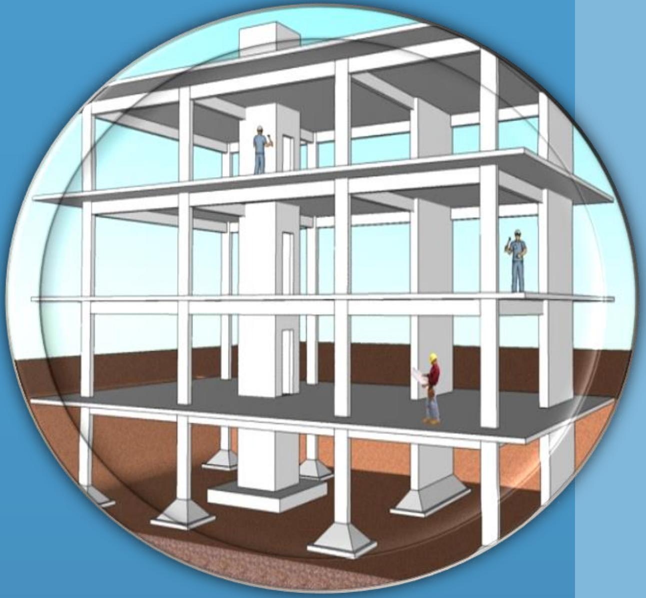
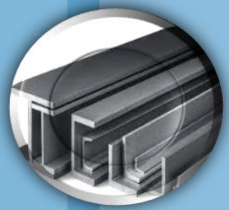
UNIVERSITY OF ZAMBIA
School of Engineering
Department of Civil & Environmental Eng.



CEE 3222: THEORY OF STRUCTURES

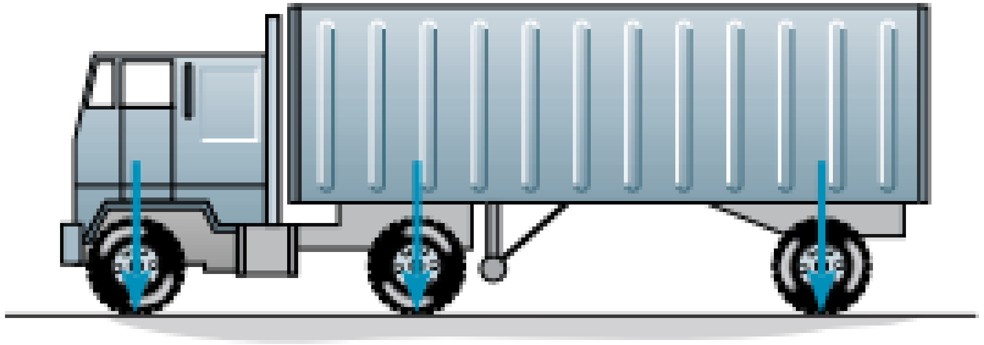
Lecture 2.4

MAXIMUM INFLUENCE AT A POINT DUE TO A SERIES OF CONCENTRATED LOADS



Mr. MWABA MSc, B.Eng., R.Eng., PEIZ,

Maximum Influence at a Point Due to a Series of Concentrated Loads



Maximum Influence at a Point Due to a Series of Concentrated Loads

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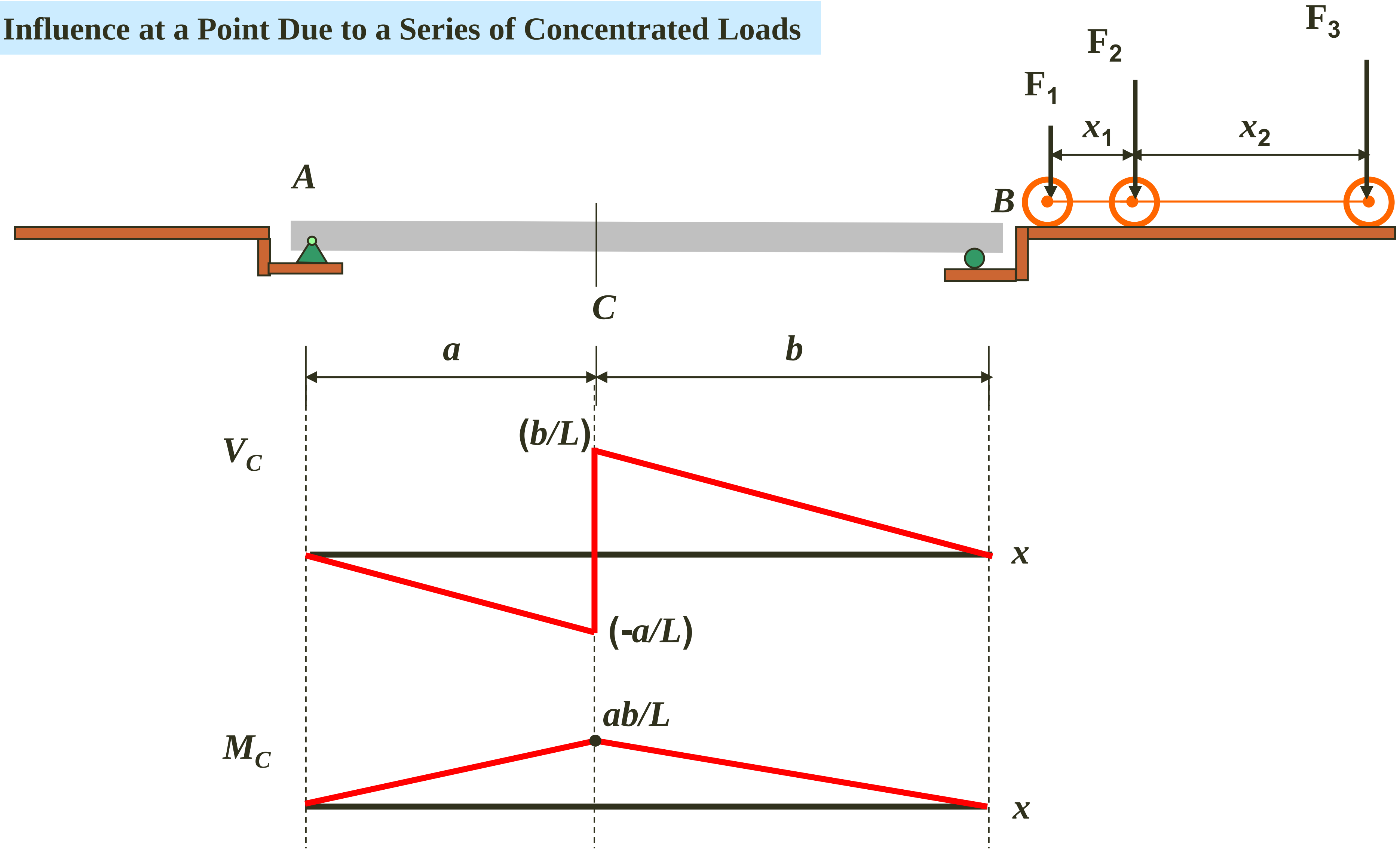
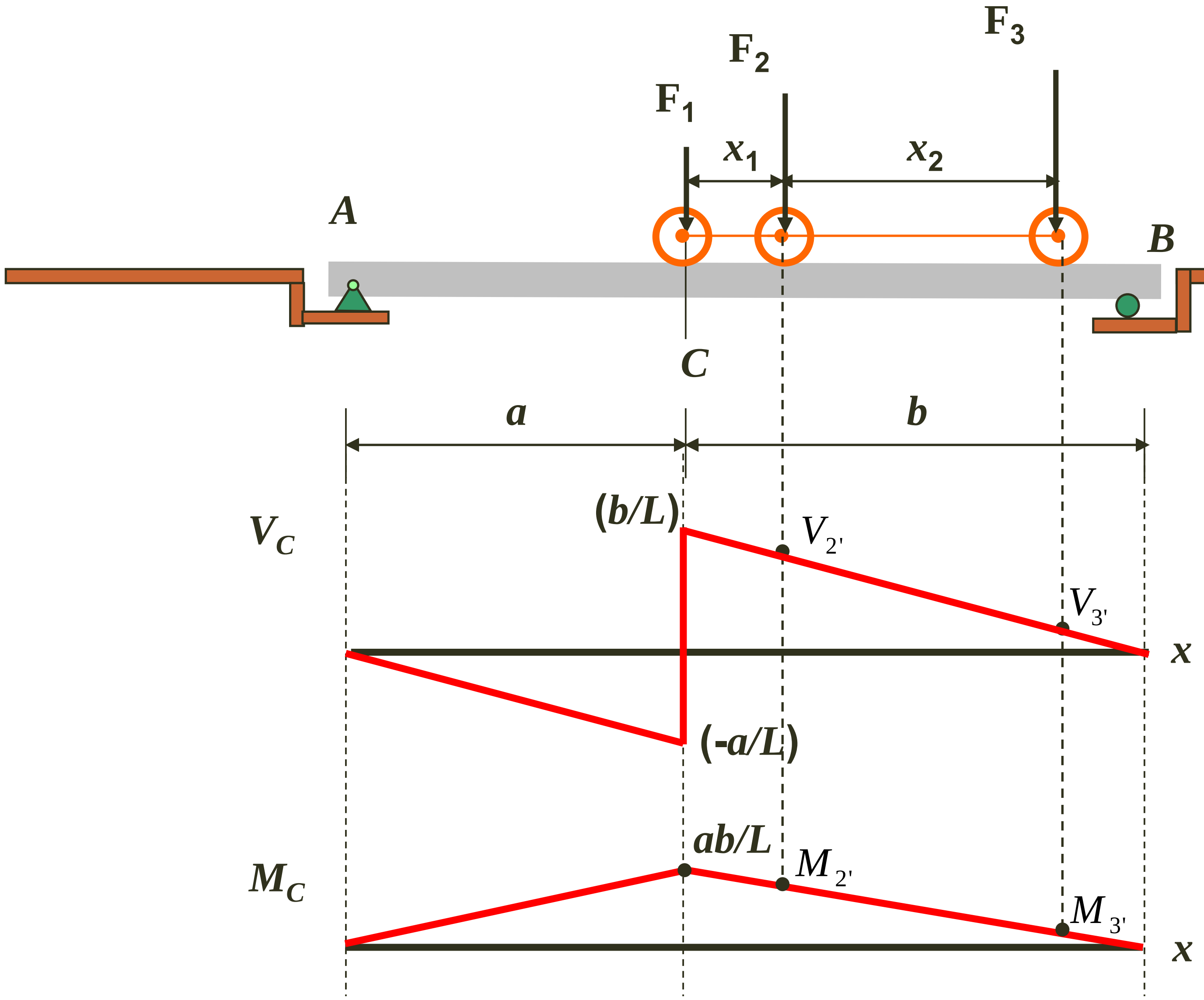


Maximum Influence at a Point Due to a Series of Concentrated Loads

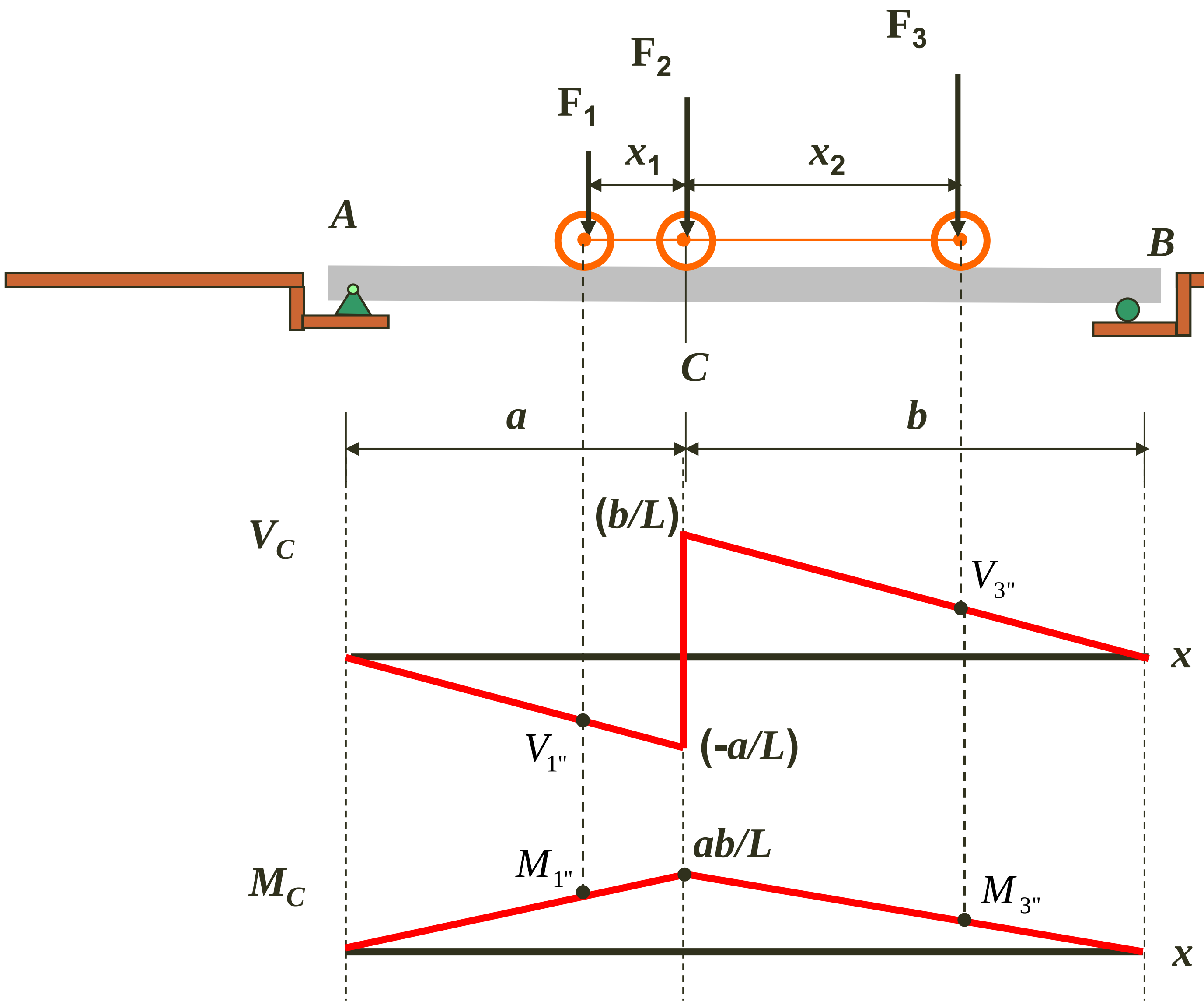


$$(V_{CR})' = F_1(b/L) + F_2(V_{2'}) + F_3(V_{3'})$$

$$(V_{CL})' = F_1(-a/L) + F_2(V_{2'}) + F_3(V_{3'})$$

$$M' = F_1(ab/L) + F_2(M_{2'}) + F_3(M_{3'})$$

Maximum Influence at a Point Due to a Series of Concentrated Loads

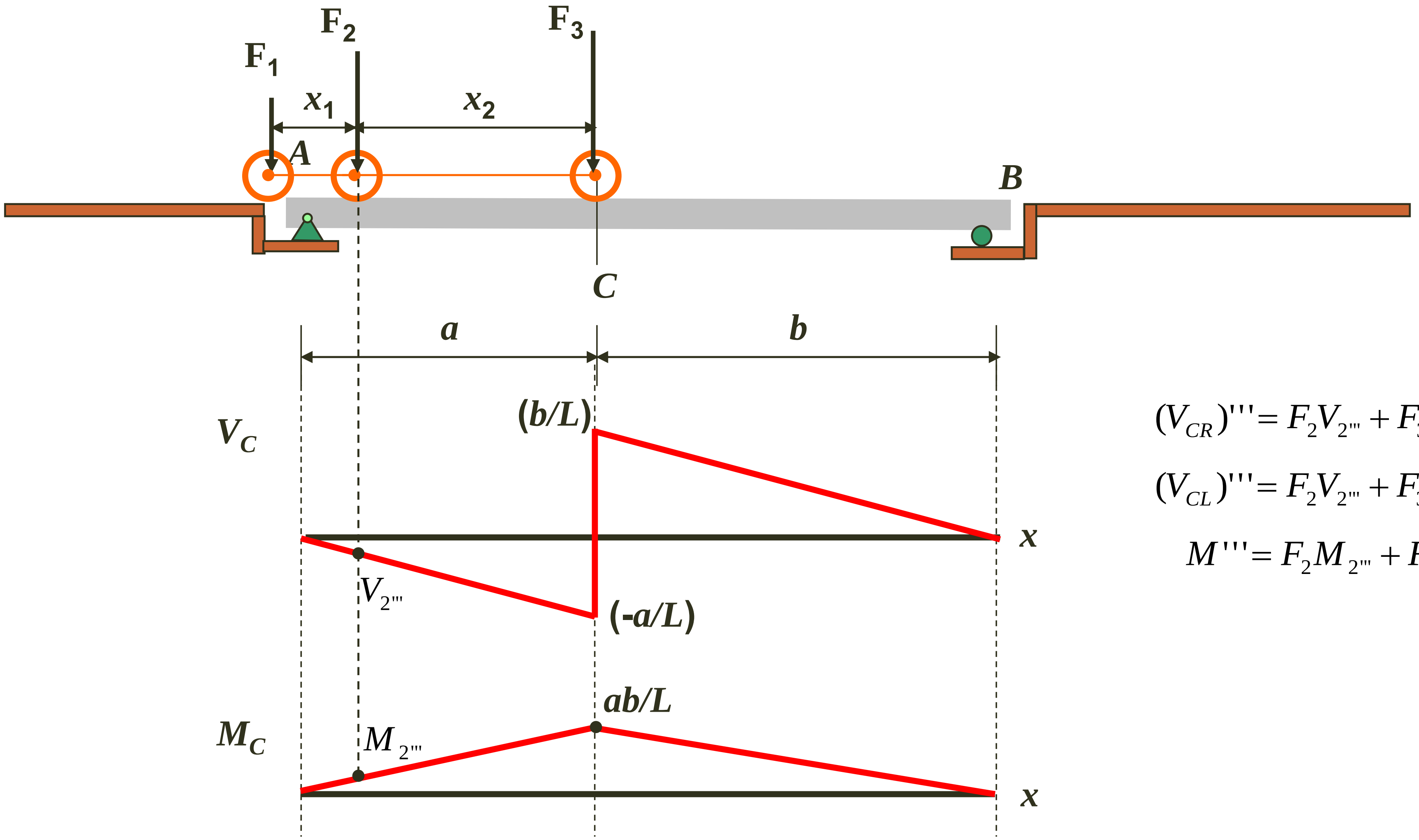


$$(V_{CR})'' = F_1 V_{1''} + F_2(b/L) + F_3(V_{3''})$$

$$(V_{CL})'' = F_1 V_{1''} + F_2(-a/L) + F_3(V_{3''})$$

$$M'' = F_1 M_{1''} + F_2(ab/L) + F_3(M_{3''})$$

Maximum Influence at a Point Due to a Series of Concentrated Loads



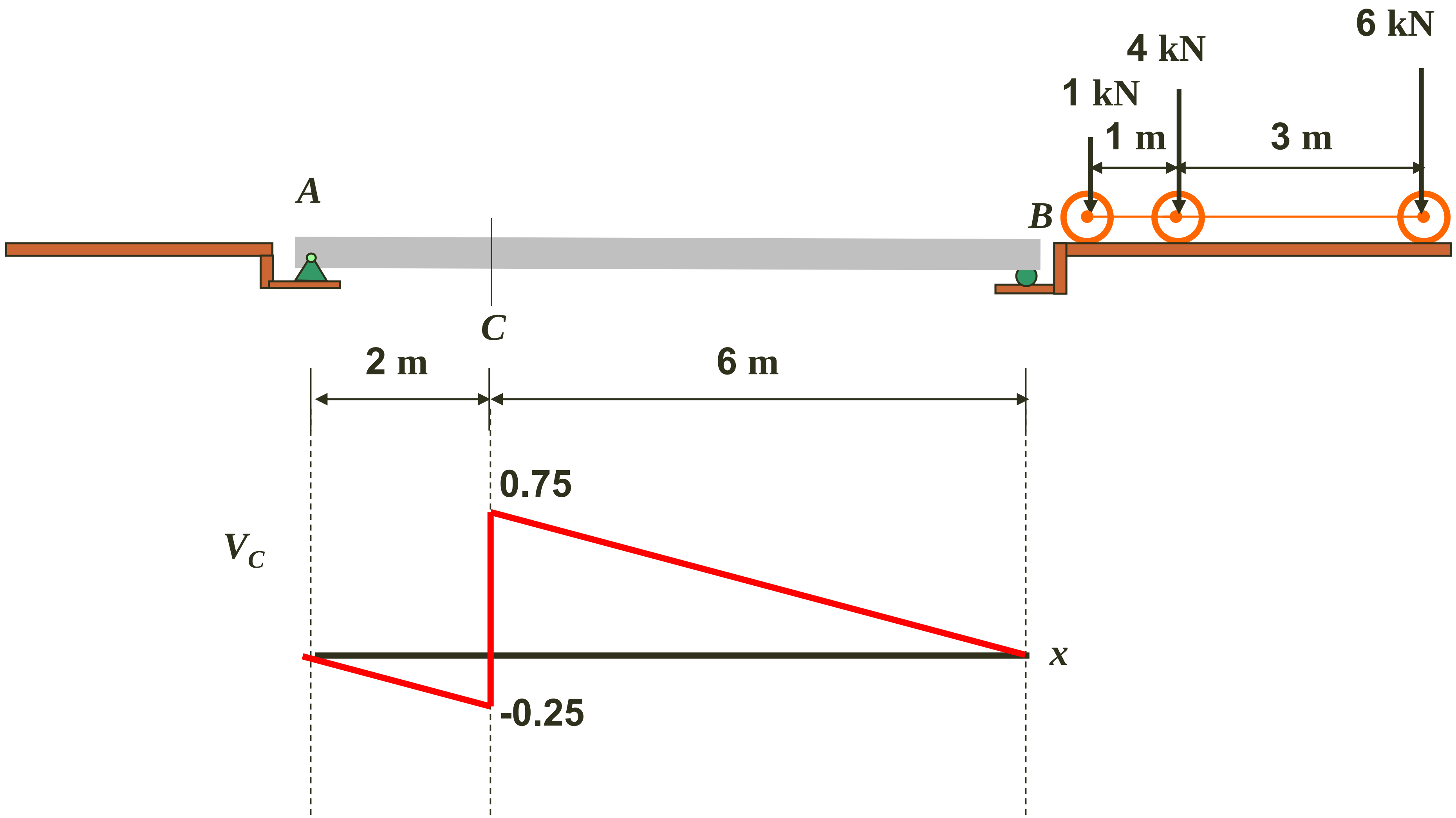
$$(V_{CR})''' = F_2 V_{2'''} + F_3 (b/L)$$

$$(V_{CL})''' = F_2 V_{2'''} + F_3 (-a/L)$$

$$M''' = F_2 M_{2'''} + F_3 (ab/L)$$

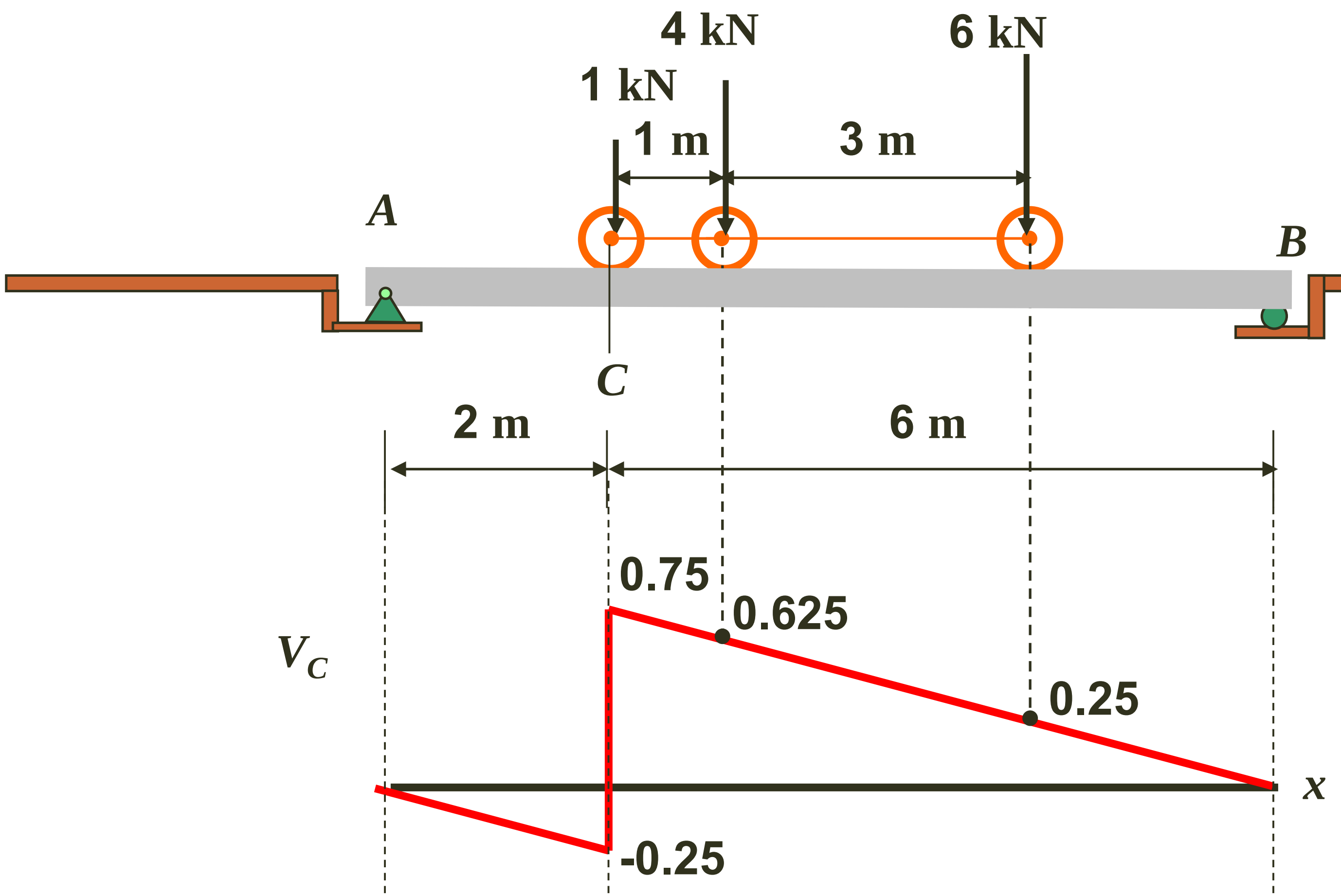
Maximum Influence at a Point Due to a Series of Concentrated Loads

