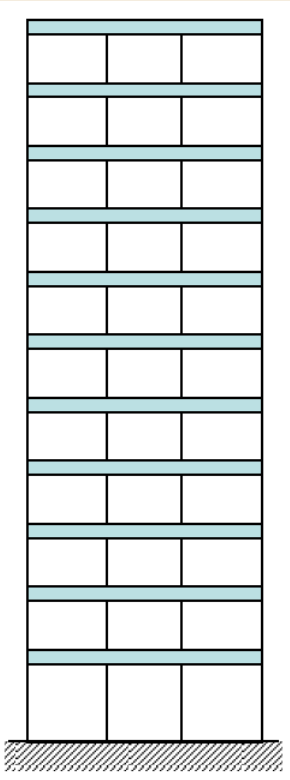
Chapter 2:
Analysis of Statically Determinate Structures

CEE 3222 THEORY OF STRUCTURES

Dr. Lenganji Simwanda

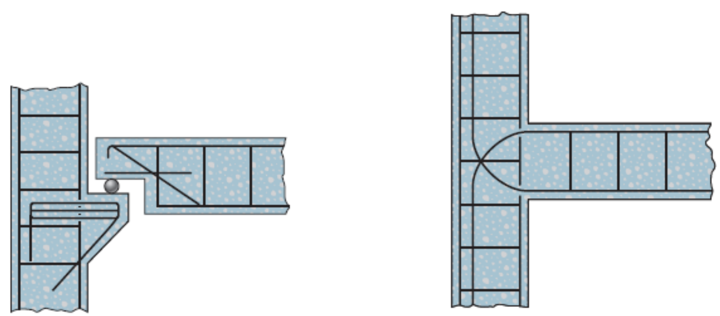
Idealized Structure

- To model or idealize a structure so that structural analysis can be performed



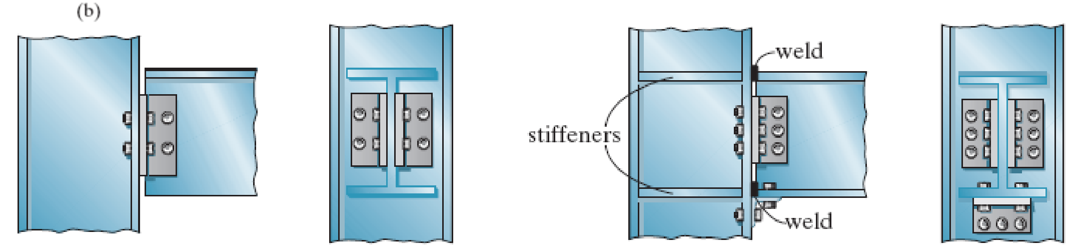
Idealized Structure

- Support Connections
  - Pin connection (allows slight rotation)
  - Roller support (allows slight rotation/translation)
  - Fixed joint (allows no rotation/translation)



typical "roller-supported" connection (concrete)
(a)

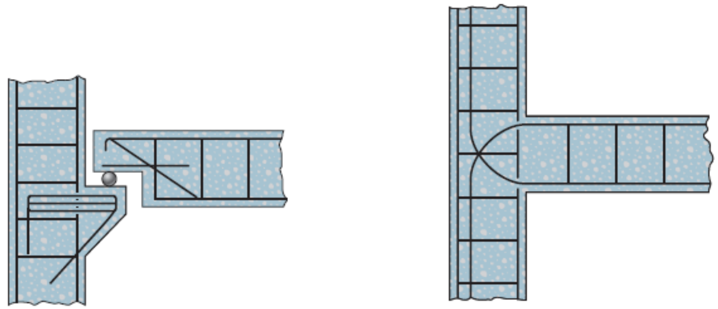
typical "fixed-supported" connection (concrete)
(b)



typical "pin-supported" connection (metal)
(a)

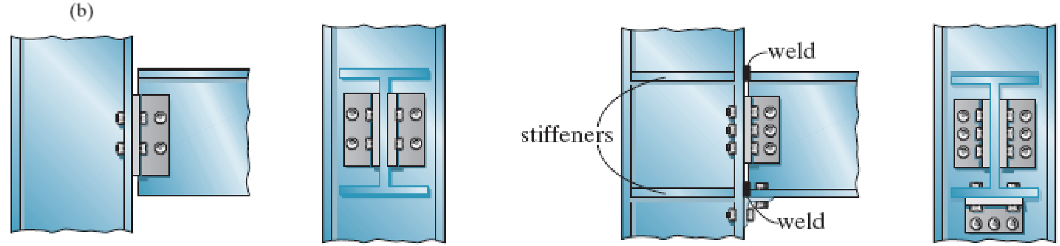
typical "fixed-supported" connection (metal)
(b)

Idealized Structure



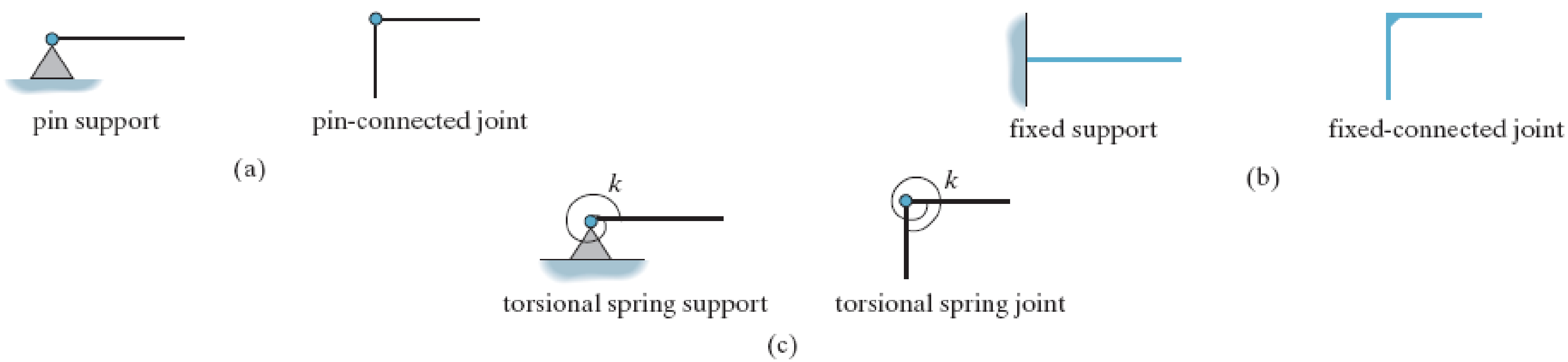
typical "roller-supported" connection (concrete)
(a)

typical "fixed-supported" connection (concrete)
(b)



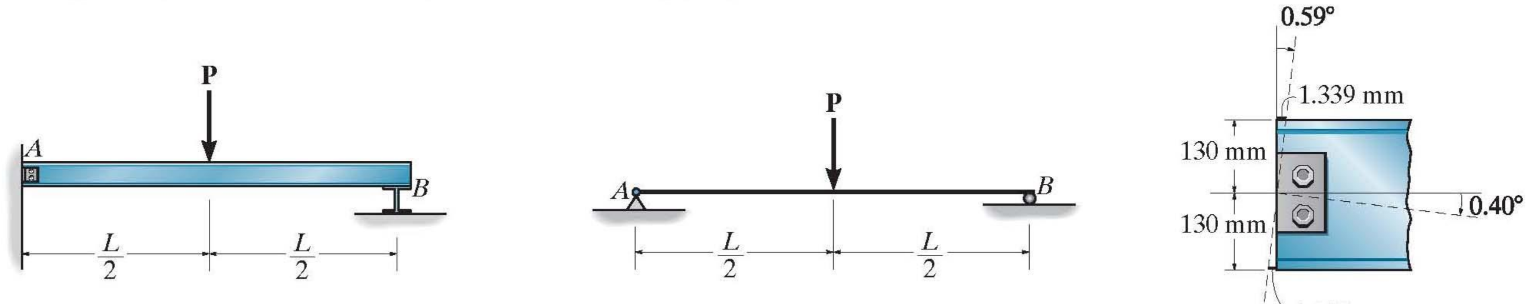
typical "pin-supported" connection (metal)
(a)

typical "fixed-supported" connection (metal)
(b)



Idealized Structure

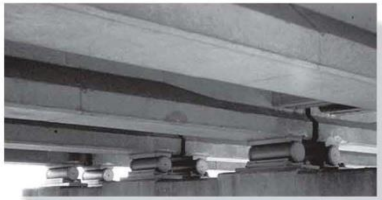
- In reality, all connections and supports are modeled with assumptions. Need to be aware how the assumptions will affect the actual performance



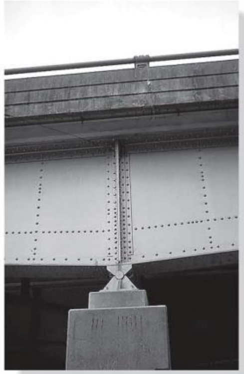
actual beam
(a)

idealized beam
(b)

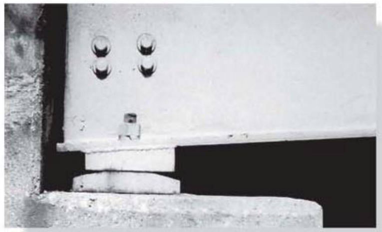
(c)



Rollers and associated bearing pads are used to support the prestressed concrete girders of a highway bridge.

