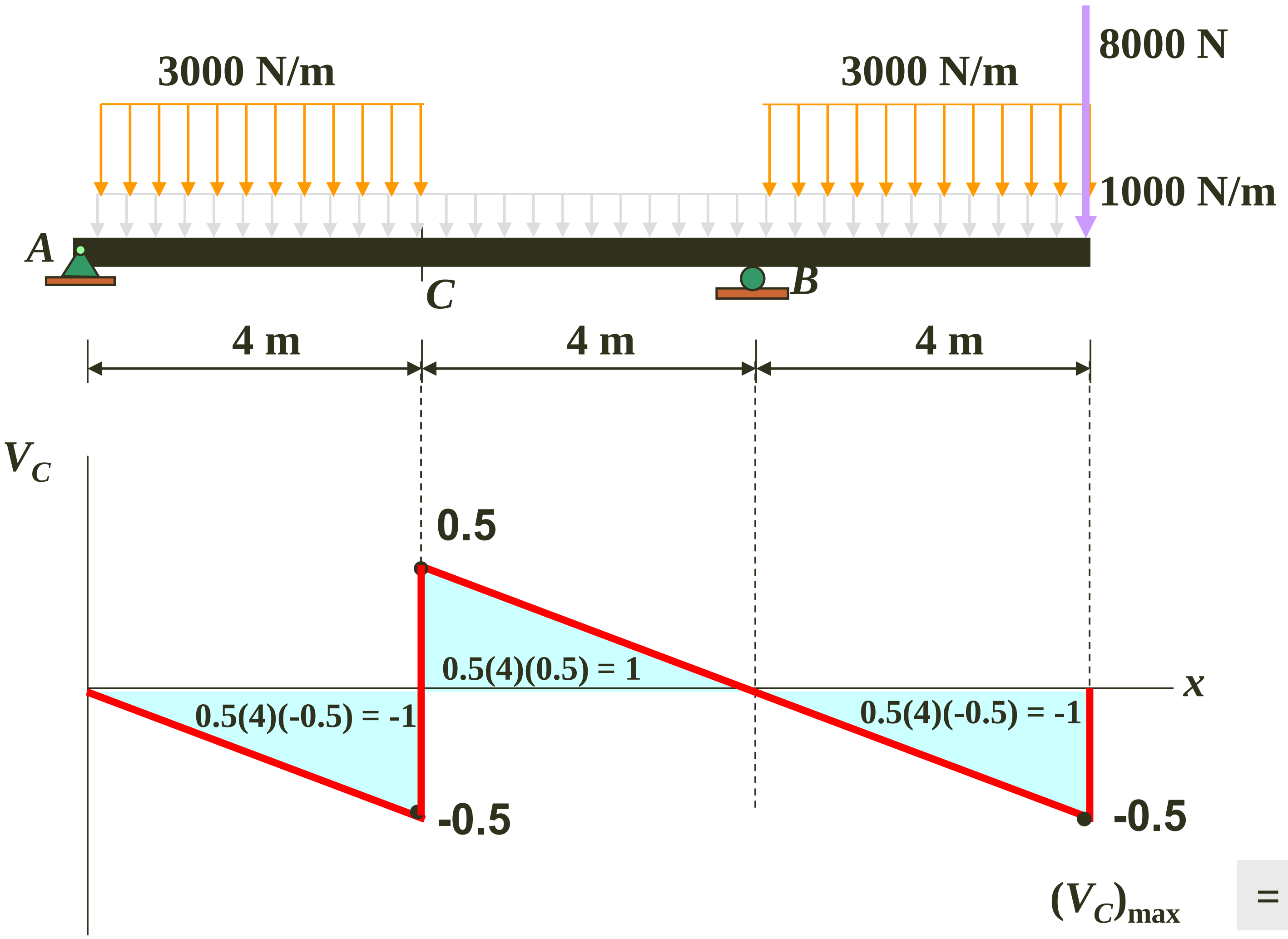
SOLUTION



$$(R_B)_{max} = (1000)(9) + (3000)(9) + (8000)(1.5)$$

$$= 48000 \text{ N} = 48 \text{ kN}$$

Influence Lines For Beams- The Use of



$(V_C)_{max}$ $= (1000)(-2+1) + (3000)(-2) + (8000)(-0.5)$