- Shear



Maximum Influence at a Point Due to a Series of Concentrated Loads

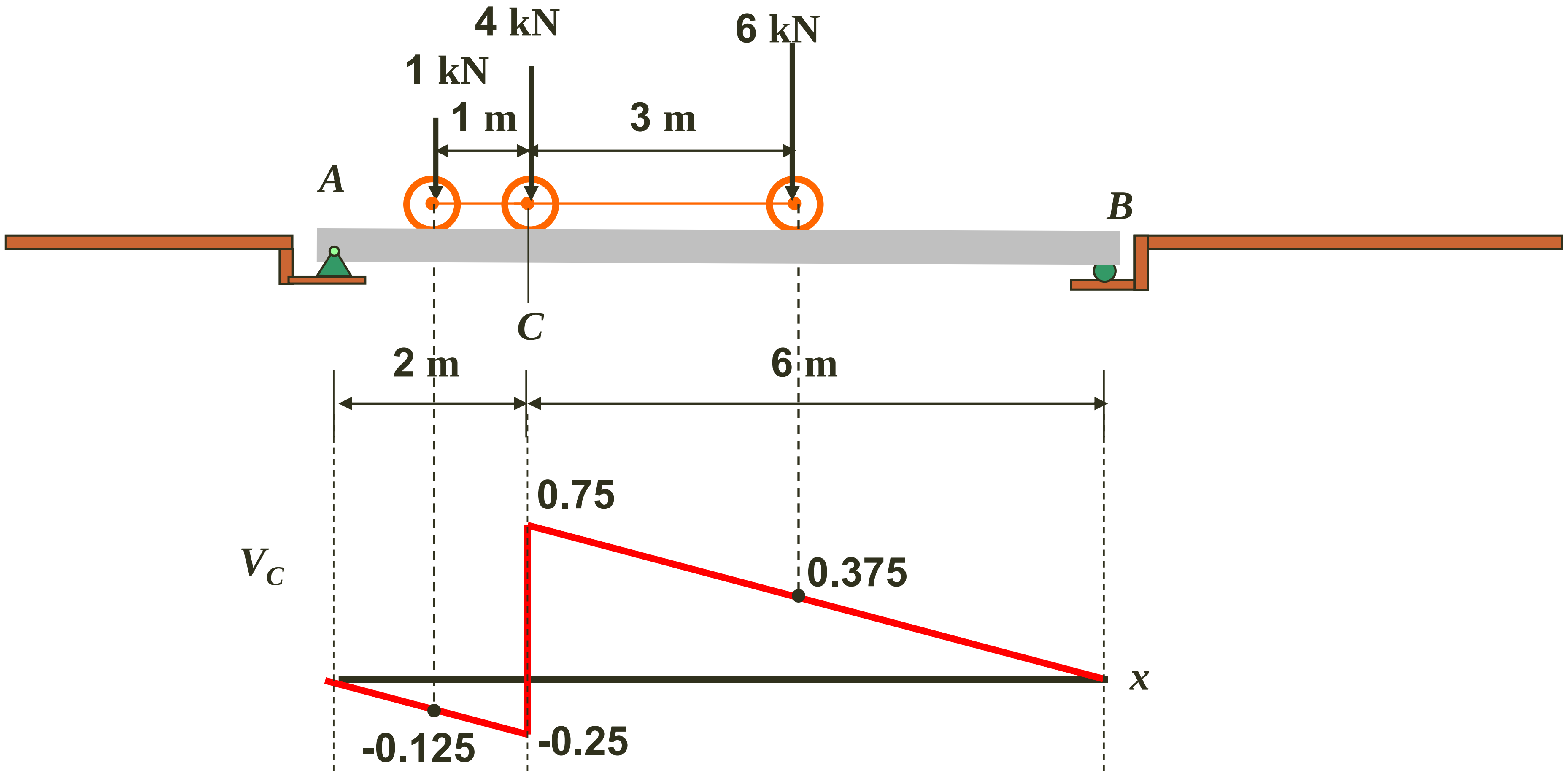
Case 1



$(V_C)_1$ = 1(0.75) + 4(0.625) + 6(0.25) = 4.75 kN

Maximum Influence at a Point Due to a Series of Concentrated Loads

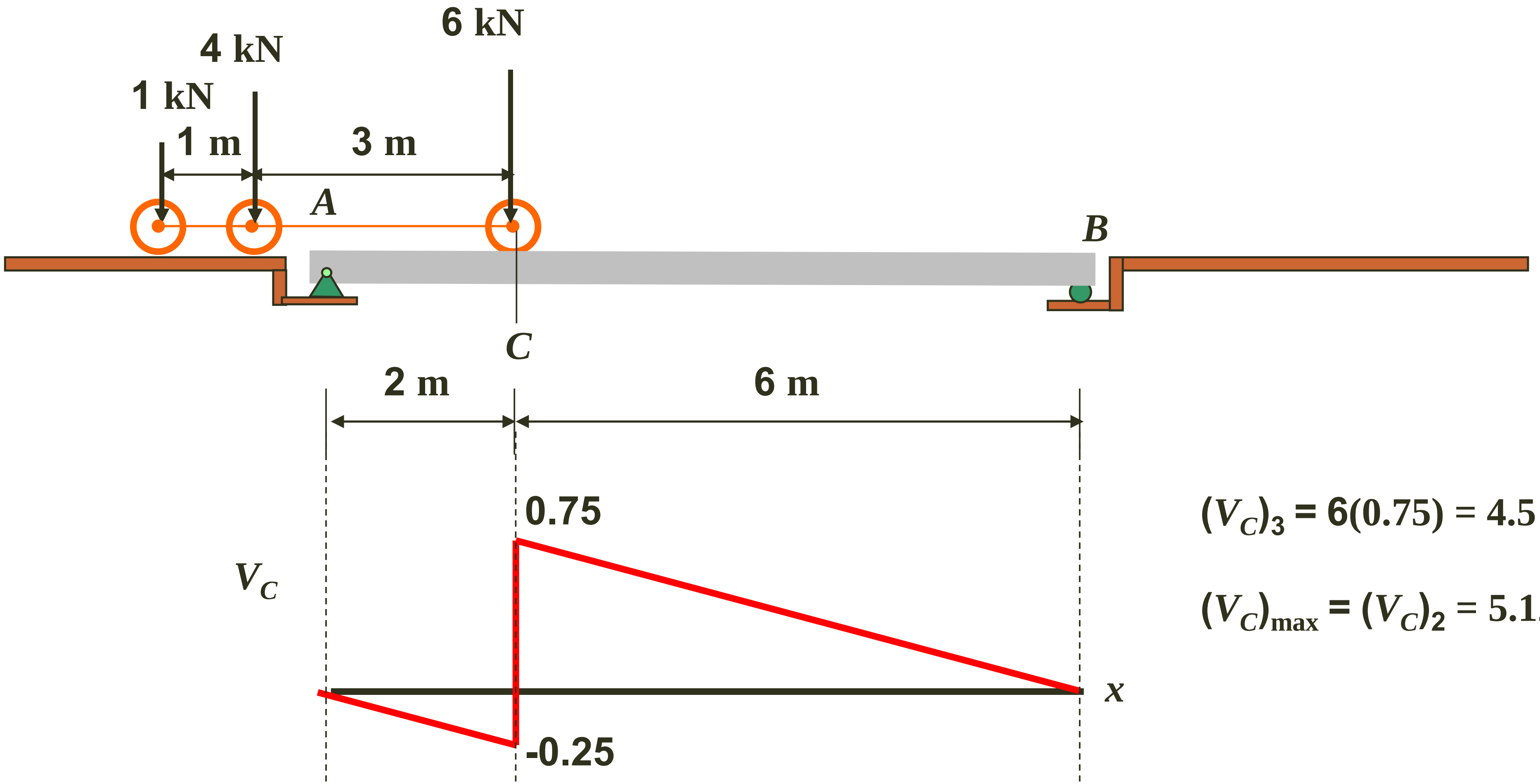
Case 2



$(V_C)_2 = 1(-0.125) + 4(0.75) + 6(0.375) = 5.125$ kN

Maximum Influence at a Point Due to a Series of Concentrated Loads

Case 3



$(V_C)_3 = 6(0.75) = 4.5$ kN

$(V_C)_{max} = (V_C)_2 = 5.125$ kN

Maximum Influence at a Point Due to a Series of Concentrated Loads. Climb and Jump Method

- The trial-and-error computations can be tedious when to many concentrated loads.
- A more direct manner by finding the change in shear, ΔV, which occurs when the loads move from Case 1 to Case 2, then from Case 2 to Case 3 and so on.
- As long as each computed ΔV is positive, the new position will yield a larger shear in the beam at C than the previous position.
- When there is a negative change in shear , the previous position of the loads will give the critical value.

$$\Delta V_1 = Ps(\boldsymbol{p}_2 - \boldsymbol{p}_1)$$

Sloping Line

$$\Delta V_2 = P(\boldsymbol{q}_2 - \boldsymbol{q}_1)$$

Jump

Maximum Influence at a Point Due to a Series of Concentrated Loads Climb and Jump Method

- The critical position of the loads can be determined in a more direct manner by finding the change in shear, DV, which occurs when the load are moved from Case 1 to Case 2, then from Case 2 to Case 3, and so

$$\Delta V1 = Ps(x_2 - x_1)$$

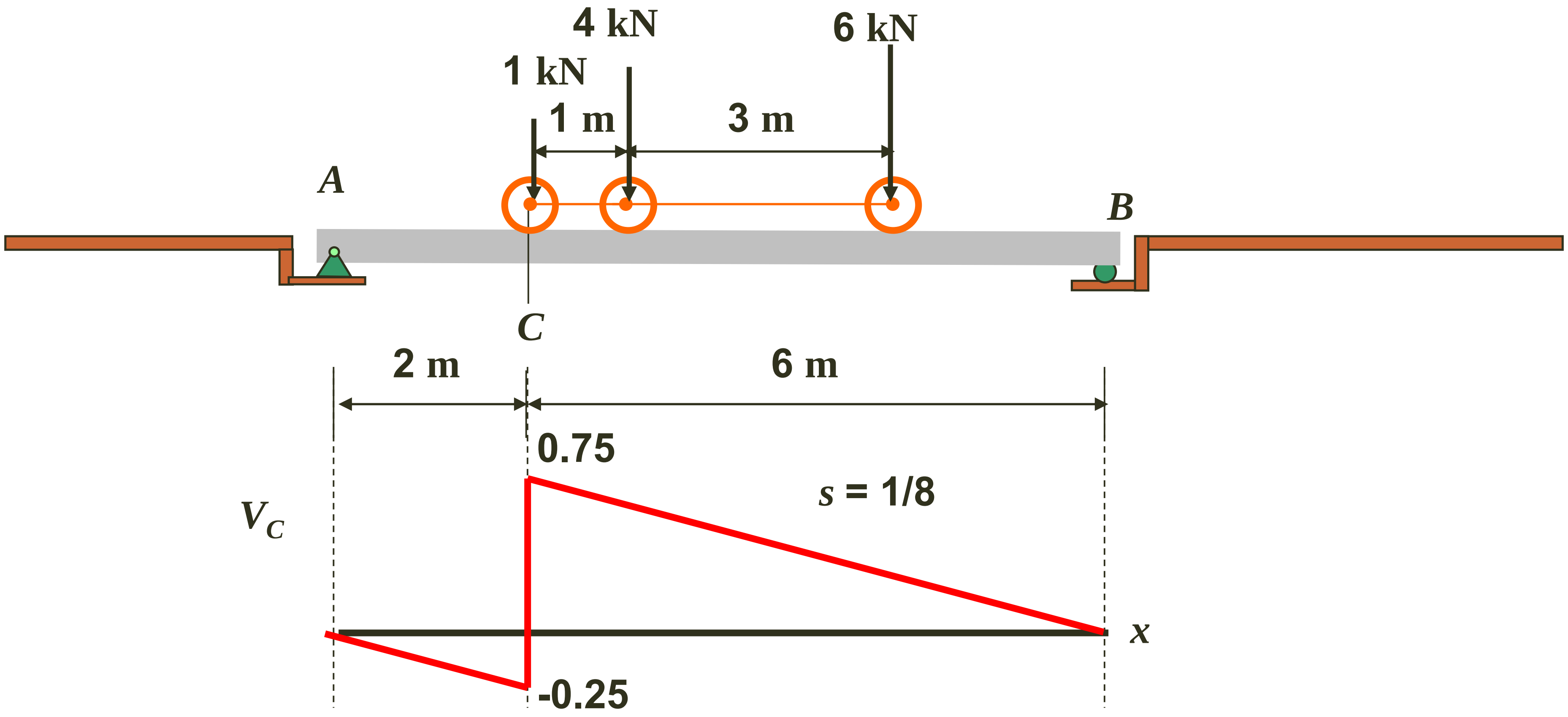
Sloping Line

- If the load moves past a point where there is a discontinuity or "jump" in the influence line, as point C, then the change in shear is simply

$$\Delta V2 = P(y_2 - y_1)$$

Jump

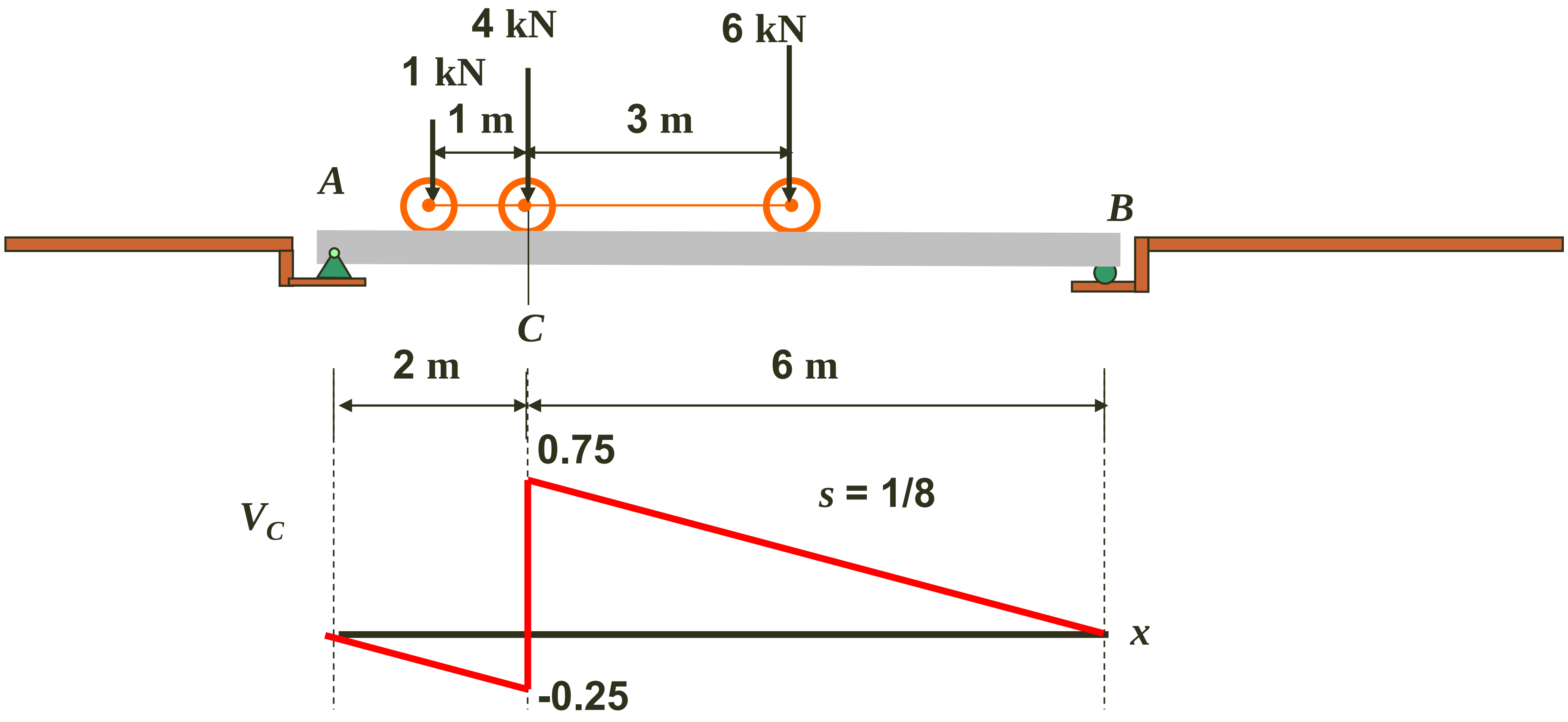
Maximum Influence at a Point Due to a Series of Concentrated Loads Climb and Jump Method



$\Delta V_{1\text{-}2}$ = 1(-0.25 - 0.75) + 1(1/8)(1) + 4(1/8)(1) + 6(1/8)(1) = 0.375 kN

The change is positive, meaning at 2 yields higher influence

Maximum Influence at a Point Due to a Series of Concentrated Loads Climb and Jump Method

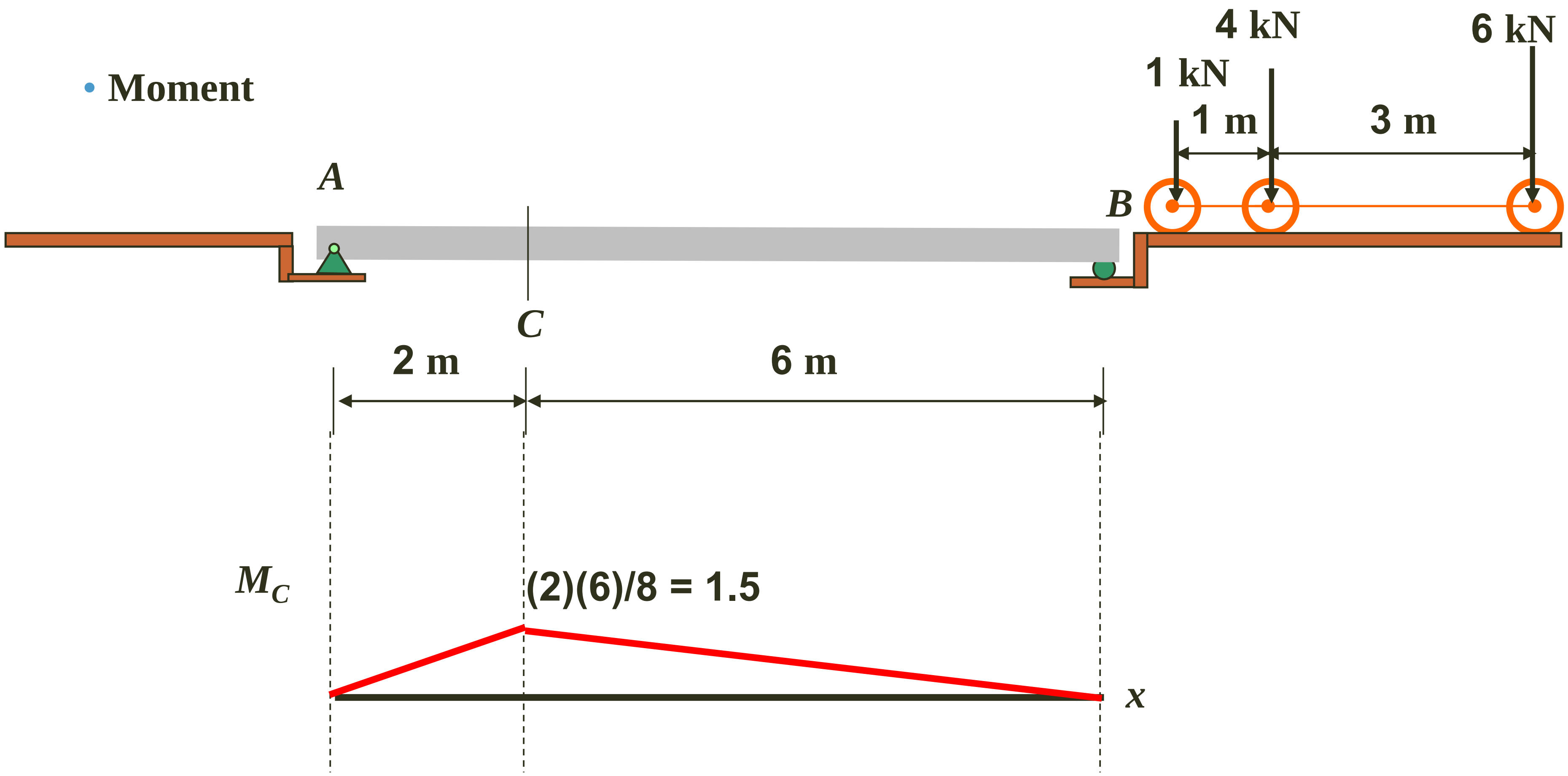


$\Delta V_{2\text{-}3}$ = 1(1/8)(1) + 4(-0.25-0.75) + 4(1/8)(2) + 6(1/8)(3) = -0.625 kN

