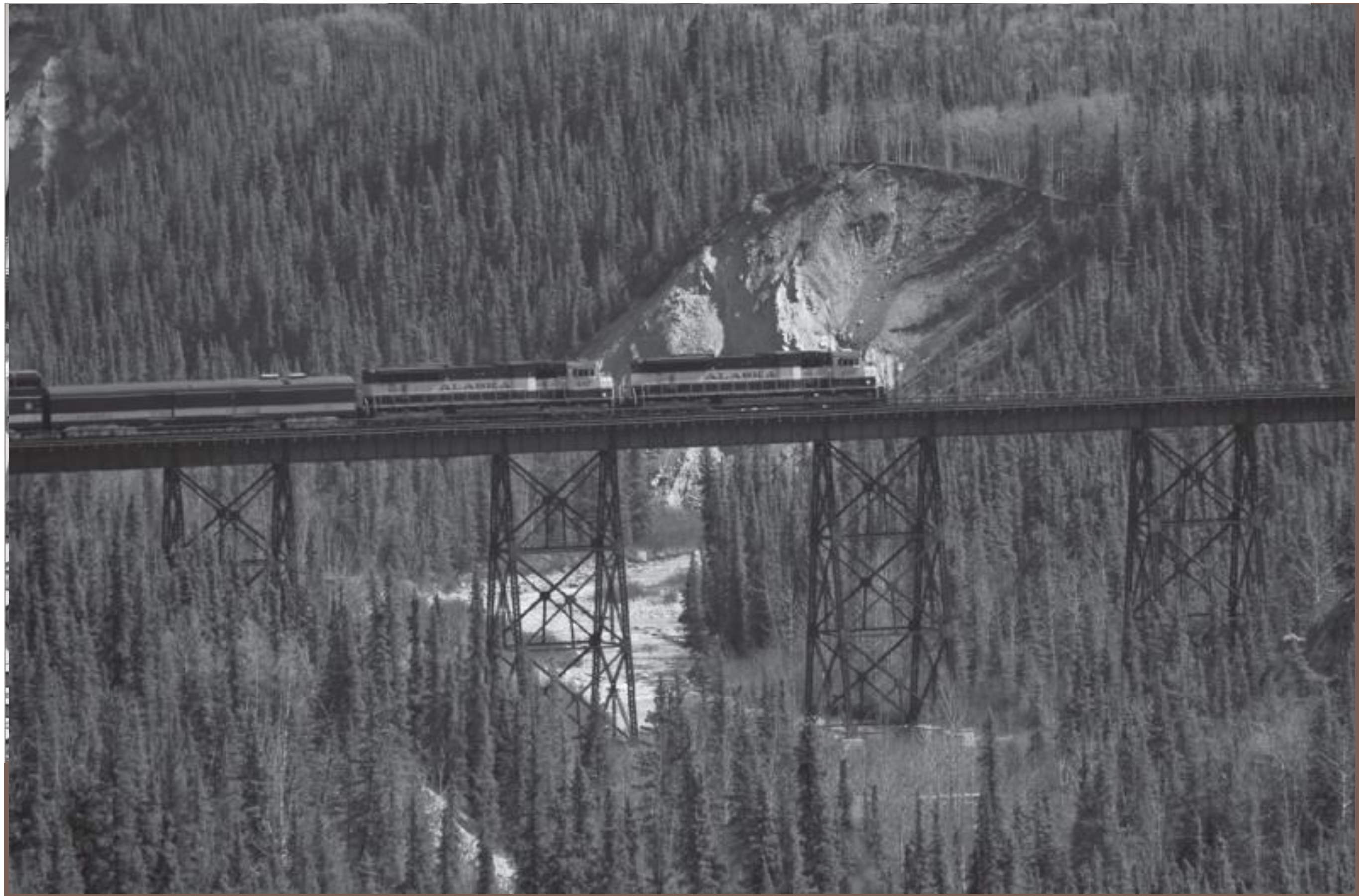


Chapter 6: Influence lines in statically determinate structures

- CEE 3222 THEORY OF STRUCTURES

- Dr. Lenganji Simwanda, PhD

Objective

- ✓ To analyze the influence line of beam
- ✓ To draw influence line using qualitative method
- ✓ To determine the maximum values of different concentrated loads

Influence Lines

- If a structure is subjected to a moving load, the variation of shear & bending moment is best described using the influence line

- One can tell at a glance, where the moving load should be placed on the structure so that it creates the greatest influence at a specified point

- The magnitude of the associated shear, moment or deflection at the point can then be calculated using the ordinates of the influence-line diagram

Influence Lines

- One should be clear of the difference between Influence Lines & shear or moment diagram

- Influence line represent the effect of a moving load only at a specific point

- Shear or moment diagrams represent the effect of fixed loads at all points along the axis of the member

- Procedure for Analysis
    - Tabulate Values
    - Influence-Line equations

Influence Lines

- Tabulate Values
  - Place a unit load at various locations, *x*, along the member
  - At each location use statics to determine the value of function at the specified point
  - If the influence line for a vertical force reaction at a point on a beam is to be constructed, consider the reaction to be +ve at the point when it acts upward on the beam
  - If a shear or moment influence line is to be drawn for a point, take the shear or moment at the point as +ve according to the same sign convention used for drawing shear & moment diagram
  - All statically determinate beams will have influence lines that consist of straight line segments

