UNIVERSITY OF ZAMBIA
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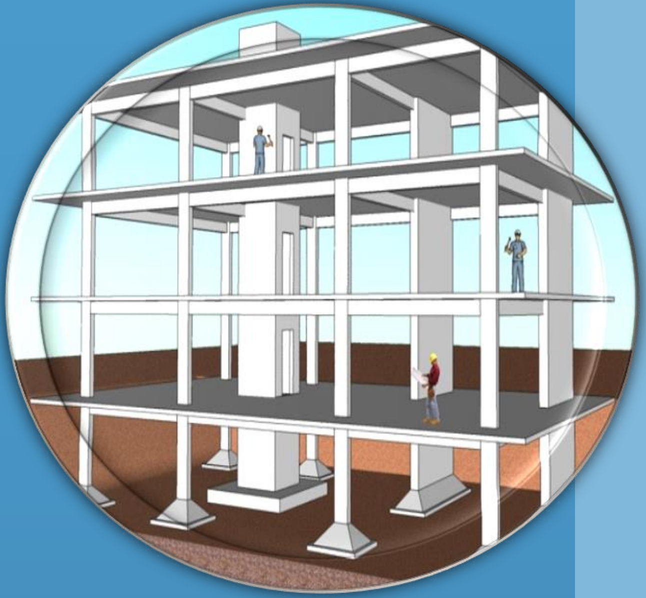

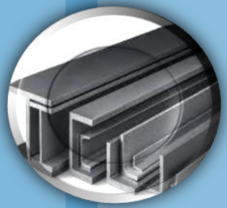

CEE 3222: THEORY OF STRUCTURES

Lecture 4.1

FORCE AND DISPLACEMENT METHODS FOR STATICALLY INDETERMINATE SSTRUCTURES

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Contents

Introduction

- For a coplanar structure there are at most three equilibrium equations for each part, so that if there is a total of n parts and r force and moment reaction components, we have

$$r = 3n, \text{ statically determinate}$$
$$r > 3n, \text{ statically indeterminate}$$

- This indeterminacy may arise as a result of:-
  - Added supports
  - Added members
  - General form of the structure (eg RC)

Introduction

Advantages

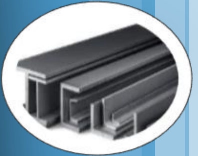
- The maximum stress and deflection of an indeterminate structure are generally smaller than those of its statically determinate
- Tendency to redistribute its load to its redundant supports in cases where faulty design or overloading occurs
- statically indeterminate structures can support a loading with thinner members and with increased stability compared to their statically determinate counterparts

Disadvantages

- Costly to fabricate joints for statically indeterminate.
- Because redundant support reactions, Great care to prevent differential displacement of the supports.
- any deformation, such as that caused by relative support displacement, or changes in member lengths caused by temperature or fabrication errors, will introduce additional stresses in the structure, which must be considered when designing indeterminate structures.

Introduction

Statically determinate	Statically indeterminate
Equilibrium equations could be directly solved, and thus forces could be calculated in an easy way	Equilibrium equations could be solved only when coupled with physical law and compatibility equations
Stress state depends only on geometry & loading	Stress state depends on rigidities
Not survivable, moderately used in modern aviation (due to damage tolerance requirement)	Survivable, widely used in modern aviation (due to damage tolerance property)
Easy to manufacture	Hard to manufacture



Introduction

- Satisfy equilibrium, compatibility, and force-displacement
  - Equilibrium:- The reactive forces hold the structure at rest.
  - Compatibility:- structure fit together without breaks or overlaps
  - Force-displacement:- Structure carries the Load without excessive displacements
- For a statically indeterminate structure, they are the **force or flexibility method**, and **the displacement or stiffness method**.

	Unknowns	Equations Used for Solution	Coefficients of the Unknowns
Force Method	Forces	Compatibility and Force Displacement	Flexibility Coefficients
Displacement Method	Displacements	Equilibrium and Force Displacement	Stiffness Coefficients

Introduction

Small degree of statical indeterminacy

Force method

Slope-deflection method

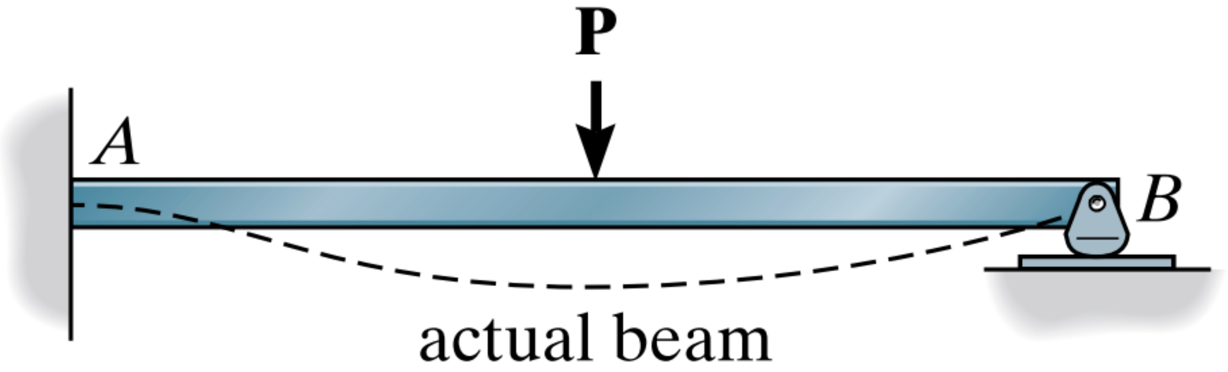
Slope-deflection method in matrix formulation

Numerical methods

Large degree of statical indeterminacy

General Compatibility (Force) method

- Originally developed by James Clerk Maxwell in 1864 and later refined by Otto Mohr and Heinrich Müller-Breslau.
- It is also called **Flexibility Method** or **Method of consistent Displacements.**
- Consider the beam below



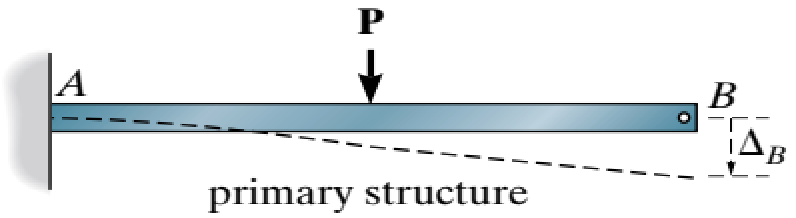
The beam has 4 unknown reactions and 3 equilibrium equations. Hence statically indeterminate to first degree.

An additional equation is necessary and is obtained by superposition consider the compatibility of displacement at one of the supports

General Compatibility (Force) method

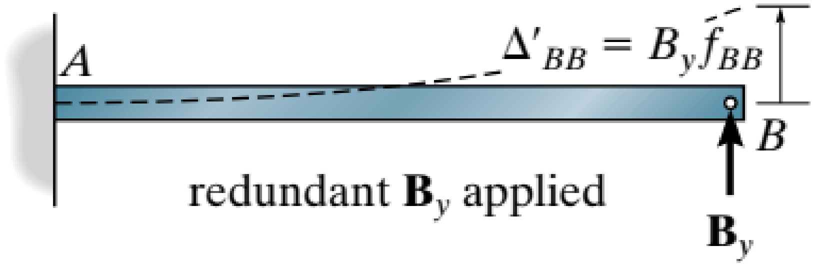
- One of the support (a rocker at B) will be temporarily removed and considered "redundant". The structure which remains is statically determinate and it's a primary structure.

=



With no rocker, the load P will displace B through Δ_B

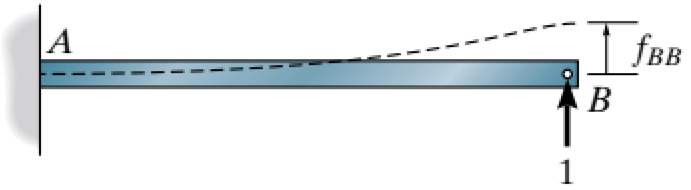
+



By superposition, the unknown reaction B_y will cause B to displace Δ'_{BB}

$$0 = -\Delta_B + \Delta'_{BB}$$

Compatibility Equation



Let B_y be unit, will cause B to displace f_{BB} (*linear flexibility coefficient* with units m/N). If linearly Elastic, Any B_y will cause a proportional increase in f_{BB}. Thus $\Delta'_{BB} = B_y f_{BB}$

Δ'_{BB}

The first letter: Where the deflection is specified

The Second letter: Where the unknown reaction acts

General Compatibility (Force) method

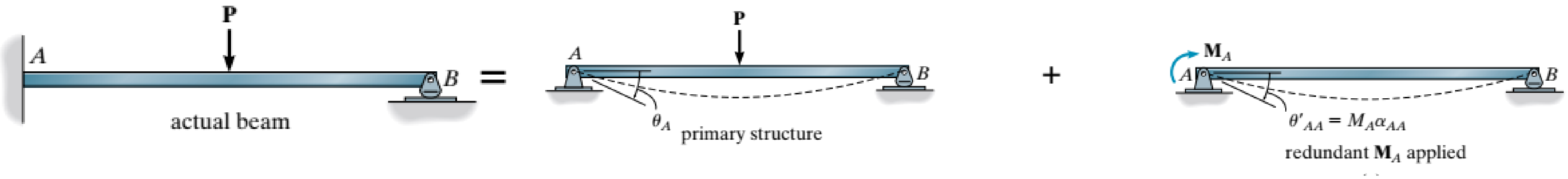
- The compatibility Equation will then become

$$0 = -\Delta_B + B_y f_{BB}$$

- Δ_B is determined through the deflection methods learnt from the energy methods.
- The reaction at B is then determined as $B_y = \Delta_B / f_{BB}$
- The other reactions of the primary structure can be determined using equilibrium equation.

General Compatibility (Force) method

- If you declare the moment at A as redundant, then you can replace the fixed joint with a pin, to remove moment resistant. Similar procedure as before.

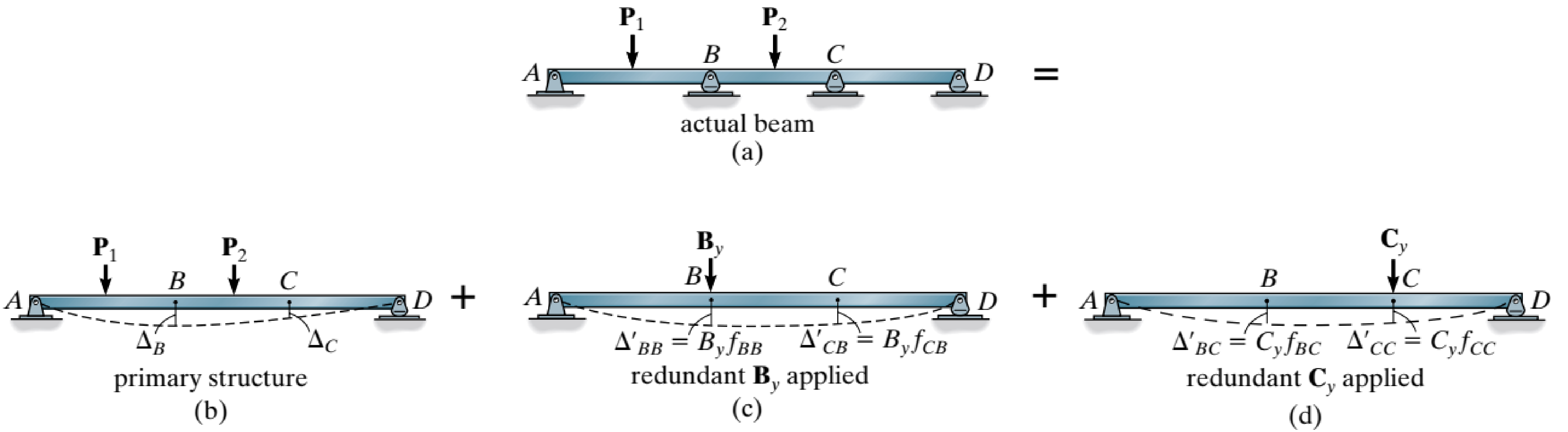


- Where α_{AA} is the angular flexibility coefficient

$$\theta'_{AA} = M_A\alpha_{AA} \qquad 0 = \theta_A + M_A\alpha_{AA} \qquad M_A = -\theta_A/\alpha_{AA}$$