Since this change is negative, then the previous case is critical

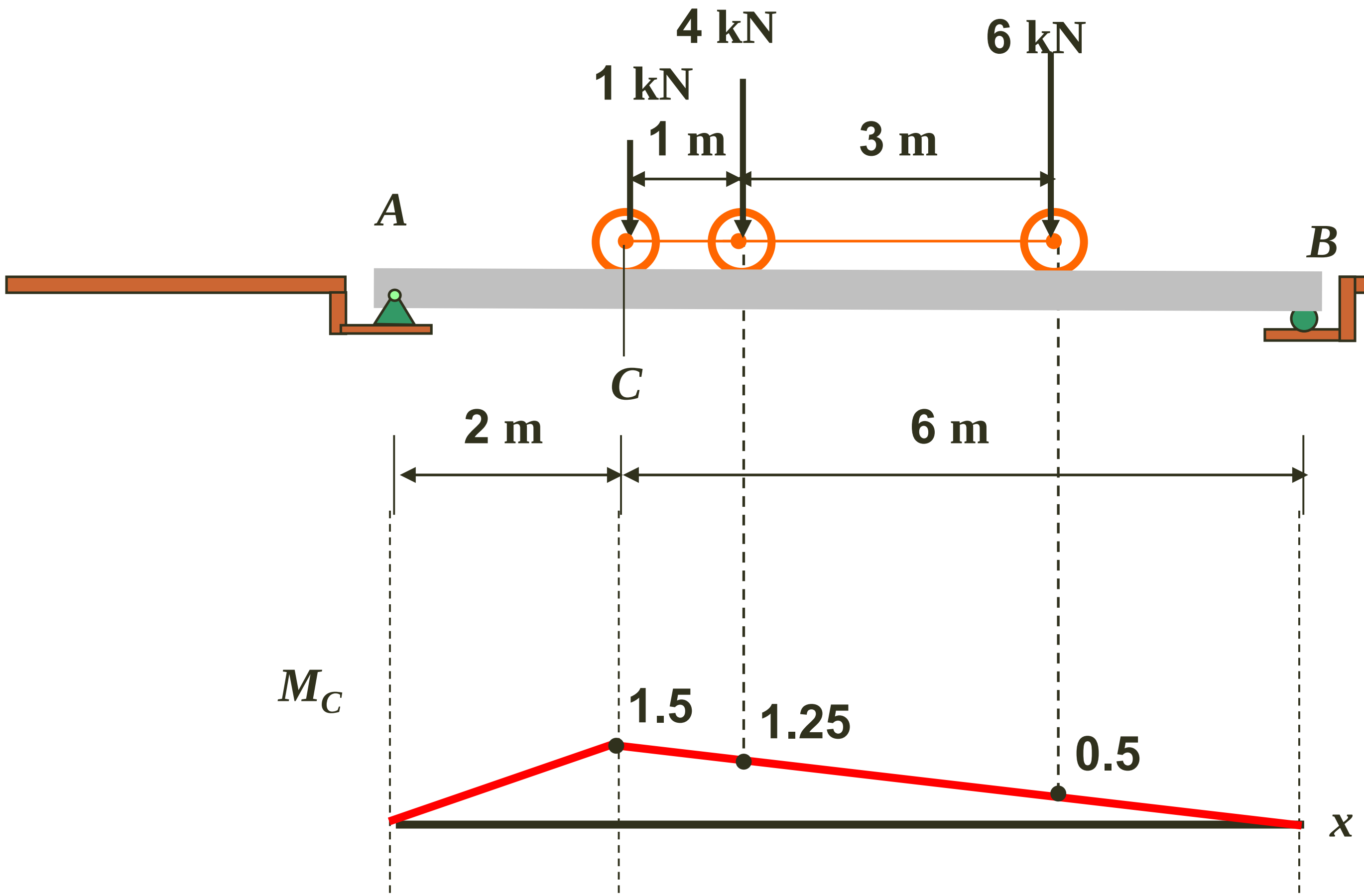
Maximum Influence at a Point Due to a Series of Concentrated Loads

- **Moment**



Maximum Influence at a Point Due to a Series of Concentrated Loads

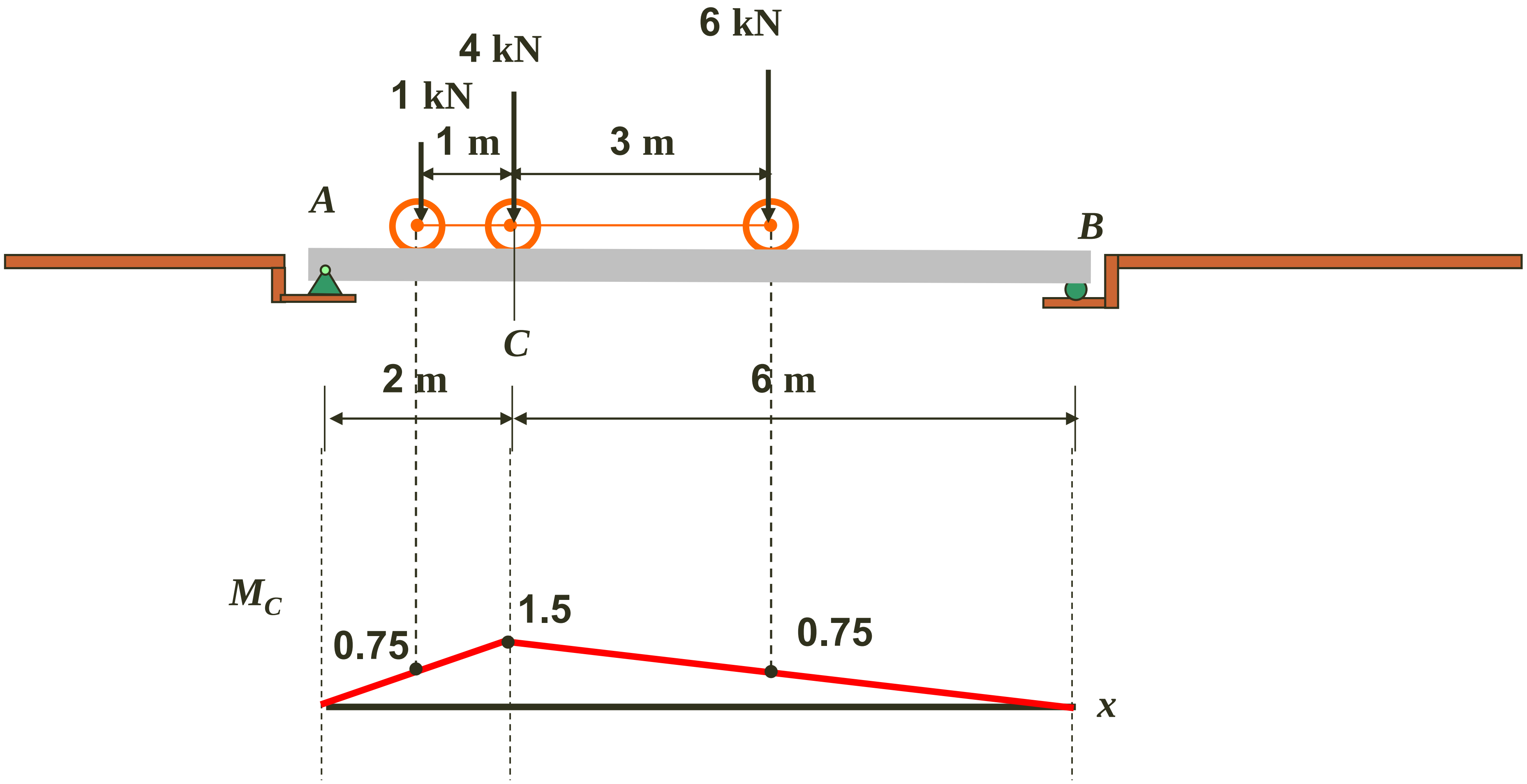
Case 1



$$(M_C)_1 = 1(1.5) + 4(1.25) + 6(0.5) = 9.5 \text{ kN}\cdot\text{m}$$

Maximum Influence at a Point Due to a Series of Concentrated Loads

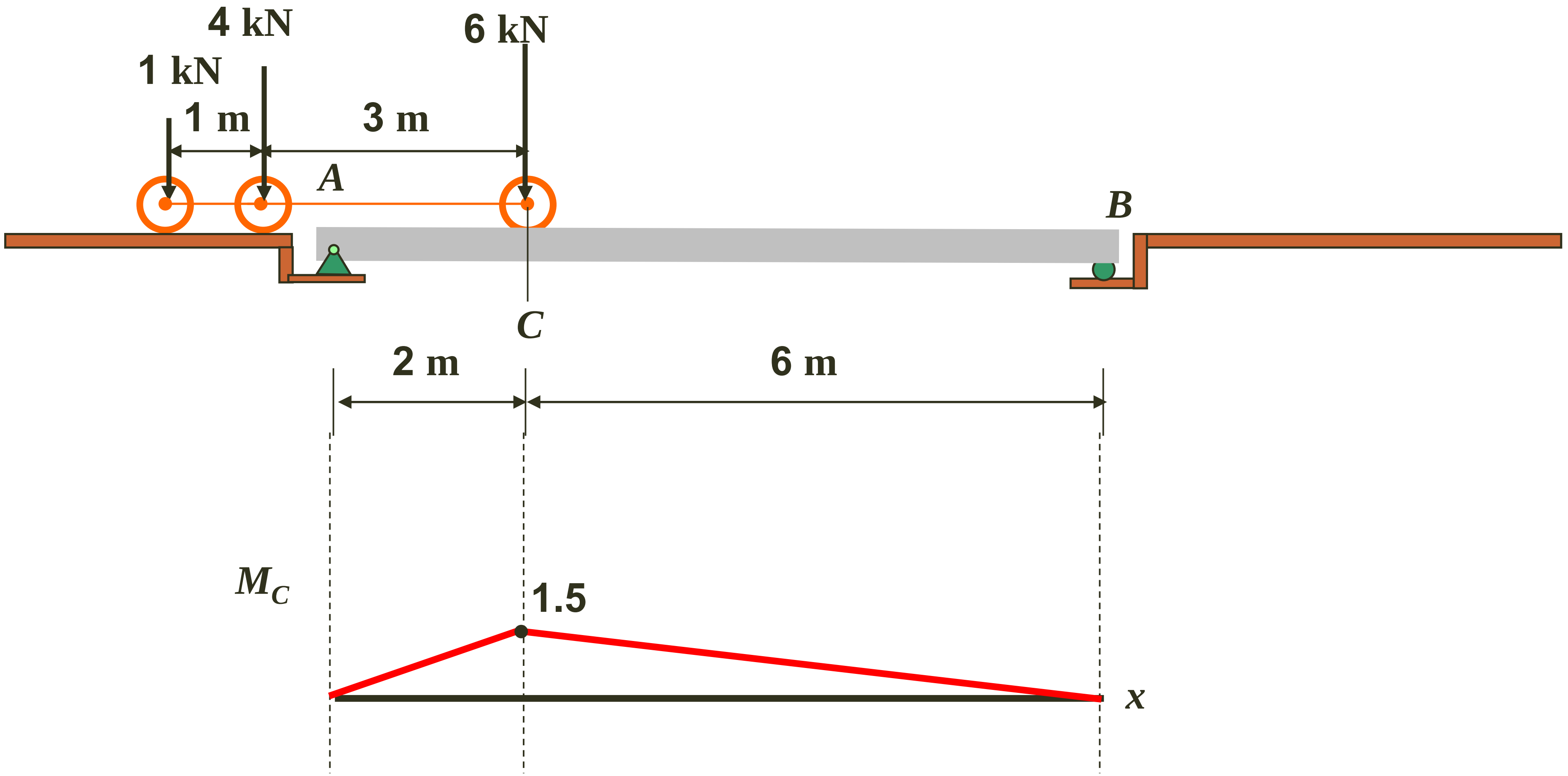
<u>Case 2</u>



$$(M_C)_2 = 1(0.75) + 4(1.5) + 6(0.75) = 11.25 \text{ kN}\cdot\text{m}$$

Maximum Influence at a Point Due to a Series of Concentrated Loads

Case 3



$(M_C)_3 = 6(1.5) = 9$ kN•m $\qquad (M_C)_{max} = (M_C)_2 = 11.25$ kN•m

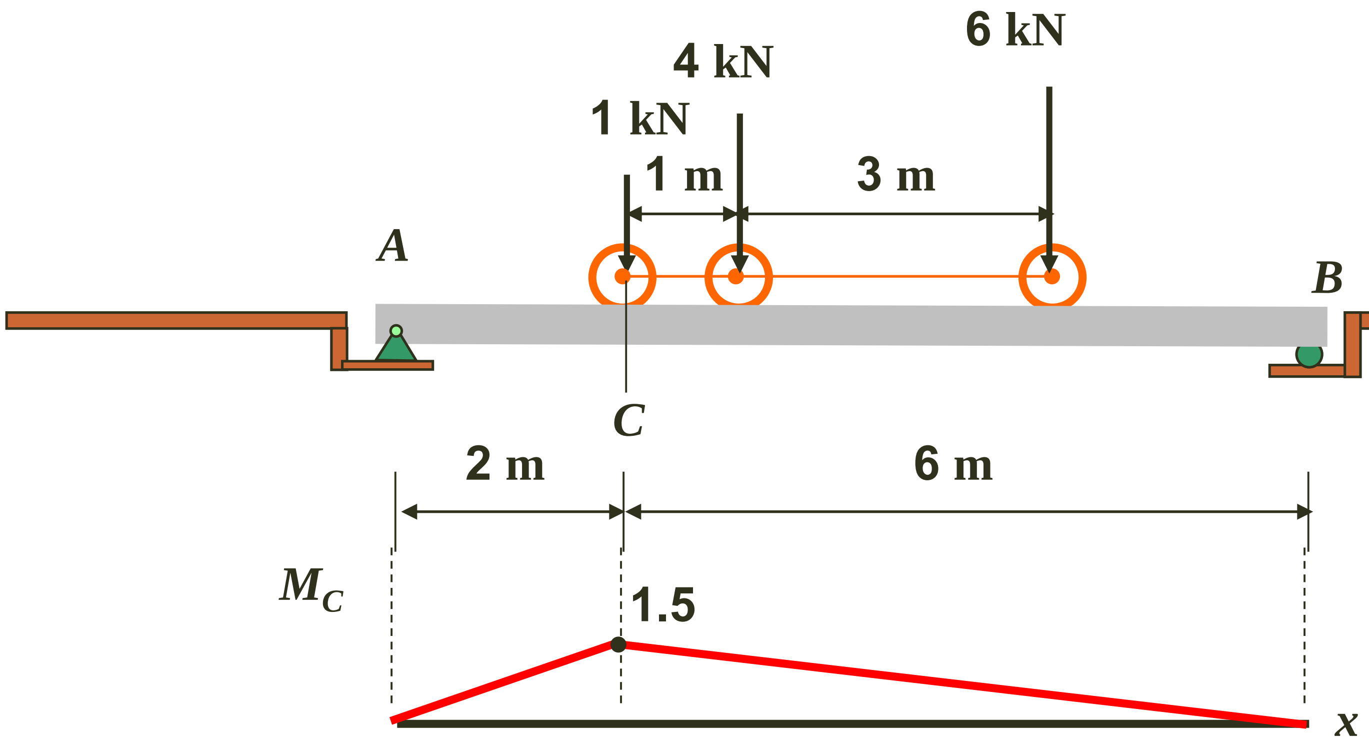
Maximum Influence at a Point Due to a Series of Concentrated Loads

- We can also use the foregoing methods to determine the critical position of a series of concentrated forces so that they create the largest internal moment at a specific point in a structure. Of course, it is first necessary to draw the influence line for the moment at the point and determine the slopes s of its line segments. For a horizontal movement (p_2 - p_1) of a concentrated force P, the change in moment, ΔM, is equivalent to the magnitude of the force times the change in the influence-line ordinate under the load, that is

$$\Delta M = Ps(p_2 - p_1) \quad \text{----------(6-3)}$$

Sloping Line

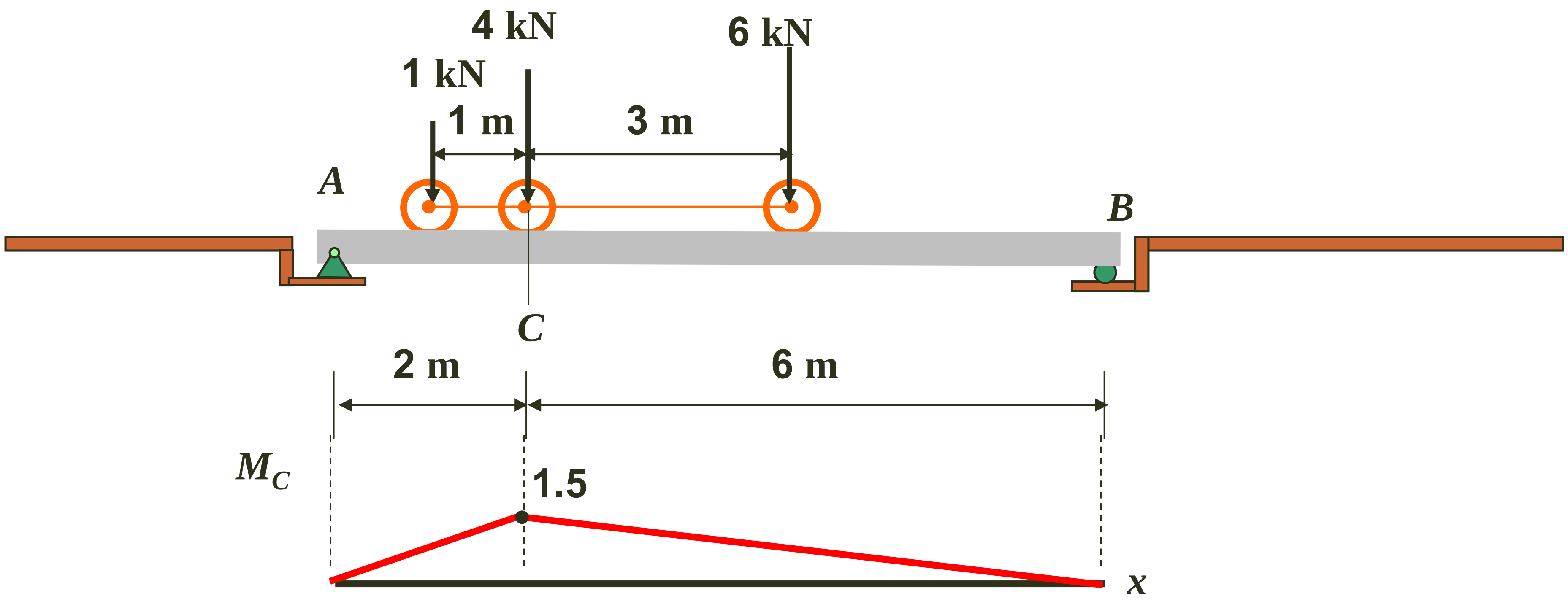
Maximum Influence at a Point Due to a Series of Concentrated Loads



$$\Delta M_{1-2} = -1\left(\frac{1.5}{2}\right)(1) + (4+6)\left(\frac{1.5}{6}\right)(1) = 1.75 \text{ kN•m}$$

$$(M_C)_2 = (M_C)_1 + \Delta M_{1-2} = 9.5 + 1.75 = 11.25 \text{ kN•m}$$

Maximum Influence at a Point Due to a Series of Concentrated Loads



$$\Delta M_{2-3} = -1\left(\frac{1.5}{2}\right)(1) + 4\left(\frac{1.5}{2}\right)(2) + 6\left(\frac{1.5}{6}\right)(3) = -2.25 \text{ kN·m}$$

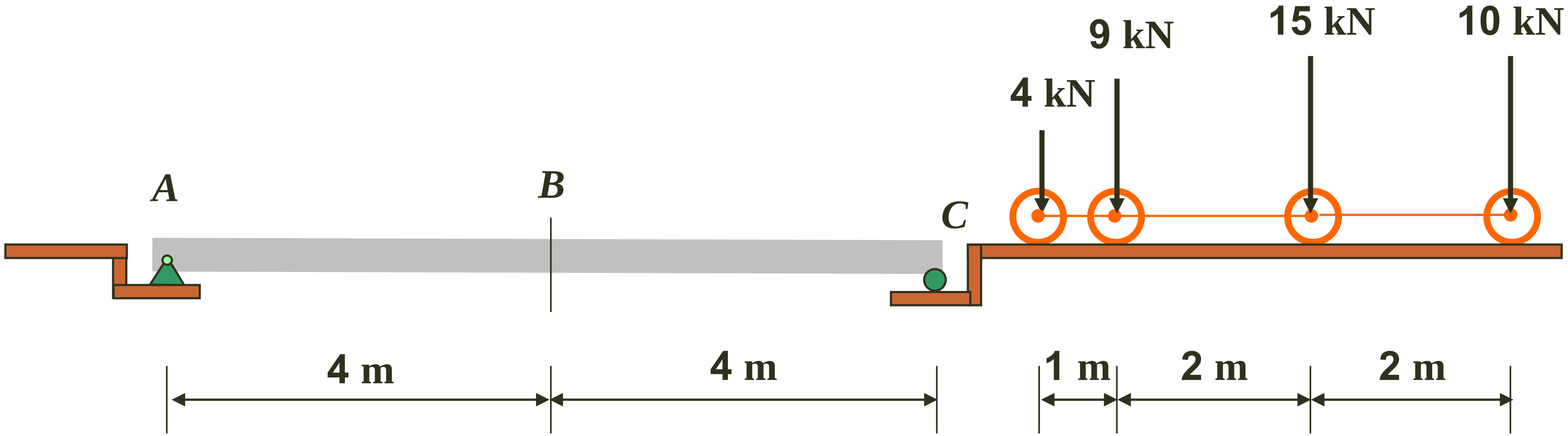
The change is negative, thus the previous yields higher influence and thus critical

$$(M_C)_3 = (M_C)_2 + \Delta M_{2-3} = 11.25 - 2.25 = 9 \text{ kN·m}$$

Maximum Influence at a Point Due to a Series of Concentrated Loads

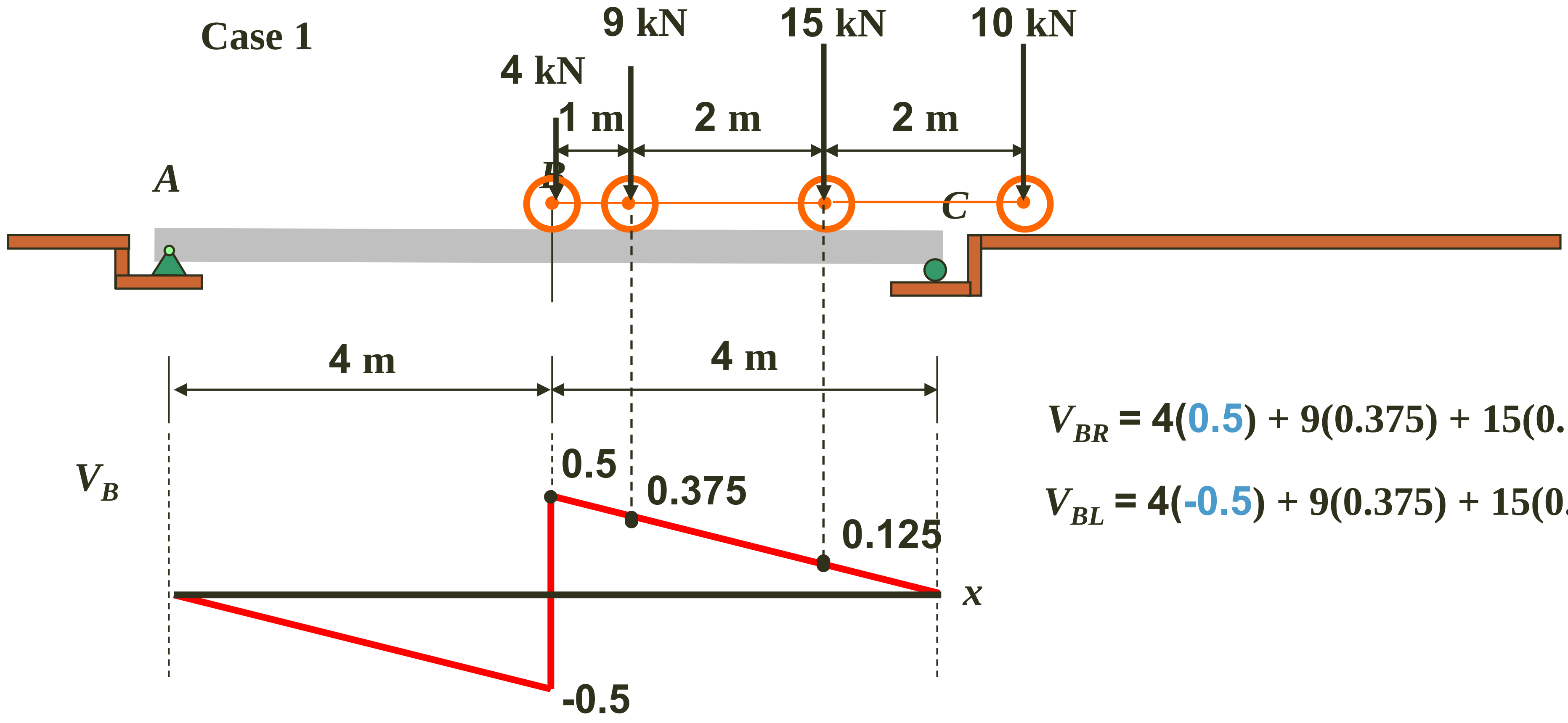
Example 6-9

Determine the maximum shear created at point *B* in the beam shown in the figure below due to the wheel loads of the moving truck.