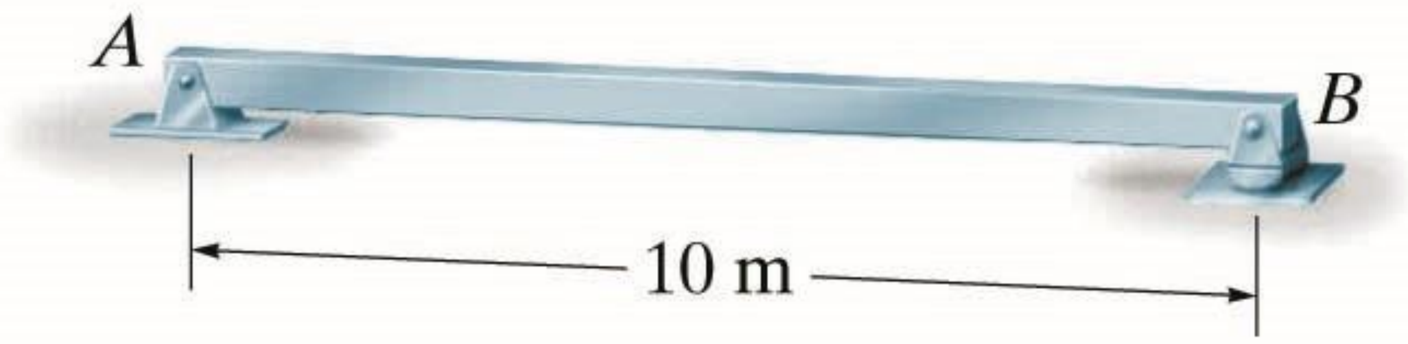
Influence Lines

- Influence-Line Eqns
  - The influence line can also be constructed by placing the unit load at a variable position, *x*, on the member & then computing the value of *R*, *V* or *M* at the point as a function of *x*
  - The eqns of the various line segments composing the influence line can be determined & plotted

Influence Lines

Example 6.1

Construct the influence line for the vertical reaction at A of the beam.

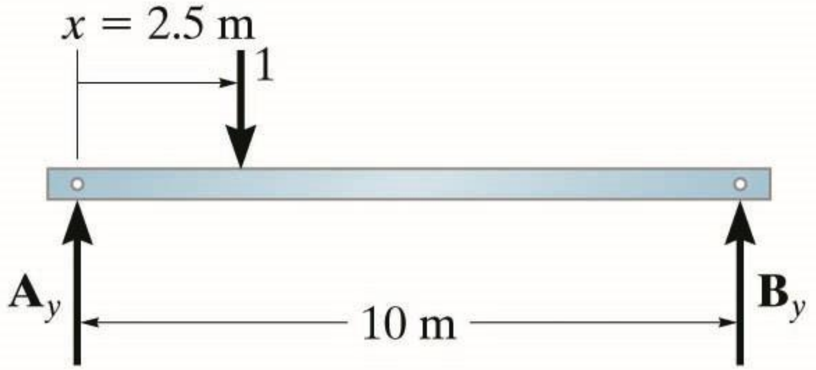


Influence Lines

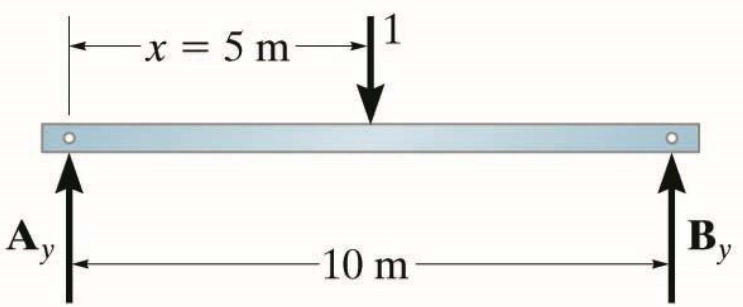
Example 6.1 (Solution)

Tabulate Values

A unit load is placed on the beam at each selected point *x* & the value of A_y is calculated by summing moments about *B*.



$$\circlearrowleft + \Sigma M_B = 0;\ -A_y\,(10) + 1\,(7.5) = 0$$
$$A_y = 0.75$$

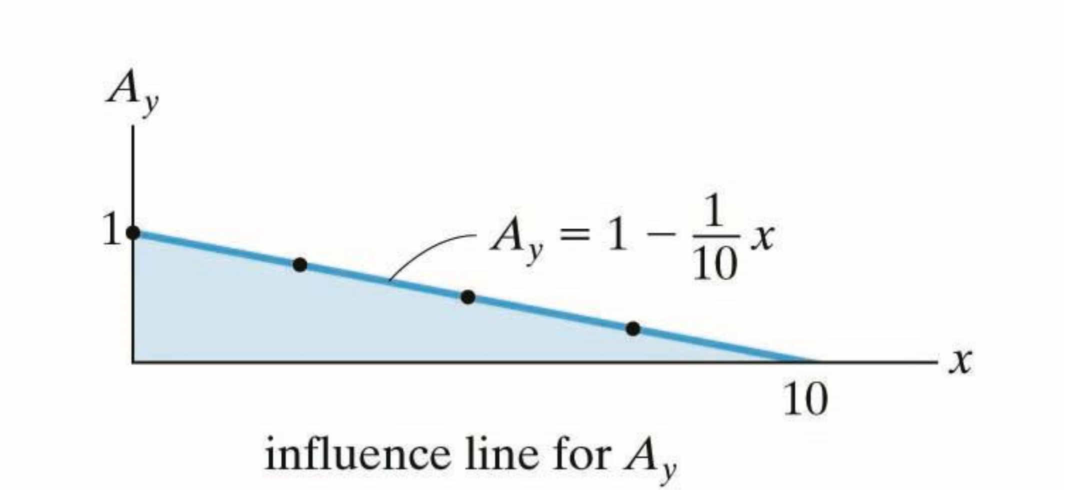


$$\circlearrowleft + \Sigma M_B = 0;\ -A_y\,(10) + 1\,(5) = 0$$
$$A_y = 0.5$$

Influence Lines

Example 6.1 (Solution)

