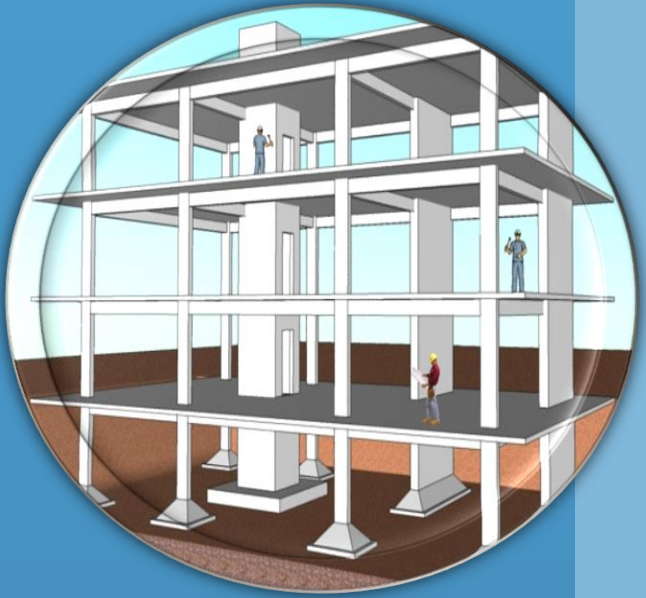
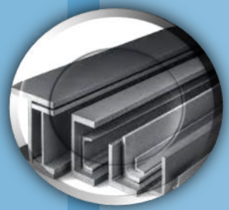
UNIVERSITY OF ZAMBIA
School of Engineering
Department of Civil & Environmental Eng.



CEE 3222: THEORY OF STRUCTURES

Lecture 5.1

INTRODUCTION TO MATRIX METHODS – FLEXIBILITY METHOD



Mr. MWABA MSc, B.Eng., R.Eng., PEIZ,

Contents

Introduction

- There are essentially two ways in which structures can be analyzed using matrix methods.
  - The flexibility method – which is a force method of analysis can also be used to analyze structures.
  - The stiffness method – which is a displacement method of analysis.

- The Stiffness method is more popular for several reasons
  - Analyze both statically determinate and indeterminate structures without much modifications
  - Yields the displacements and forces directly.
  - It is generally easier

Matrix Force Method-flexibility Method

- The method, which was introduced by James C. Maxwell in 1864
- The systematic development of consistent deformation method in the matrix form has lead to flexibility matrix method.
- The method is also called force method. Since the basic unknowns are the redundant forces in the structure.
- This method is exactly opposite to stiffness matrix method. The flexibility matrix equation is given by

$$[P]\,[F] = \{[\Delta] - [\Delta_L]\}$$

$$[P] = [F]^{-1}\{[\Delta] - [\Delta_L]\}$$

Where,

$[P]$ = Redundant in matrix form

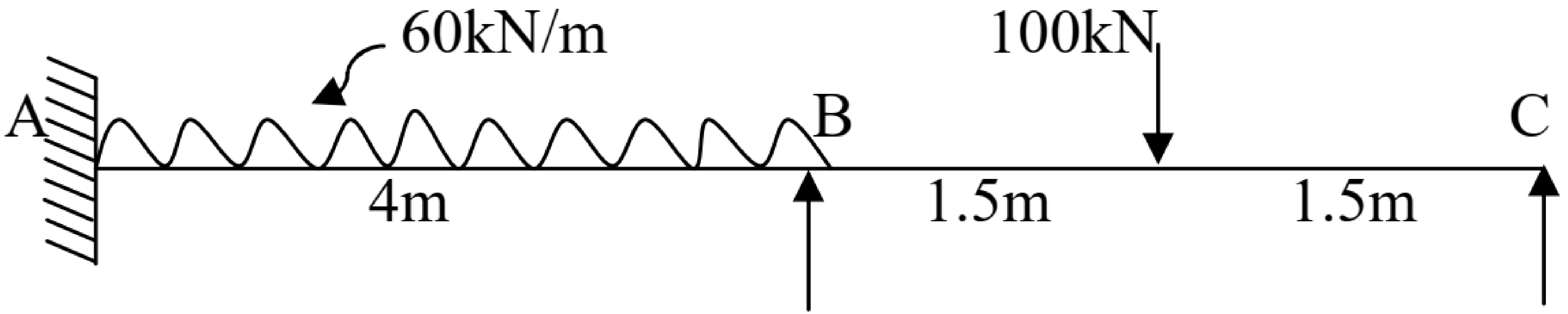
$[F]$ = Flexibility matrix

$[\Delta]$ = Displacement at supports

$[\Delta_L]$= Displacement due to load

Matrix Force Method-flexibility Method

- Analyze the continuous beam shown in the figure by flexibility matrix method, draw BMD