Maximum Influence at a Point Due to a Series of Concentrated Loads

SOLUTION

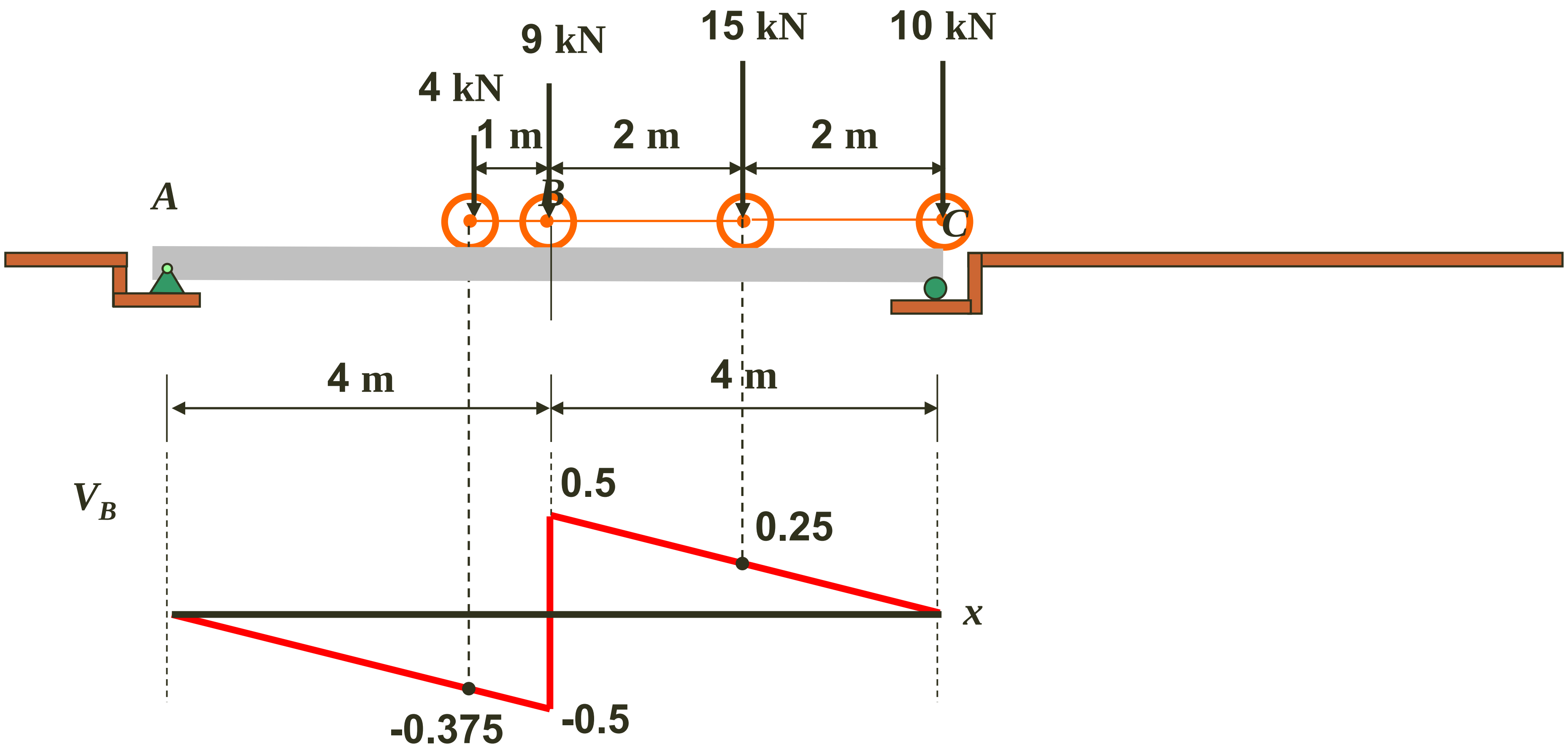


V_{BR} = 4(0.5) + 9(0.375) + 15(0.125) = 7.25 kN

V_{BL} = 4(-0.5) + 9(0.375) + 15(0.125) = 3.25 kN

Maximum Influence at a Point Due to a Series of Concentrated Loads

Case 2

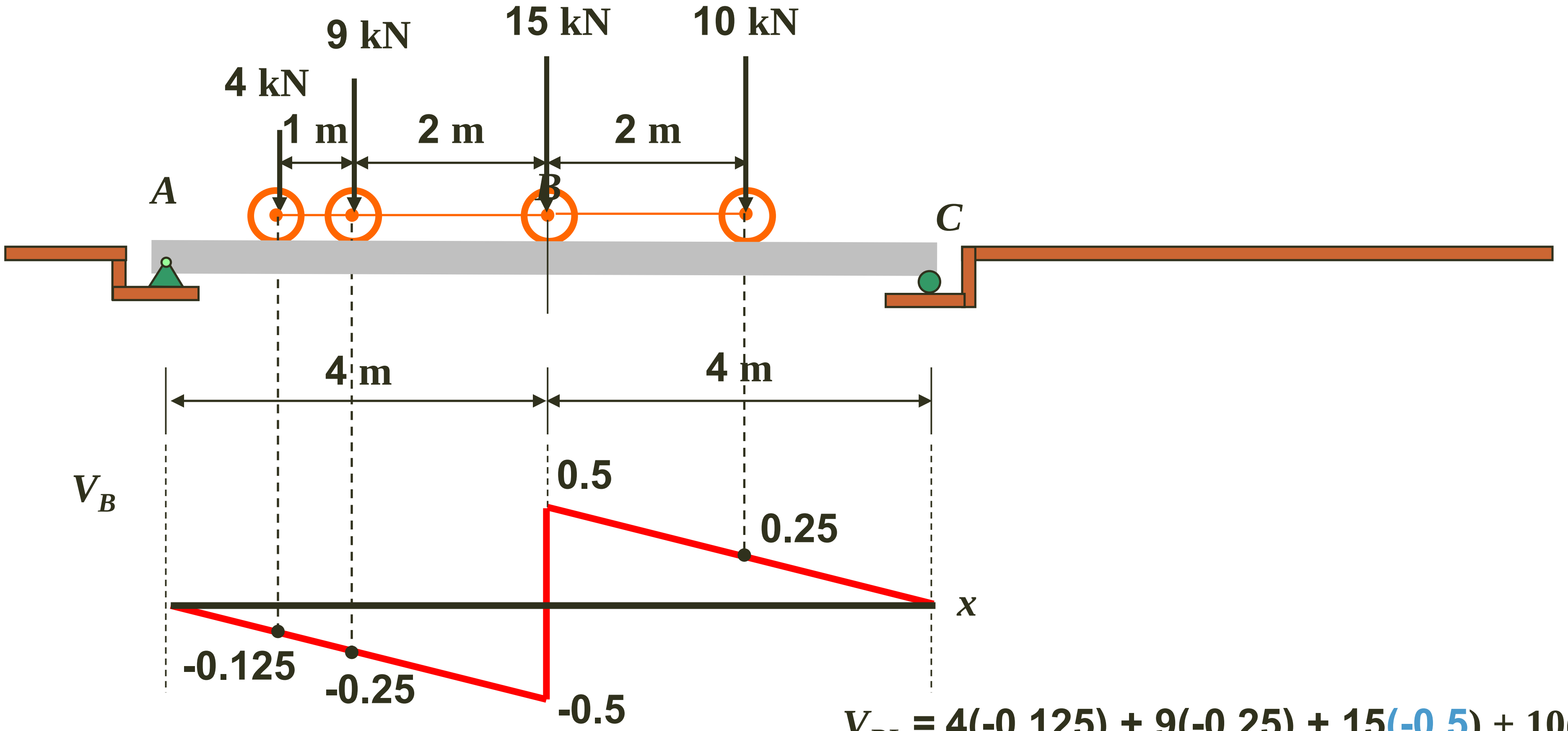


V_{BR} = 4(-0.375) + 9(0.5) + 15(0.25) + 10(0) = 6.75 kN

V_{BL} = 4(-0.375) + 9(-0.5) + 15(0.25) + 10(0) = -2.25 kN

Maximum Influence at a Point Due to a Series of Concentrated Loads

Case 3

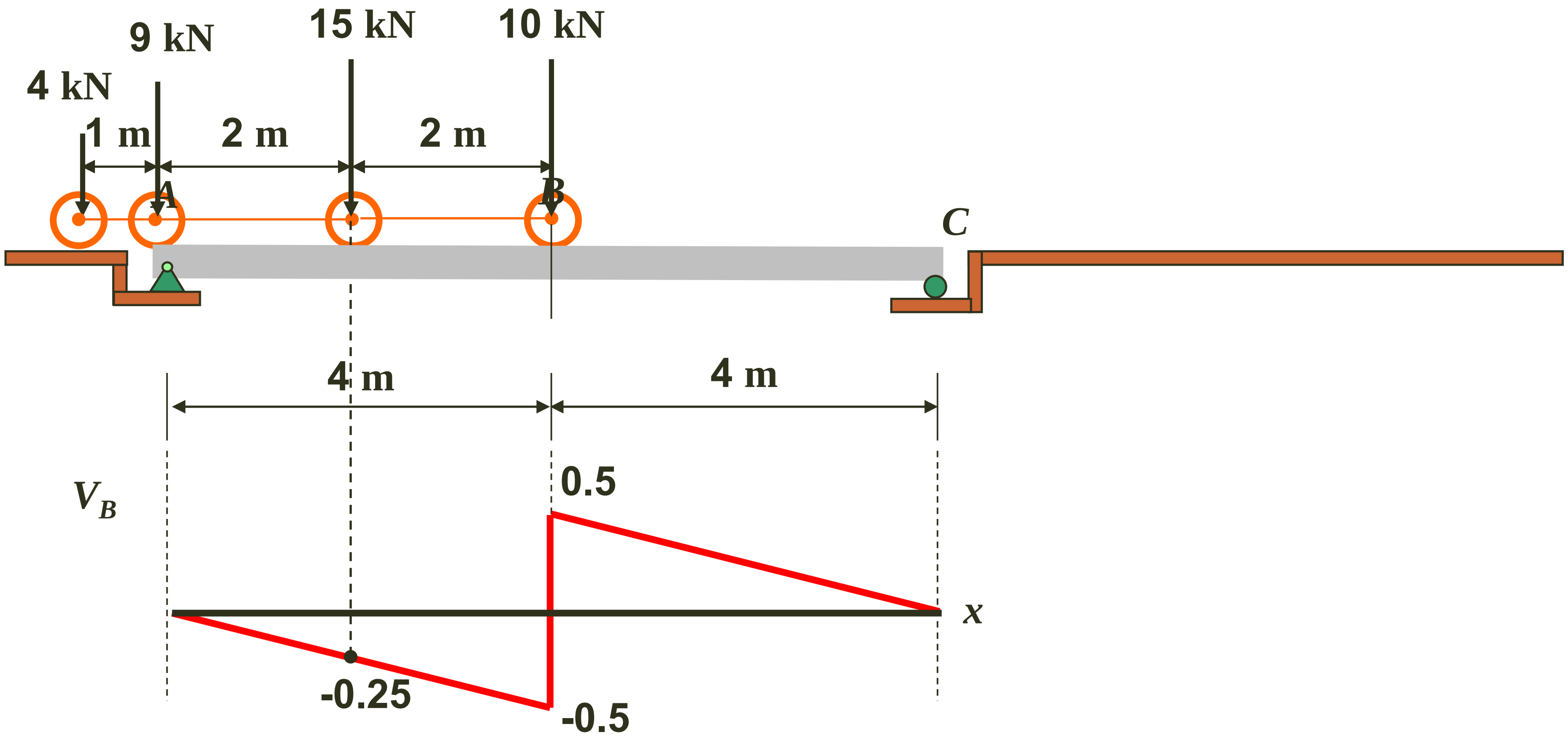


V_{BL} = 4(-0.125) + 9(-0.25) + 15(-0.5) + 10(0.25) = -7.75 kN

V_{BR} = 4(-0.125) + 9(-0.25) + 15(0.5) + 10(0.25) = 7.25 kN

Maximum Influence at a Point Due to a Series of Concentrated Loads

Case 4



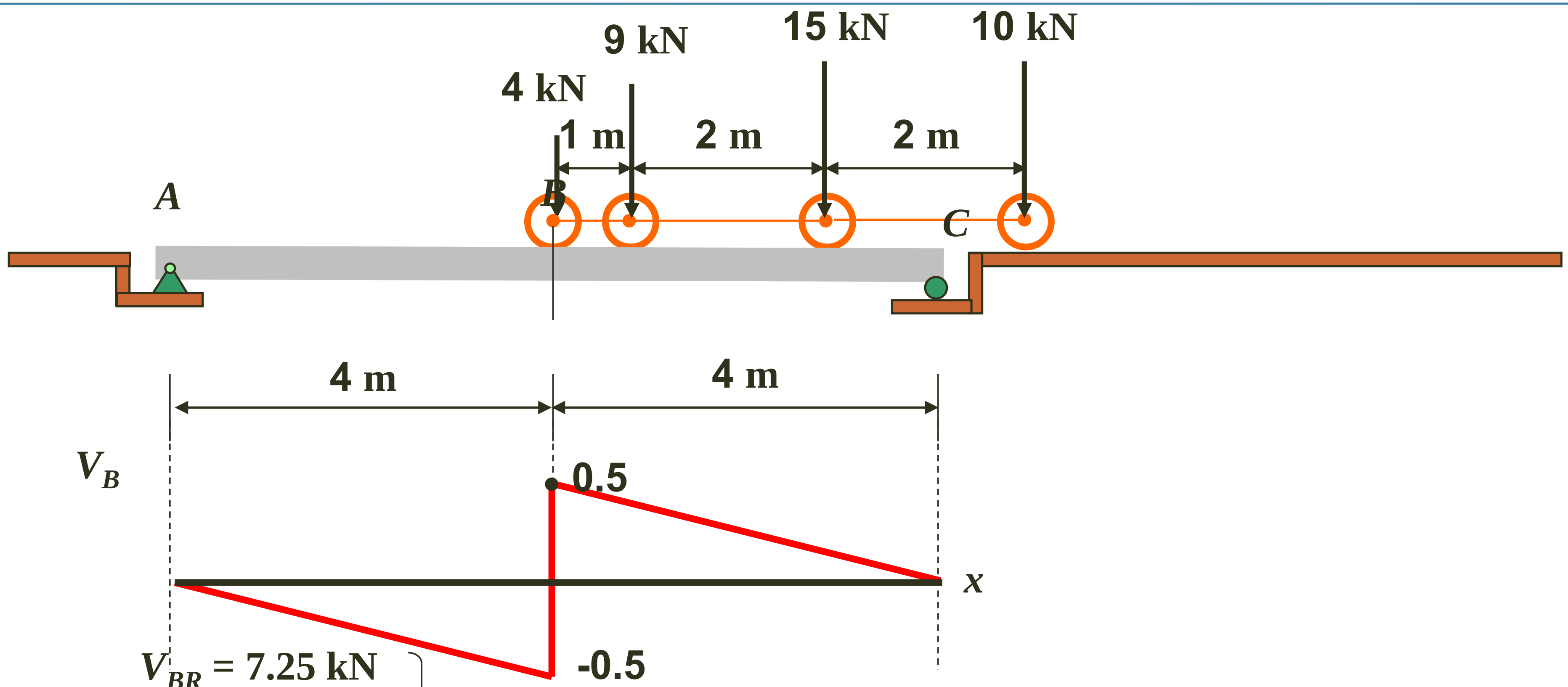
V_{BR} = 9(0) + 15(-0.25) + 10(0.5) = 1.25 kN