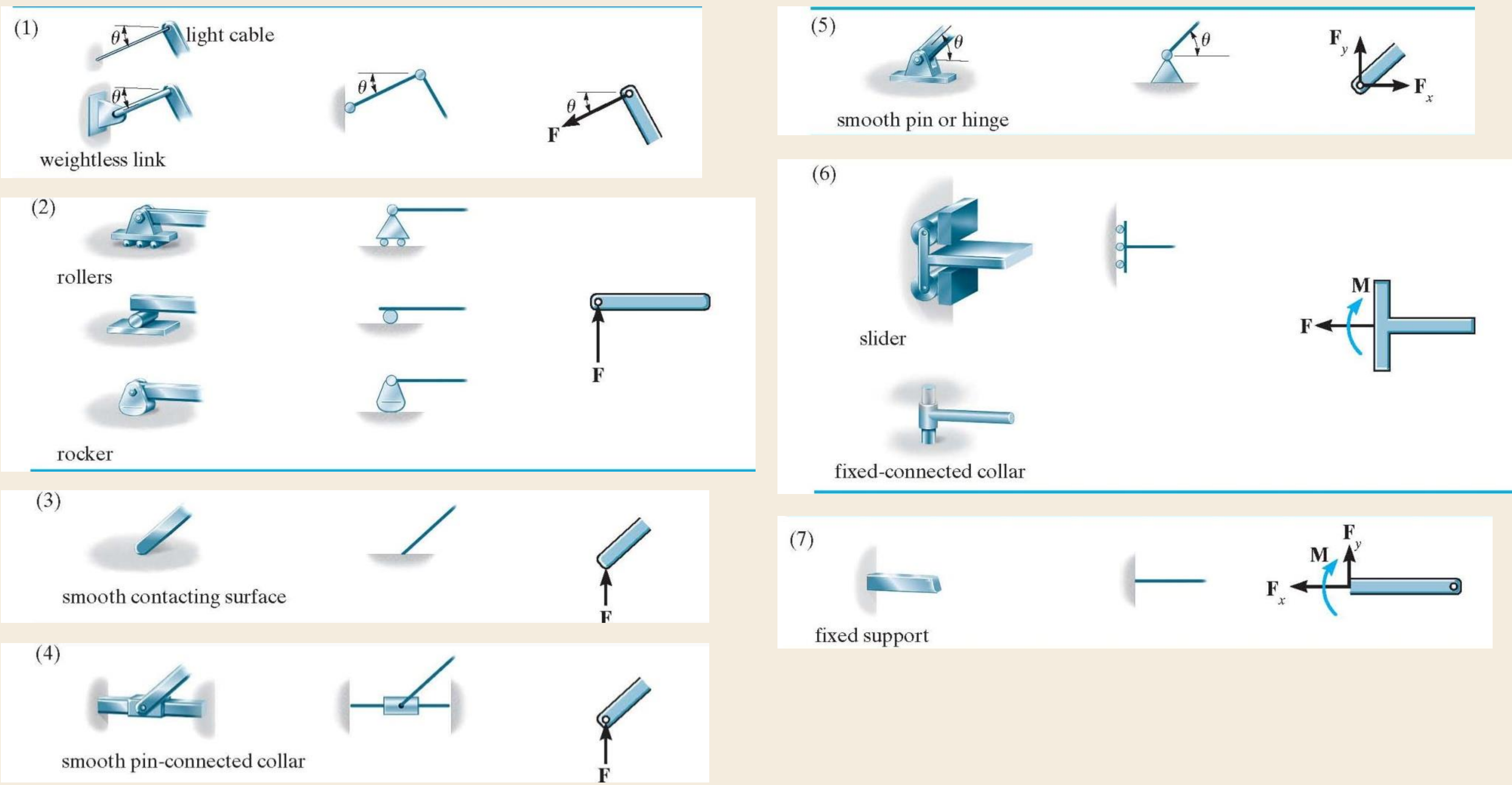


Typical pin used to support the steel girder of a railroad bridge.



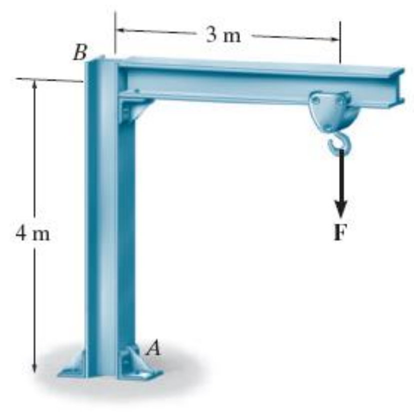
A typical rocker support used for a bridge girder.

Idealized Structure



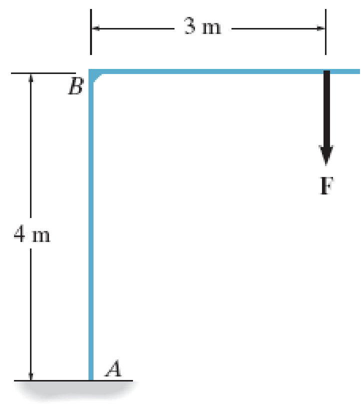
Idealized Structure

- Idealized Structure
  - No thickness for the components
  - The support at A can be modeled as a fixed support



actual structure

(a)

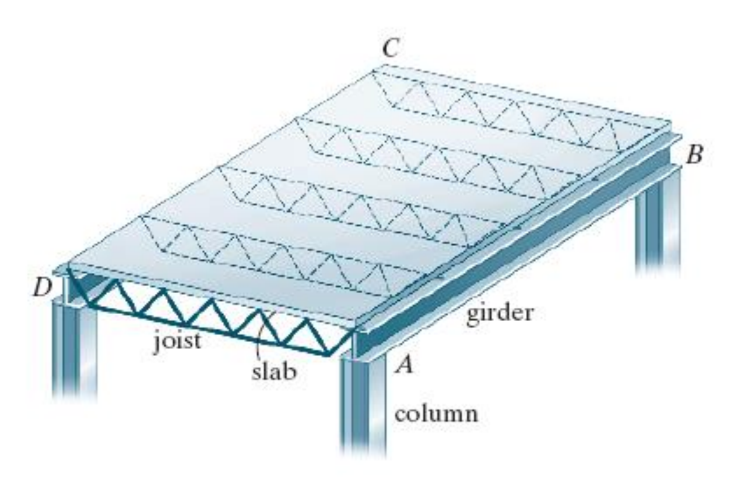


idealized structure

(b)

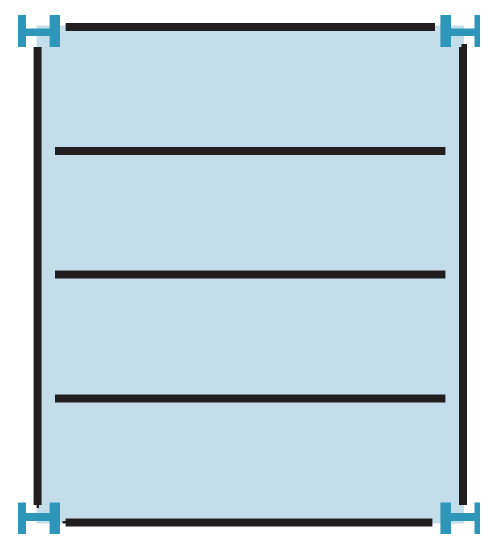
Idealized Structure

- Idealized Structure
  - Consider the framing used to support a typical floor slab in a building
  - The slab is supported by floor joists located at even intervals
  - These are in turn supported by 2 side girders AB & CD