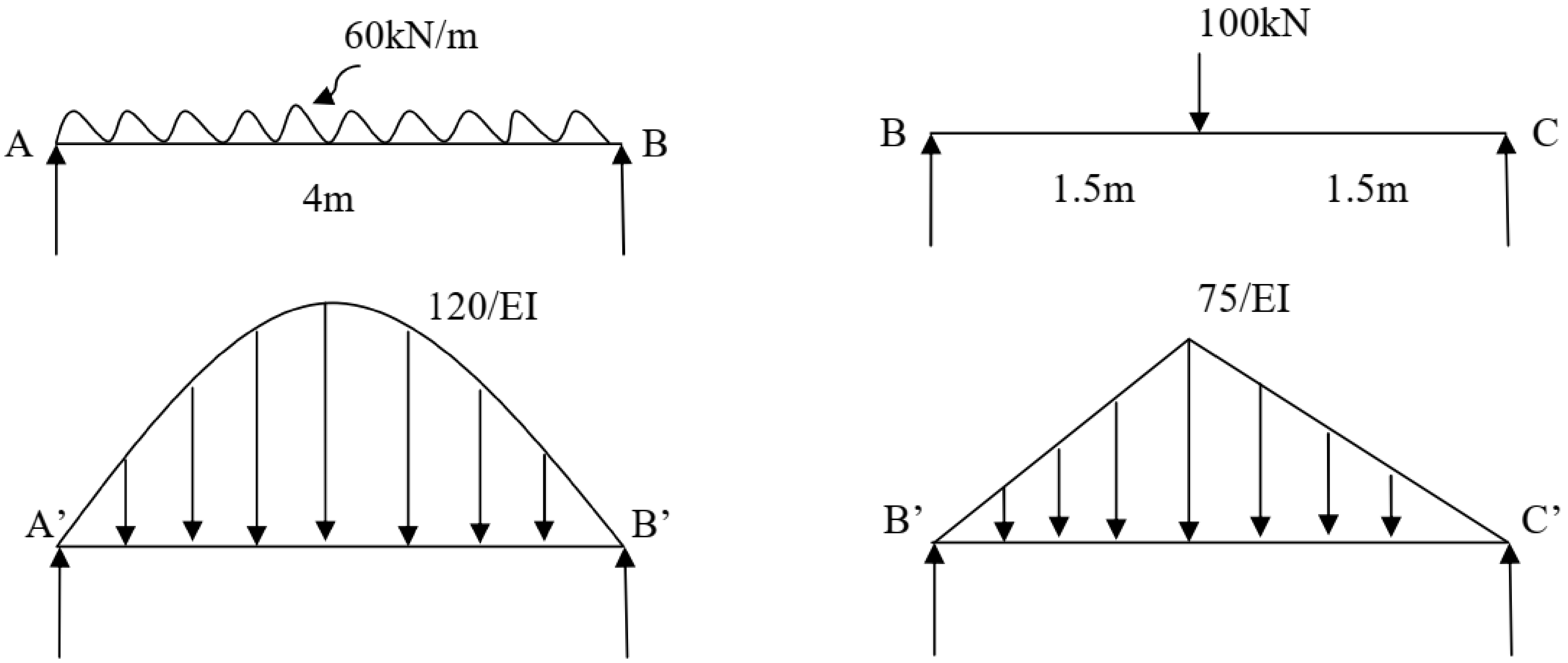


Static Indeterminacy SI = 2 (M_A and M_B)

M_A and M_B are the redundant

Matrix Force Method-flexibility Method

- Let us remove the redundant to get primary determinate structure



Matrix Force Method-flexibility Method

$$[\Delta_L] = \begin{bmatrix} \Delta_{1L} \\ \Delta_{2L} \end{bmatrix}$$

Δ_{1L} = Rotation at A = SF at A'

$\Delta_{1L} = ½\ [2/3 \text{ X } 4 \text{ X } \frac{120}{EI}]$

$\Delta_{1L} = \frac{160}{EI}$

Δ_{2L} = Rotation at A = SF at B'

$= V_{B1}' + V_{B2}'$

$\Delta_{2L} = ½\ [2/3 \text{ X } 4 \text{ X } \frac{120}{EI}] + ½\ [1/2 \text{ X } 3 \text{ X } \frac{75}{EI}]$

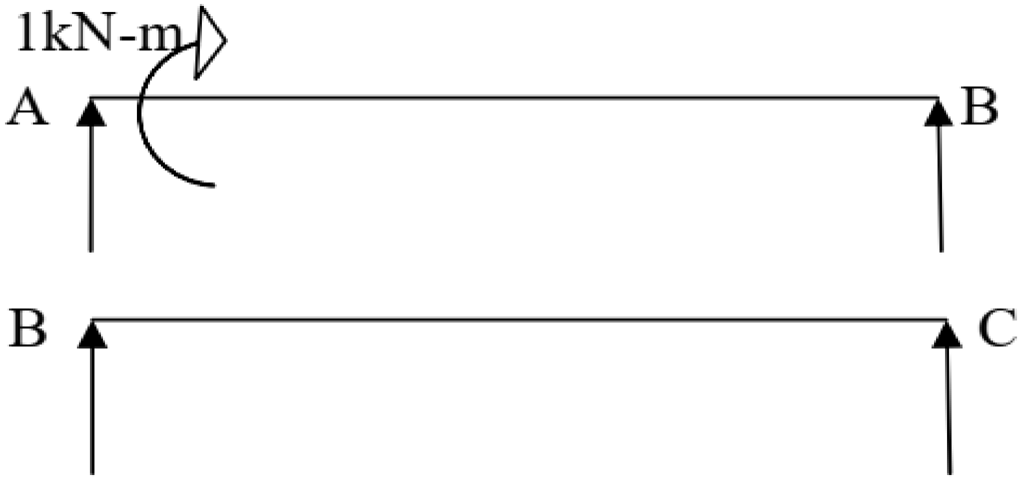
$\Delta_{2L} = \frac{216.25}{EI}$

$$[\Delta_L] = \frac{1}{EI}\begin{bmatrix} 160 \\ 216.25 \end{bmatrix}$$

- Note: The rotation due to sagging is taken as positive. The moments producing due to sagging are also taken as positive.