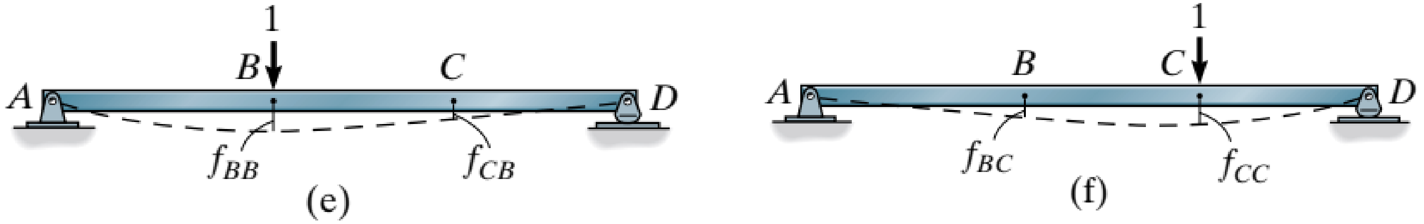
General Compatibility (Force) method

- The beam is indeterminate to the second degree and therefore two compatibility equations will be necessary for the solution.



$$0 = \Delta_B + B_y f_{BB} + C_y f_{BC}$$

$$0 = \Delta_C + B_y f_{CB} + C_y f_{CC}$$



General Compatibility (Force) method

Procedure for Analysis

The following procedure provides a general method for determining the reactions or internal loadings of statically indeterminate structures using the force or flexibility method of analysis.

Principle of Superposition

Determine the number of degrees n to which the structure is indeterminate. Then specify the n unknown redundant forces or moments that must be removed from the structure in order to make it statically determinate and stable. Using the principle of superposition, draw the statically indeterminate structure and show it to be equal to a series of corresponding statically *determinate* structures. The primary structure supports the same external loads as the statically indeterminate structure, and each of the other structures added to the primary structure shows the structure loaded with a separate redundant force or moment. Also, sketch the elastic curve on each structure and indicate symbolically the displacement or rotation at the point of each redundant force or moment.

Compatibility Equations

Write a compatibility equation for the displacement or rotation at each point where there is a redundant force or moment. These equations should be expressed in terms of the unknown redundants and their corresponding flexibility coefficients obtained from unit loads or unit couple moments that are collinear with the redundant forces or moments.

Determine all the deflections and flexibility coefficients using the table on the inside front cover or the methods of Chapter 8 or 9.* Substitute these load-displacement relations into the compatibility equations and solve for the unknown redundants. In particular, if a numerical value for a redundant is negative, it indicates the redundant acts opposite to its corresponding unit force or unit couple moment.

Equilibrium Equations

Draw a free-body diagram of the structure. Since the redundant forces and/or moments have been calculated, the remaining unknown reactions can be determined from the equations of equilibrium.

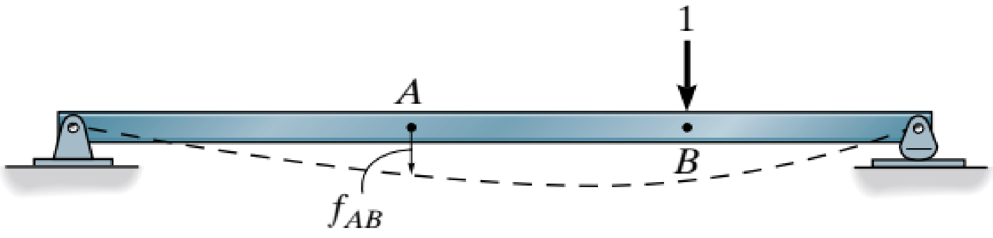
It should be realized that once all the support reactions have been obtained, the shear and moment diagrams can then be drawn, and the deflection at any point on the structure can be determined using the same methods outlined previously for statically determinate structures.

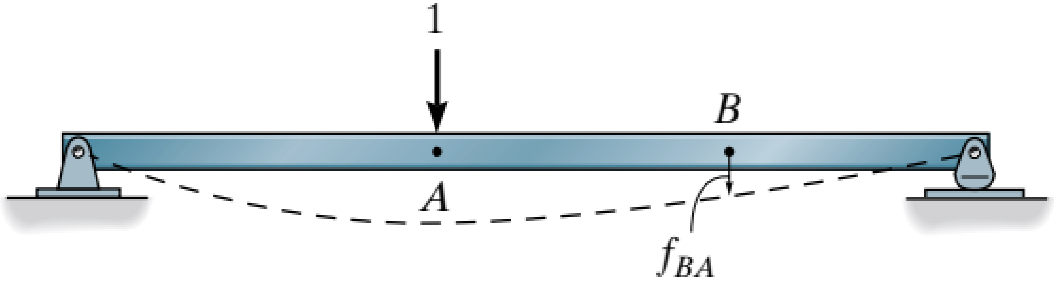
General Compatibility (Force) method

- **Classification of the problem** — Redundant constraints are removed
- **Basic system** — In loaded state, external load is applied. In unity states, unit force is applied instead of constraint.
- **Loaded and unity states** — Displacements corresponding to removed constraints are determined for each state
- **Canonical equations** — Forces in removed constraints are determined
- **Total stress state**

Maxwell's Theorem of Reciprocal Displacements; Betti's Law

- The **theorem of reciprocal displacements** states that *the displacement of a point B on a structure due to a unit load acting at point A is equal to the displacement of point A when the unit load is acting at point B, that is,* $f_{BA} = f_{AB}$





$$f_{AB} = \int \frac{m_A m_B}{EI} dx = \int \frac{m_B m_A}{EI} dx = f_{BA}$$

- The theorem also applies for reciprocal rotations, and may be stated as follows: the rotation at point B on a structure due to a unit couple moment acting at point A is equal to the rotation at point A when the unit couple moment is acting at point B.
- Furthermore, using a unit force and unit couple moment, applied at separate points on the structure, we may also state: The rotation in radians at point B on a structure due to a unit load acting at point A is equal to the displacement at point A when a unit couple moment is acting at point B.

Maxwell's Theorem of Reciprocal Displacements; Betti's Law

- When the theorem of reciprocal displacements is formalized in a more general sense, it is referred to as ***Betti's law***.
- Briefly stated:
- *The virtual work* dU_{AB} *done by a system of forces* $\sum P_B$ *that undergo a displacement caused by a system of forces* $\sum P_A$ *is equal to the virtual work* dU_{BA} *caused by the forces* $\sum P_A$ *when the structure deforms due to the system of forces* $\sum P_B$. *In other words,* $dU_{AB} = dU_{BA}$.

General Compatibility (Force) method-Generalised

- When computing the flexibility coefficients, f_{ij} ($or\ \alpha_{ij}$), for the structure, it will be noticed that they depend only on the material and geometrical properties of the members and not on the loading of the primary structure. Hence these values, once determined, can be used to compute the reactions for any loading.
- For a structure having n redundant reactions, R_n, we can write n compatibility equations, namely:

$$\begin{aligned}
\Delta_1 + f_{11}R_1 + f_{12}R_2 + \cdots + f_{1n}R_n &= 0\\
\Delta_2 + f_{21}R_1 + f_{22}R_2 + \cdots + f_{2n}R_n &= 0\\
&\vdots\\
\Delta_n + f_{n1}R_1 + f_{n2}R_2 + \cdots + f_{nn}R_n &= 0
\end{aligned}$$

In particular, note that $f_{ij} = f_{ji}$ ($f_{12} = f_{21}$, etc.)

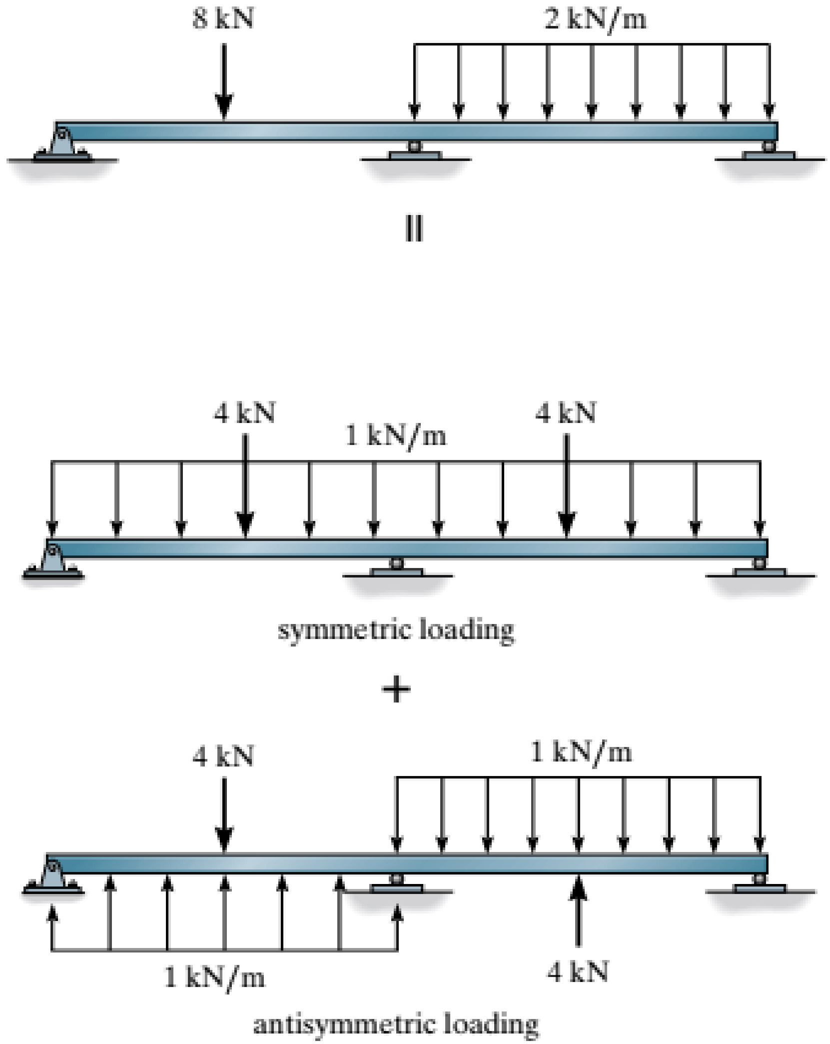
$$\begin{bmatrix} f_{11} & f_{12} & \cdots & f_{1n} \\ f_{21} & f_{22} & \cdots & f_{2n} \\ & & \vdots & \\ f_{n1} & f_{n2} & \cdots & f_{nn} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{bmatrix} = - \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_n \end{bmatrix}$$

or simply

$$\mathbf{fR} = -\mathbf{\Delta}$$

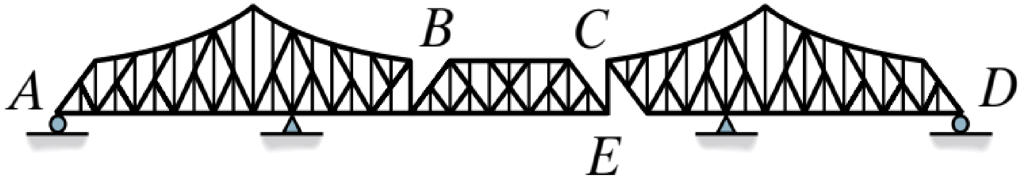
General Compatibility (Force) method- symmetry

- The analysis of a statically indeterminate structure can be simplified if the structure has symmetry of material, geometry, and loading about its central axis. In particular, structures having an asymmetric loading can be replaced with a superposition of a symmetric and antisymmetric load.