Idealized Structure

- Idealized Structure
  - For analysis, it is reasonable to assume that the joints are pin and/or roller connected to girders & the girders are pin and/or roller connected to columns



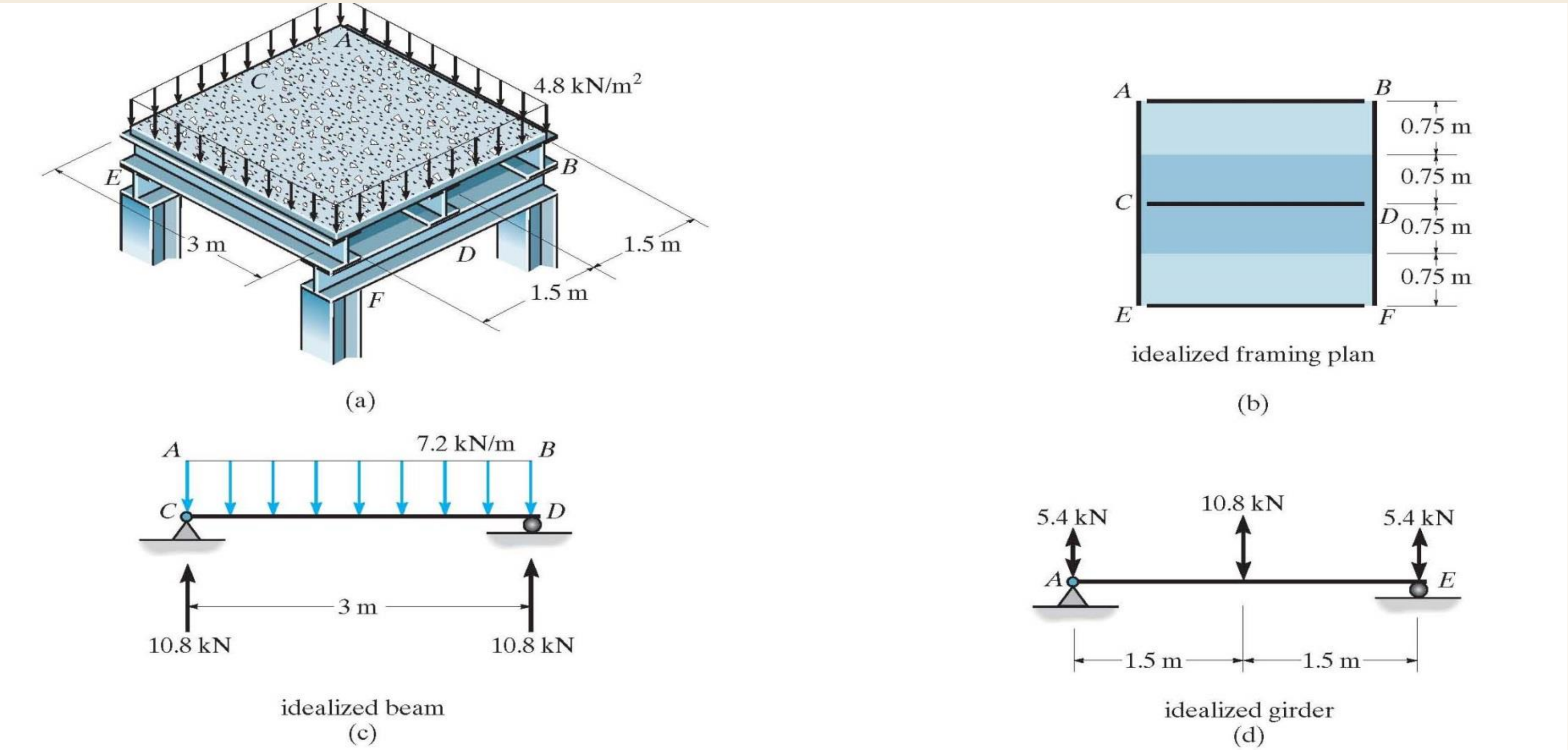
idealized framing plan

Idealized Structure

- Tributary Loadings
  - There are 2 ways in which the load on surfaces can transmit to various structural elements
  - 1-way system
  - 2-way system