To get Flexibility Matrix

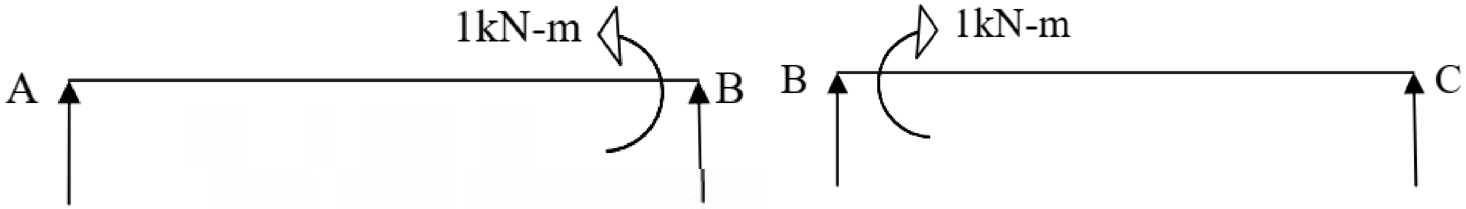
Apply unit moment to joint A



Matrix Force Method-flexibility Method

$$[F] = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix}$$

Apply unit moment at joint B



$$\delta_{11} = \frac{ml}{3EI} = \frac{1 X 4}{3EI} = \frac{1.33}{EI}$$

$$\delta_{12} = \frac{ml}{6EI} = \frac{1 X 4}{6EI} = \frac{0.67}{EI}$$

$$\delta_{21} = \frac{ml}{6EI} = \frac{1 X 4}{6EI} = \frac{0.67}{EI}$$

$$\delta_{22} = \frac{ml}{3EI} + \frac{ml}{3EI} = \frac{1 X 4}{3EI} + \frac{1 X 3}{EI} = \frac{2.33}{EI}$$

$$[F] = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix} = \frac{1}{EI}\begin{pmatrix} 1.33 & 0.67 \\ 0.67 & 1.33 \end{pmatrix}$$

Matrix Force Method-flexibility Method

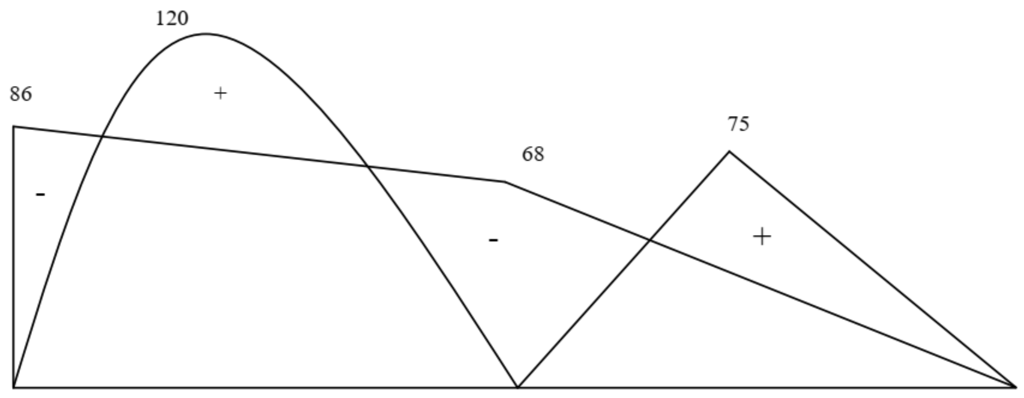
Apply the flexibility equation

$$[P] = [F]^{-1}\{[\Delta] - [\Delta_L]\}$$

$$[\Delta] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$[P] = EI\begin{pmatrix} 1.33 & 0.67 \\ 0.67 & 1.33 \end{pmatrix}^{-1} \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \frac{1}{EI}\begin{bmatrix} 160 \\ 216.25 \end{bmatrix} \right\}$$

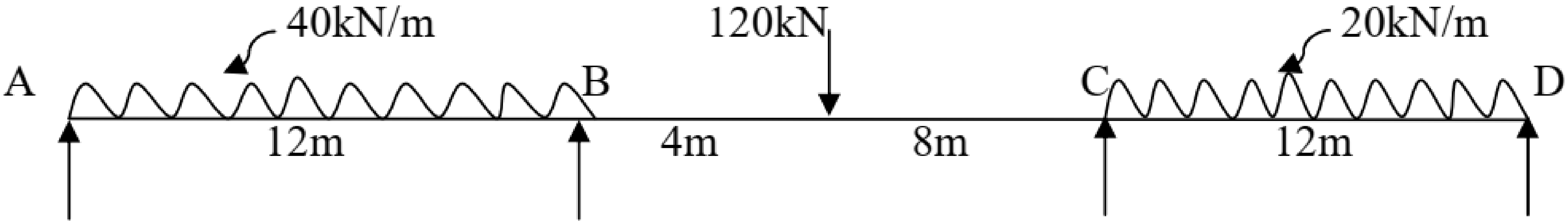
$$[P] = \begin{bmatrix} M_{AB} \\ M_{BA} \end{bmatrix} = \begin{bmatrix} -86.00 \\ -68.08 \end{bmatrix} \text{kN-m}$$



BMD

Matrix Force Method-flexibility Method

- Analyze the continuous beam shown in the figure by flexibility matrix method, draw BMD