V_{BL} = 9(0) + 15(-0.25) + 10(-0.5) = -8.75 kN

The *maximum shear* created at point *B* is -8.75 kN

Maximum Influence at a Point Due to a Series of Concentrated Loads

Summary



Case 1 V_{BR} = 7.25 kN

1-m Movement of 4 kN, Δ_{VB} = 6.75-7.25 = - 0.5 kN

Case 2 V_{BR} = 6.75 kN

2-m Movement of 9 kN, Δ_{VB} = 7.25-6.75 = + 0.5 kN

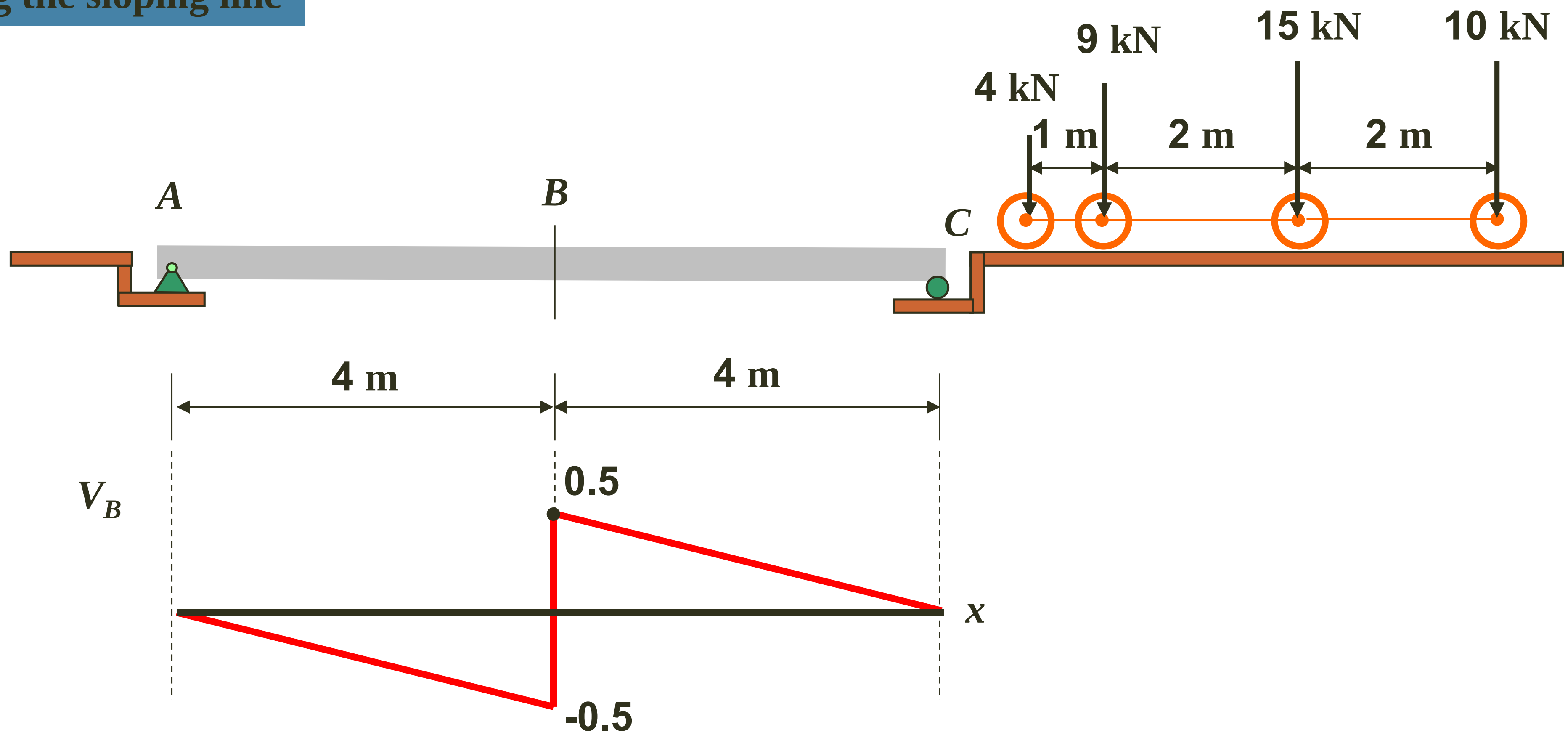
Case 3 V_{BR} = 7.25 kN

2-m Movement of 15 kN, Δ_{VB} = 1.25-7.25 = - 6 kN

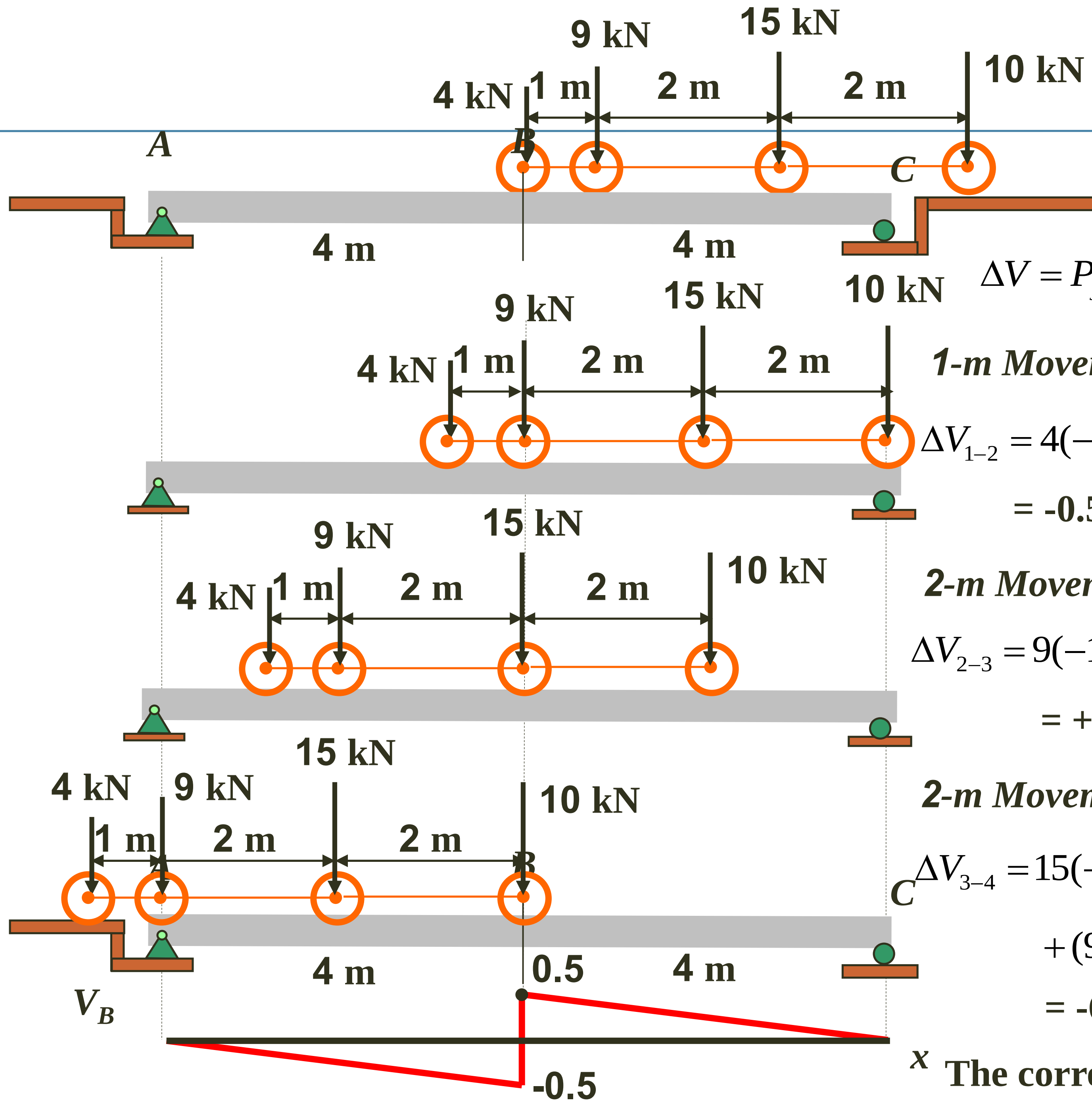
Case 4 V_{BR} = 1.25 kN

Maximum Influence at a Point Due to a Series of Concentrated Loads

Or using the sloping line



$$s = \frac{0.5}{4}$$



$$\Delta V = P_{jump}(y_2 - y_1) + P_n s(x_2 - x_1)$$

1-m Movement of 4 kN

$$\Delta V_{1-2} = 4(-1) + (4+9+15)(\frac{0.5}{4})(1)$$

= -0.5 kN

2-m Movement of 9 kN

$$\Delta V_{2-3} = 9(-1) + (4+9+15+10)(\frac{0.5}{4})(2)$$

= +0.5 kN

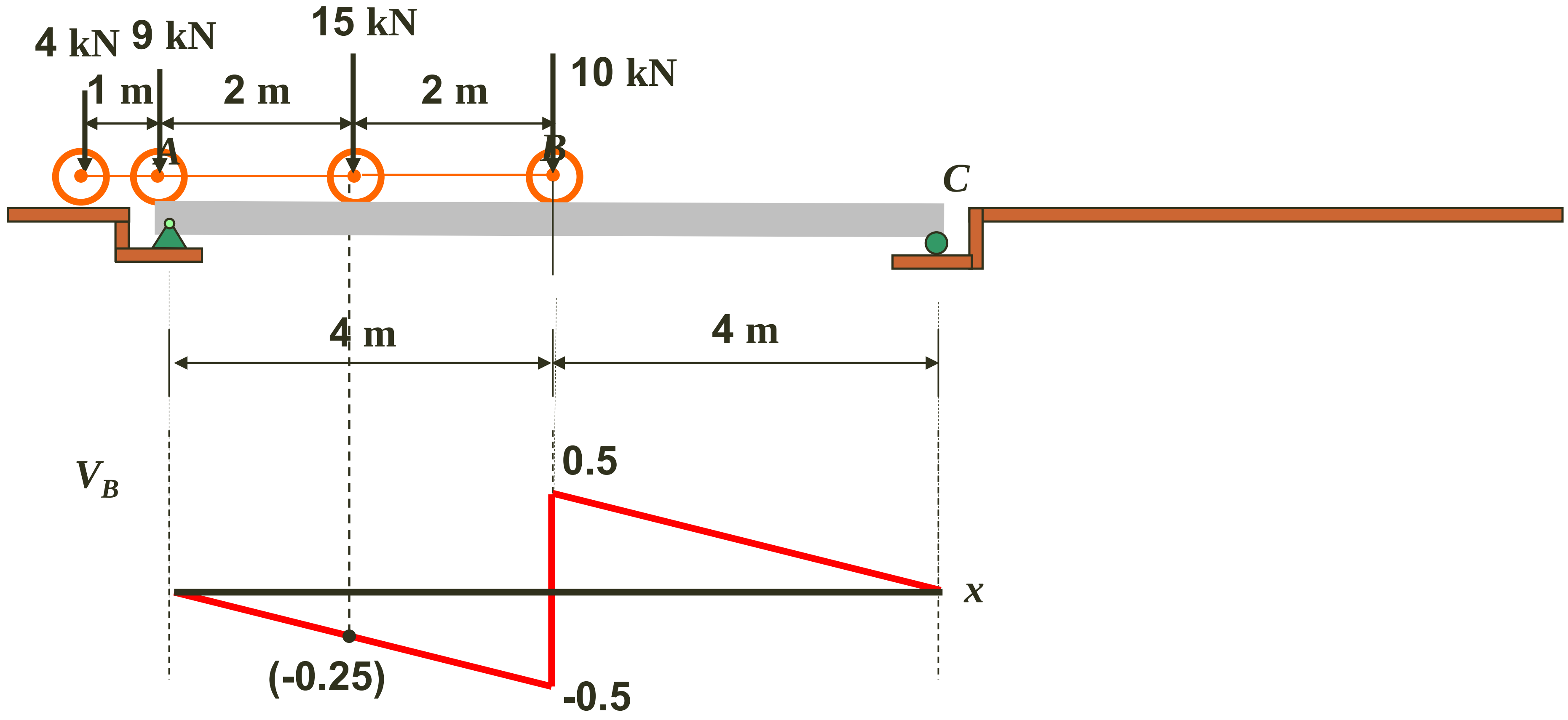
2-m Movement of 15 kN

$$\Delta V_{3-4} = 15(-1) + 4(\frac{0.5}{4})(1) + (9+15+10)(\frac{0.5}{4})(2)$$

= -6 kN

The correct position of the loads.

Maximum Influence at a Point Due to a Series of Concentrated Loads



V_{BR} = 9(0) + 15(-0.25) + 10(0.5) = 1.25 kN

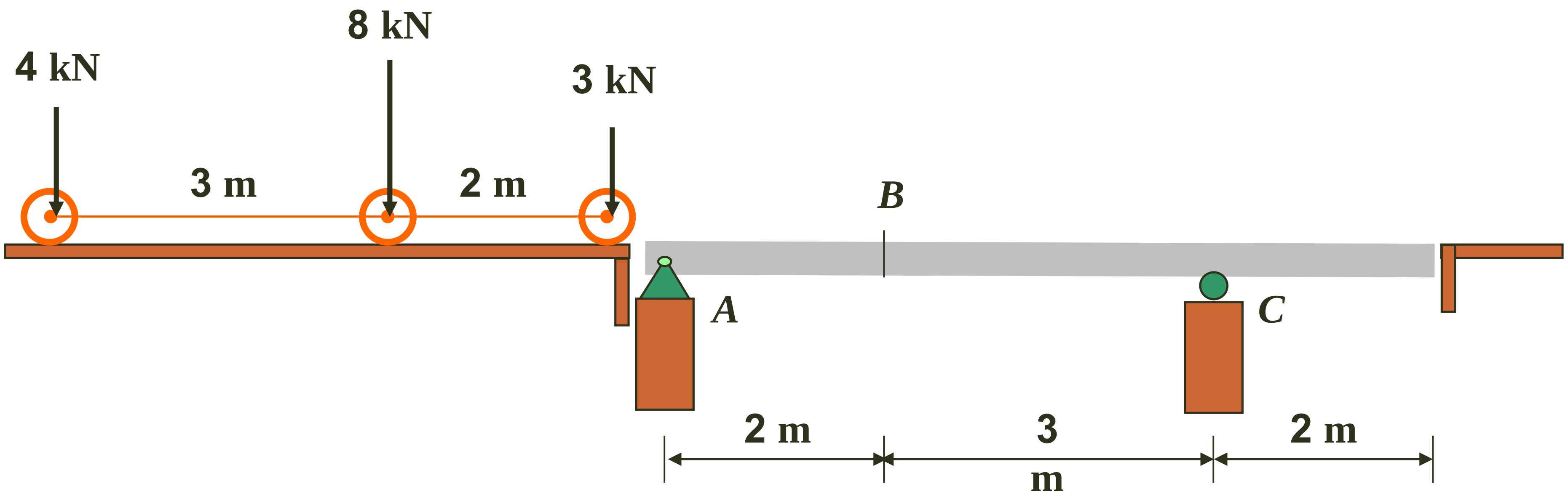
V_{BL} = 9(0) + 15(-0.25) + 10(-0.5) = -8.75 kN

The *maximum shear* created at point *B* is -8.75 kN

Maximum Influence at a Point Due to a Series of Concentrated Loads

Example 6-10

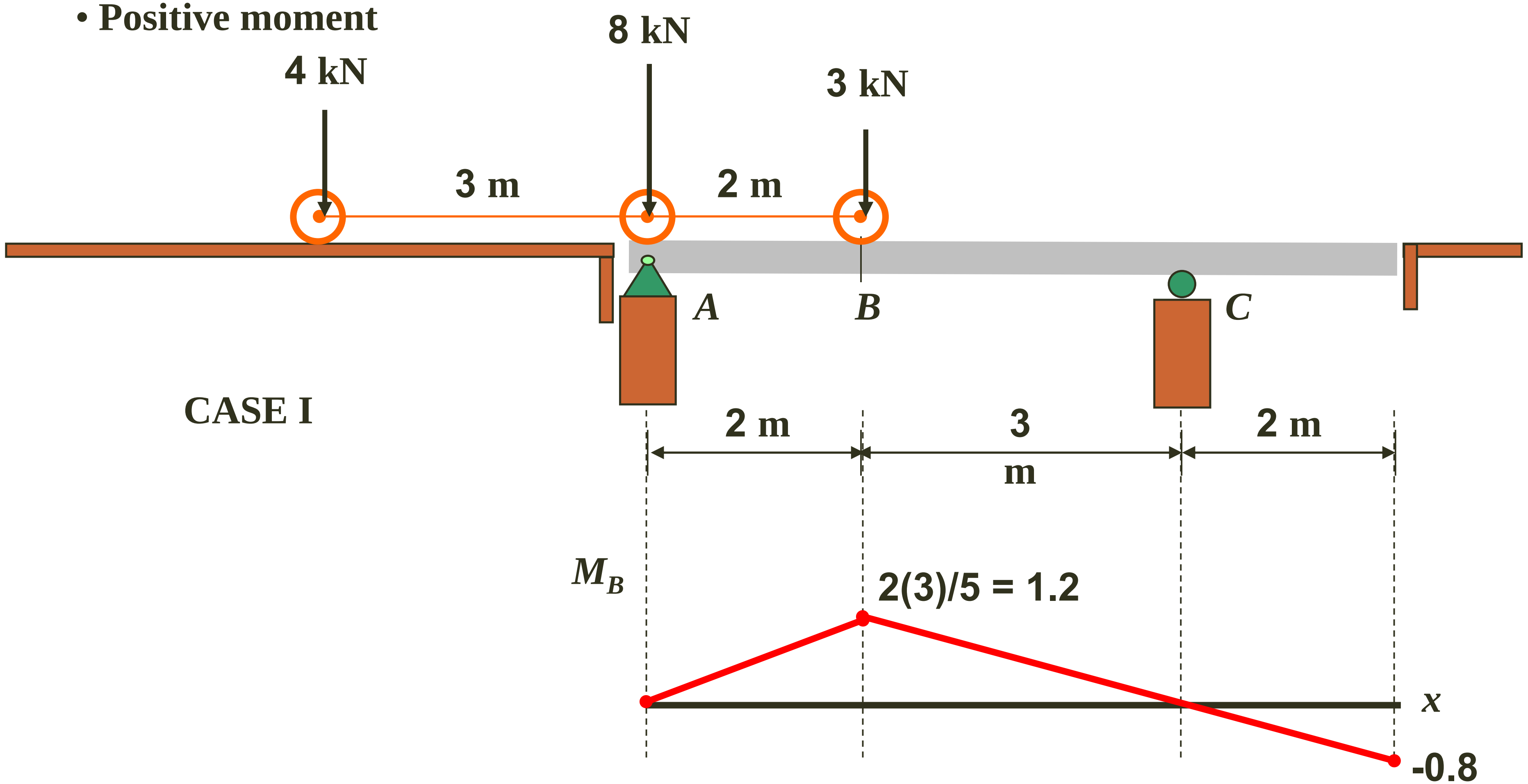
Determine the maximum positive moment and negative moment created at point *B* in the beam shown in the figure below due to the wheel loads of the crane.



Maximum Influence at a Point Due to a Series of Concentrated Loads

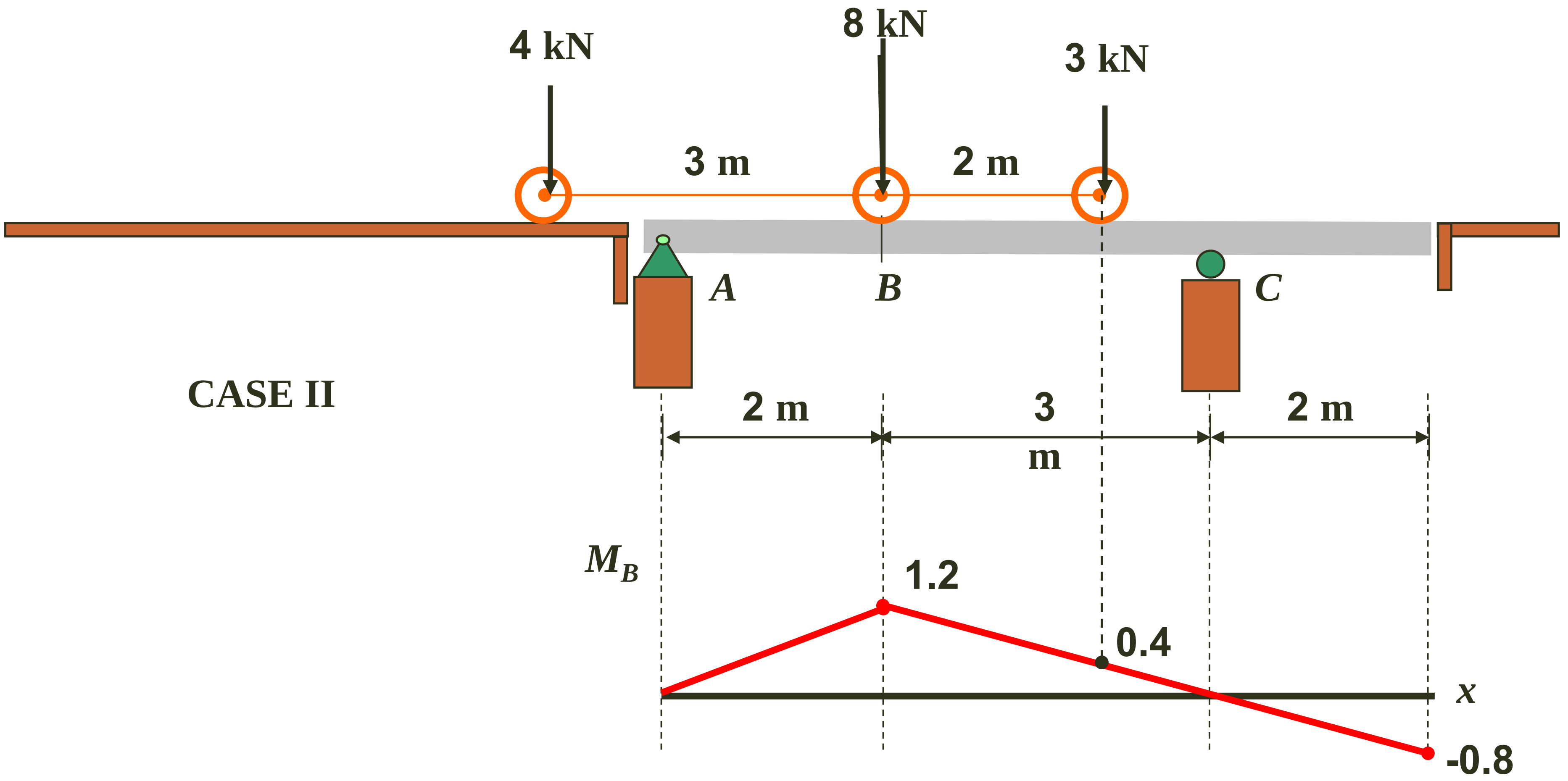
SOLUTION

- Positive moment



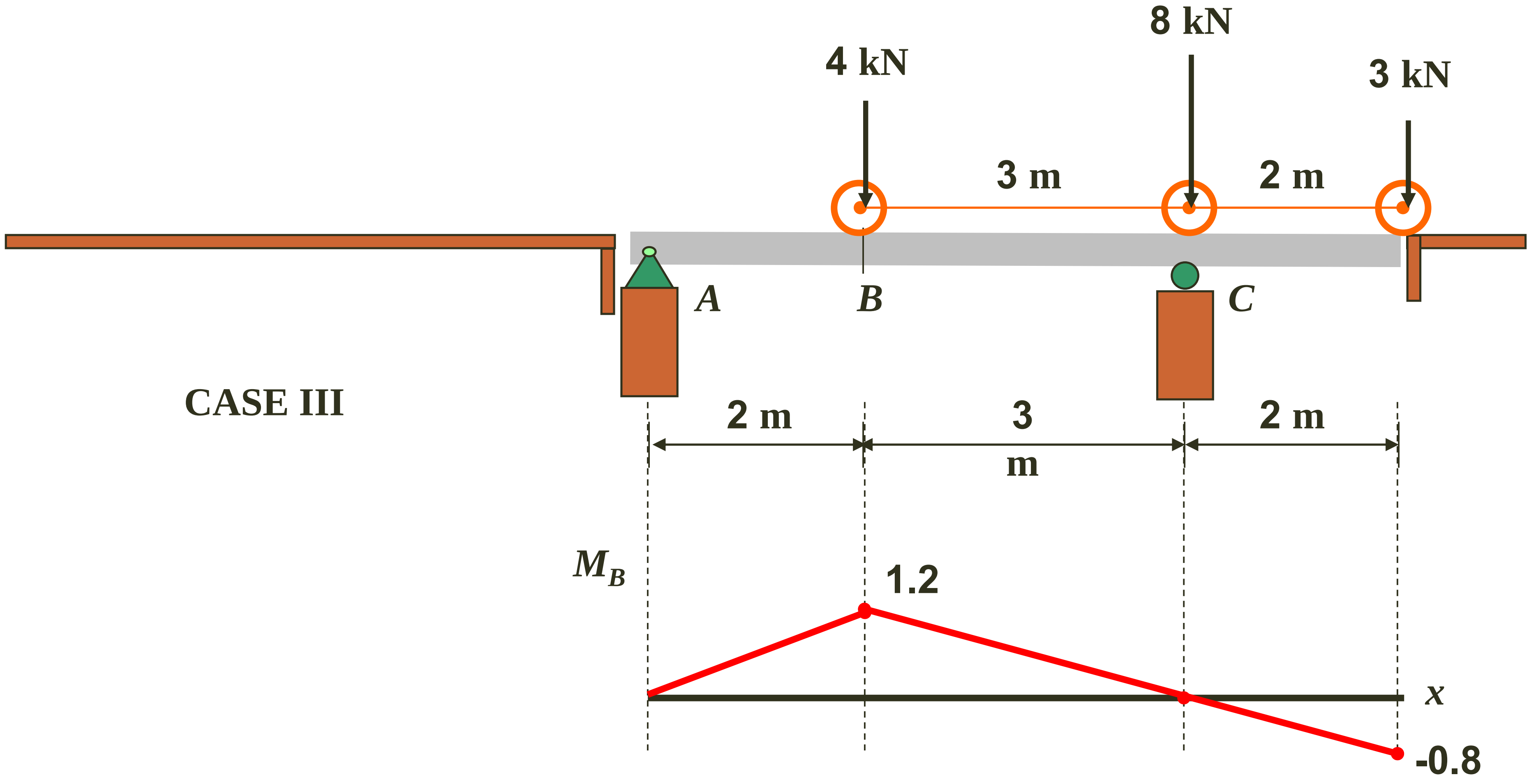
$$M_B = 3(1.2) + 8(0) = 3.6 \text{ kN}\cdot\text{m}$$