Tabulate Values



x	A_y
0	1
2.5	0.75
5	0.5
7.5	0.25
10	0

Influence Lines

Example 6.1 (Solution)

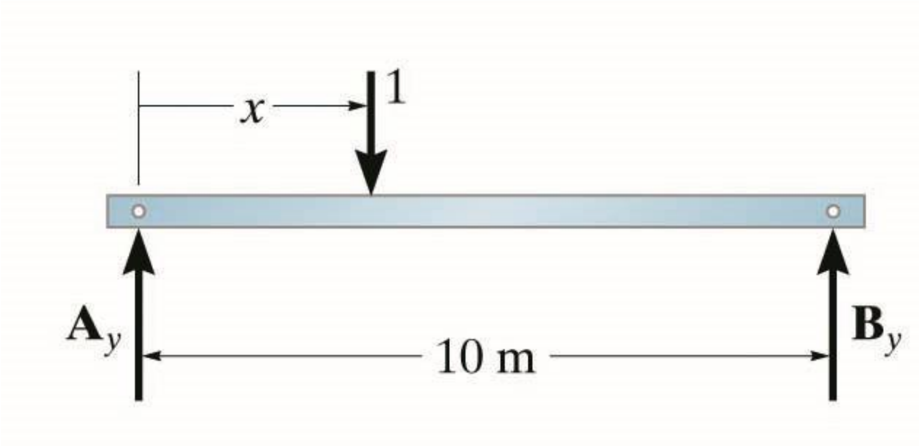
Influence-Line Equation

The reaction as a function of *x* can be determined from

$$\Sigma M_B = 0$$

$$-A_y(10) + (10 - x)(1) = 0$$

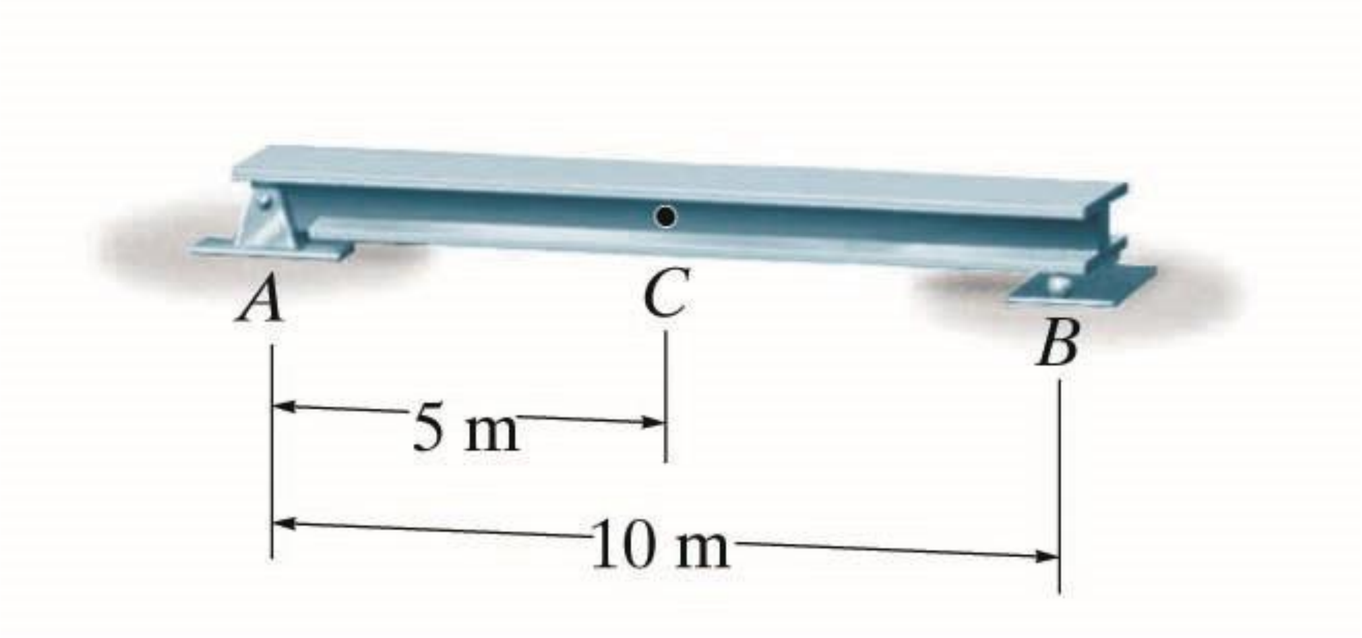
$$A_y = 1 - \frac{1}{10}x$$



Influence Lines

Example 6.5

Construct the influence line for the moment at *C* of the beam.