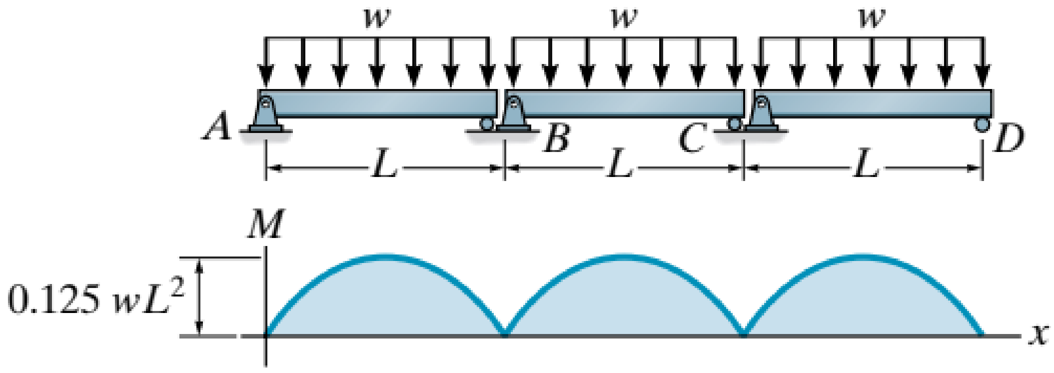


Force Method for beams

- Consider a trussed cantilever bridge, such as the one shown in the photo.
- It consists of two cantilevered sections AB and CD and a center suspended span BC that was floated out and lifted in place.
- The span BC was pinned at B and suspended from C by a primary vertical member CE.

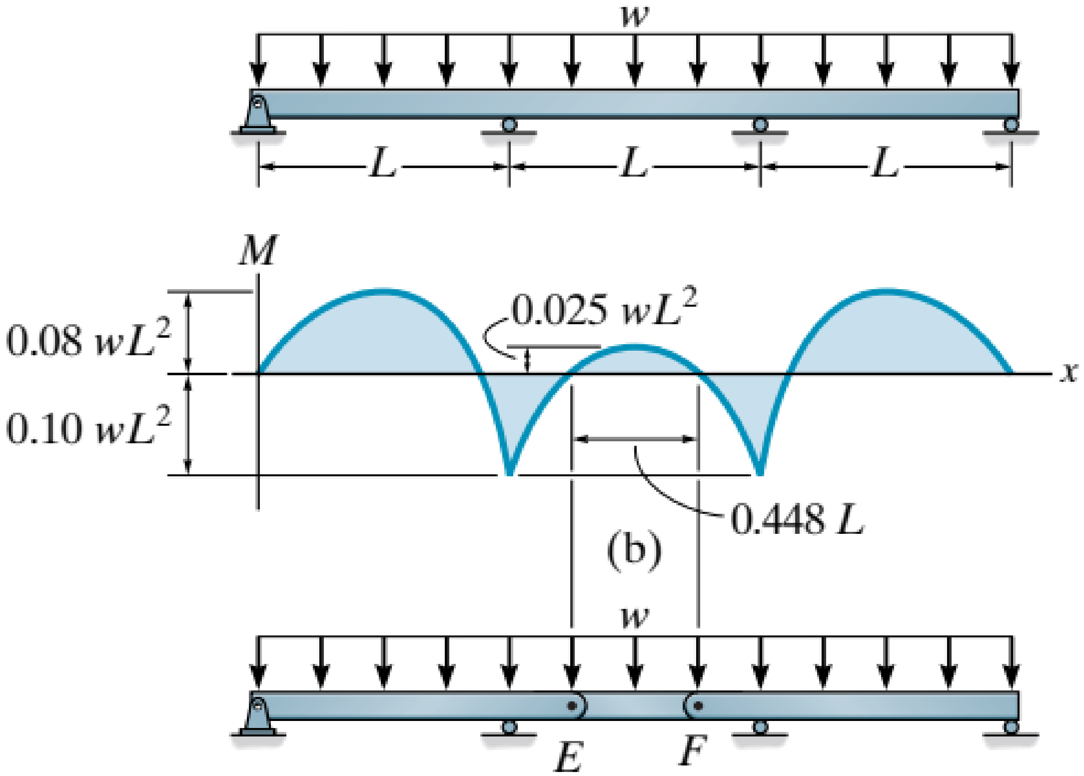


Force Method for beams



(a)

- Although this is a 25% reduction in the maximum moment, unfortunately any slight settlement of one of the bridge piers would introduce larger reactions at the supports, and also larger moments in the beam

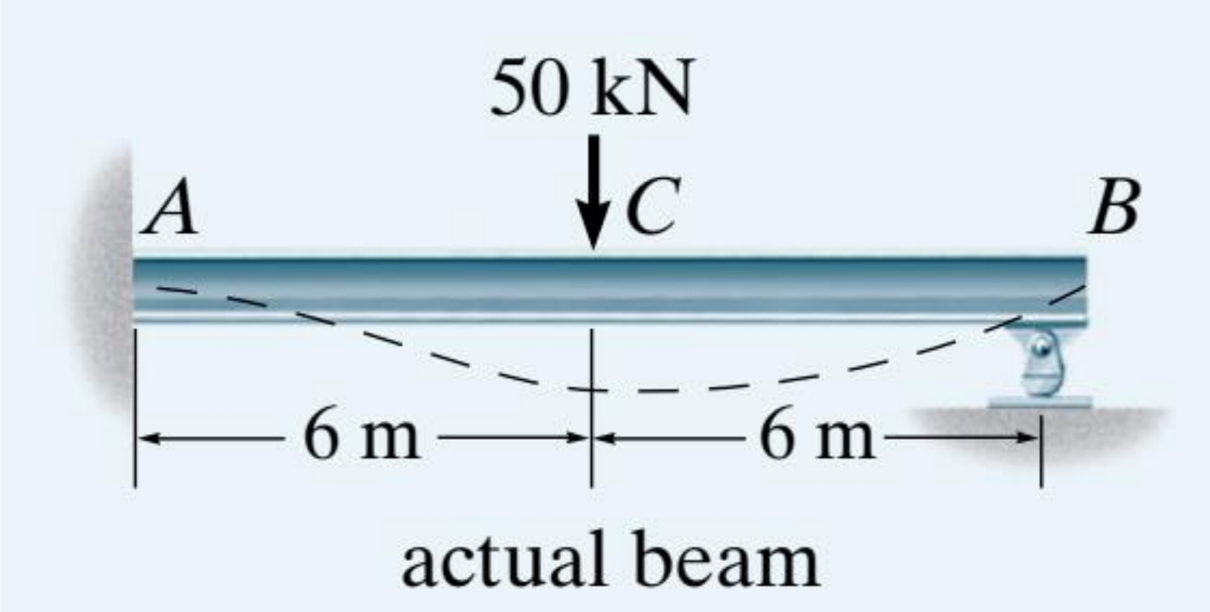


(b)

- The beam then becomes statically determinate by introducing pins in the span at point E and F as shown and yet continuity of the span is maintained.
- In this case any settlement of a support would not affect the reactions. The cantilevered bridge span works on the same principle.

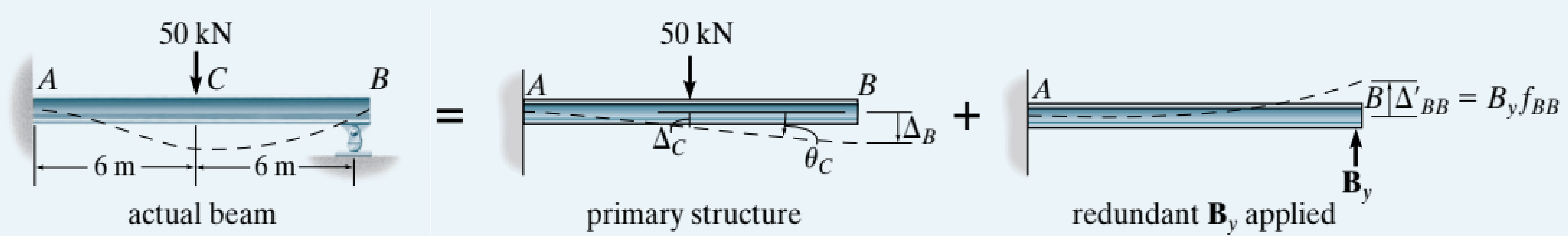
Example 1

- Determine the reaction at the roller support B of the beam shown in the figure. EI is constant.



Solution 1

By inspection, the beam is statically indeterminate to the first degree. The redundant will be taken as By so that this force can be determined directly



$$0 = -\Delta_B + B_y f_{BB}$$

Solution 1

By inspection, the beam is statically indeterminate to the first degree. The redundant will be taken as By so that this force can be determined directly

Compatibility Equation. Taking positive displacement as upward, Fig. 10–9*b*, we have

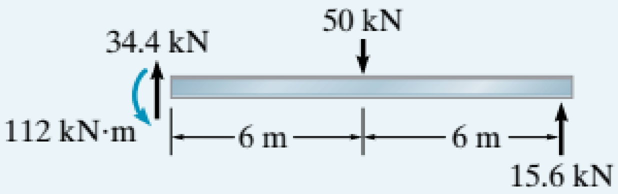
$$(+\uparrow) \qquad 0 = -\Delta_B + B_y f_{BB} \qquad (1)$$

The terms Δ_B and f_{BB} are easily obtained using the table on the inside front cover. In particular, note that $\Delta_B = \Delta_C + \theta_C(6\text{ m})$. Thus,

$$\Delta_B = \frac{P(L/2)^3}{3EI} + \frac{P(L/2)^2}{2EI}\left(\frac{L}{2}\right)$$

$$= \frac{(50\text{ kN})(6\text{ m})^3}{3EI} + \frac{(50\text{ kN})(6\text{ m})^2}{2EI}(6\text{ m}) = \frac{9000\text{ kN}\cdot\text{m}^3}{EI}\downarrow$$

$$f_{BB} = \frac{PL^3}{3EI} = \frac{1(12\text{ m})^3}{3EI} = \frac{576\text{ m}^3}{EI}\uparrow$$



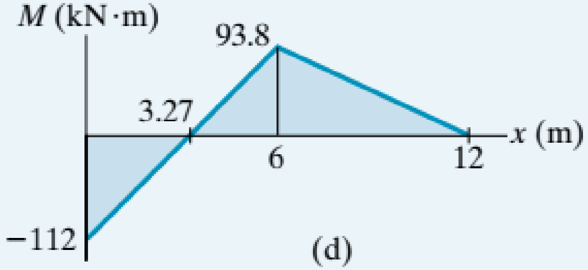
(c)

Substituting these results into Eq. (1) yields

$$(+\uparrow) \qquad 0 = -\frac{9000}{EI} + B_y\left(\frac{576}{EI}\right) \qquad B_y = 15.6\text{ kN} \qquad \textit{Ans.}$$

If this reaction is placed on the free-body diagram of the beam, the reactions at *A* can be obtained from the three equations of equilibrium, Fig. 10–9*c*.