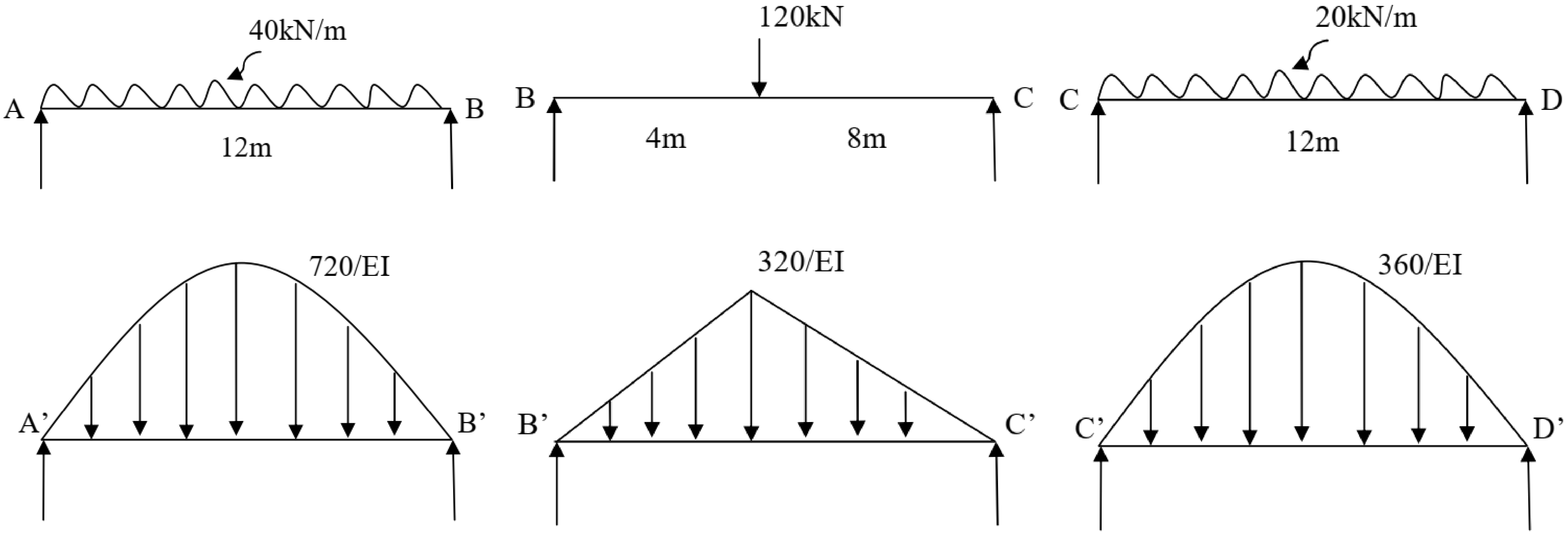


Static Indeterminacy SI = 2 (M_B and M_C)

M_B and M_C are the redundant

Matrix Force Method-flexibility Method

- Let us remove the redundant to get primary determinate structure



Matrix Force Method-flexibility Method

$$[\Delta_L] = \begin{bmatrix} \Delta_{1L} \\ \Delta_{2L} \end{bmatrix}$$

Δ_{1L} = Rotation at B = SF at B'

$= V_{B1}' + V_{B2}'$

$$\Delta_{1L} = \frac{3946.67}{EI}$$

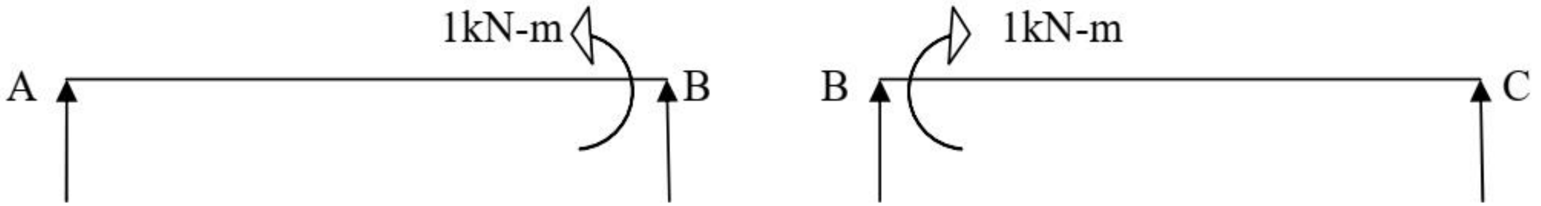
Δ_{2L} = Rotation at C = SF at C'

$= V_{C1}' + V_{C2}'$

$$\Delta_{2L} = \frac{2293.33}{EI}$$

$$[\Delta_L] = \frac{1}{EI}\begin{bmatrix} 3946.67 \\ 2293.33 \end{bmatrix}$$

<u>To get Flexibility Matrix</u> Apply unit moment at joint B



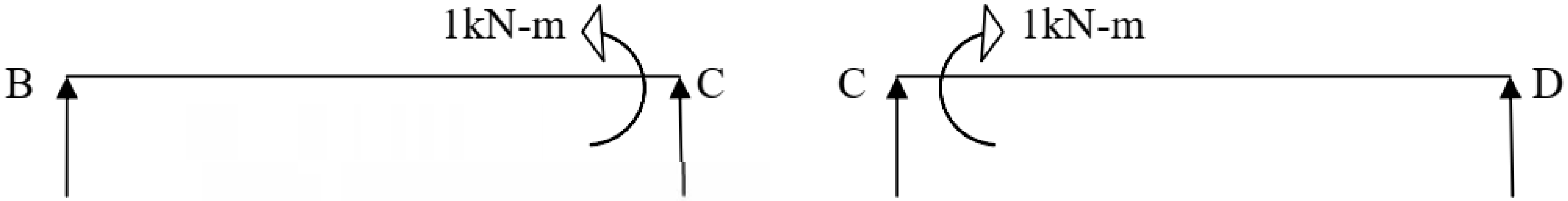
$$[F] = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}$$