$= -11000\ \text{N} = 11\ \text{kN}$

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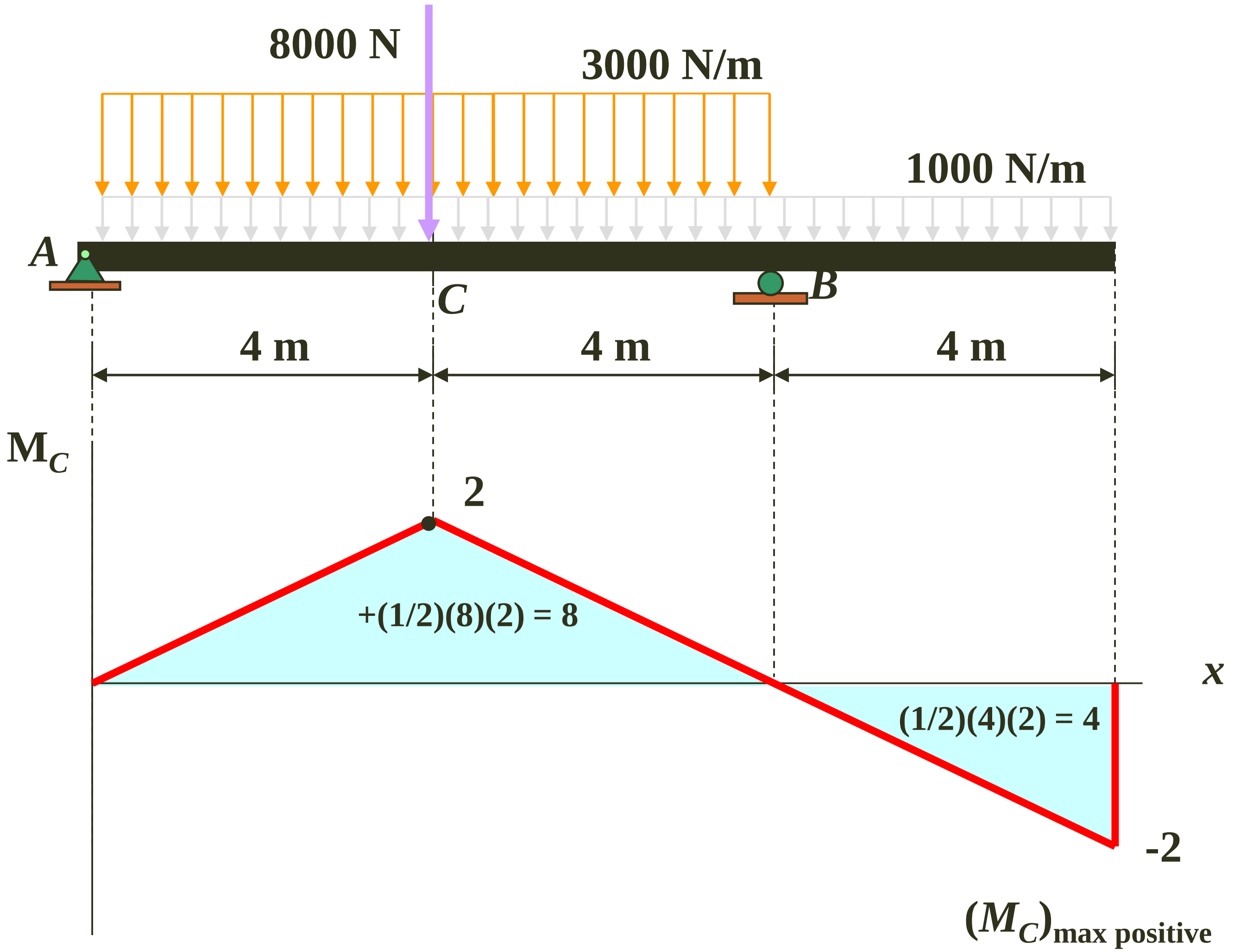

$(M_C)_{max\ positive}$ = (8000)(2) + (3000)(8) + (8-4)(1000)

= 44000 N•m = 44 kN•m