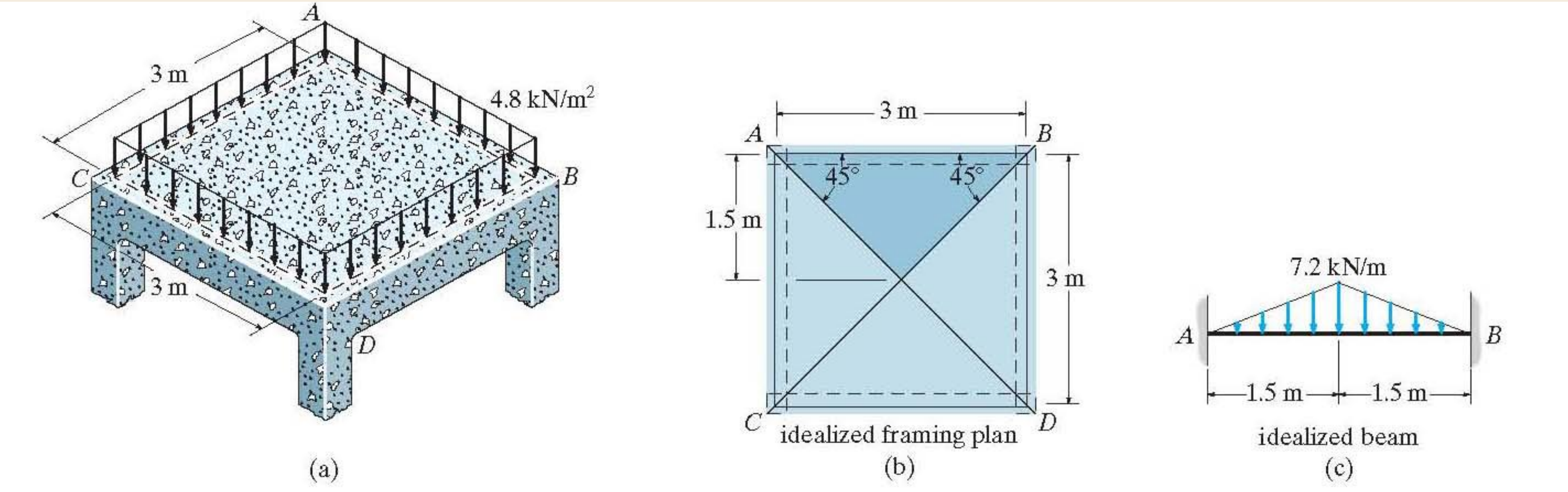
Idealized Structure

- Tributary Loadings
  - 1-way system

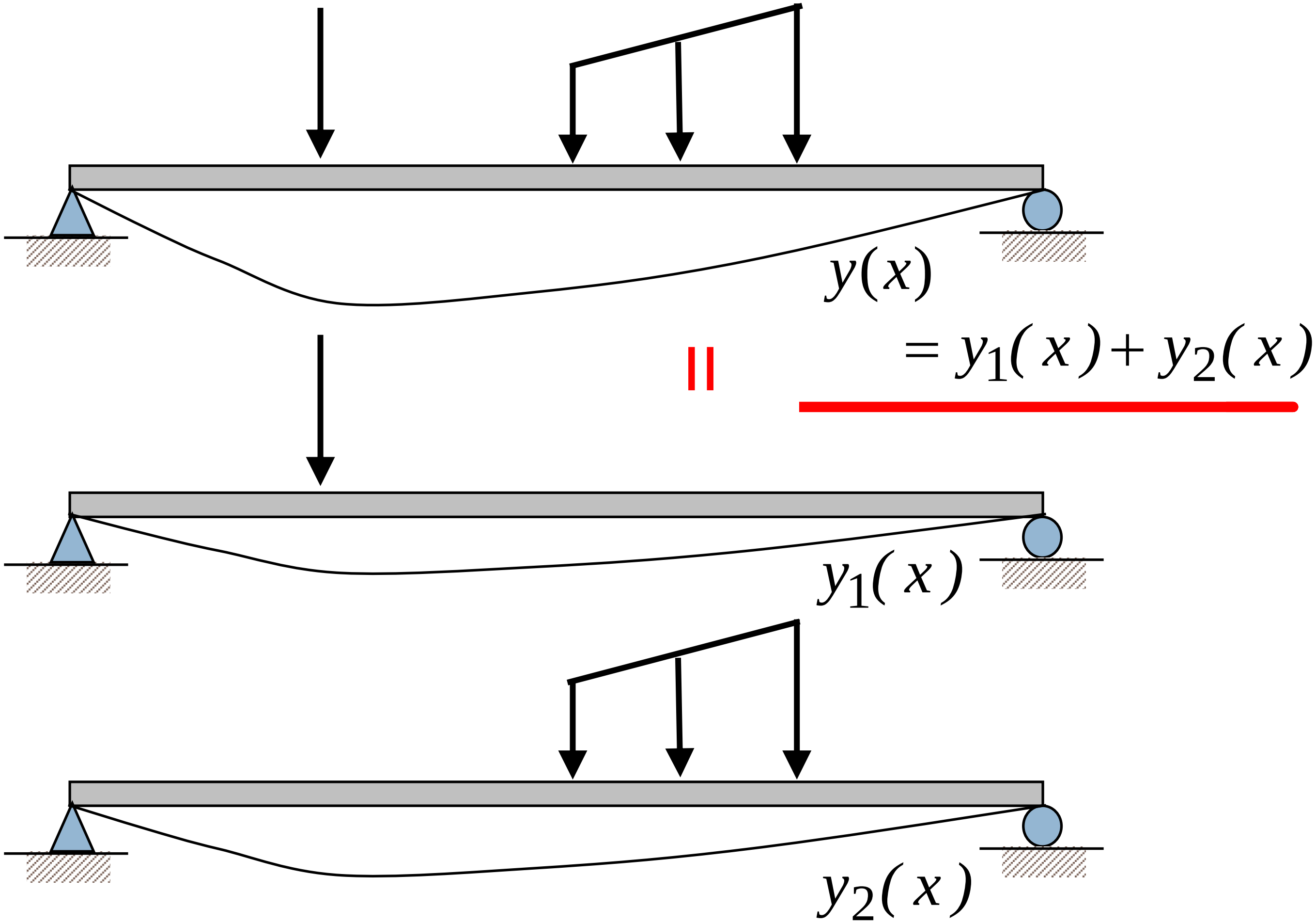


Idealized Structure

- Tributary Loadings
  - 2-way system



Principle of Superposition



Principle of Superposition

- Total disp. (or internal loadings, stress) at a point in a structure subjected to several external loadings can be determined by adding together the displacements (or internal loadings, stress) caused by each of the external loads acting separately
- Linear relationship exists among loads, stresses & displacements
- 2 requirements for the principle to apply:
  - Material must behave in a linear-elastic manner, Hooke's Law is valid
  - The geometry of the structure must not undergo significant change when the loads are applied, small displacement theory

Equilibrium and Determinacy

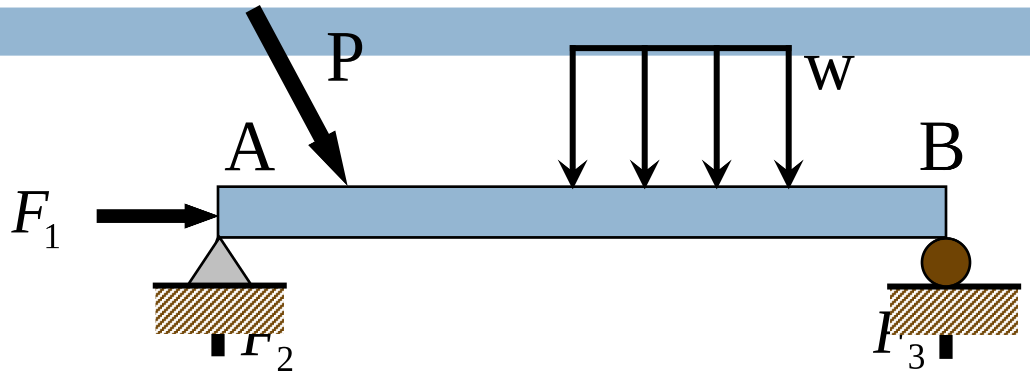
- For general 3D equilibrium:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$
$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

- For 2D structures, it can be reduced to:

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum M_o = 0$$

Equilibrium and Determinacy

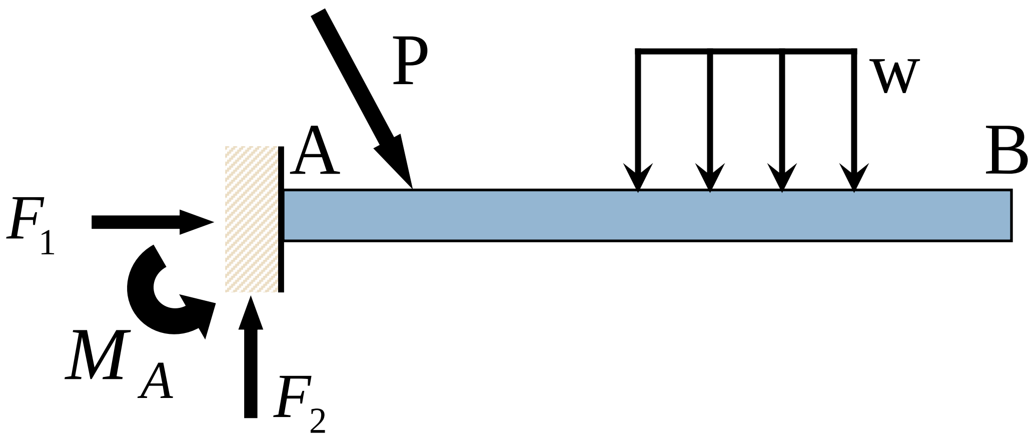


$$\sum F_x = 0$$

$$\sum F_y = 0$$

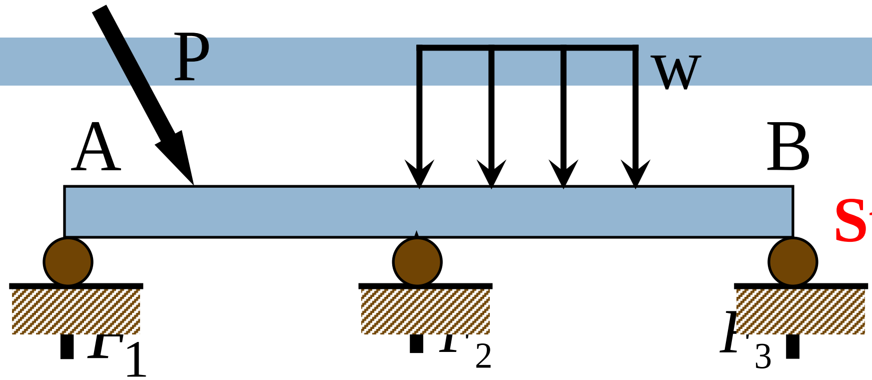
$$\sum M_o = 0$$

3 EQs → 3 unknown reactions



Stable Structures!

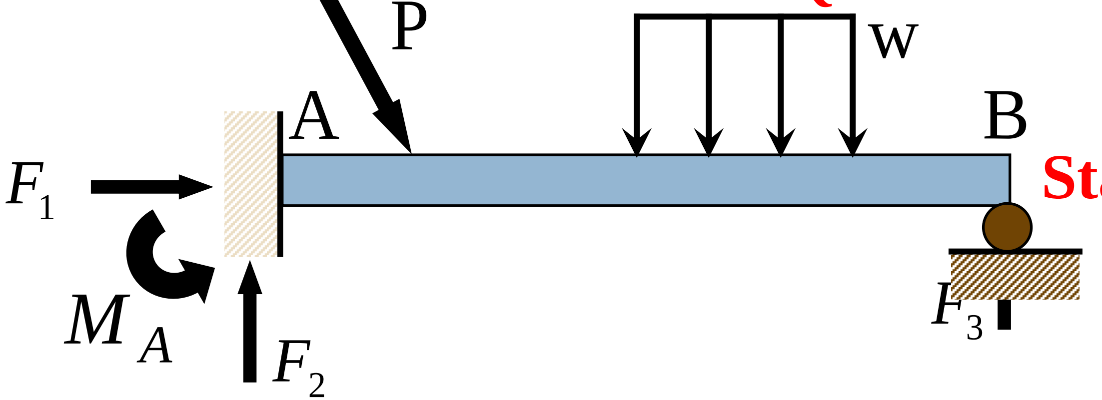
Equilibrium and Determinacy



Stable Structures?