Maximum Influence at a Point Due to a Series of Concentrated Loads



$M_B = 8(1.2) + 3(0.4) = 10.8$ kN•m

Maximum Influence at a Point Due to a Series of Concentrated Loads



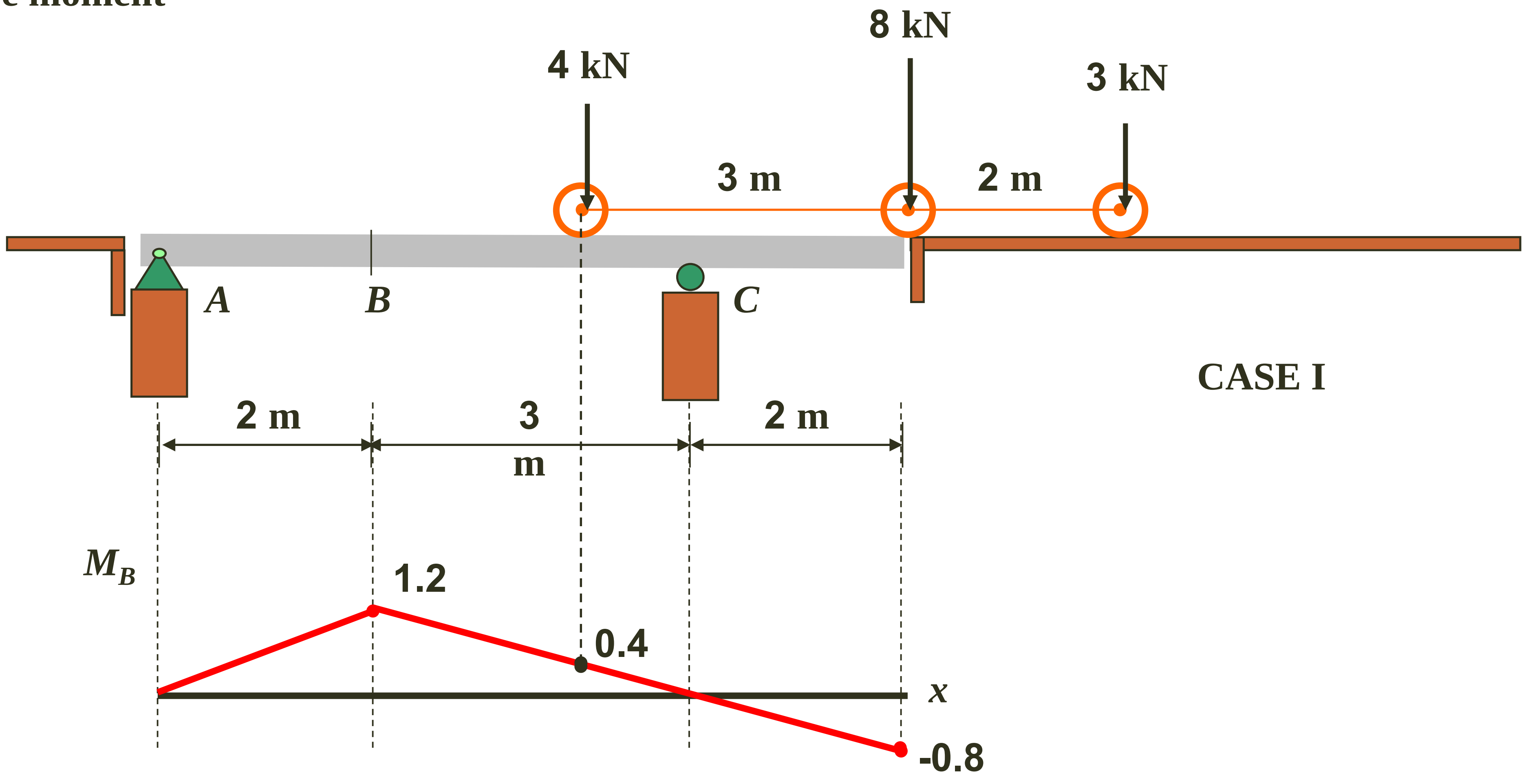
CASE III

M_B = 4(1.2) + 8(0) + 3(-0.8) = 2.4 kN•m

The *maximum positive moment* created at point *B* is 10.8 kN•m

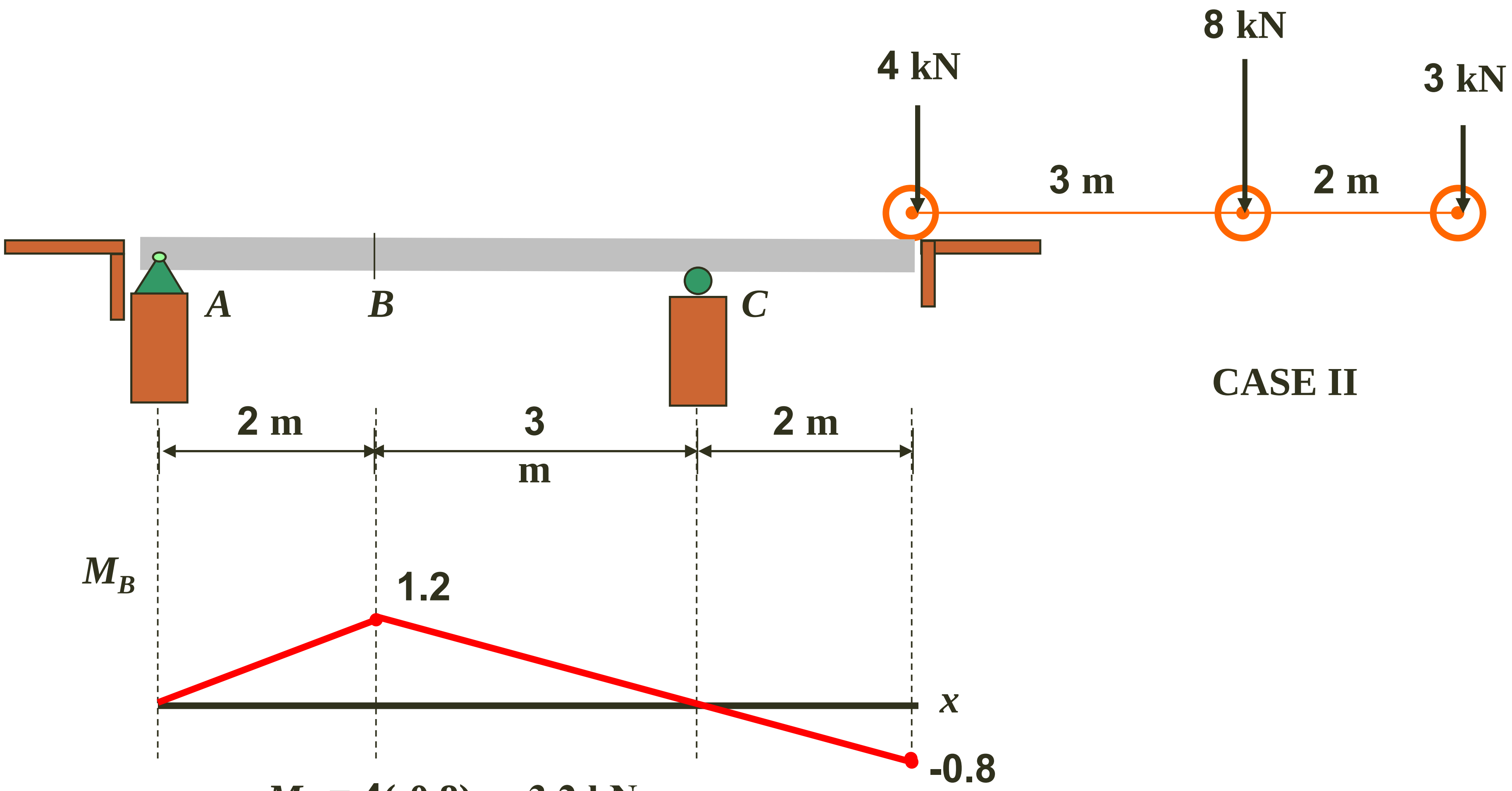
Maximum Influence at a Point Due to a Series of Concentrated Loads

- Negative moment



$M_B = 8(-0.8) + 4(0.4) = -4.8$ kN•m

Maximum Influence at a Point Due to a Series of Concentrated Loads



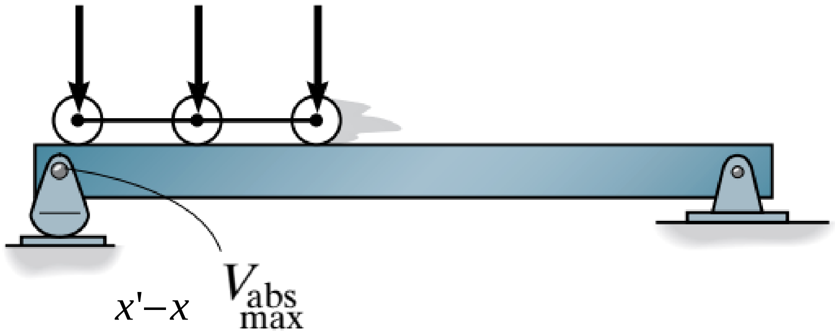
M_B = 4(-0.8) = -3.2 kN•m

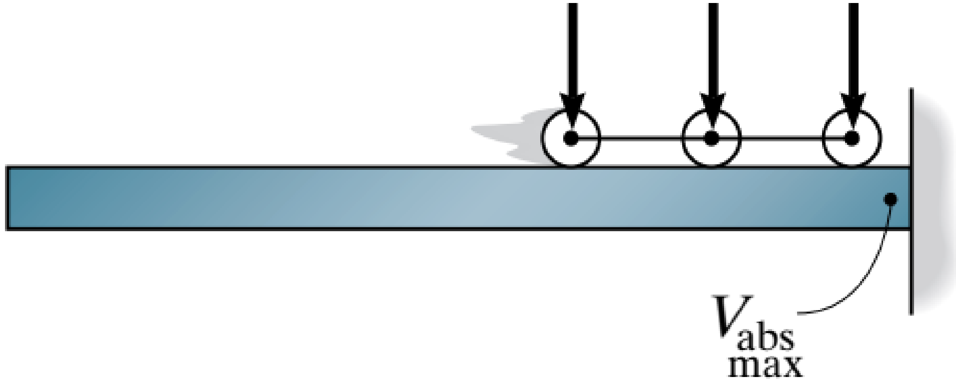
The *maximum negative moment* created at point *B* is 4.8 kN•m

Maximum Influence at a Point Due to a Series of Concentrated Loads

Absolute Maximum Shear and Moment

- A more general problem involves the determination of both the location of the point in the beam and the position of the loading on the beam so that one can obtain the absolute maximum shear and moment caused by the loads.
- If the beam is cantilevered or simply supported, this problem can be readily solved.



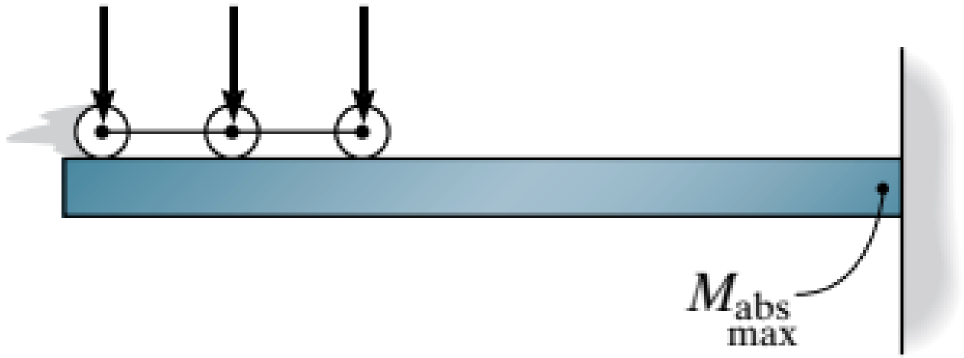


- For a cantilevered beam the absolute maximum shear will occur at a point located just next to the fixed support. The maximum shear is found by the method of sections, with the loads positioned anywhere on the span.
- For simply supported beams the absolute maximum shear will occur just next to one of the supports. For example, if the loads are equivalent, they are positioned so that the first one in sequence is placed close to the support.

Maximum Influence at a Point Due to a Series of Concentrated Loads

Absolute Maximum Shear and Moment

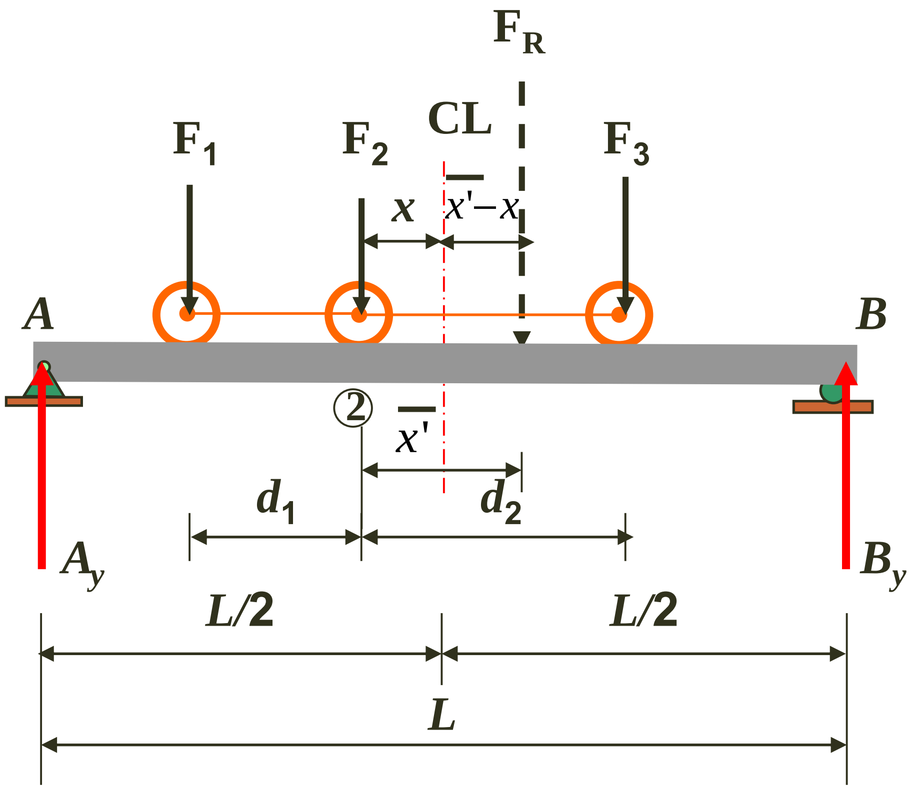
- The absolute maximum moment for a cantilevered beam occurs at the same point where absolute maximum shear occurs, although in this case the concentrated loads should be positioned at the far end of the beam



Maximum Influence at a Point Due to a Series of Concentrated Loads

Absolute Maximum Shear and Moment

- For a simply supported beam the critical position of the loads and the associated absolute maximum moment cannot, in general, be determined by inspection. We can, however, determine the position analytically. For purposes of discussion, consider a beam subjected to the forces F_1, F_2, F_3 as shown in the figure. Since the moment diagram for a series of concentrated forces consists of straight line segments having peaks at each force, the absolute maximum moment will occur under one of the forces. Assume this maximum moment occurs under F_2. The position of the loads F_1, F_2, F_3 on the beam will be specified by the distance x, measured from F_2 to the beam's centerline as shown. To determine a specific value of x, we first obtain the resultant force of the system, F_R, and its distance $\overline{x}'$ measured from F_2. Once this is done, moments are summed about B, which yields the beam's left reaction, A_y, that is,



$$\sum M_B = 0: \qquad A_y = \frac{1}{L}(F_R)\left[\frac{L}{2} - (\overline{x}' - x)\right]$$

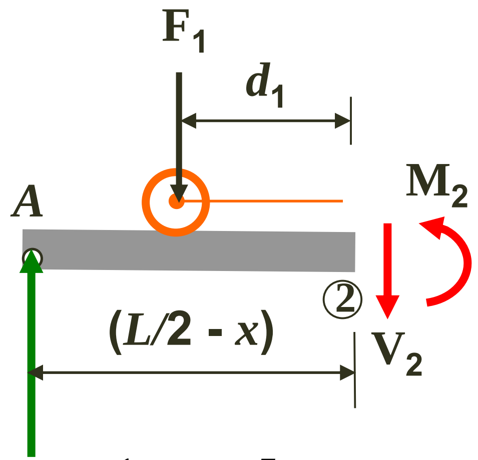
Maximum Influence at a Point Due to a Series of Concentrated Loads

If the beam is sectioned just to the left of $\mathbf{F}_2$, the resulting free-body diagram is shown in figure below. The moment $\mathbf{M}_2$ under $\mathbf{F}_2$ is therefore

$$\sum M_2 = 0: \qquad M_2 = A_y\left(\frac{L}{2} - x\right) - F_1 d_1$$

$$= \frac{1}{L}(F_R)\left[\frac{L}{2} - (\bar{x}' - x)\right]\left(\frac{L}{2} - x\right) - F_1 d_1$$

$$M_2 = \frac{F_R L}{4} - \frac{F_R \bar{x}'}{2} - \frac{F_R x^2}{L} + \frac{F_R x \bar{x}'}{L} - F_1 d_1$$



$$A_y = \frac{1}{L}(F_R)\left[\frac{L}{2} - (\bar{x}' - x)\right]$$

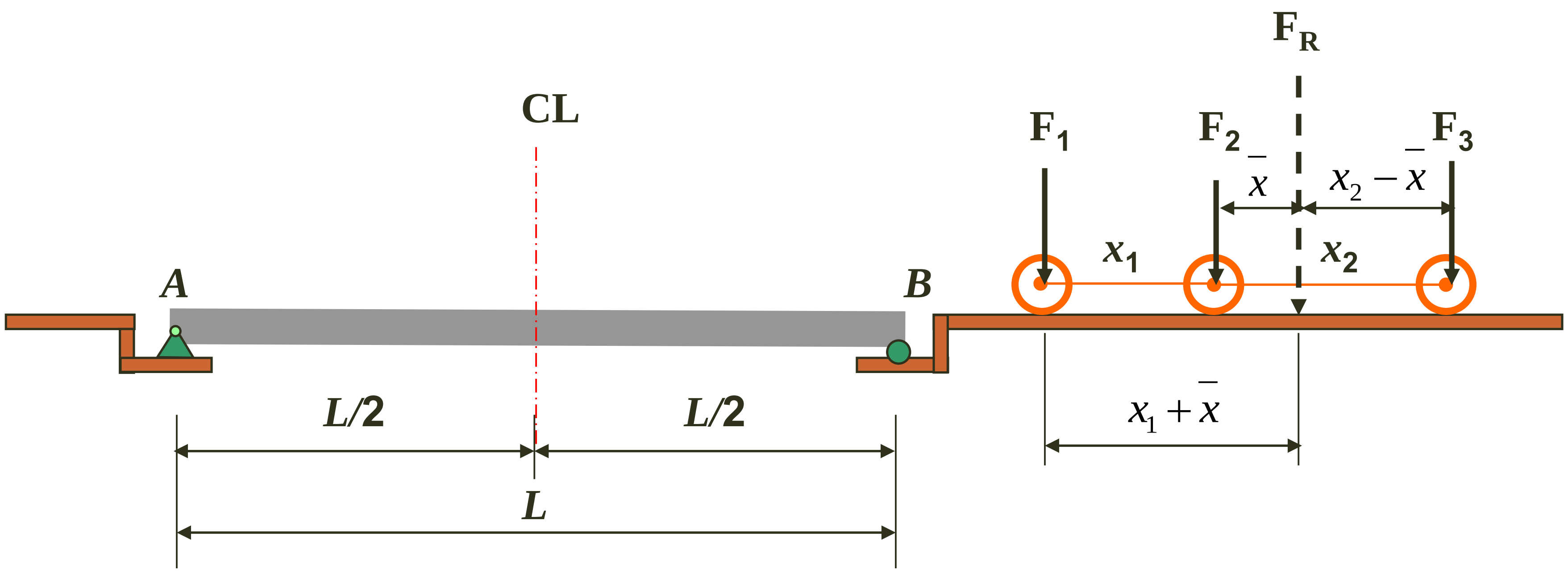
For maximum M_2 we require

$$\frac{dM_2}{dx} = -\frac{2F_R x}{L} + \frac{F_R \bar{x}'}{L} = 0$$

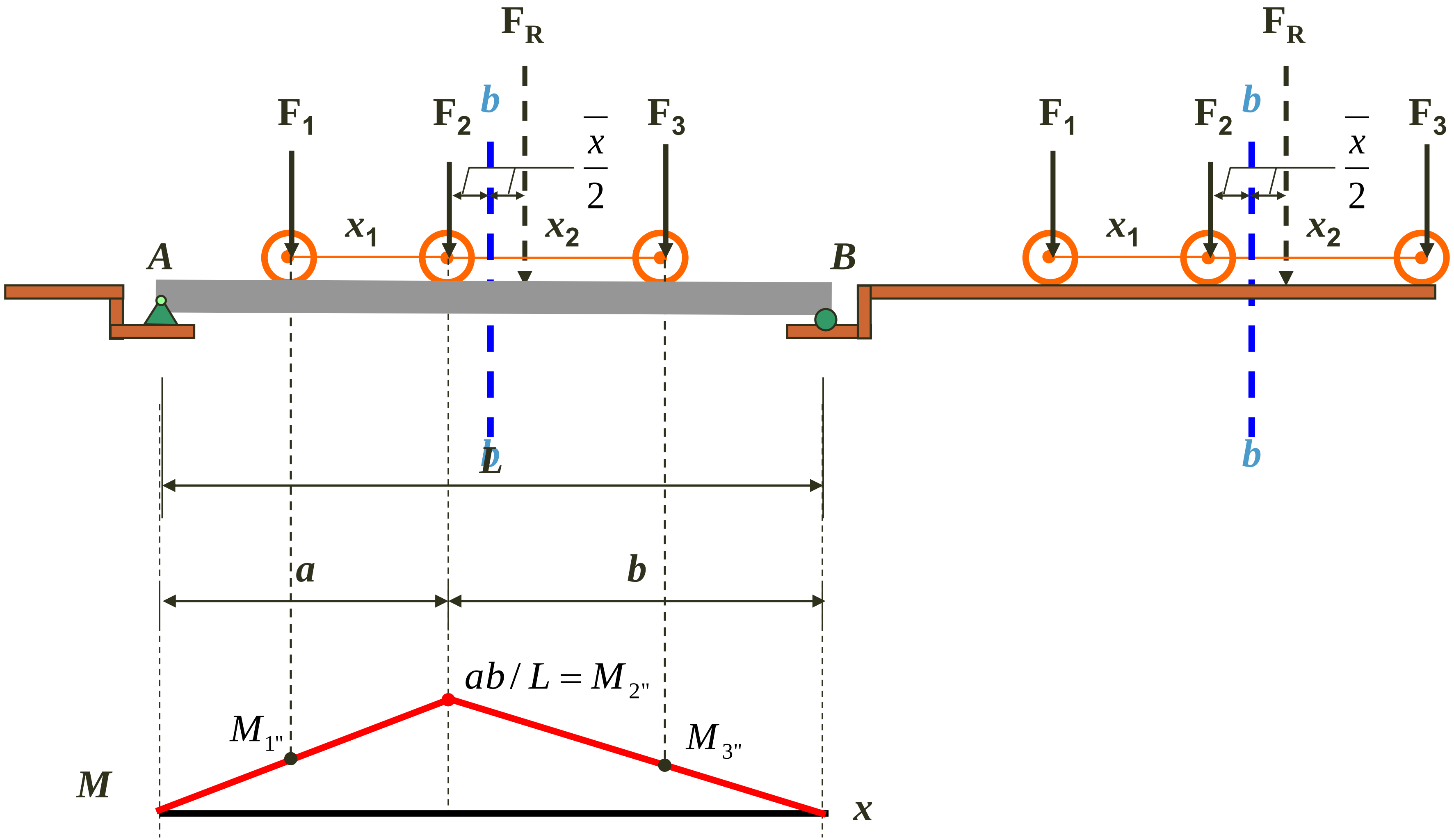
$$x = \frac{\bar{x}'}{2}$$

Hence, we may conclude that the absolute maximum moment in a simply supported beam occurs under one of the concentrated forces, such that this force is positioned on the beam so that it and the resultant force of the system are equidistant from the beam's centerline.