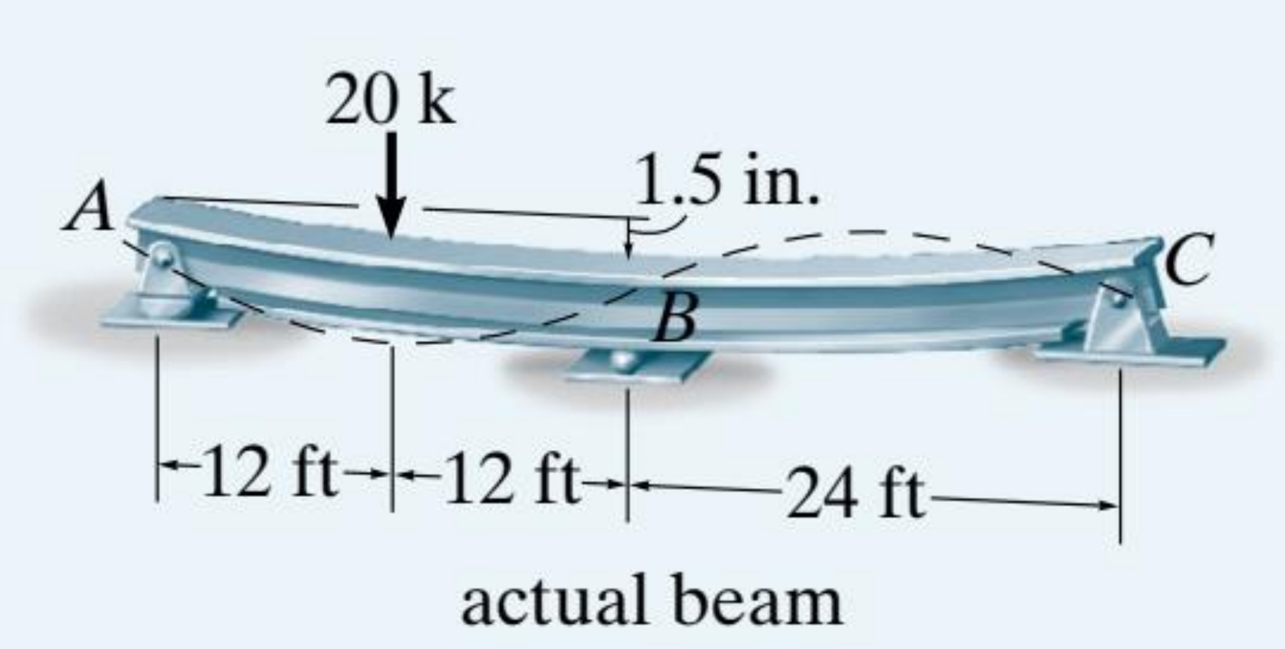
Having determined all the reactions, the moment diagram can be constructed as shown in Fig. 10–9*d*.



(d)

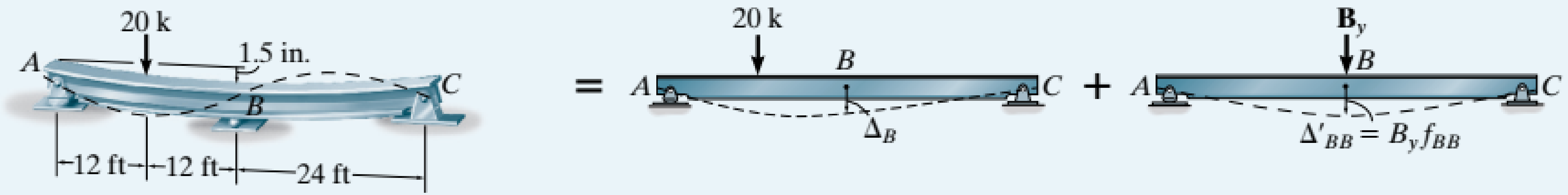
Example 2

- Draw the shear and moment diagrams for the beam shown in the figure. The support at B settles 1.5 in. Take $E = 29(10^3)\ ksi, I = 750\ in^4$.



Solution 2

- By inspection, the beam is indeterminate to the first degree. The center support B will be chosen as the redundant, so that the roller at B is removed. Here B_y is assumed to act downward on the beam.



actual beam | primary structure | redundant $\mathbf{B}_y$ applied

Compatibility Equation. With reference to point *B* in Fig. using units of inches, we require

$$(+\downarrow) \qquad 1.5 \text{ in.} = \Delta_B + B_y f_{BB} \qquad (1)$$

We will use the table on the inside front cover. Note that for Δ_B the equation for the deflection curve requires $0 < x < a$. Since $x = 24$ ft, then $a = 36$ ft. Thus,

Solution 2

$$\Delta_B = \frac{Pbx}{6LEI}(L^2 - b^2 - x^2) = \frac{20(12)(24)}{6(48)EI}\left[(48)^2 - (12)^2 - (24)^2\right]$$

$$= \frac{31{,}680 \text{ k}\cdot\text{ft}^3}{EI}$$

$$f_{BB} = \frac{PL^3}{48EI} = \frac{1(48)^3}{48\,EI} = \frac{2304 \text{ ft}^3}{EI}$$

Substituting these values into Eq. (1), we get

$$1.5 \text{ in.}\,(29(10^3)\text{ k/in}^2)(750 \text{ in}^4)$$

$$= 31{,}680 \text{ k}\cdot\text{ft}^3(12\text{ in./ft})^3 + B_y(2304 \text{ ft}^3)(12 \text{ in./ft})^3$$

$$B_y = -5.56 \text{ k}$$

The negative sign indicates that $\mathbf{B}_y$ acts *upward* on the beam.

Equilibrium Equations. From the free-body diagram shown in Fig. 10–10*c* we have

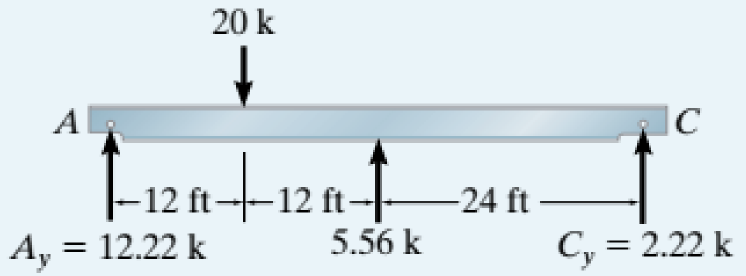
$\circlearrowleft + \Sigma M_A = 0;$ $\quad -20(12) + 5.56(24) + C_y(48) = 0$

$$C_y = 2.22 \text{ k}$$

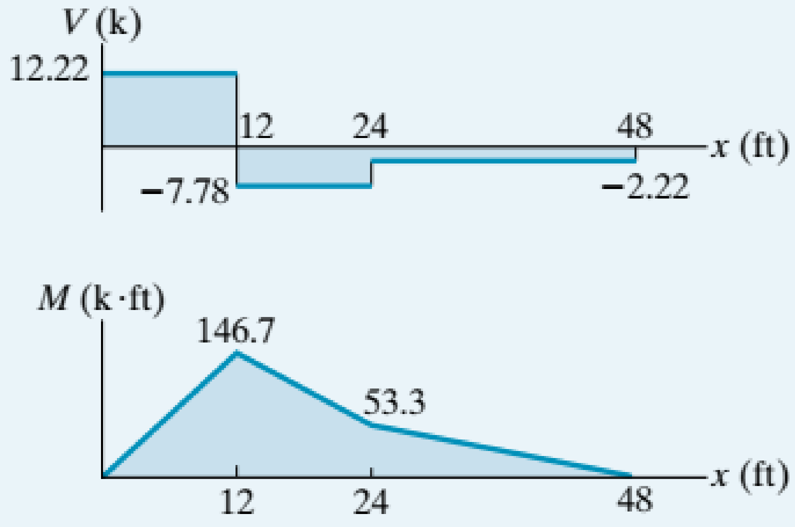
$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 20 + 5.56 + 2.22 = 0$

$$A_y = 12.22 \text{ k}$$

Using these results, verify the shear and moment diagrams shown in Fig. 10–10*d*.



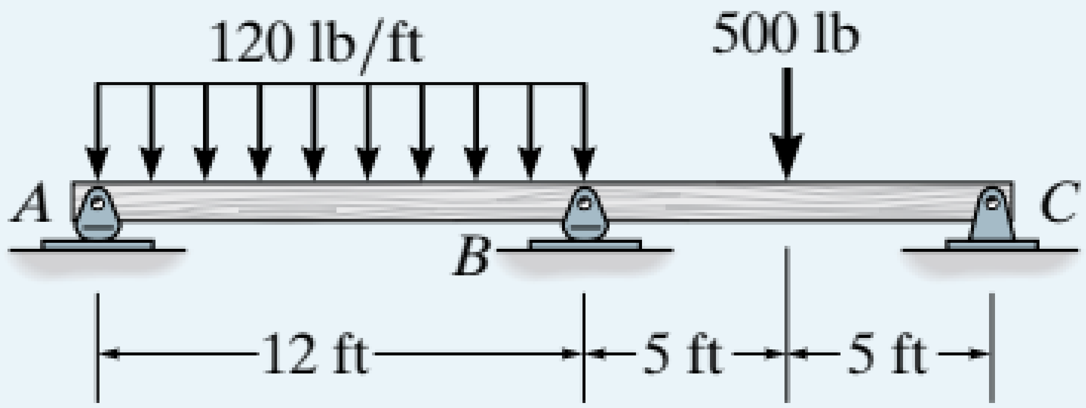
(c)



(d)

Example 3

- Determine the reactions at the supports for the beam shown in Figure. EI is constant.



actual beam

Solution 3

Principle of Superposition. By inspection, the beam is indeterminate to the first degree. Here, for the sake of illustration, we will choose the internal moment at support *B* as the redundant. Consequently, the beam is cut open and end pins or an internal hinge are placed at *B* in order to release *only* the capacity of the beam to resist moment at this point, Fig. 10–12*b*. The internal moment at *B* is applied to the beam in Fig. 10–12*c*.

Compatibility Equations. From Fig. 10–12*a* we require the relative rotation of one end of one beam with respect to the end of the other beam to be zero, that is,

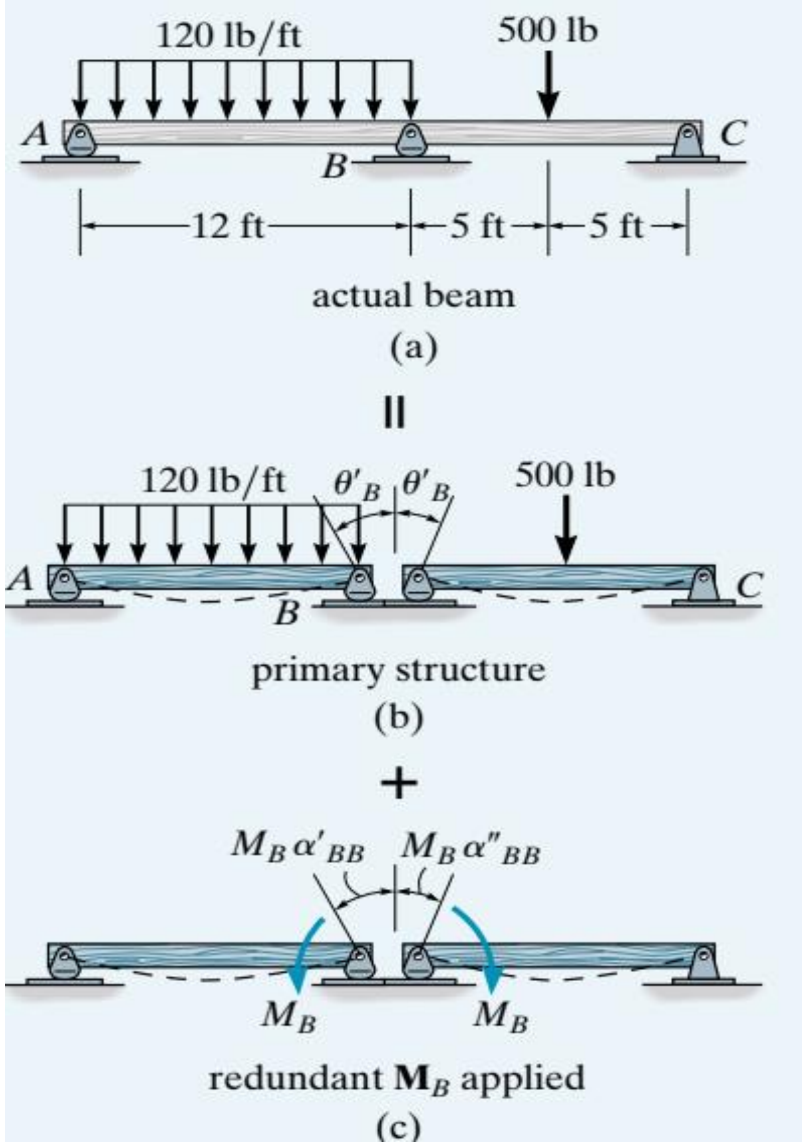
$$(\circlearrowright +) \qquad \theta_B + M_B\alpha_{BB} = 0$$