3 EQs → 3 unknown reactions

Not properly supported



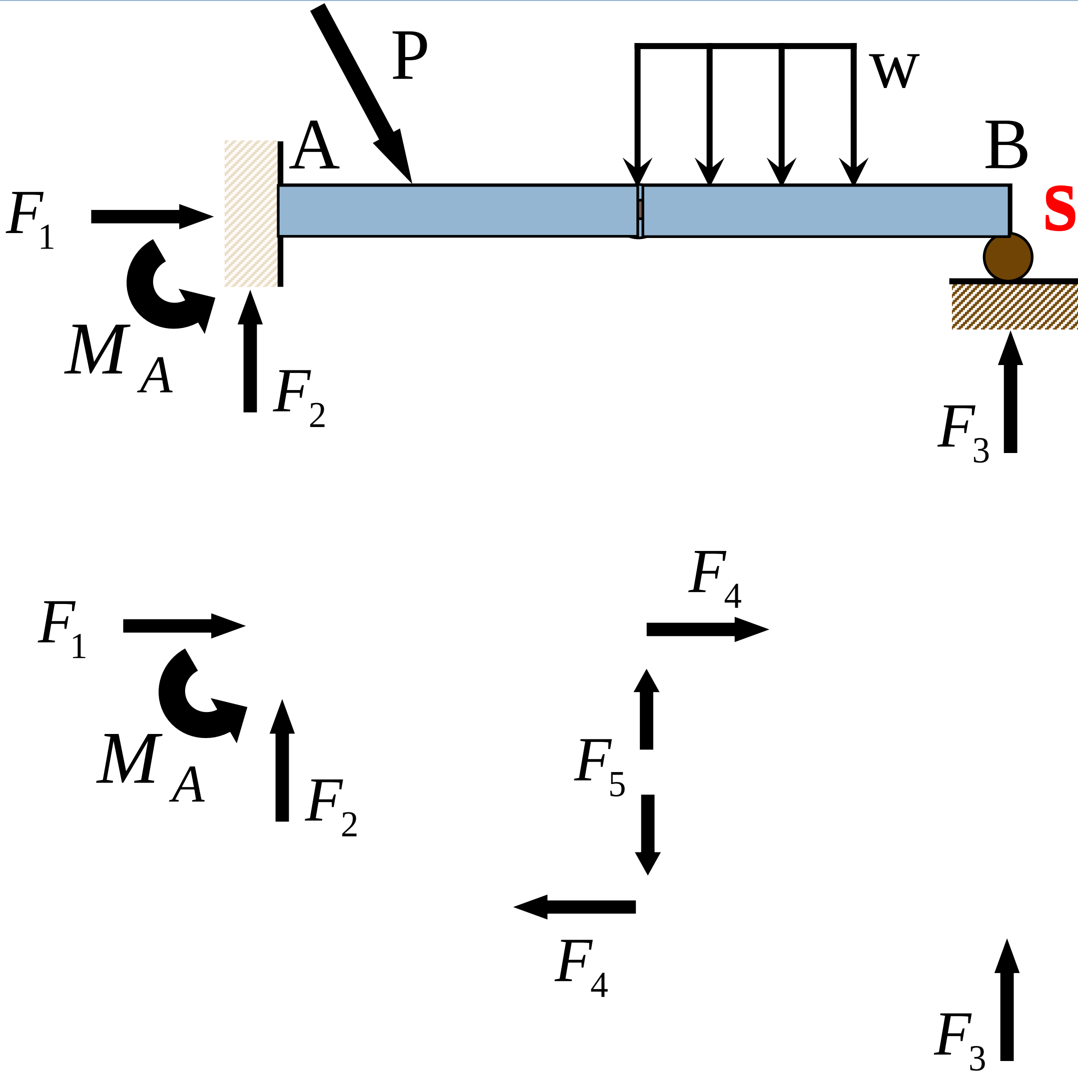
Stable Structures?

3 EQs → 4 unknown reactions

Indeterminate stable

1 degree indeterminancy

Equilibrium and Determinacy



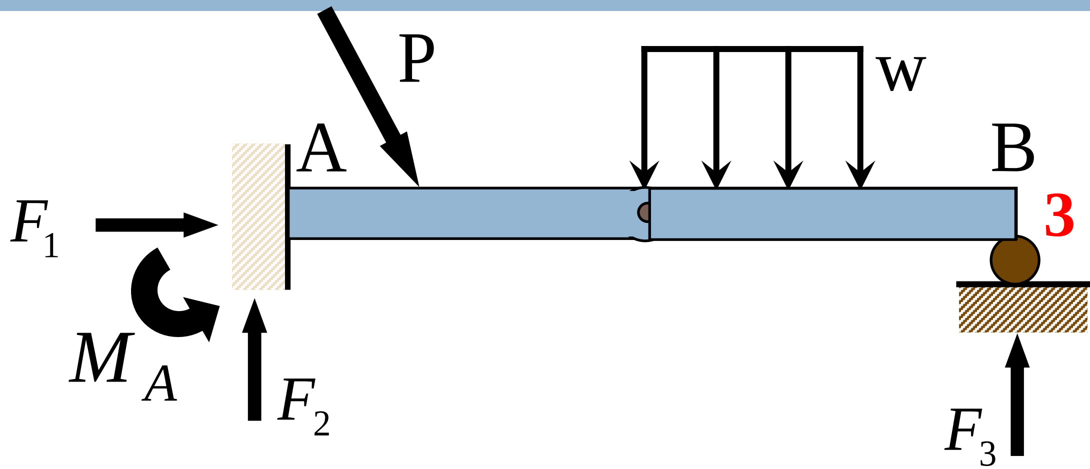
Stable Structures ?

6 equilibrium conditions

||

6 unknown forces

Equilibrium and Determinacy



3 equilibrium conditions

+

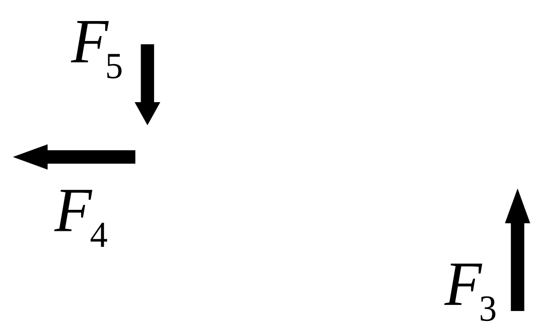
$$\sum M_c^{cb} = 0$$

C

4 equilibrium conditions

||

4 unknown forces



Equilibrium and Determinacy

- Equilibrium and Determinacy
  - If the reaction forces can be determined solely from the equilibrium EQs → STATICALLY DETERMINATE STRUCTURE
  - No. of unknown forces > equilibrium EQs → STATICALLY INDETERMINATE
  - Can be viewed globally or locally (via free body diagram)

Equilibrium and Determinacy

- Determinacy and Indeterminacy
  - For a 2D structure

No. of components

$$r = 3n, \quad \text{statically determinate}$$
$$r > 3n, \quad \text{statically indeterminate}$$

No. of unknown forces

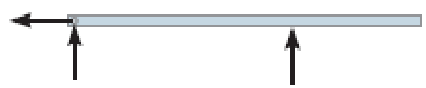
$\mathrm{r} - 3\mathrm{n}$: degree of indeterminacy

  - The additional EQs needed to solve for the unknown forces are referred to as compatibility EQs

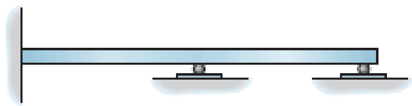
Discuss the Determinacy

(a)

$r = 3,\ n = 1,\ 3 = 3(1)$

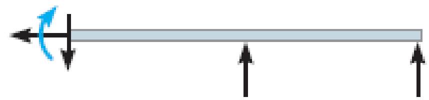


Statically determinate

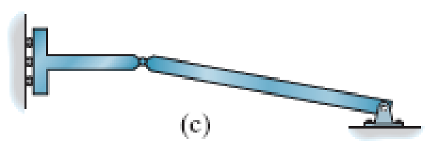


(b)