Maximum Influence at a Point Due to a Series of Concentrated Loads

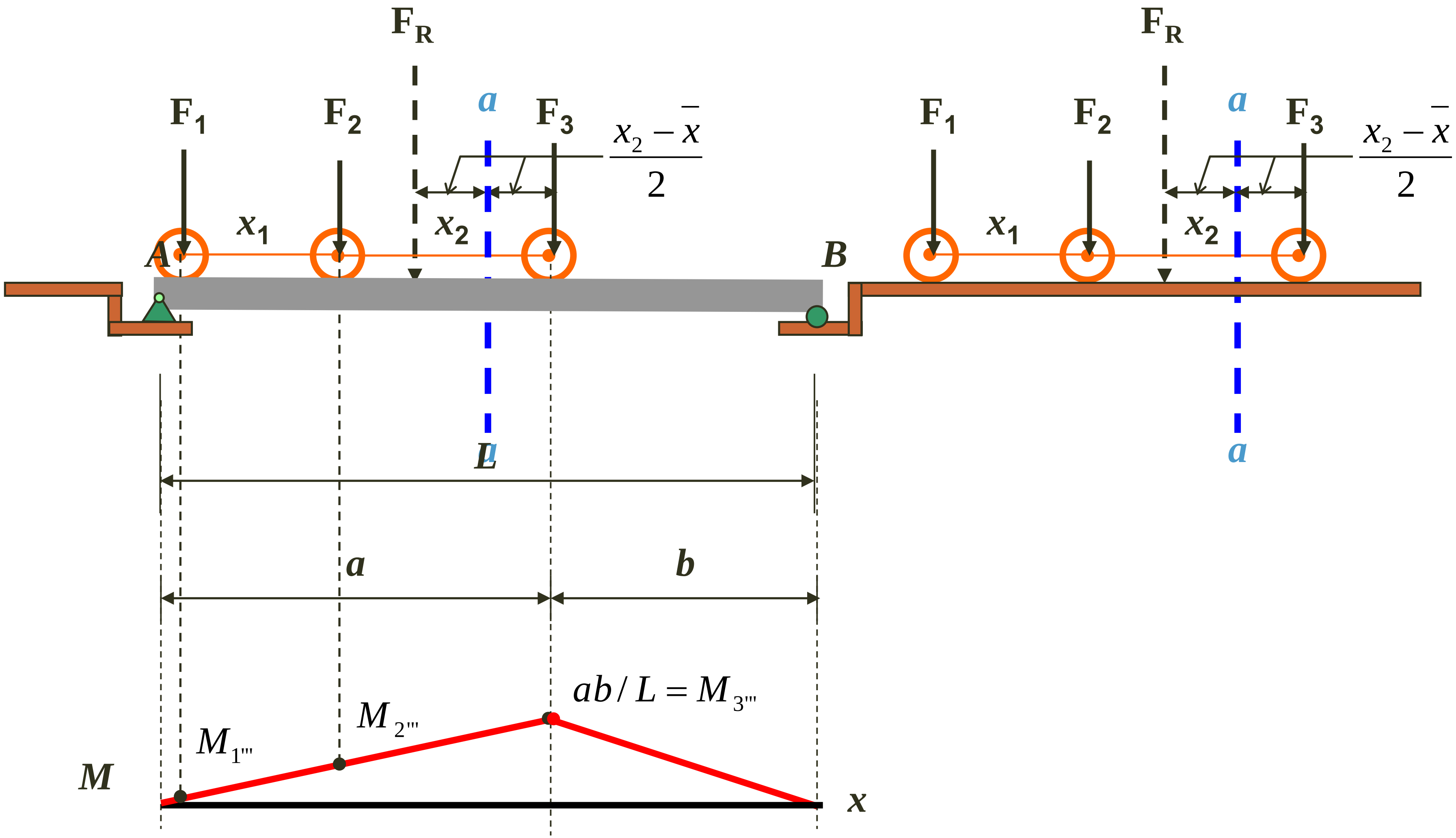


Maximum Influence at a Point Due to a Series of Concentrated Loads



$$M_{S1} = F_1 M_{1"} + F_2 M_{2"} + F_3 M_{3"}$$

Maximum Influence at a Point Due to a Series of Concentrated Loads



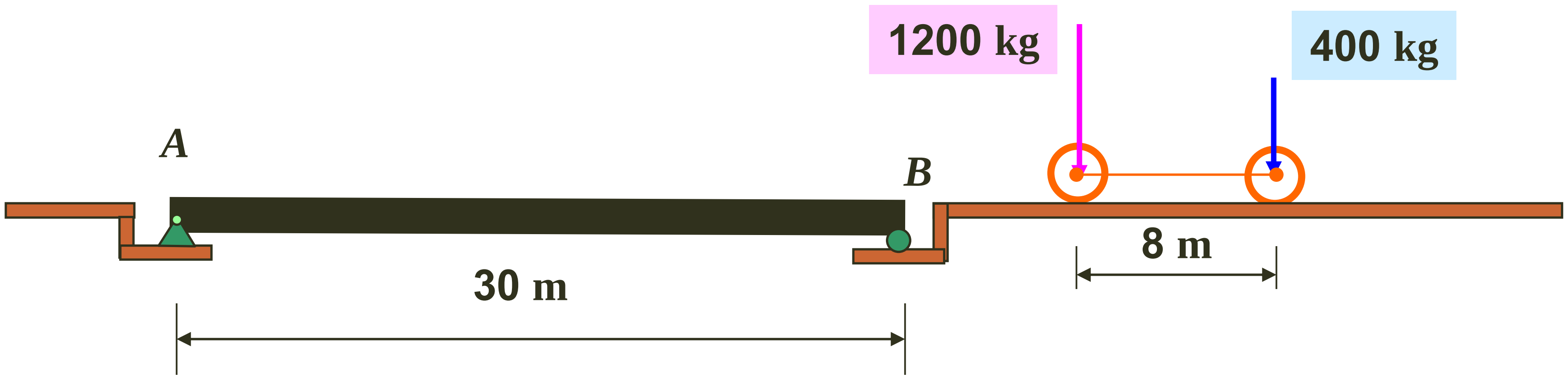
$$M_{S2} = F_1 M_{1'''} + F_2 M_{2'''} + F_3 M_{3'''}$$

The *absolute maximum moment* is comparison by M_{S1} and M_{S2}.

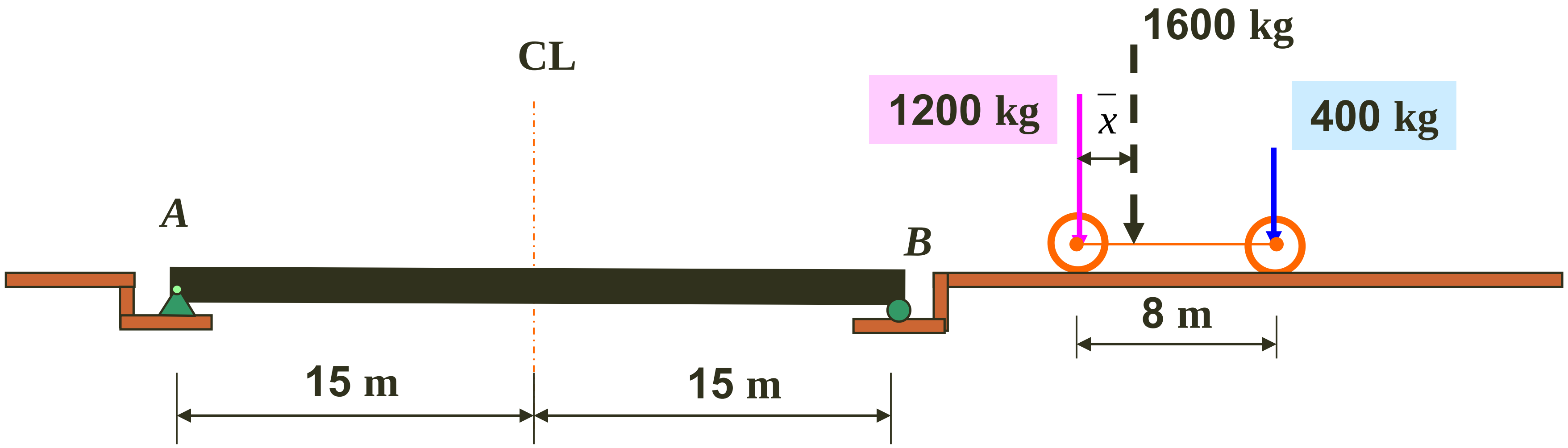
Maximum Influence at a Point Due to a Series of Concentrated Loads

Example 6-11

Determine the absolute maximum moment on the simply supported beam cased by the wheel loads.



Maximum Influence at a Point Due to a Series of Concentrated Loads

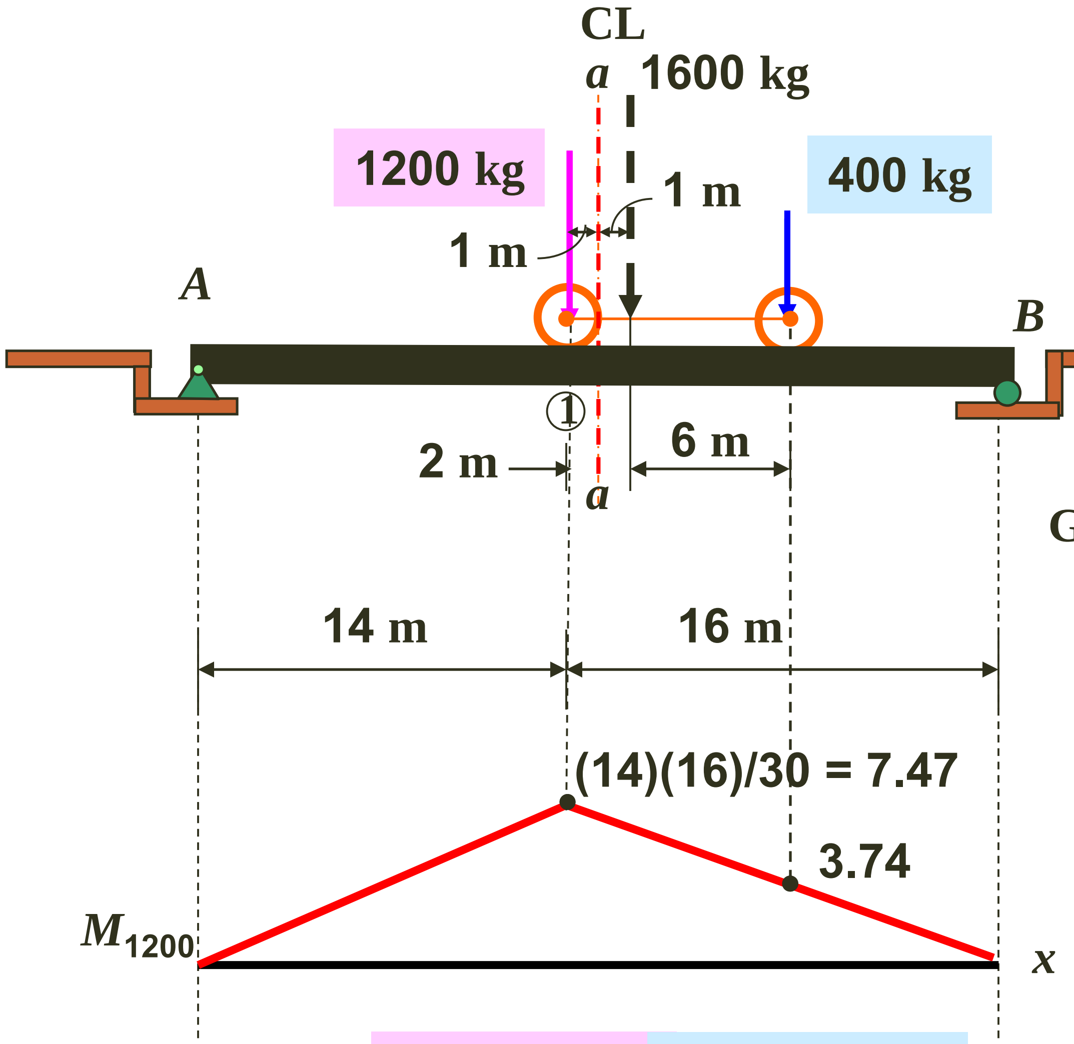


$$\Sigma M_{@1200\ kg} = 0 :$$

$$1600\bar{x} = 1200(0) + 400(8)$$

$$\bar{x} = 2$$

Maximum Influence at a Point Due to a Series of Concentrated Loads



M_S = (1200)(7.47) + (400)(3.74)

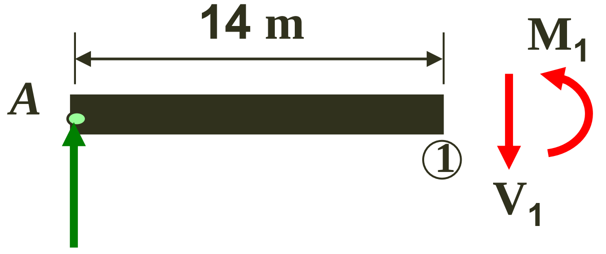
= 10460 kg•m

Or using equilibrium conditions:

Global: $+\circlearrowleft \Sigma M_B = 0$:

$$1600(14) - A_y(30) = 0$$

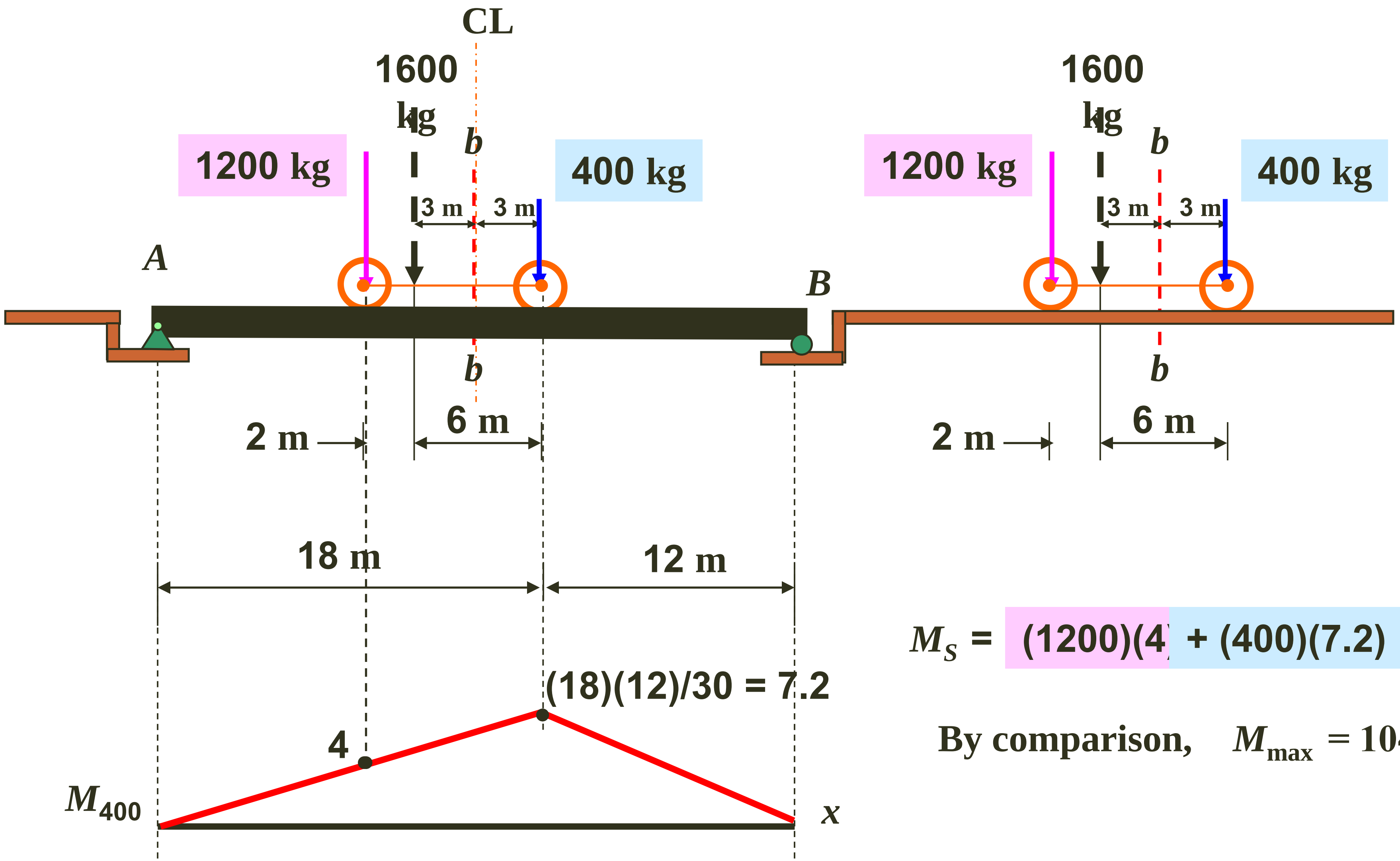
$$A_y = 746.67 \text{ kg}$$



$A_y = 746.67$ kg

$+\circlearrowleft \Sigma M_1 = 0$: $M_1 = 10453.38$ kg•m

Maximum Influence at a Point Due to a Series of Concentrated Loads



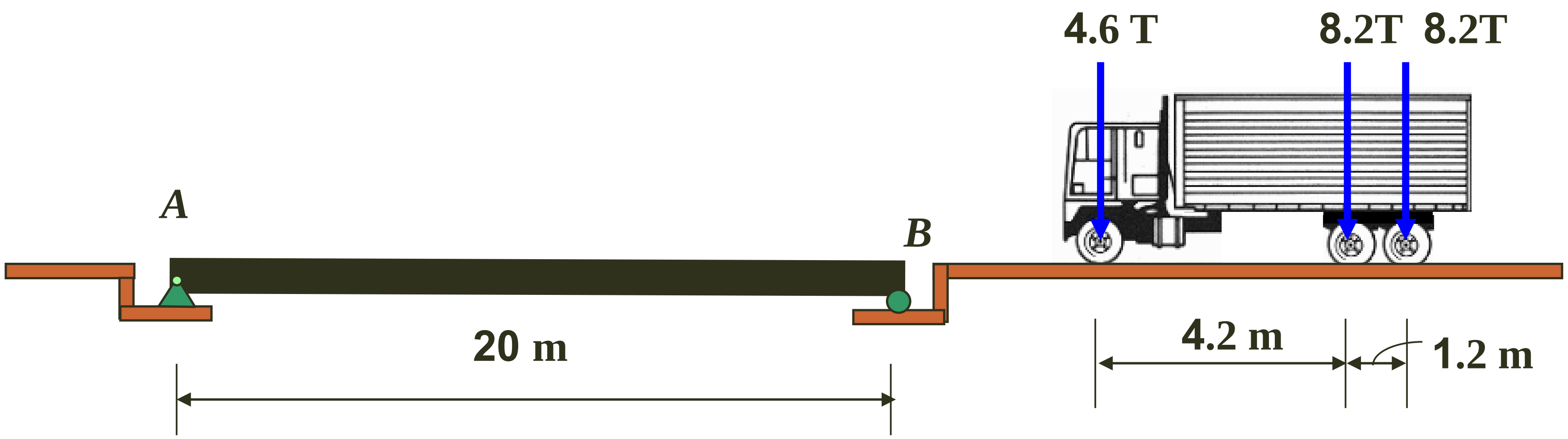
M_S = (1200)(4 + (400)(7.2) = 7680 kg•m

By comparison, M_{max} = 10460 kg•m

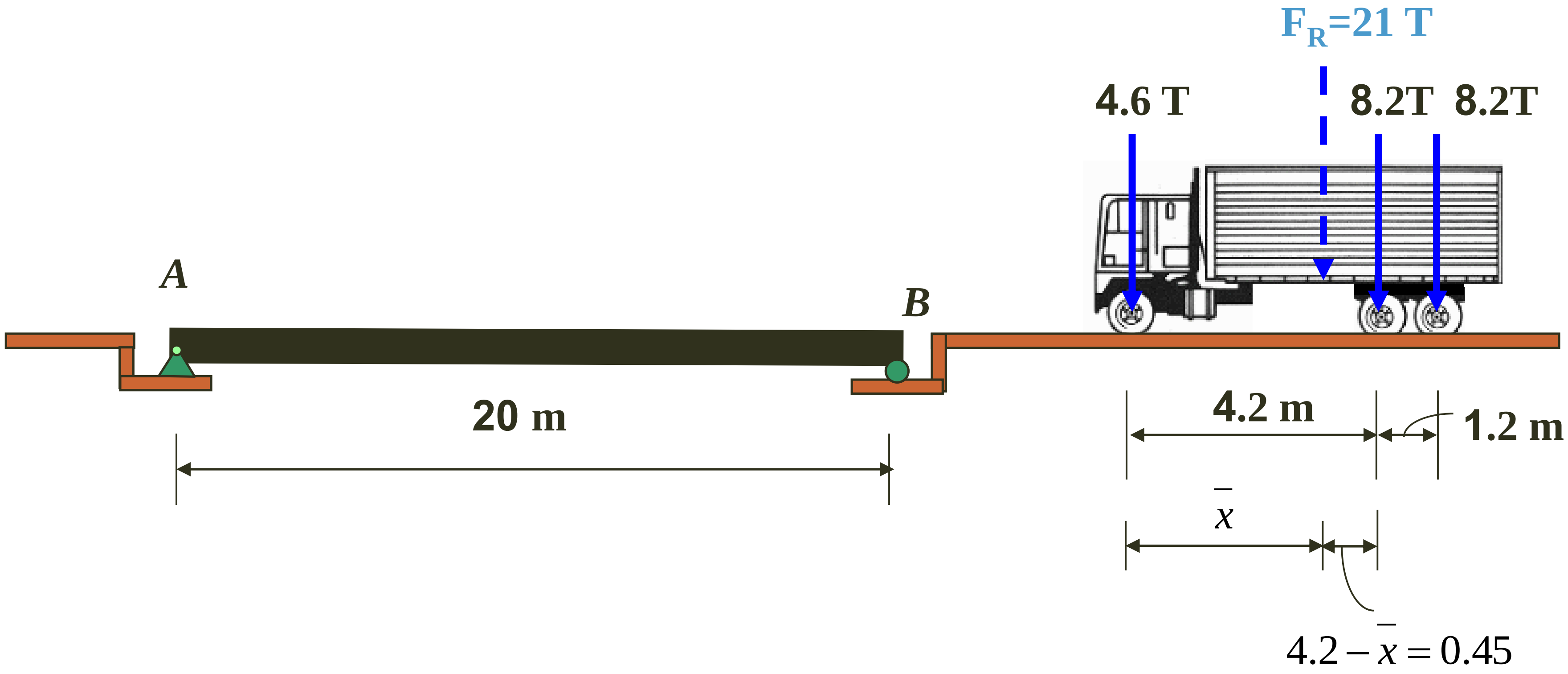
Maximum Influence at a Point Due to a Series of Concentrated Loads

Example 6-12

Determine the absolute maximum moment on the simply supported beam cased by the wheel loads.