$$\delta_{11} = \frac{ml}{3EI} + \frac{ml}{3EI} = \frac{1\ X\ 12}{3EI} + \frac{1\ X\ 12}{3EI} = \frac{8}{EI}$$

$$\delta_{21} = \frac{ml}{6EI} = \frac{1\ X\ 12}{6EI} = \frac{2}{EI}$$

Matrix Force Method-flexibility Method

To get Flexibility Matrix   Apply unit moment at joint C



$$\delta_{12} = \frac{ml}{6EI} = \frac{1\ X\ 12}{6EI} = \frac{2}{EI}$$

$$\delta_{22} = \frac{ml}{3EI} + \frac{ml}{3EI} = \frac{1\ X\ 12}{3EI} + \frac{1\ X\ 12}{EI} = \frac{8}{EI}$$

$$[F] = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix} = \frac{1}{EI}\begin{pmatrix} 8 & 2 \\ 2 & 8 \end{pmatrix}$$

Matrix Force Method-flexibility Method

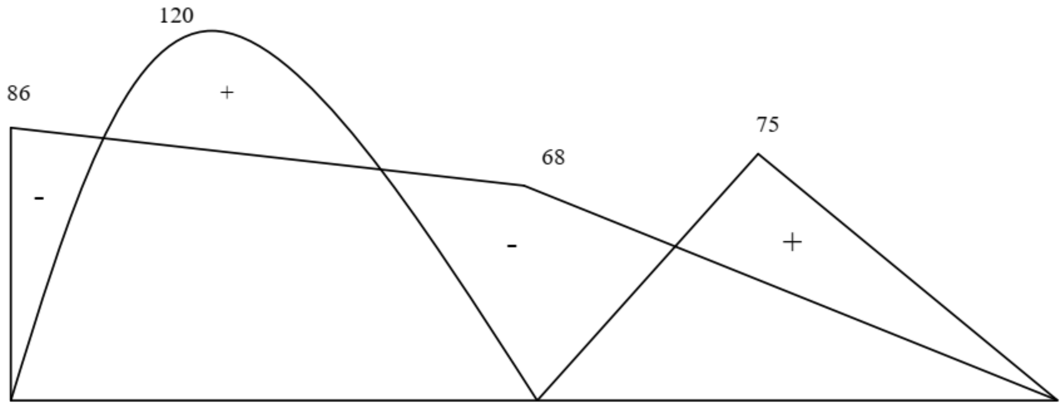
Apply the flexibility equation

$$[P] = [F]^{-1}\{[\Delta] - [\Delta_L]\}$$

$$[\Delta] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$[P] = EI\begin{pmatrix} 8 & 2 \\ 2 & 8 \end{pmatrix}^{-1}\left\{\begin{bmatrix} 0 \\ 0 \end{bmatrix} - \frac{1}{EI}\begin{bmatrix} 3946 \\ 2293 \end{bmatrix}\right\}$$

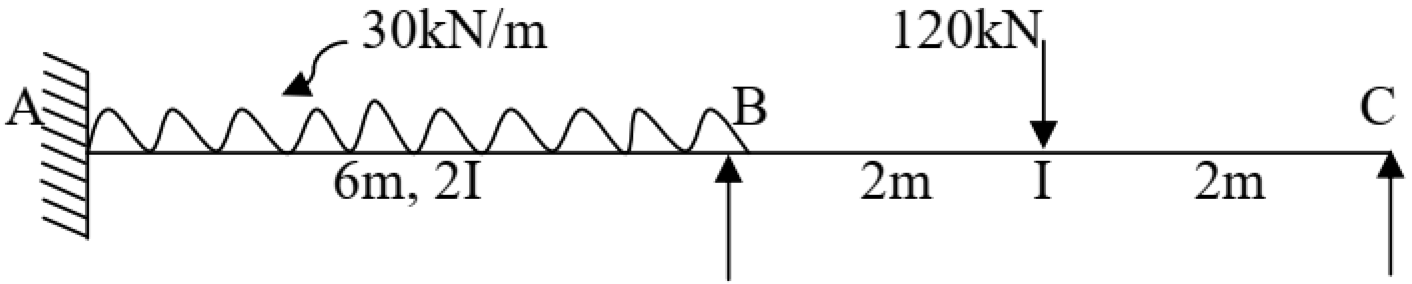
$$[P] = \begin{bmatrix} M_{AB} \\ M_{BA} \end{bmatrix} = \begin{bmatrix} -449.97 \\ -174.22 \end{bmatrix} \text{kN-m}$$