$r = 5,\ n = 1,\ 5 > 3(1)$

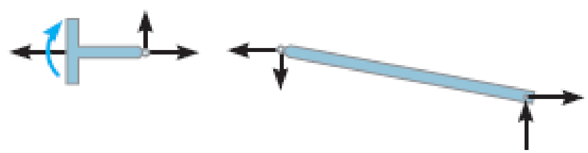


Statically indeterminate to the second degree

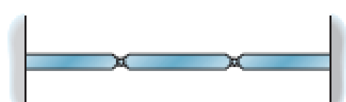


(c)

$r = 6,\ n = 2,\ 6 = 3(2)$



Statically determinate

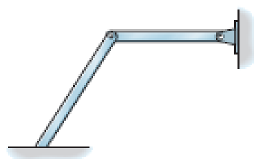


(d)

$r = 10,\ n = 3,\ 10 > 3(3)$

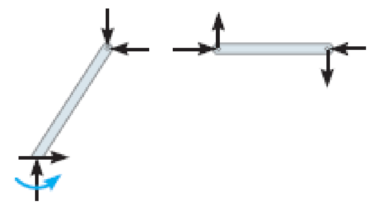
Statically indeterminate to the first degree

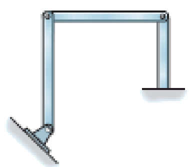
Discuss the Determinacy



(a)

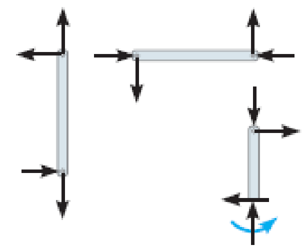
$r = 7,\ n = 2,\ 7 > 6$
Statically indeterminate to the first degree **Ans.**

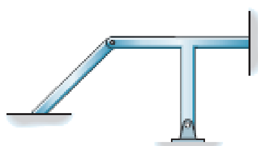




(b)

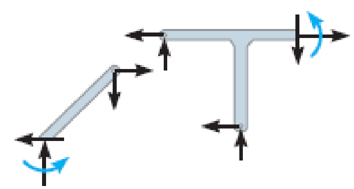
$r = 9,\ n = 3,\ 9 = 9,$
Statically determinate **Ans.**

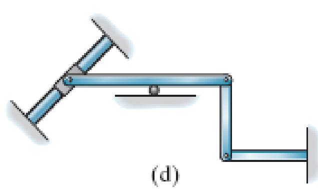




(c)

$r = 10,\ n = 2,\ 10 > 6,$
Statically indeterminate to the fourth degree **Ans.**





(d)

$r = 9,\ n = 3,\ 9 = 9,$
Statically determinate **Ans.**

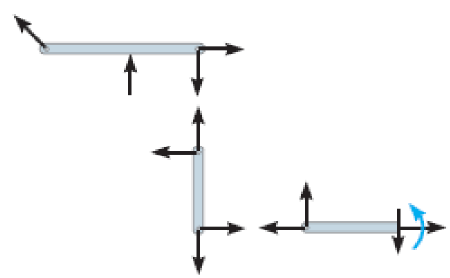


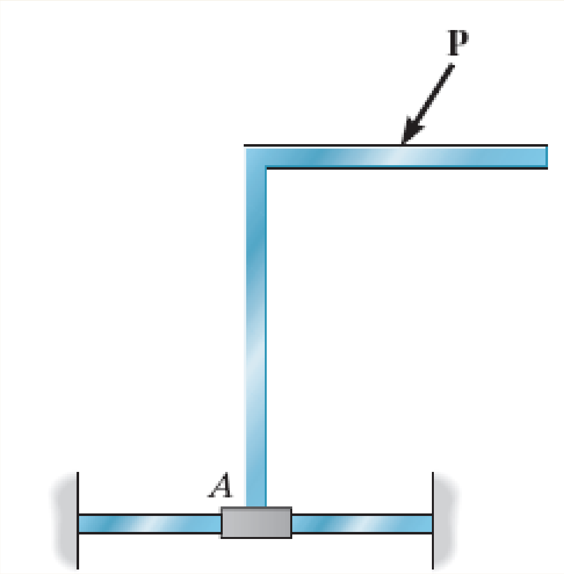
Fig. 2–19

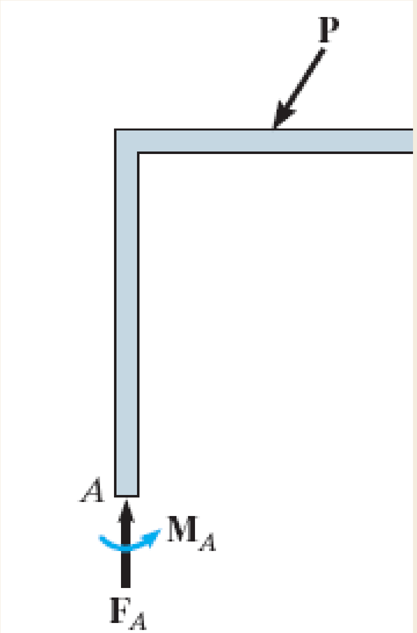
Stability

- To ensure equilibrium (stability) of a structure or its members:
  - Must satisfy equilibrium EQs
  - Members must be properly held or constrained by their supports
  - There is a unique set of values for reaction forces and internal forces

Determinacy and Stability

- Partial constraints
  - Fewer reactive forces than equilibrium EQs
  - Some equilibrium EQs can not be satisfied
  - Structure or Member is unstable