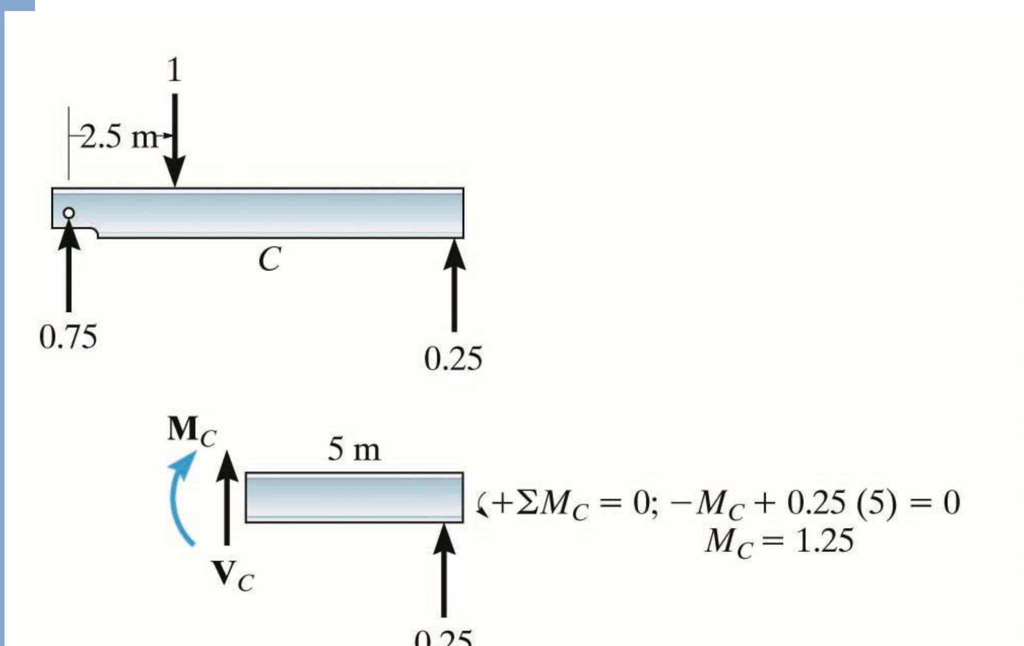
Influence Lines

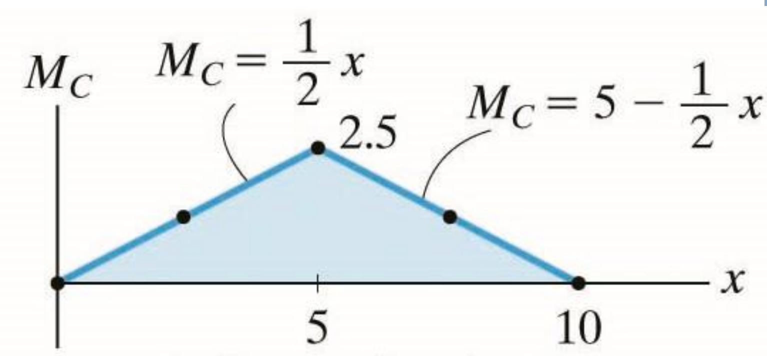
Example 6.5 (Solution)

Tabulate Values

At each selected position of the unit load, the value of
calculated using the method of sections.

x	M_C
0	0
2.5	1.25
5	2.5
7.5	1.25
10	0





influence line for M_C

Influence Lines

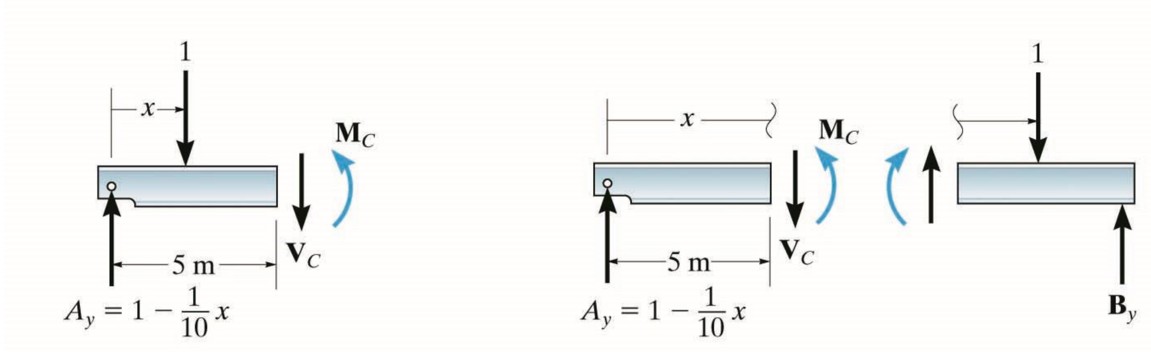
Example 6.5 (Solution)

Influence-Line Equations

$$\Sigma M_C = 0$$

$$M_C + 1(5 - x) - (1 - \frac{1}{10}x)5 = 0$$

$$M_C = \frac{1}{2}x \quad \text{for } 0 \le x < 5\,\text{m}$$



$$\Sigma M_C = 0$$

$$M_C - (1 - \frac{1}{10}x)5 = 0$$

$$M_C = 5 - \frac{1}{2}x \quad \text{for } 5\,\text{m} < x \le 10\,\text{m}$$

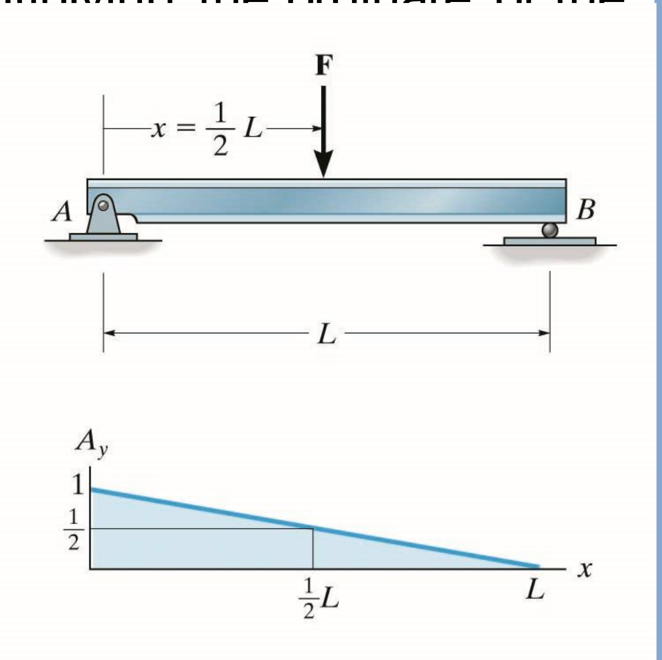
Influence Lines for Beams

- Once the influence line for a function has been constructed, it will be possible to position live loads on the beam which will produce the max value of the function