where

$$\theta_B = \theta'_B + \theta''_B$$

and

$$\alpha_{BB} = \alpha'_{BB} + \alpha''_{BB}$$



Solution 3

The slopes and angular flexibility coefficients can be determined from the table on the inside front cover, that is,

$$\theta'_B = \frac{wL^3}{24EI} = \frac{120(12)^3}{24EI} = \frac{8640\ \text{lb}\cdot\text{ft}^2}{EI}$$

$$\theta''_B = \frac{PL^2}{16EI} = \frac{500(10)^2}{16EI} = \frac{3125\ \text{lb}\cdot\text{ft}^2}{EI}$$

$$\alpha'_{BB} = \frac{ML}{3EI} = \frac{1(12)}{3EI} = \frac{4\ \text{ft}}{EI}$$

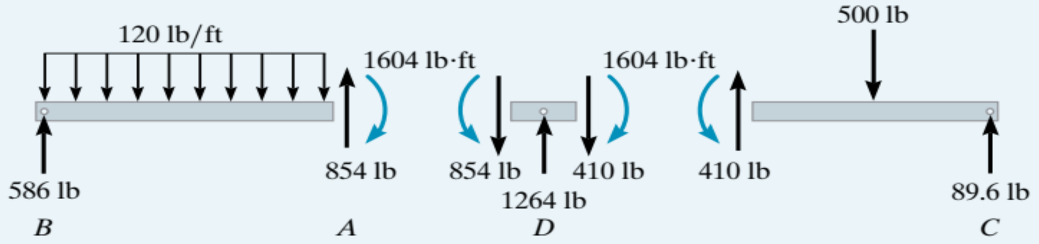
$$\alpha''_{BB} = \frac{ML}{3EI} = \frac{1(10)}{3EI} = \frac{3.33\ \text{ft}}{EI}$$

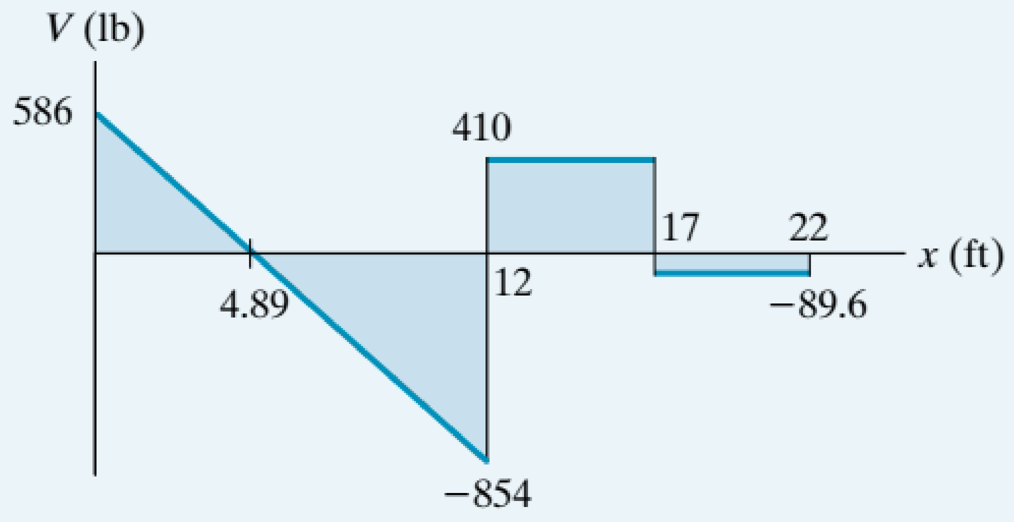
Thus

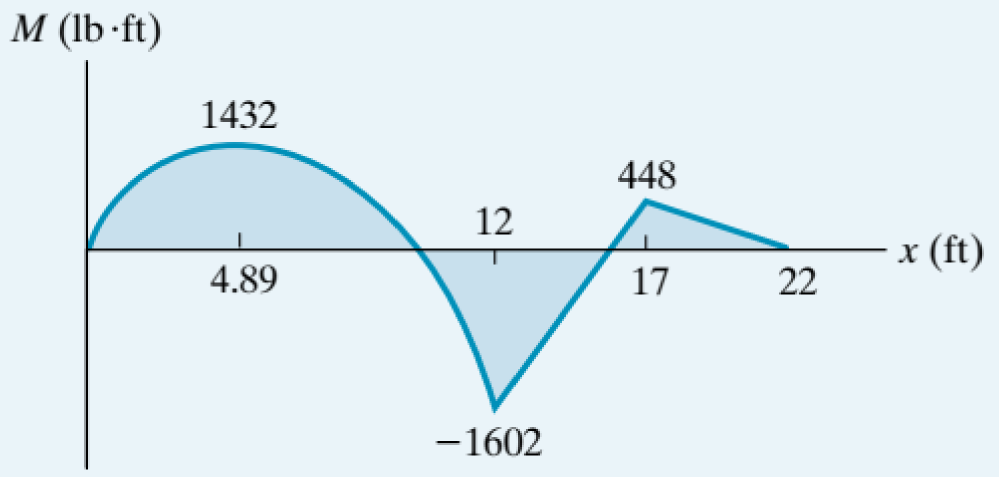
$$\frac{8640\ \text{lb}\cdot\text{ft}^2}{EI} + \frac{3125\ \text{lb}\cdot\text{ft}^2}{EI} + M_B\left(\frac{4\ \text{ft}}{EI} + \frac{3.33\ \text{ft}}{EI}\right) = 0$$

$$M_B = -1604\ \text{lb}\cdot\text{ft}$$

The negative sign indicates M_B acts in the opposite direction to that shown in Fig. 10–12*c*. Using this result, the reactions at the supports are calculated as shown in Fig. 10–12*d*. Furthermore, the shear and moment diagrams are shown in Fig. 10–12*e*.





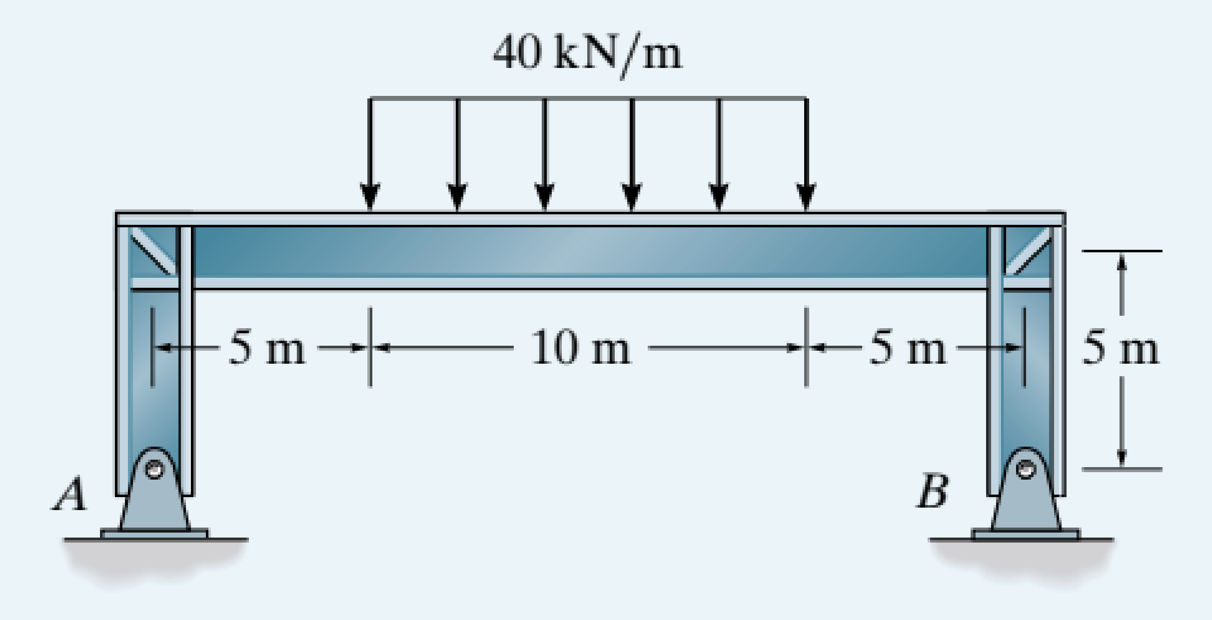


Force Method of Analysis: Frames

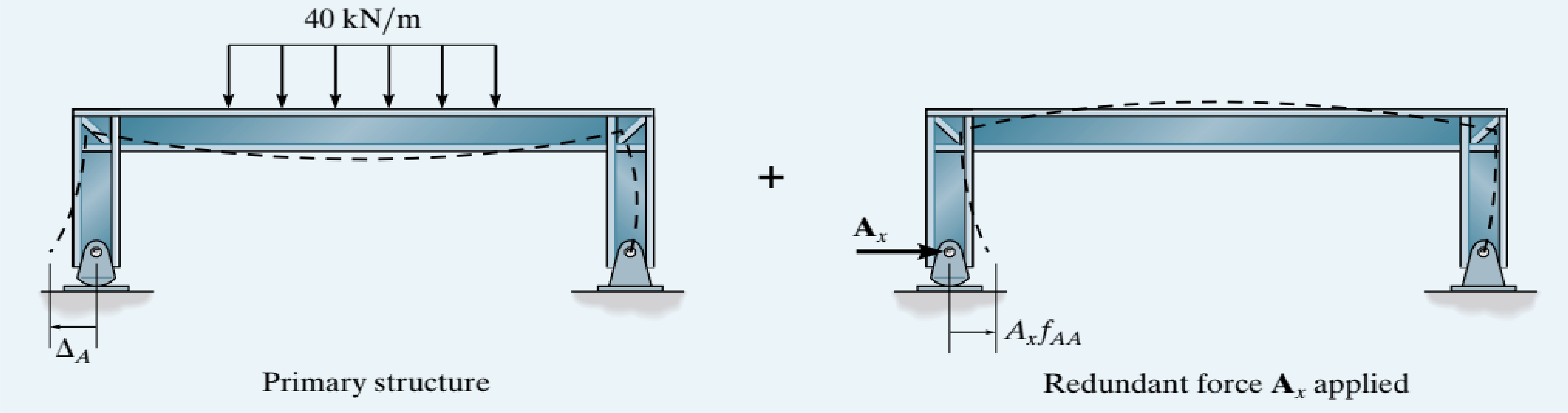
- The force method is very useful for solving problems involving statically indeterminate frames that have a single story and unusual geometry, such as gabled frames. Problems involving multistory frames, or those with a high degree of indeterminacy, are best solved using the slope-deflection, moment-distribution, or the stiffness method discussed in later chapters.

Example 4

- The saddle bent shown in the photo is used to support the bridge deck. Assuming EI is constant, a drawing of it along with the dimensions and loading is shown in the figure. Determine the horizontal support reaction at A.