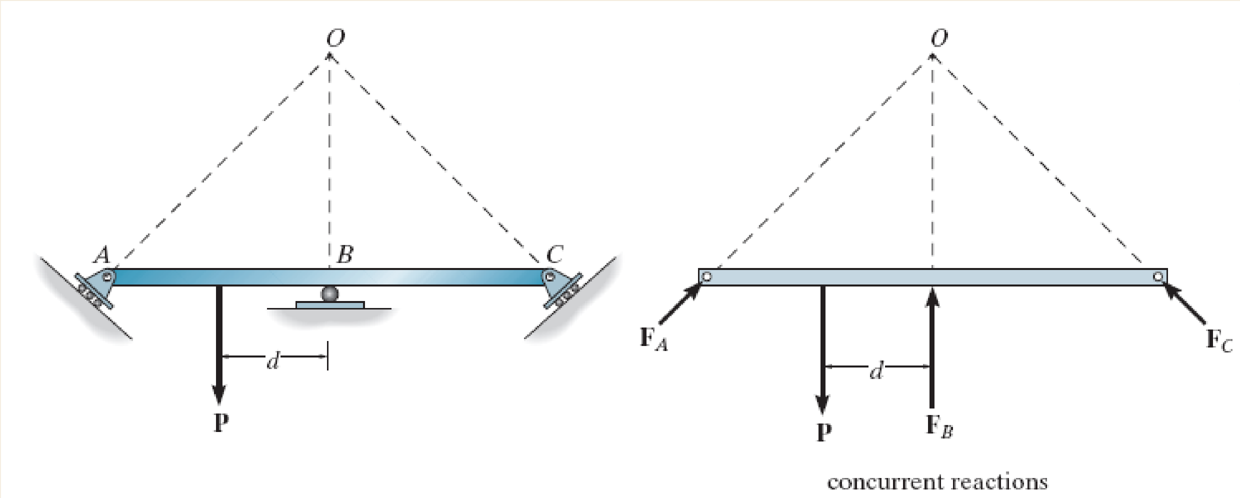




partial constraints

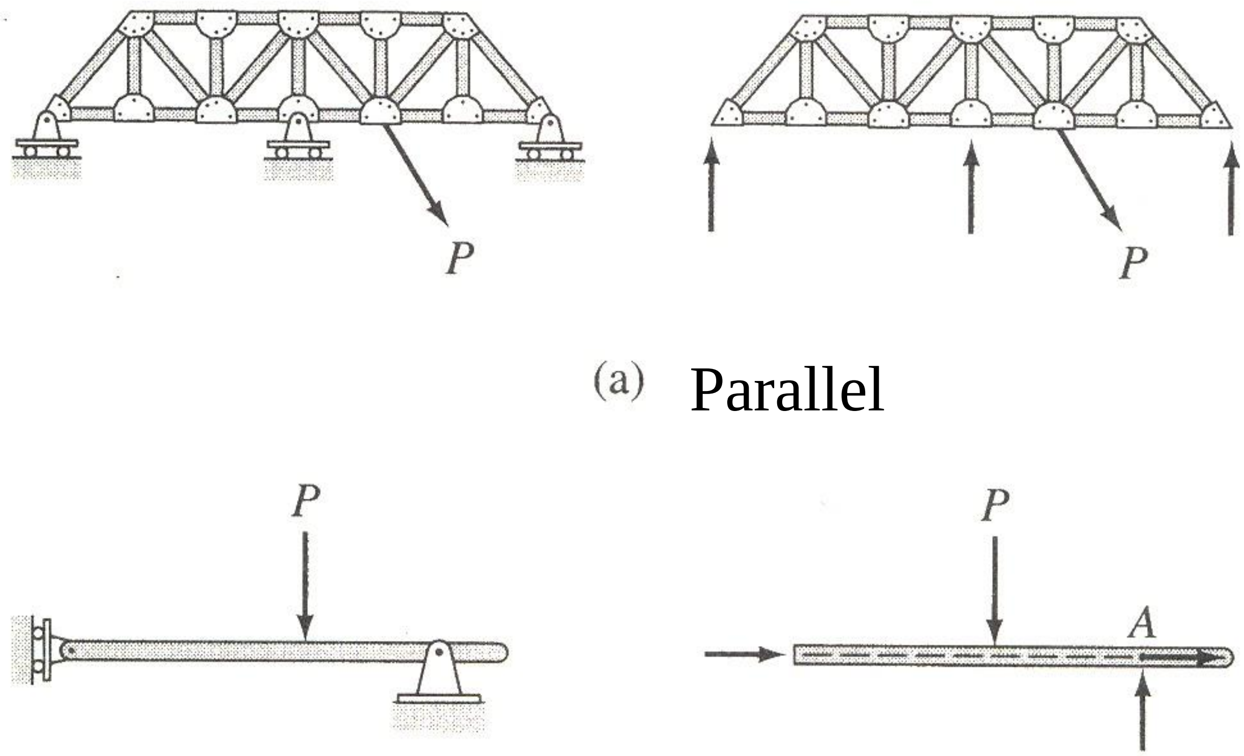
Determinacy and Stability

- Improper constraints
  - In some cases, unknown forces may equal equilibrium EQs
  - However, instability or movement of structure could still occur if support reactions are concurrent at a point



Determinacy and Stability

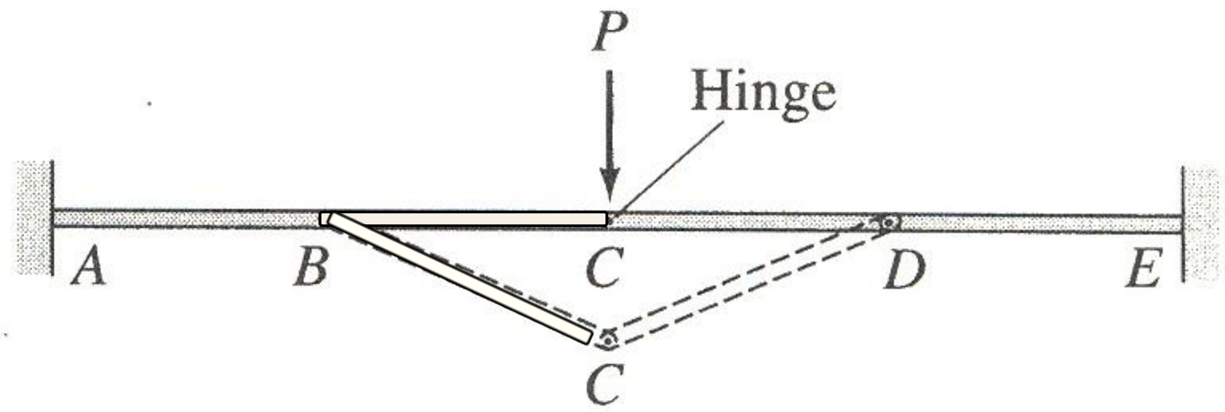
- Improper constraints



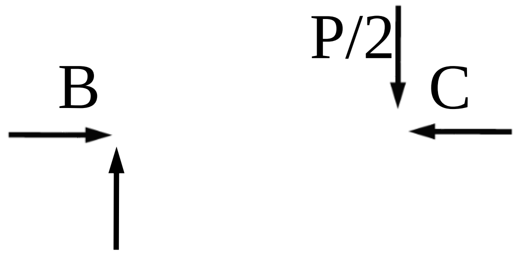
(a) Parallel

(b) Concurrent

Determinacy and Stability



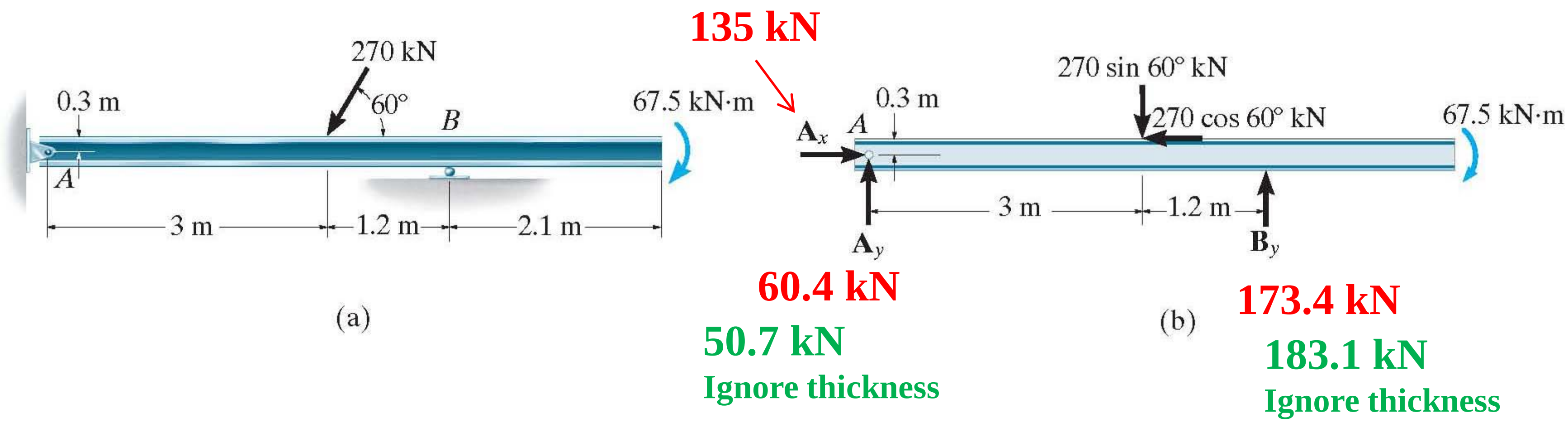
6 Reactions – 6 Conditions



unstable **stable**

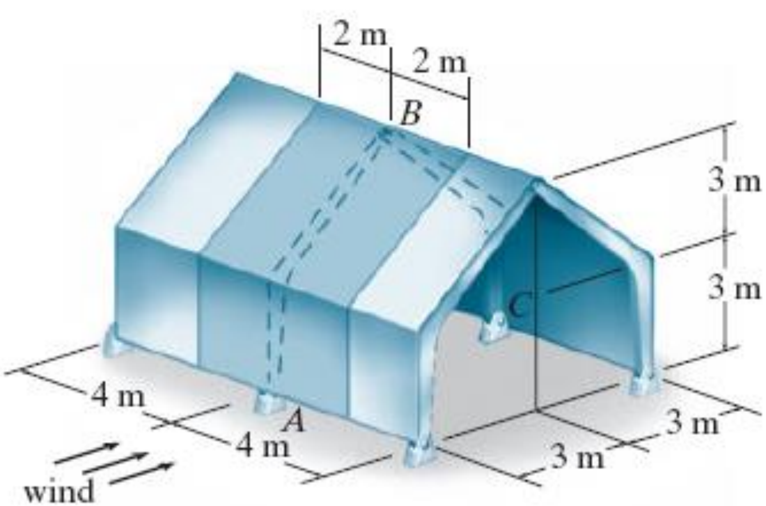
Solving Determinate Structures

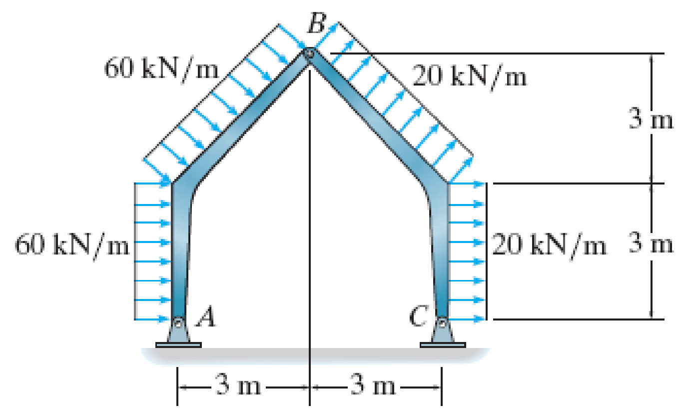
Determine the reactions on the beam as shown.