BMD

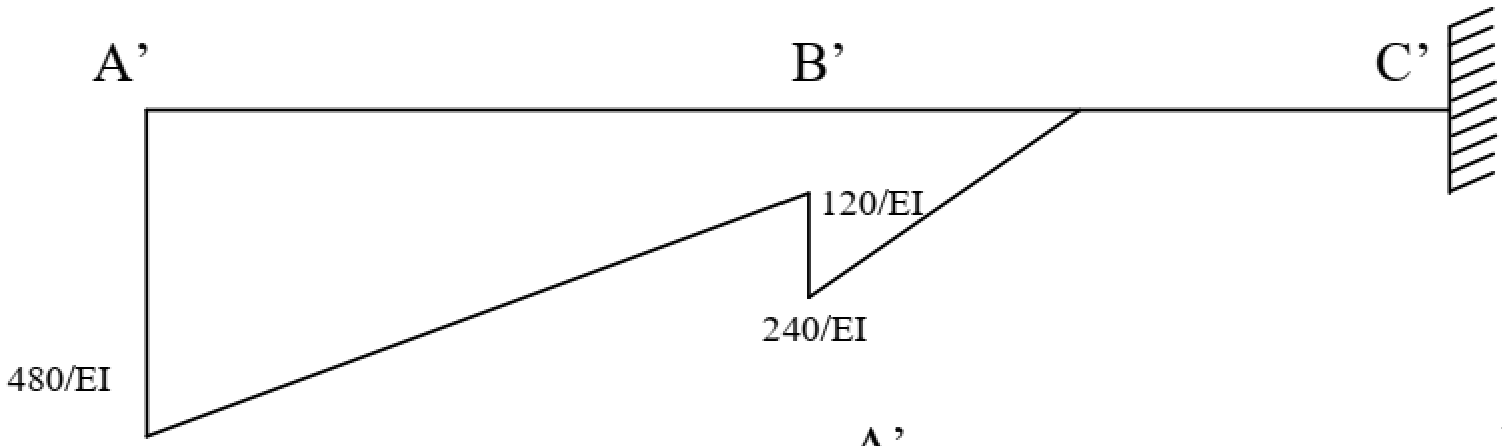
SINKING OF SUPPORT

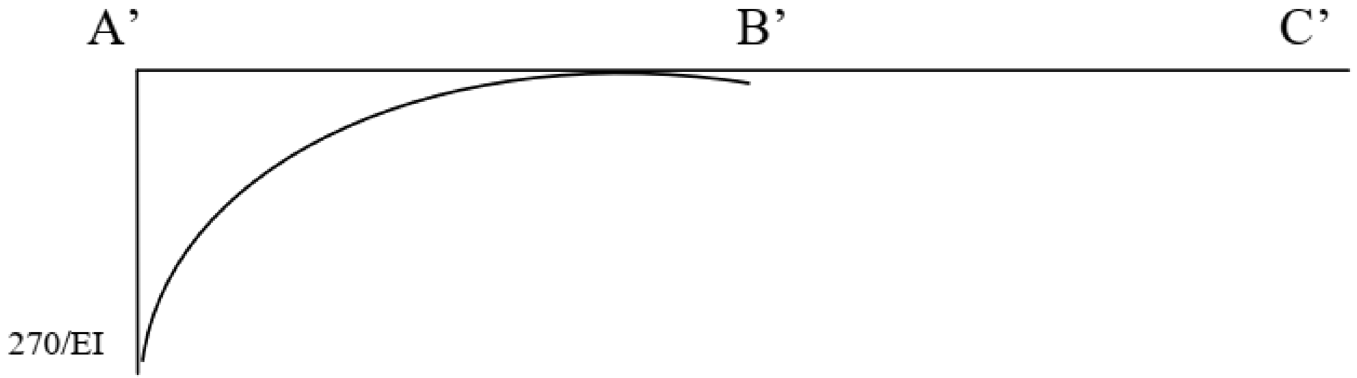
1. Analyse the continuous beam by flexibility method, support B sinks by 5mm. Sketch the BMD and EC given $EI = 15 \times 10^3$ kN-m^2



NOTE: In this case of example with sinking of supports, the redundant should be selected as the vertical reaction.
Static indeterminacy is equal to 2. Let V_B and V_C be the redundant, remove the redundant to get the primary structure.

Matrix Force Method-flexibility Method





$$[\Delta_L] = \begin{bmatrix} \Delta_{1L} \\ \Delta_{2L} \end{bmatrix}$$

Matrix Force Method-flexibility Method

Δ_{1L} = Displacement at B in primary determinate structure = BM at B' in conjugate beam

$$\Delta_{1L} = [\,\tfrac{1}{2} \times 6 \times \frac{360}{EI} \times (2/3 \times 6)\,] + (6 \times \frac{120}{EI} \times 6/2) + [\,\tfrac{1}{3} \times 6 \times \frac{270}{EI} \times (3/4 \times 6)\,]$$

$$\Delta_{1L} = \frac{8910}{EI}$$

Δ_{2L} = Displacement at C in primary determinate structure = BM at C' in conjugate beam

$$\Delta_{2L} = [\,\tfrac{1}{2} \times 6 \times \frac{360}{EI} \times (2/3 \times 6 + 4)\,] + (6 \times \frac{120}{EI} \times 6/2 + 4) + [\,\tfrac{1}{3} \times 6 \times \frac{270}{EI} \times (3/4 \times 6 + 4)\,]$$