- 2 types of loadings will be considered:
    - Concentrated force
    - Uniform load

Influence Lines for Beams

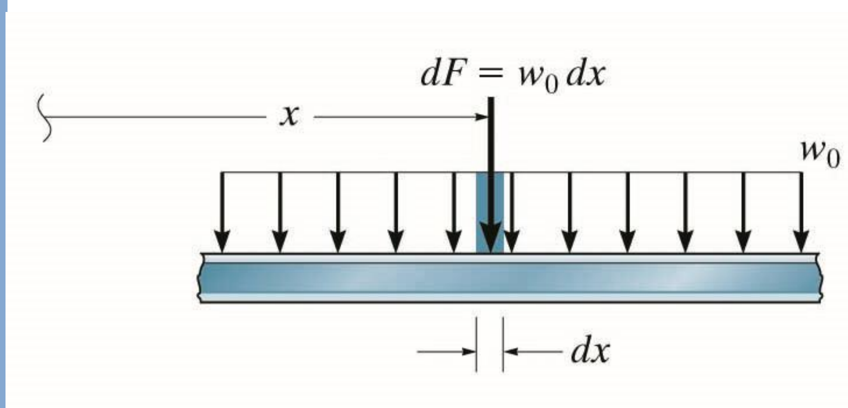
- Concentrated force
  - For any concentrated force, **F** acting on the beam, the value of the function can be found by multiplying the ordinate of the influence line at position *x* by magr
  - Consider Fig 6.7, influence line for
  - For unit load, $A_y = ½$
  - For a force of F, $A_y = (½)\,F$

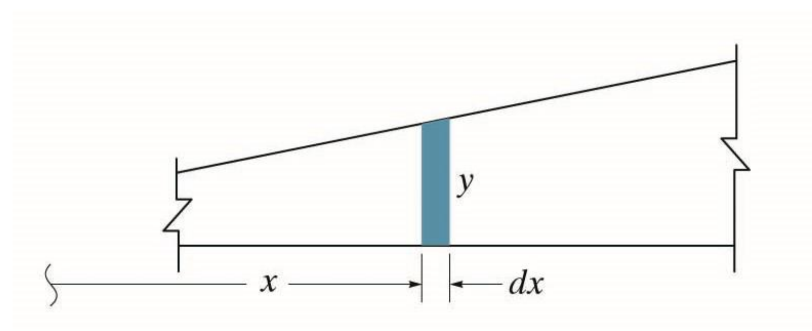


influence line for A_y

Influence Lines for Beams

- Uniform load
  - Each dx segment of this load creates a concentrated force of $dF = w_0 dx$
  - If d**F** is located at x, where the influence-line ordinate is y, the value of the function is $(dF)(y) = (w_0 dx)y$
  - The effect of all concentrated forces is determined by





influence line for function

Influence Lines for Beams

- Uniform load

$$\int w_o y dx = w_o \int y dx$$

  - Since $\int y dx$ is equivalent to the area under the influence line, in general:

  - value of the function caused by a uniform load = the area under the influence line *x* intensity of the uniform load

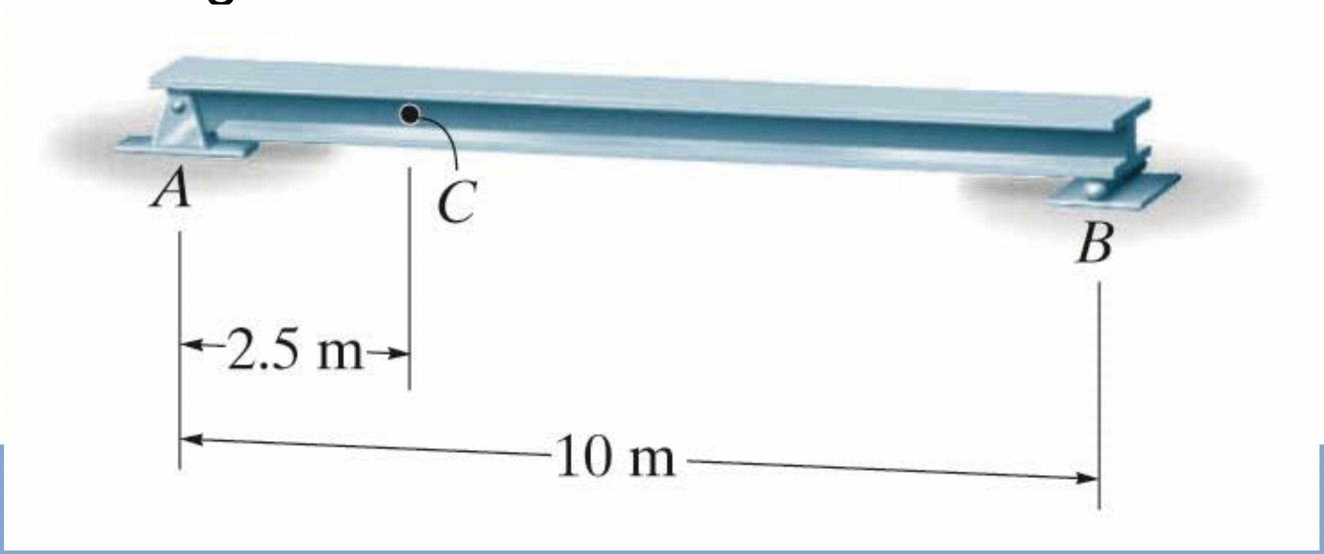
Influence Lines for Beams

Example 6.7

Determine the max +ve shear that can be developed at point *C* in the beam due to:

➔A concentrated moving load of 4 kN, and

➔A uniform moving load of 2 kN/m



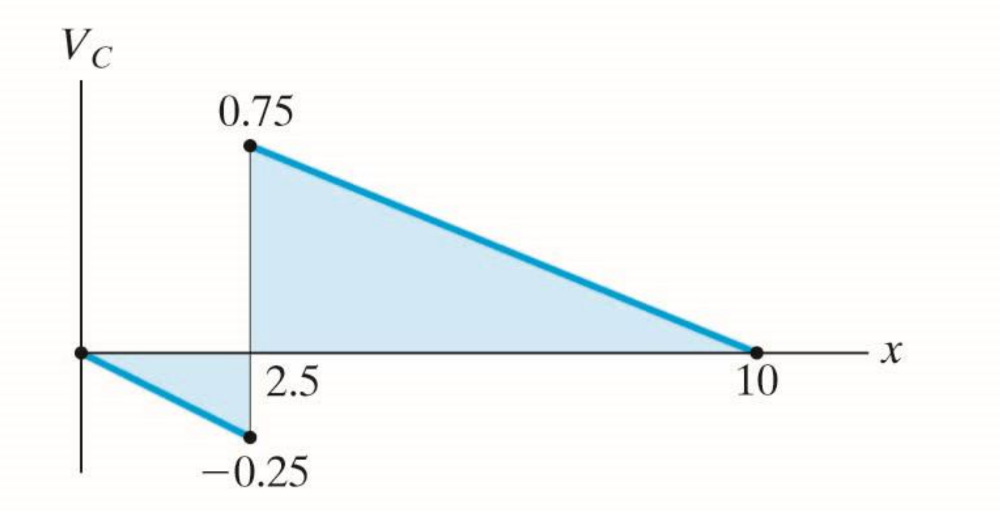
Influence Lines for Beams

Example 6.7 (Solution)

Concentrated force

The max +ve positive shear at *C* will occur when the 4 kN force is located at x = 2.5 m. The ordinate at this peak is +0.75, hence:

$$V_C = 0.75(4kN) = 3kN$$



Influence Lines for Beams

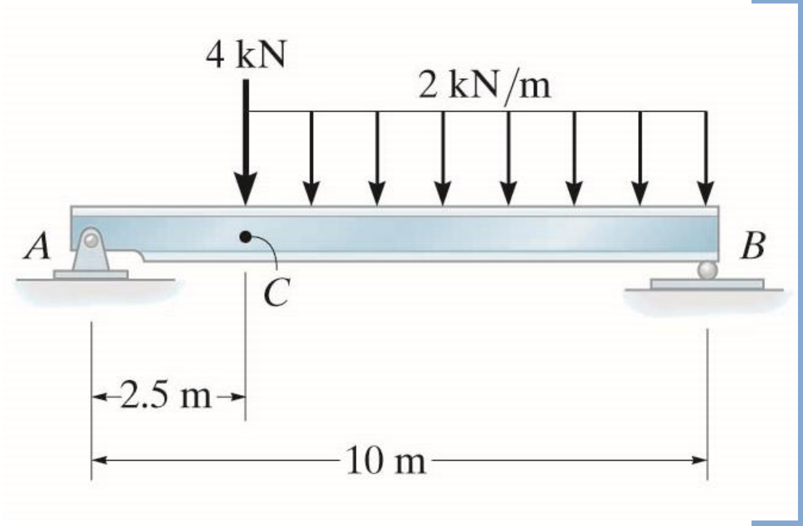
Example 6.7 (Solution)

Uniform load

The uniform moving load creates the max +ve influence for V_C when the load acts on the beam between x = 2.5 m and x = 10 m The magnitude of $\mathbf{V}_C$ due to this loading is:

$$V_C = \left[\frac{1}{2}(10\,\text{m} - 2.5\,\text{m})(0.75)\right](2\,\text{kN/m})$$

$$= 5.625\,\text{kN}$$



Total max shear at C:

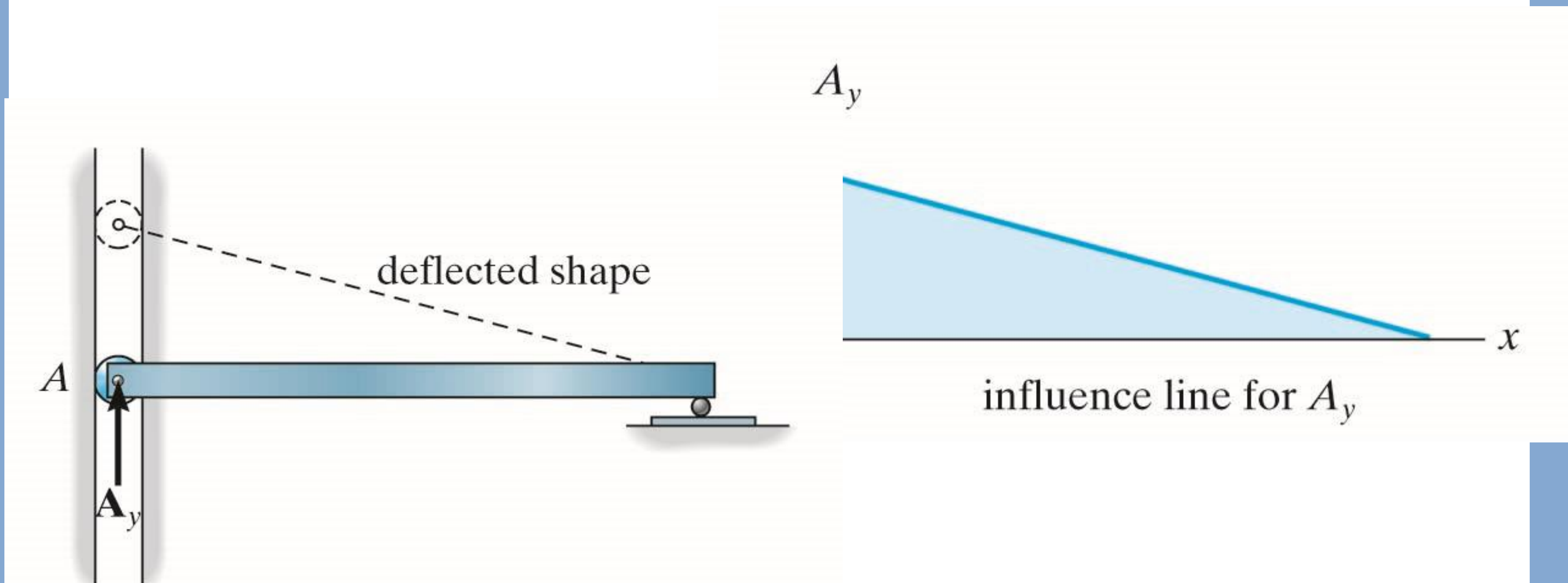
$$(V_C)_{\max} = 3\,\text{kN} + 5.625\,\text{kN} = 8.625\,\text{kN}$$

Qualitative Influence Lines

- The Müller-Breslau Principle states that the influence line for a function is to the same scale as the deflected shape of the beam when the beam is acted upon by the function
- If the shape of the influence line for the vertical reaction at *A* is to be determined, the pin is first replaced by a roller guide

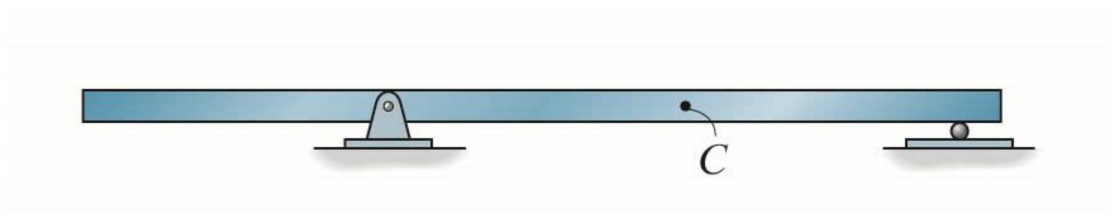
Qualitative Influence Lines

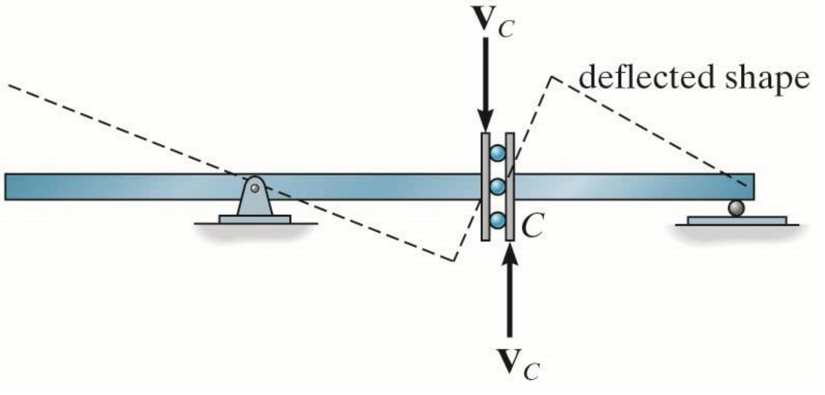
- When the +ve force $\mathbf{A}_y$ is applied at A, the beam deflects to the dashed position which rep the general shape of the influence line

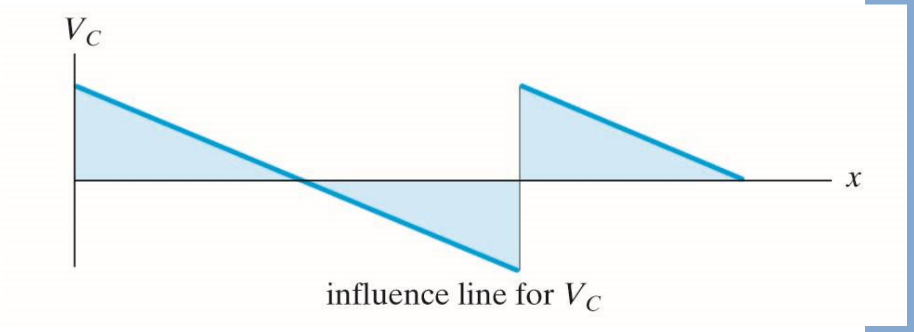


Qualitative Influence Lines

- If the shape of the influence line for shear at *C* is to be determined, the connection at *C* may be symbolized by a roller guide
- Applying a +ve shear force $\mathbf{V}_c$ to the beam at *C* & allowing the beam to deflect to the dashed position