Maximum Influence at a Point Due to a Series of Concentrated Loads

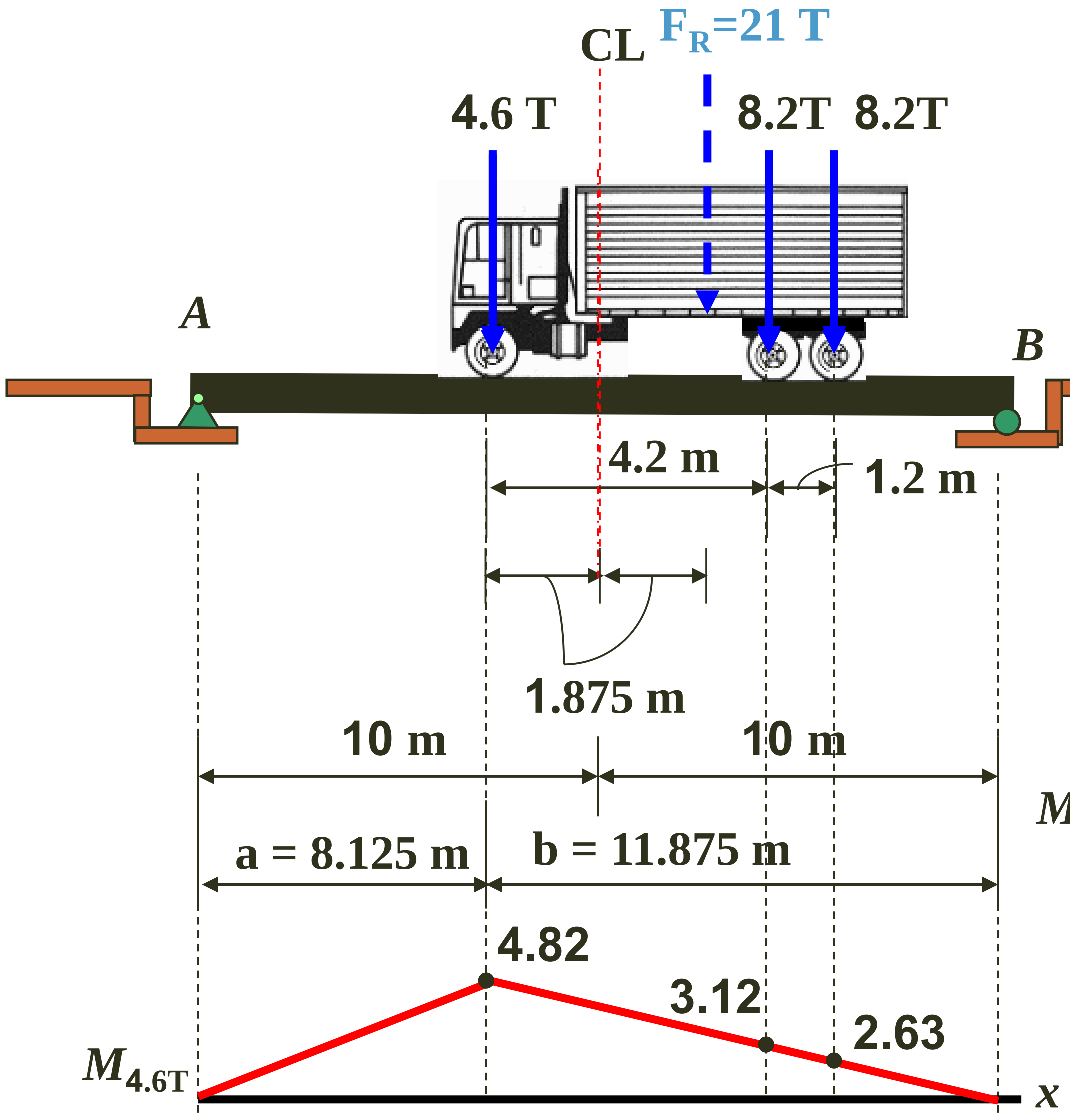


$\Sigma M_{@\,4.6\,T} = 0:$

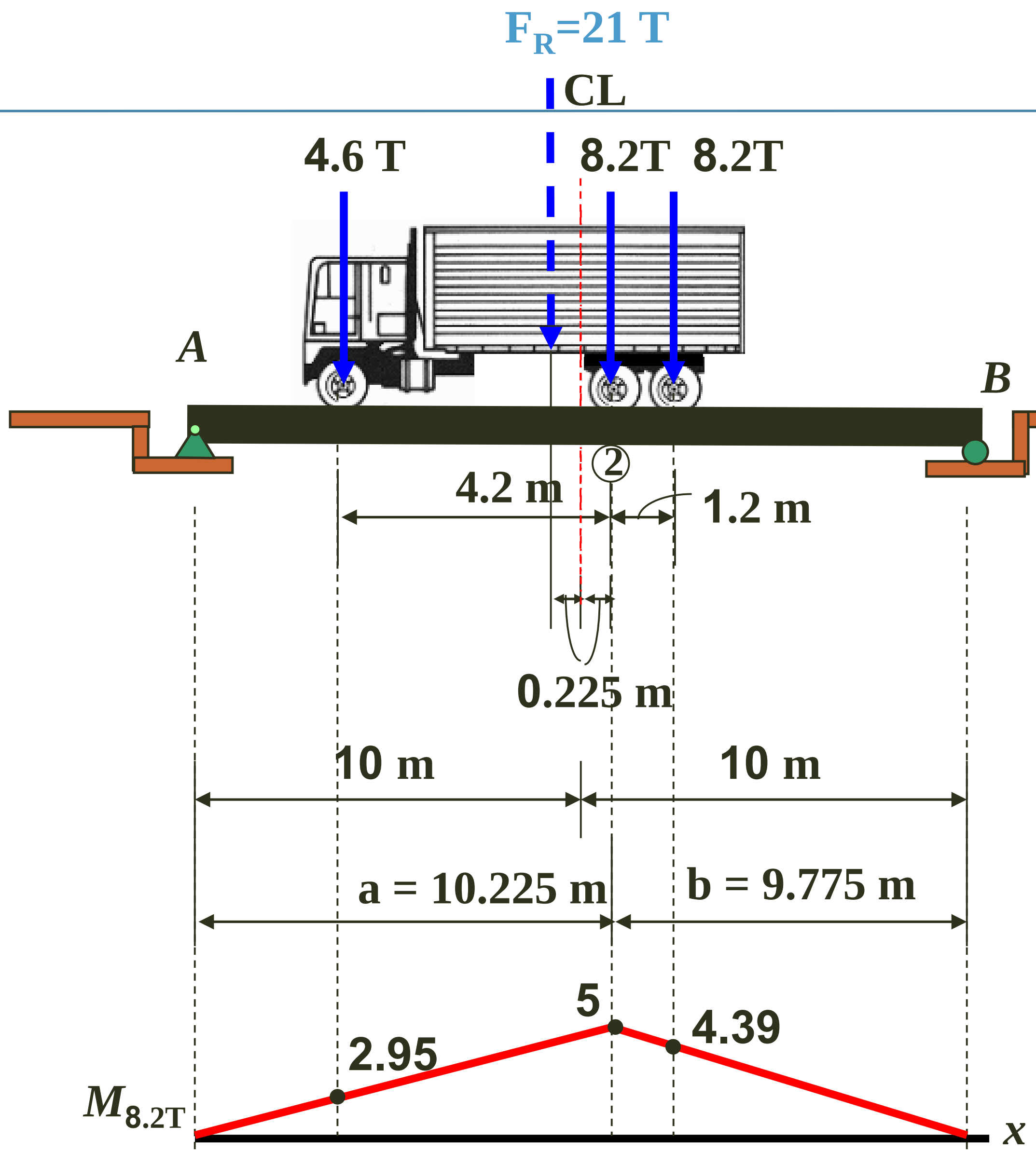
$$21\bar{x} = 4.6(0) + 8.2(4.2) + 8.2(5.4)$$

$$\bar{x} = 3.75$$

Maximum Influence at a Point Due to a Series of Concentrated Loads



M_S = (4.6)(4.82)+ (8.2)(3.12)+ (8.2)(2.63) = 69.32 T•m



M_S = (4.6)(2.95)+ (8.2)(5)+ (8.2)(4.39) = 90.57 T•m

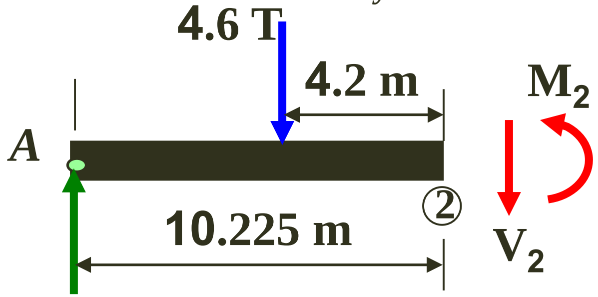
By comparison, M_{max} = 90.57 T•m

Or using equilibrium conditions:

Global: $+\circlearrowleft \Sigma M_B = 0$:

$$21(10.225) - A_y(20) = 0$$

A_y = 10.74 T

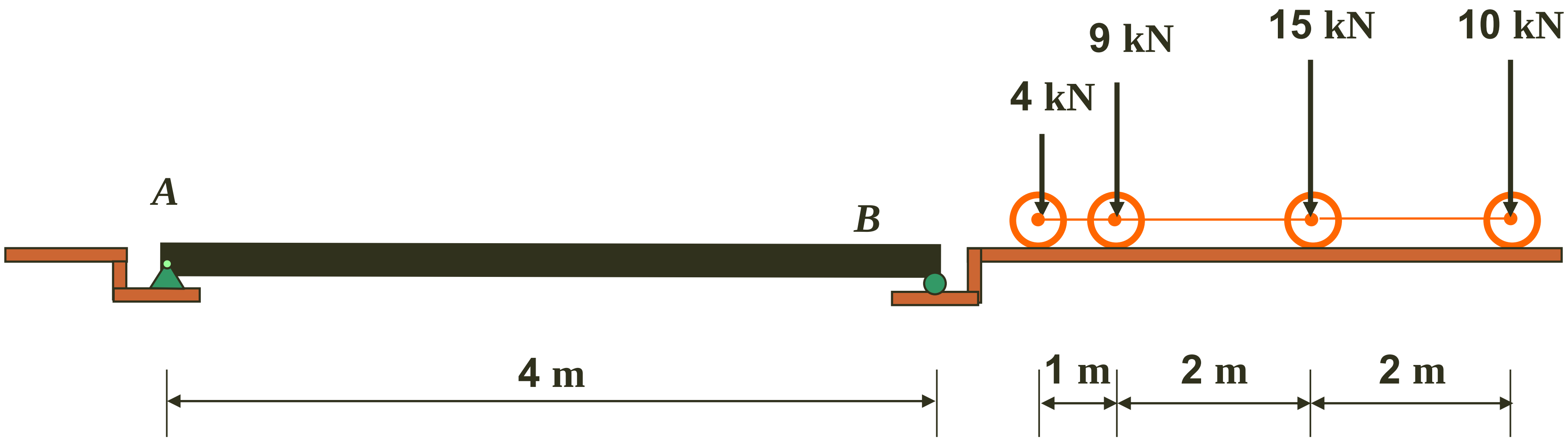


A_y = 10.74 T $\quad +\circlearrowleft \Sigma M_2 = 0$: $\quad M_2$ = 90.50 T•m

Maximum Influence at a Point Due to a Series of Concentrated Loads

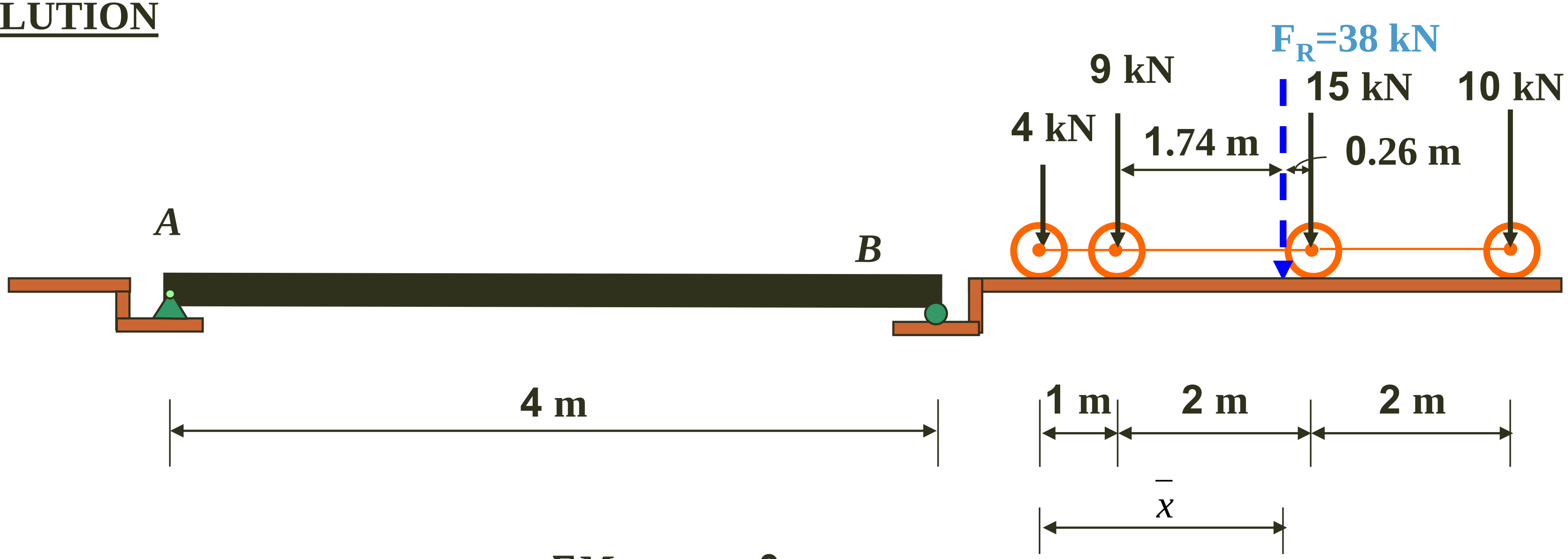
Example 6-13

Determine the absolute maximum moment on the simply supported beam cased by the wheel loads.



Maximum Influence at a Point Due to a Series of Concentrated Loads

SOLUTION

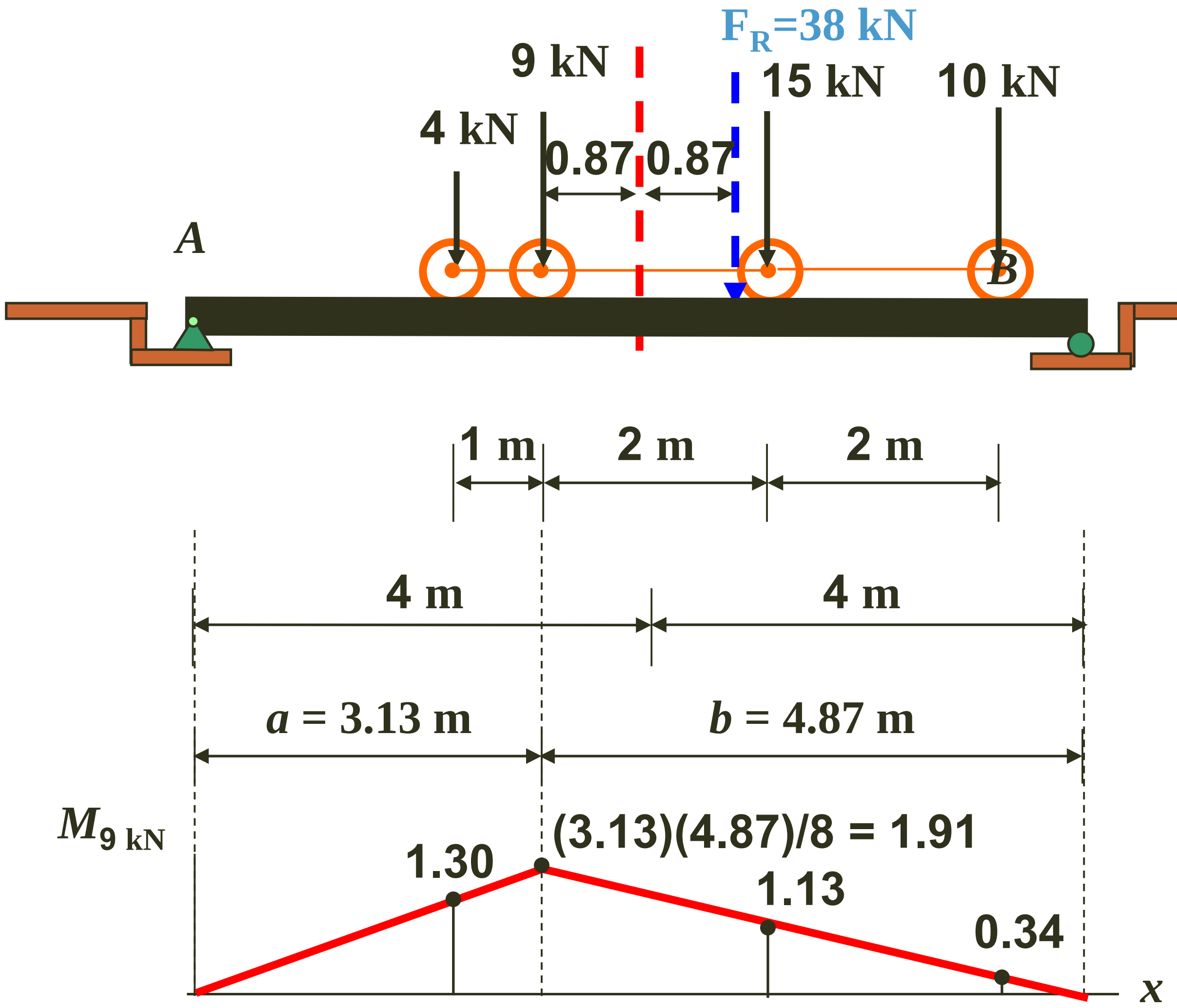


$\Sigma M_{@\,4\,kN} = 0:$

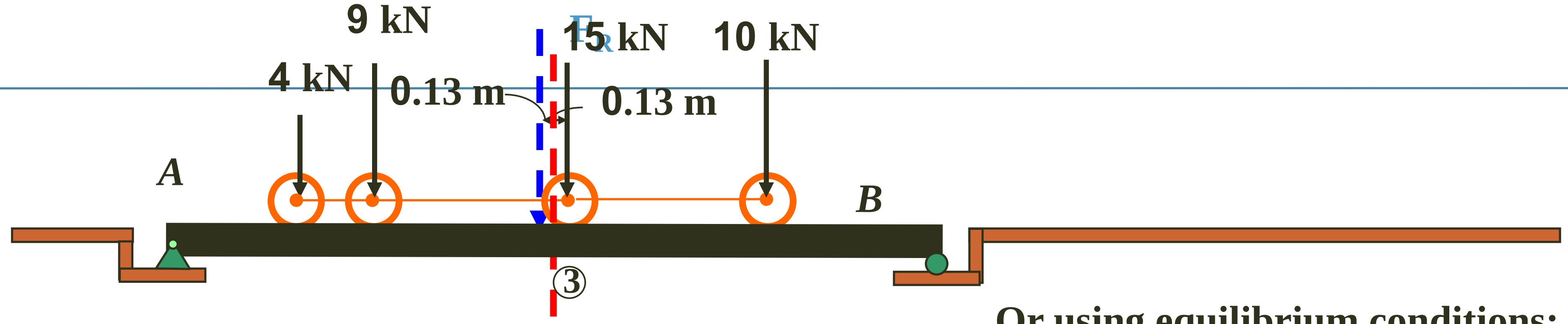
$$38\bar{x} = 4(0) + 9(1) + 15(3) + 10(5)$$

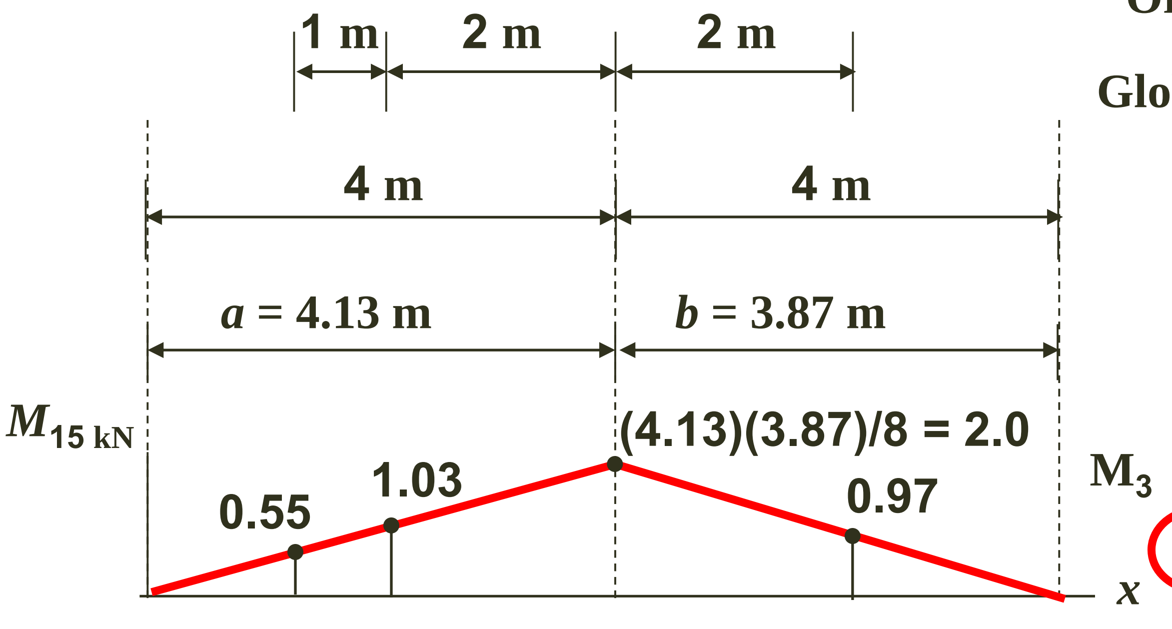
$$\bar{x} = 2.74$$

Maximum Influence at a Point Due to a Series of Concentrated Loads



$$M_S = 4(1.30) + 9(1.91) + 15(1.13) + 10(0.34) = 42.74 \text{ kN•m}$$



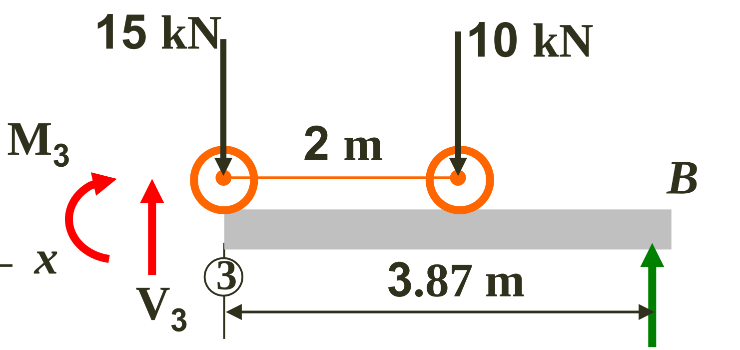


Or using equilibrium conditions:

Global: $+\circlearrowright \Sigma M_A = 0$:

$$-38(3.87) + B_y(8) = 0$$

$$B_y = 18.38 \text{ kN}$$



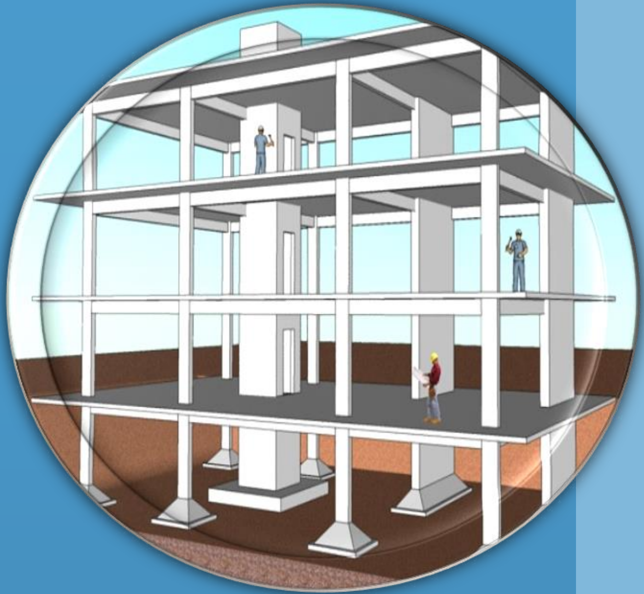
$+\circlearrowright \Sigma M_3 = 0$:

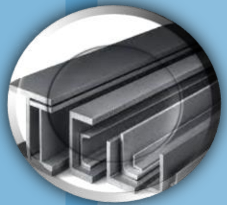
$$-M_3 - 10(2) + 18.38(3.87) = 0$$

$$M_3 = 51.13 \text{ kN}\cdot\text{m}$$

$$M_S = 4(0.55) + 9(1.03) + 15(2.0) + 10(0.97) = 51.17 \text{ kN}\cdot\text{m}$$

By comparison, $M_{max} = 51.17 \text{ kN}\cdot\text{m}$





Thank You!