Solution 4



Principle of Superposition. By inspection the frame is statically indeterminate to the first degree. To obtain a direct solution for A_x we will choose this reaction to be the redundant. Consequently, the pin at A is replaced by a rocker, since a rocker will not constrain A in the horizontal direction. The principle of superposition applied to the idealized model of the frame is shown in Fig. 10–13*b*. Notice how the frame deflects in each case.

Solution 4

Compatibility Equation. Reference to point A in Fig. 10–13b requires

$$(\overset{+}{\rightarrow}) \qquad 0 = \Delta_A + A_x f_{AA} \qquad (1)$$

The terms Δ_A and f_{AA} will be determined using the method of virtual work. Because of symmetry of geometry *and* loading we need only three x coordinates. These and the internal moments are shown in Figs. 10–13c and 10–13d. It is important that each x coordinate be the *same* for both the real and virtual loadings. Also, the positive directions for **M** and **m** must be the *same*.

For Δ_A we require application of real loads, Fig. 10–13c, and a virtual unit load at A, Fig. 10–13d. Thus,

$$\Delta_A = \int_0^L \frac{Mm}{EI}dx = 2\int_0^5 \frac{(0)(1x_1)dx_1}{EI} + 2\int_0^5 \frac{(200x_2)(-5)dx_2}{EI}$$
$$+ 2\int_0^5 \frac{(1000 + 200x_3 - 20x_3^2)(-5)dx_3}{EI}$$
$$= 0 - \frac{25\,000}{EI} - \frac{66\,666.7}{EI} = -\frac{91\,666.7}{EI}$$

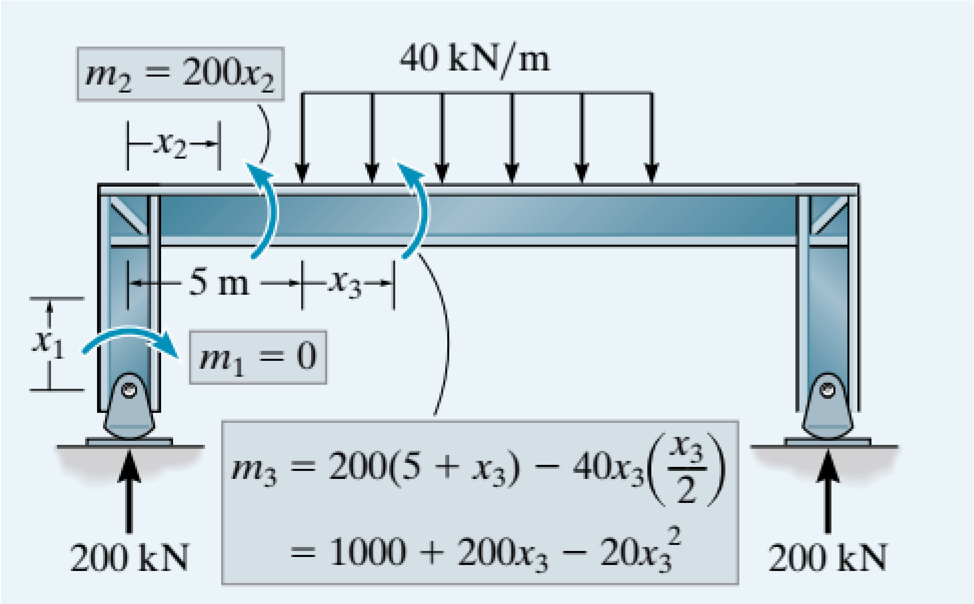
For f_{AA} we require application of a real unit load and a virtual unit load acting at A, Fig. 10–13d. Thus,

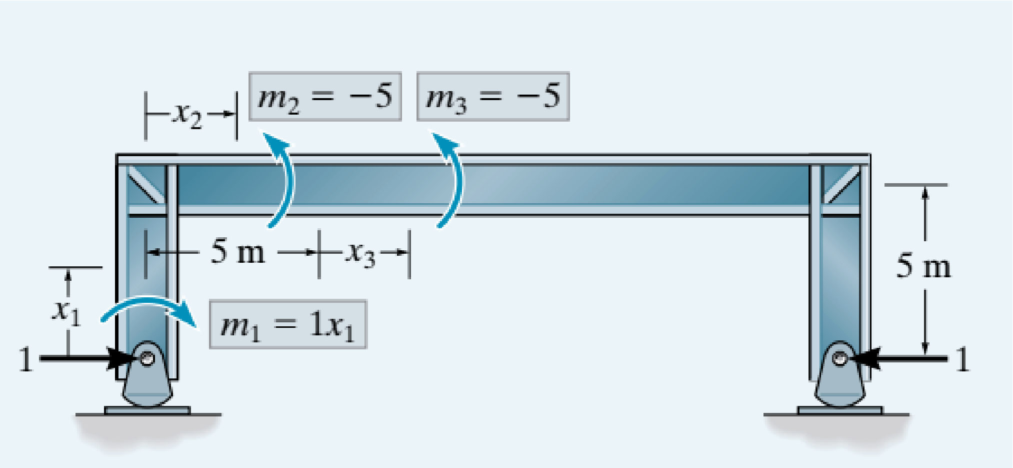
$$f_{AA} = \int_0^L \frac{mm}{EI}dx = 2\int_0^5 \frac{(1x_1)^2dx_1}{EI} + 2\int_0^5 (5)^2dx_2 + 2\int_0^5 (5)^2 d$$
$$= \frac{583.33}{EI}$$

Substituting the results into Eq. (1) and solving yields

$$0 = \frac{-91\,666.7}{EI} + A_x\left(\frac{583.33}{EI}\right)$$
$$A_x = 157 \text{ kN} \qquad \textit{Ans.}$$

Draw the free-body diagram for the bent and show that $A_y = B_y = 200$ kN, and $B_x = A_x = 157$ kN.





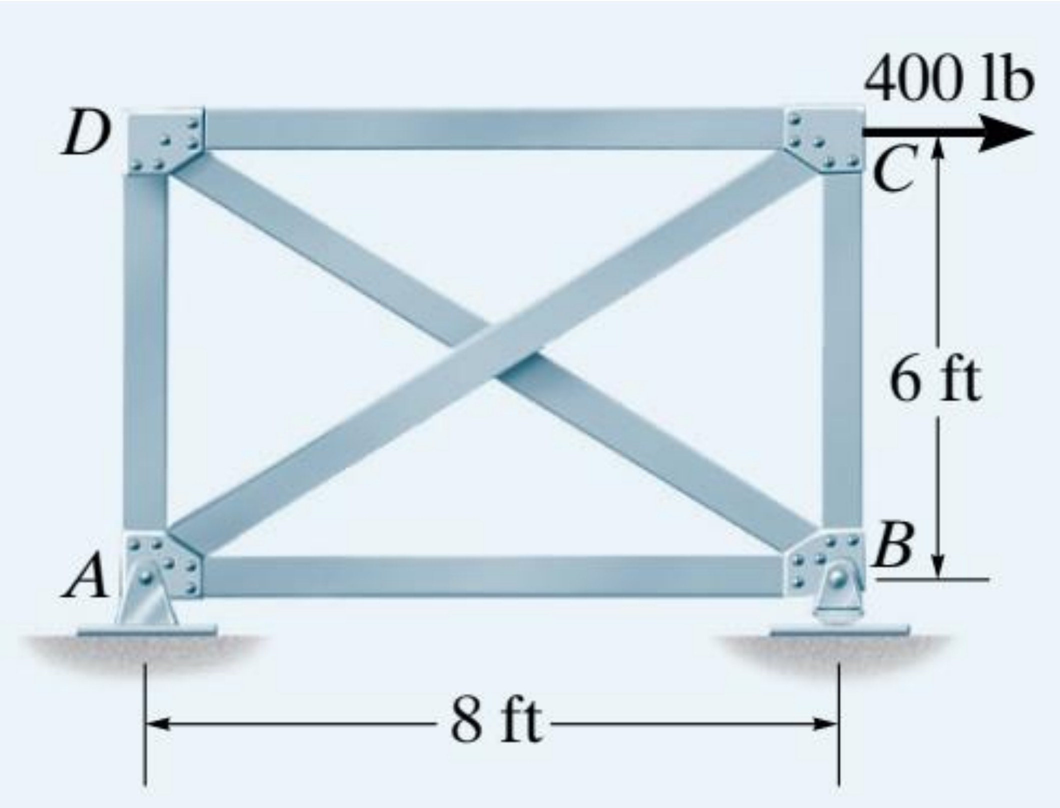
Force Method of Analysis: Trusses

- The degree of indeterminacy of a truss can usually be determined by inspection; however, if this becomes difficult, $b + r > 2j$.

- The force method is quite suitable for analyzing trusses that are statically indeterminate to the first or second degree.

Example 5

- Determine the force in member AC of the truss shown in the figure below. AE is the same for all the members



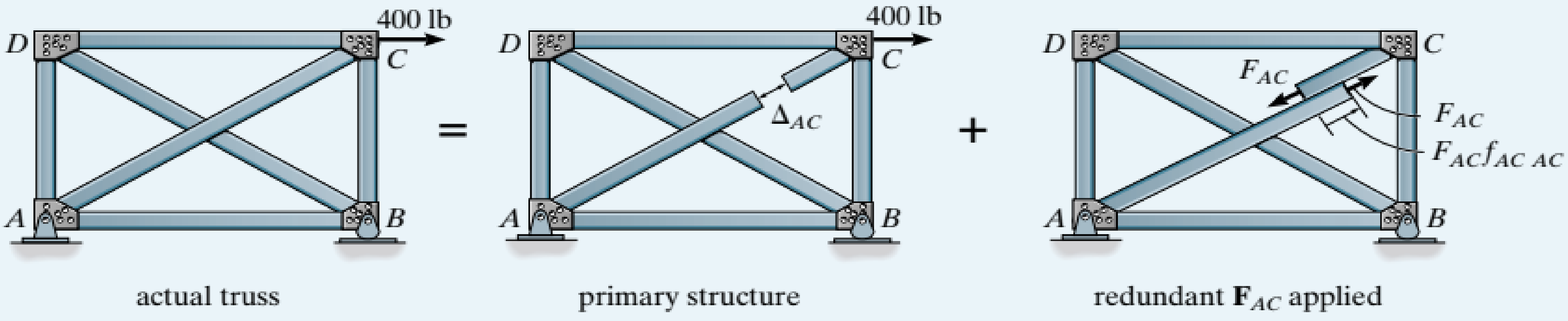
Solution 5

Principle of Superposition. By inspection the truss is indeterminate to the first degree.* Since the force in member AC is to be determined, member AC will be chosen as the redundant. This requires "cutting" this member so that it cannot sustain a force, thereby making the truss statically determinate and stable. The principle of superposition applied to the truss is shown in Fig. 10–15b.

*Applying Eq. 3–1, $b + r > 2j$ or $6 + 3 > 2(4)$, $9 > 8$, $9 - 8 = 1$st degree.

Compatibility Equation. With reference to member AC in Fig. 10–15b, we require the relative displacement Δ_{AC}, which occurs at the ends of the cut member AC due to the 400-lb load, plus the relative displacement $F_{AC} f_{AC\,AC}$ caused by the redundant force acting alone, to be equal to zero, that is,

$$0 = \Delta_{AC} + F_{AC} f_{AC\,AC}$$



Solution 5

Here the flexibility coefficient f_{ACAC} represents the relative displacement of the cut ends of member AC caused by a "real" unit load acting at the cut ends of member AC. This term, f_{ACAC}, and Δ_{AC} will be computed using the method of virtual work. The force analysis, using the method of joints, is summarized in Figs. 10–15*c* and 10–15*d*.