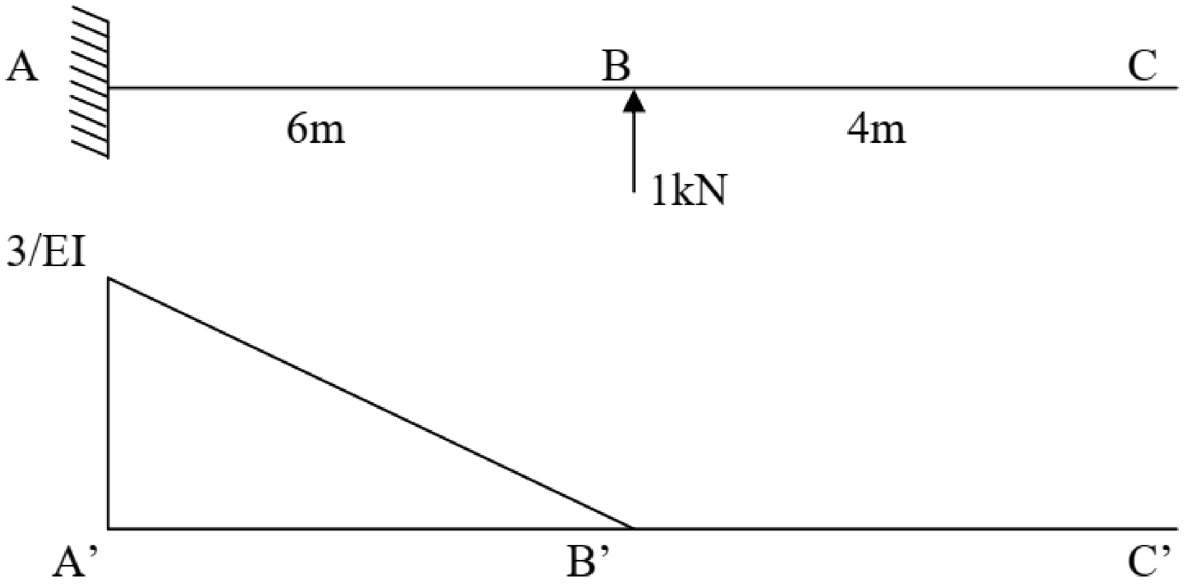
$$\Delta_{2L} = \frac{19070}{EI}$$

$$[\Delta_L] = \frac{1}{EI}\begin{bmatrix} 8910 \\ 19070 \end{bmatrix}$$

Matrix Force Method-flexibility Method

<u>To get Flexibility Matrix</u>

Apply unit Load at B



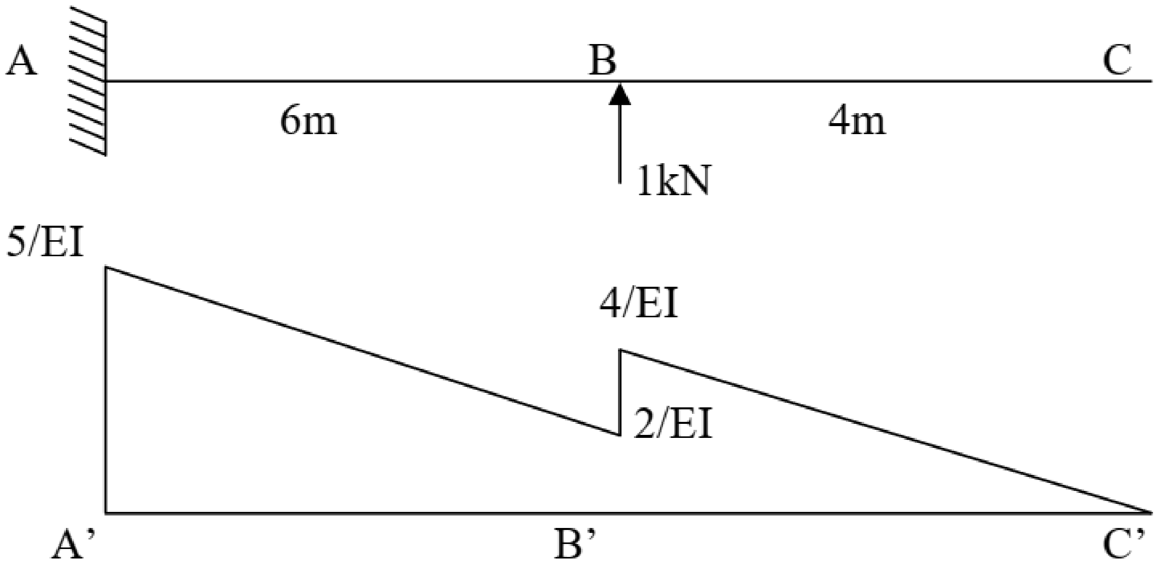
$$[F] = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix}$$

$$\delta_{11} = -\frac{1}{2} X\ 6\ X \frac{3}{EI} X\ (2/3\ X\ 6) = \frac{-36}{EI}$$

$$\delta_{21} = -\frac{1}{2} X\ 6\ X \frac{3}{EI} X\ (2/3\ X\ 6 + 4) = \frac{-72}{EI}$$

Matrix Force Method-flexibility Method

Apply unit load at C



$$[F] = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix} = \frac{1}{EI}\begin{pmatrix} -36 & -72 \\ -72 & -177.33 \end{pmatrix}$$

$$\delta_{12} = -\frac{1}{2} X\, 6\, X \frac{3}{EI} X\, (2/3\, X\, 6) - [6\, X \frac{2}{EI} X\, (6/2)] = \frac{-72}{EI}$$

$$\delta_{22} = -\frac{1}{2} X\, 6\, X \frac{3}{EI} X\, (2/3\, X\, 6 + 4) - [6\, X \frac{2}{EI} X\, (6/2 + 4)] - \frac{1}{2} X\, 4\, X \frac{4}{EI} X\, (2/3\, X\, 4) = \frac{-177.33}{EI}$$

Matrix Force Method-flexibility Method

Apply the flexibility equation

$$[P] = [F]^{-1}\{[\Delta] - [\Delta_L]\}$$

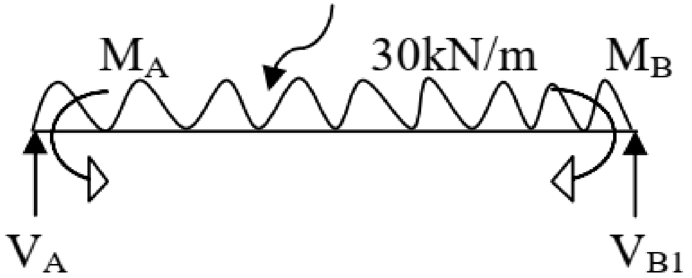
$$[\Delta] = \begin{bmatrix} 0.005 \\ 0 \end{bmatrix}$$