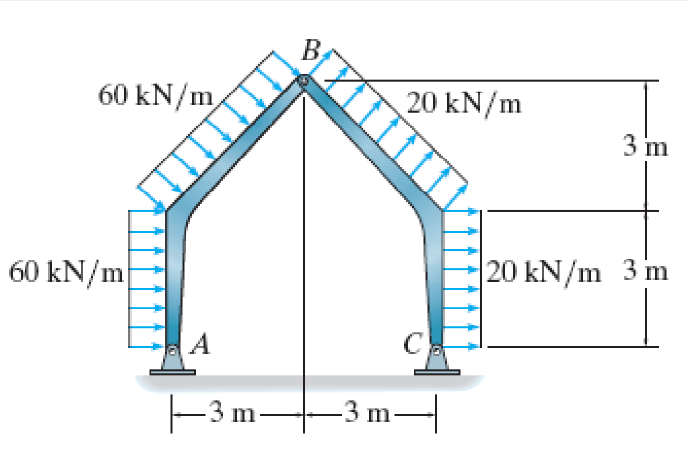
Example 2.13

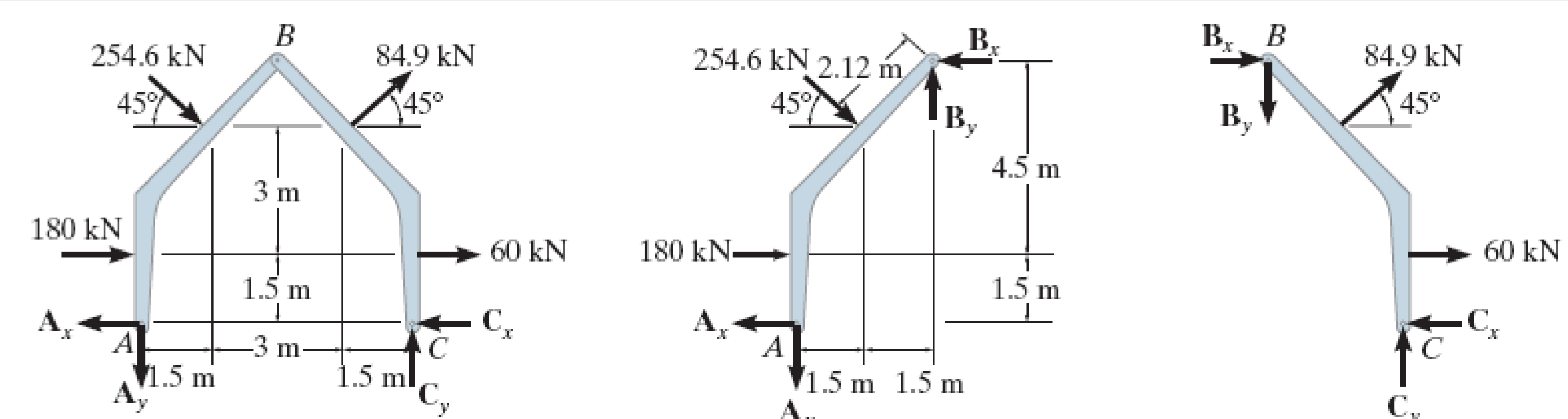
The side of the building subjected to a wind loading that creates a uniform normal pressure of 15kPa on the windward side & a suction pressure of 5kPa on the leeward side. Determine the horizontal & vertical components of reaction at the pin connections A, B & C of the supporting gable arch.





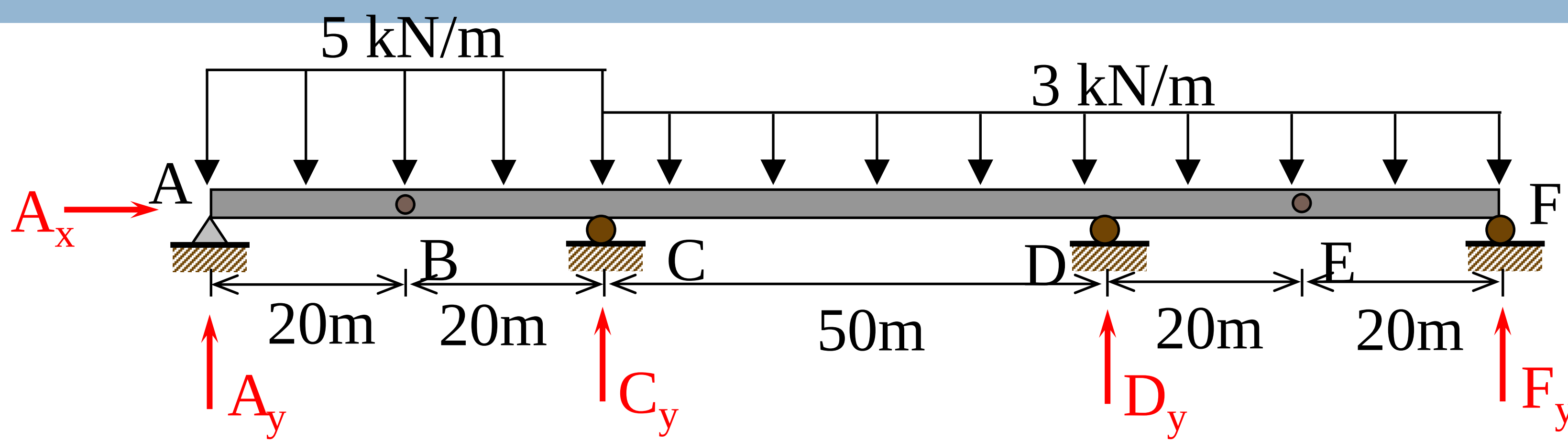
Solution


B
60 kN/m
20 kN/m
3 m
60 kN/m
20 kN/m
3 m
A
C
3 m
3 m


254.6 kN
B
84.9 kN
45°
45°
3 m
180 kN
60 kN
1.5 m
A_x
C_x
A
3 m
C
1.5 m
1.5 m
A_y
C_y
254.6 kN
2.12 m
B_x
45°
B_y
4.5 m
180 kN
1.5 m
A_x
A
1.5 m
1.5 m
A_y
B_x
B
84.9 kN
B_y
45°
60 kN
C_x
C
C_y

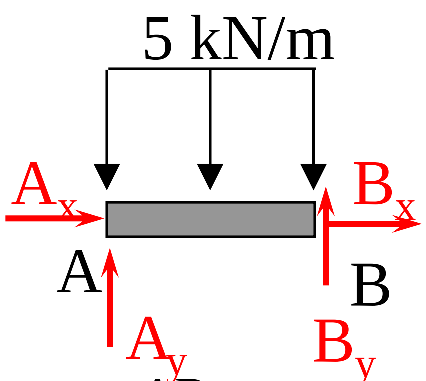
(c)

Example

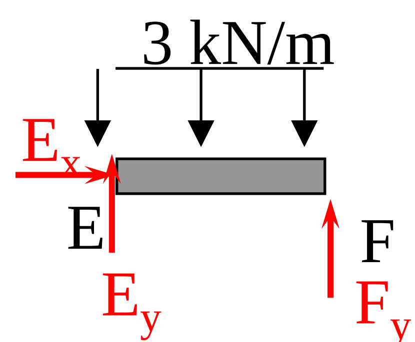


5 unknown forces

Needs 5 equations (equilibrium conditions)



- 3 global equilibriums
- 2 hinge conditions



$$\sum M_B^{AB} = 20A_y - 100 \times 10 = 0$$

$$\sum M_E^{EF} = 20F_y - 60 \times 10 = 0$$

Summary

- Difference between an actual structure and its idealized model
- Principle of superposition
- Equilibrium, determinacy and stability
- Analyzing statically determinate structures