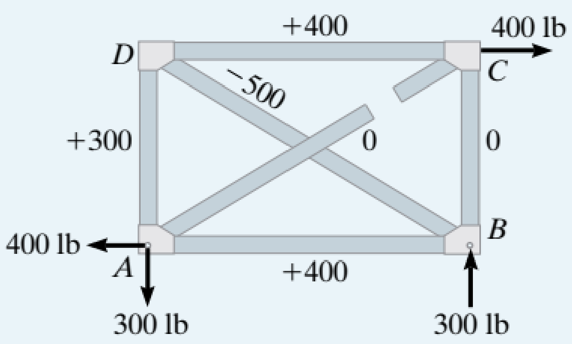
For Δ_{AC} we require application of the real load of 400 lb, Fig. 10–15*c*, and a virtual unit force acting at the cut ends of member AC, Fig. 10–15*d*. Thus,

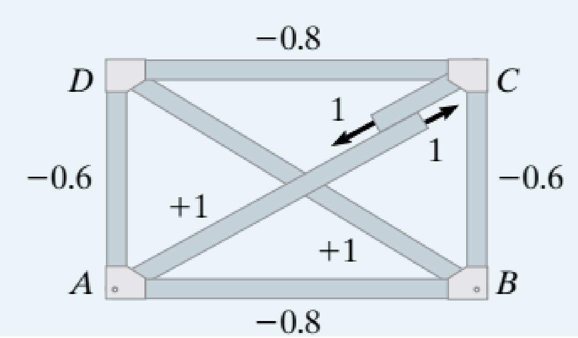
$$\Delta_{AC} = \sum \frac{nNL}{AE}$$

$$= 2\left[\frac{(-0.8)(400)(8)}{AE}\right] + \frac{(-0.6)(0)(6)}{AE} + \frac{(-0.6)(300)(6)}{AE}$$

$$+ \frac{(1)(-500)(10)}{AE} + \frac{(1)(0)(10)}{AE}$$

$$= -\frac{11\,200}{AE}$$





For f_{ACAC} we require application of real unit forces and virtual unit forces acting on the cut ends of member AC, Fig. 10–15*d*. Thus,

$$f_{ACAC} = \sum \frac{n^2 L}{AE}$$

$$= 2\left[\frac{(-0.8)^2(8)}{AE}\right] + 2\left[\frac{(-0.6)^2(6)}{AE}\right] + 2\left[\frac{(1)^2 10}{AE}\right]$$

$$= \frac{34.56}{AE}$$

Substituting the data into Eq. (1) and solving yields

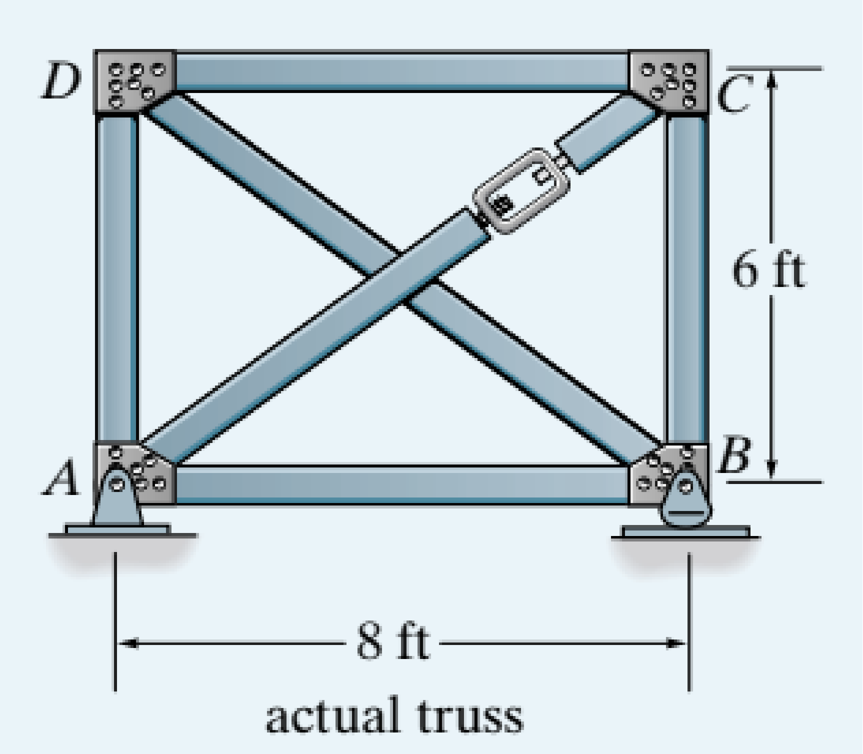
$$0 = -\frac{11\,200}{AE} + \frac{34.56}{AE} F_{AC}$$

$$F_{AC} = 324 \text{ lb (T)} \qquad \textit{Ans.}$$

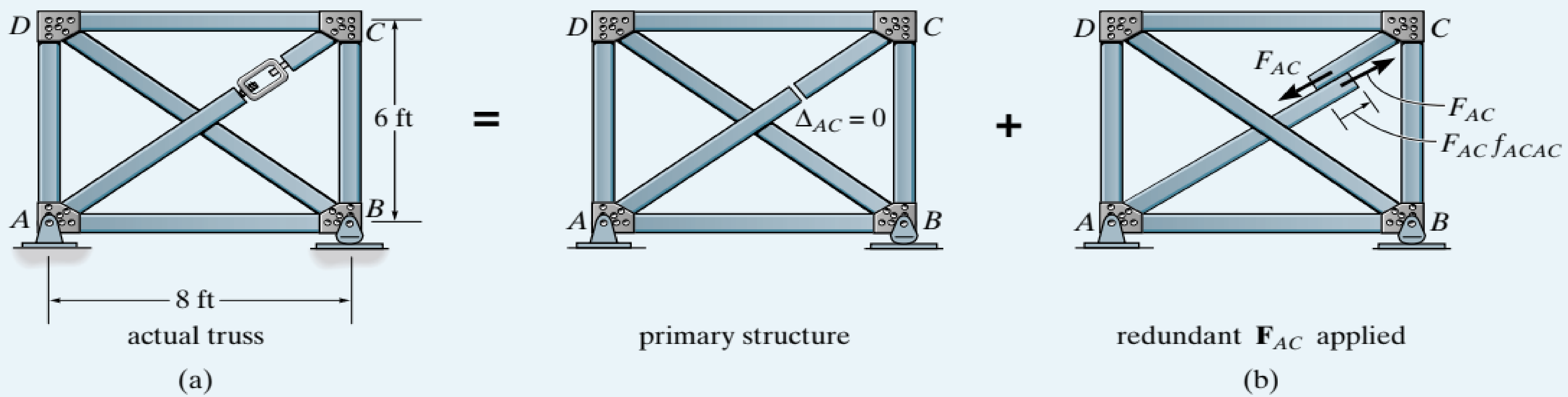
Since the numerical result is positive, AC is subjected to tension as assumed, Fig. 10–15*b*. Using this result, the forces in the other members can be found by equilibrium, using the method of joints.

Example 6

Determine the force in each member of the truss shown in the figure below. if the turnbuckle on member AC is used to shorten the member by 0.5 in. Each bar has a cross-sectional area of 0.2 in^2, and $E = 29(10^6)$ psi.



Solution 6



Principle of Superposition. This truss has the same geometry as that in Example 10.7. Since *AC* has been shortened, we will choose it as the redundant, Fig. 10–16*b*.

Compatibility Equation. Since no external loads act on the primary structure (truss), there will be no relative displacement between the ends of the sectioned member caused by load; that is, $\Delta_{AC} = 0$. The flexibility coefficient f_{ACAC} has been determined in Example 10.7, so

$$f_{ACAC} = \frac{34.56}{AE}$$

Assuming the amount by which the bar is shortened is positive, the compatibility equation for the bar is therefore

$$0.5 \text{ in.} = 0 + \frac{34.56}{AE} F_{AC}$$

Realizing that f_{ACAC} is a measure of displacement per unit force, we have

$$0.5 \text{ in.} = 0 + \frac{34.56 \text{ ft}(12 \text{ in./ft})}{(0.2 \text{ in}^2)\left[29(10^6) \text{ lb/in}^2\right]} F_{AC}$$

Thus,

$$F_{AC} = 6993 \text{ lb} = 6.99 \text{ k (T)} \qquad \textit{Ans.}$$

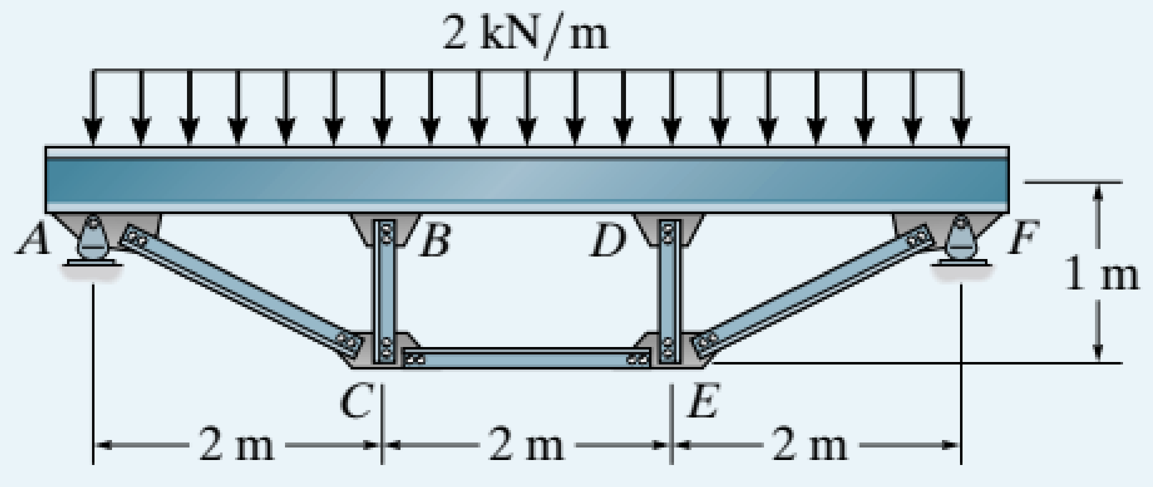
Since no external forces act on the truss, the external reactions are zero. Therefore, using F_{AC} and analyzing the truss by the method of joints yields the results shown in Fig. 10–16*c*.

Composite Structures

- Composite structures are composed of some members subjected only to axial force, while other members are subjected to bending.
- If the structure is statically indeterminate, the force method can conveniently be used for its analysis. The following example illustrates the procedure.

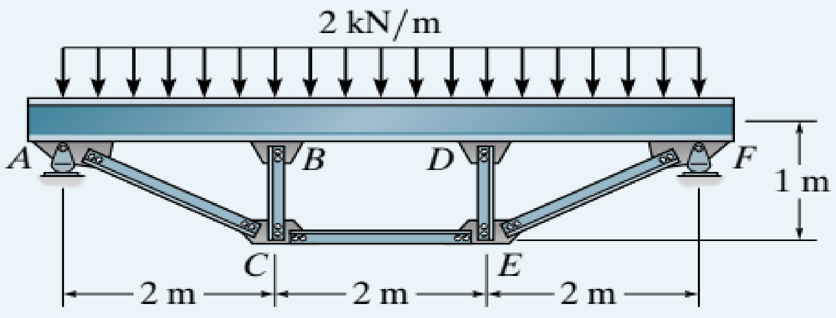
Example 7

The simply supported queen-post trussed beam shown in the photo is to be designed to support a uniform load of $2\ kN/m$. The dimensions of the structure are shown in the figure below. Determine the force developed in member CE. Neglect the thickness of the beam and assume the truss members are pin connected to the beam. Also, neglect the effect of axial compression and shear in the beam. The cross-sectional area of each strut is 400 mm2, and for the beam $I = 20(10^6)\ mm^4$. Take $E = 200\ GPa$.