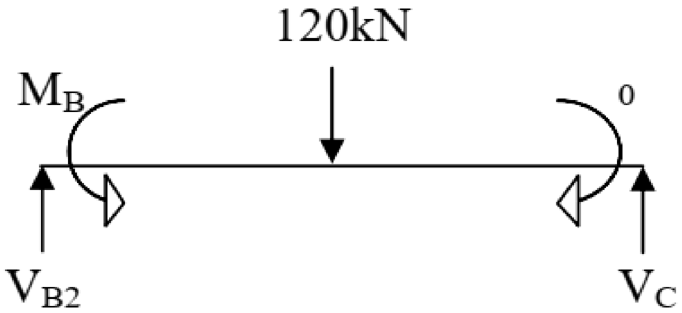
$$[P] = EI \begin{bmatrix} -36 & -72 \\ -72 & -177.33 \end{bmatrix}^{-1} \left\{ \begin{bmatrix} 0.005 \\ 0 \end{bmatrix} - \frac{1}{EI} \begin{bmatrix} 8910 \\ 19070 \end{bmatrix} \right\}$$

$$[P] = \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 161.43 \\ 41.98 \end{bmatrix} \text{kN-m}$$

Matrix Force Method-flexibility Method

Support Reaction

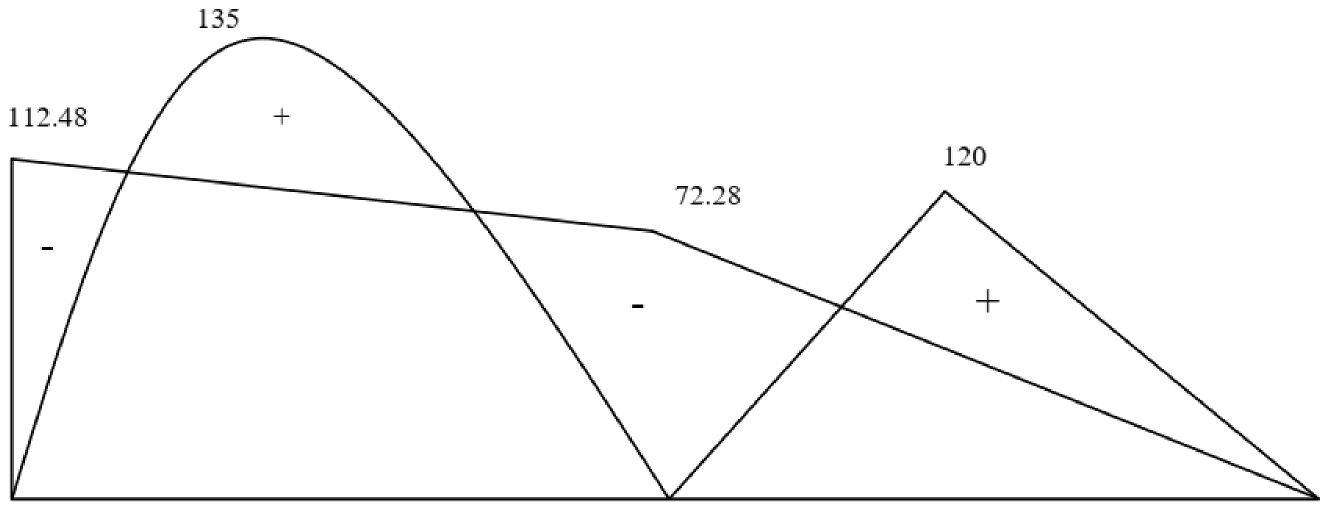




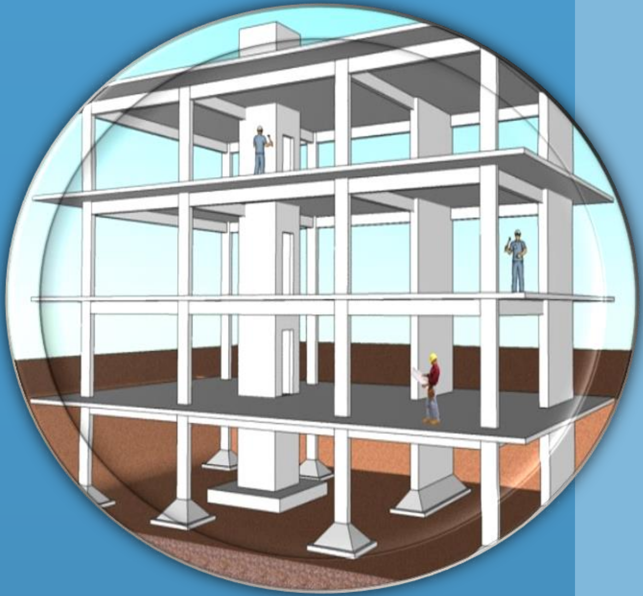
$V_A = 96.64kN$, $V_{B1} = 83.36kN$, $V_{B2} = 78.07kN$, $V_C = 41.98kN$

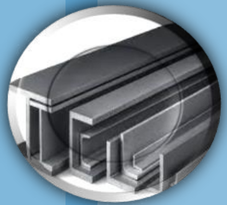
$V_B = V_{B1} + V_{B2} = 161.43kN$

$$\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \begin{bmatrix} 112.48 \\ 72.28 \end{bmatrix} \text{kN-m}$$



BMD





Thank You!