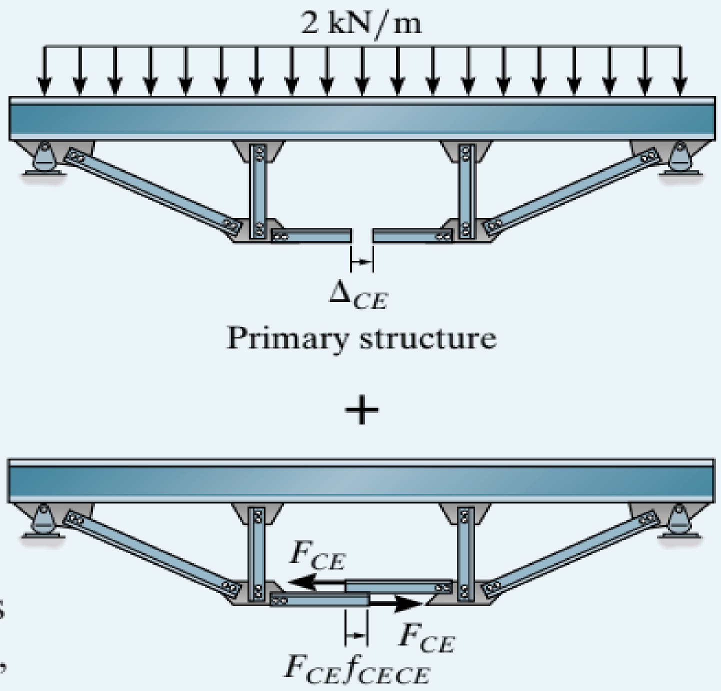
Actual structure

Solution 7



Actual structure

(a)



Redundant $\mathbf{F}_{CE}$ applied

(b)

Fig. 10–17

SOLUTION

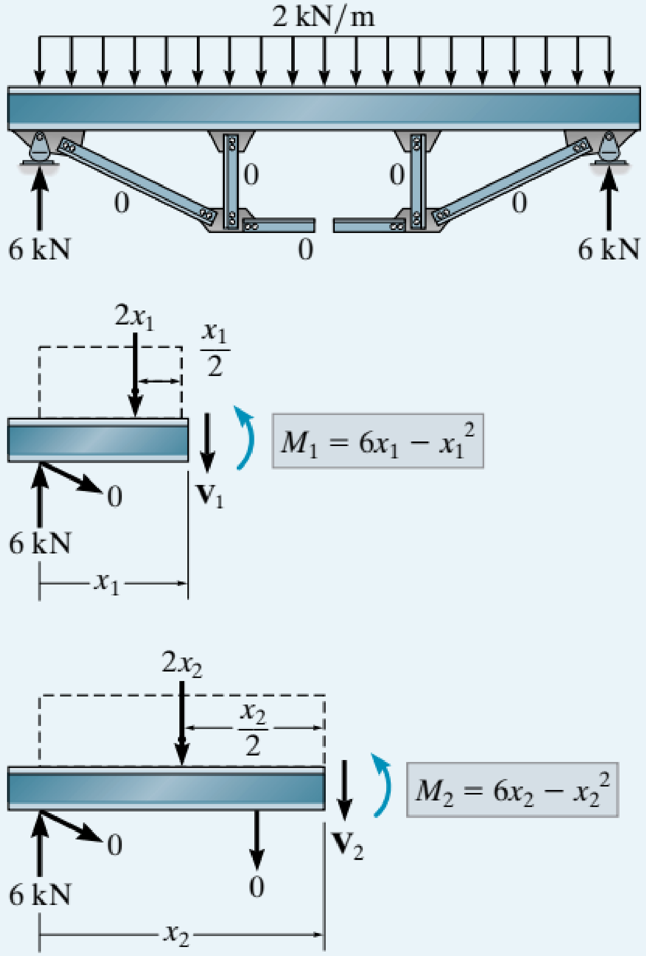
Principle of Superposition. If the force in one of the truss members is known, then the force in all the other members, as well as in the beam, can be determined by statics. Hence, the structure is indeterminate to the first degree. For solution the force in member *CE* is chosen as the redundant. This member is therefore sectioned to eliminate its capacity to sustain a force. The principle of superposition applied to the structure is shown in Fig. 10–17*b*.

Compatibility Equation. With reference to the relative displacement of the cut ends of member *CE*, Fig. 10–17*b*, we require

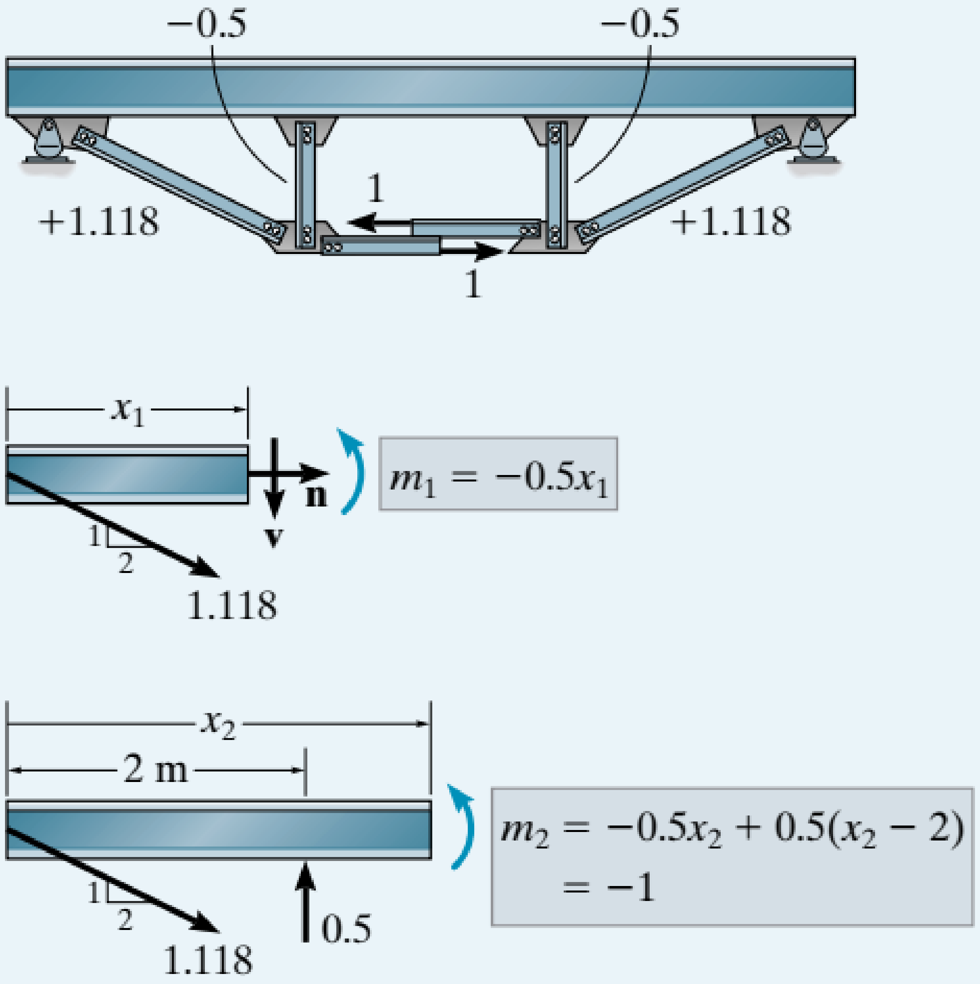
$$0 = \Delta_{CE} + F_{CE} f_{CE\,CE} \qquad (1)$$

Solution 7

The method of virtual work will be used to find Δ_{CE} and $f_{CE\,CE}$. The necessary force analysis is shown in Figs. 10–17*c* and 10–17*d*.



(c)



(d)

Solution 7

For Δ_{CE} we require application of the real loads, Fig. 10–17*c*, and a virtual unit load applied to the cut ends of member *CE*, Fig. 10–17*d*. Here we will use symmetry of *both* loading and geometry, and only consider the bending strain energy in the beam and, of course, the axial strain energy in the truss members. Thus,

$$\Delta_{CE} = \int_0^L \frac{Mm}{EI}dx + \sum \frac{nNL}{AE} = 2\int_0^2 \frac{(6x_1 - x_1^2)(-0.5x_1)dx_1}{EI}$$

$$+ 2\int_2^3 \frac{(6x_2 - x_2^2)(-1)dx_2}{EI} + 2\left(\frac{(1.118)(0)(\sqrt{5})}{AE}\right)$$

$$+ 2\left(\frac{(-0.5)(0)(1)}{AE}\right) + \left(\frac{1(0)2}{AE}\right)$$

$$= -\frac{12}{EI} - \frac{17.33}{EI} + 0 + 0 + 0$$

$$= \frac{-29.33(10^3)}{200(10^9)(20)(10^{-6})} = -7.333(10^{-3})\text{ m}$$

For $f_{CE\,CE}$ we require application of a real unit load and a virtual unit load at the cut ends of member *CE*, Fig. 10–17*d*. Thus,

$$f_{CE\,CE} = \int_0^L \frac{m^2dx}{EI} + \sum \frac{n^2L}{AE} = 2\int_0^2 \frac{(-0.5x_1)^2dx_1}{EI} + 2\int_2^3 \frac{(-1)^2dx_2}{EI}$$

$$+ 2\left(\frac{(1.118)^2(\sqrt{5})}{AE}\right) + 2\left(\frac{(-0.5)^2(1)}{AE}\right) + \left(\frac{(1)^2(2)}{AE}\right)$$

$$= \frac{1.3333}{EI} + \frac{2}{EI} + \frac{5.590}{AE} + \frac{0.5}{AE} + \frac{2}{AE}$$

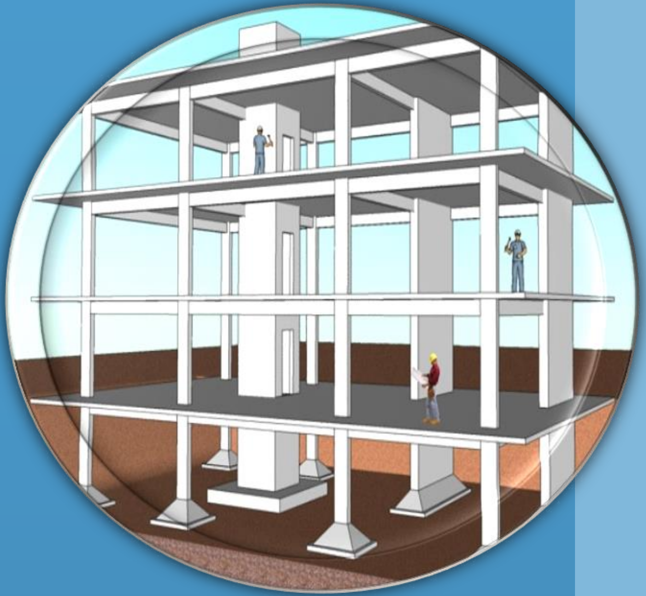
$$= \frac{3.333(10^3)}{200(10^9)(20)(10^{-6})} + \frac{8.090(10^3)}{400(10^{-6})(200(10^9))}$$

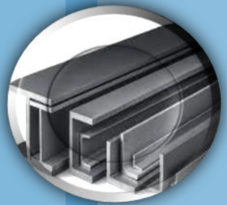
$$= 0.9345(10^{-3})\text{ m/kN}$$

Substituting the data into Eq. (1) yields

$$0 = -7.333(10^{-3})\text{ m} + F_{CE}(0.9345(10^{-3})\text{ m/kN})$$

$$F_{CE} = 7.85\text{ kN}$$ *Ans.*





Thank You!