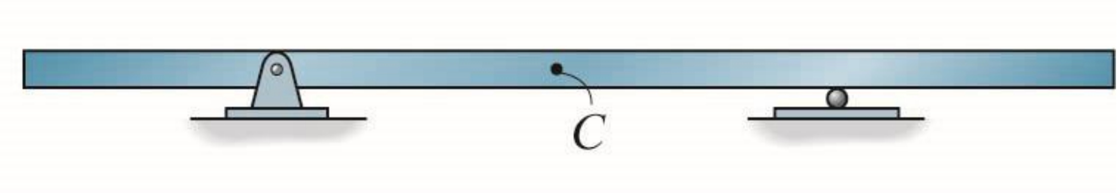


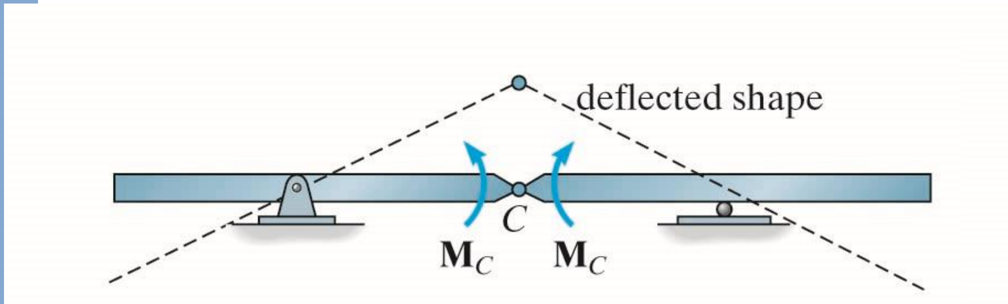


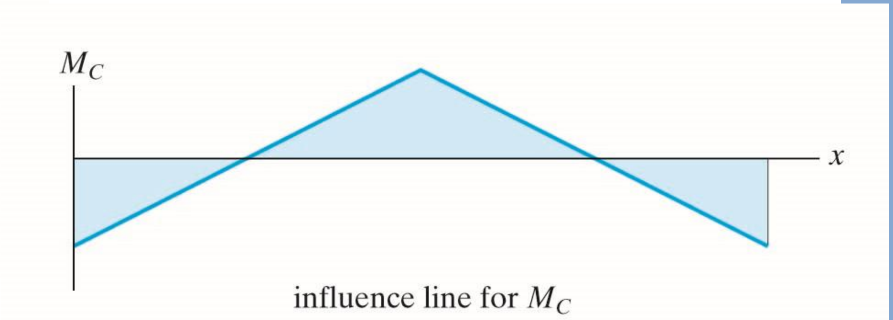


Qualitative Influence Lines

- If the shape of influence line for the moment at *C* is to be determined, an internal hinge or pin is placed at *C*
- Applying +ve moment $\mathbf{M}_c$ to the beam, the beam deflects to the dashed line





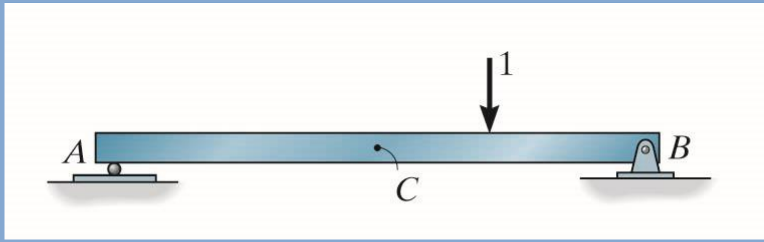


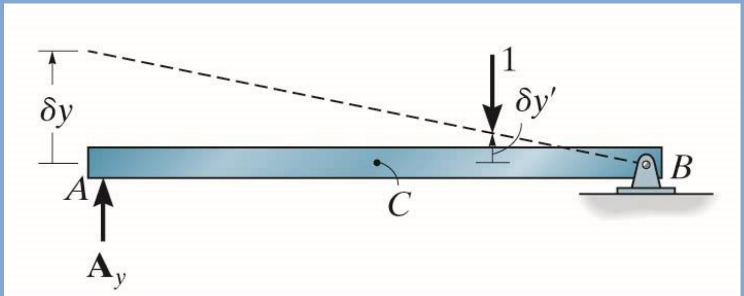
•24

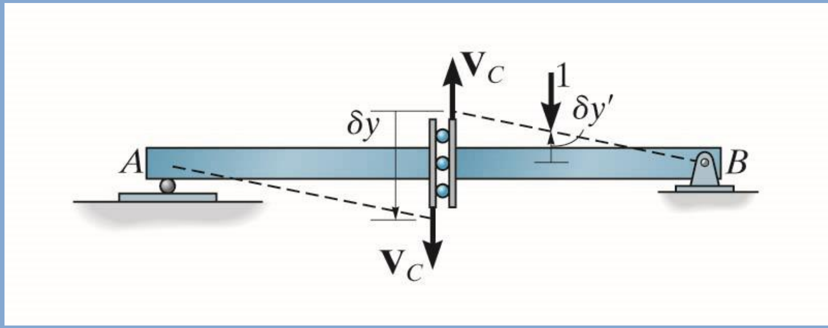
Qualitative Influence Lines

- The proof of the Müller-Breslau Principal can be established using the principle of virtual work
- Work = a linear disp x force in the direction of disp
- Or work = rotational disp x moment if the direction of the disp
- If a rigid body is in equilibrium, the sum of all the forces & moments on it must be equal to zero
- If the body is given an imaginary or virtual disp, work done by all these forces & couple moments must also be equal to zero
- If the beam shown is given a virtual disp δy at the support A, then only $\mathbf{A}_y$ & unit load do virtual work
- A_y does +ve work = $A_y\ \delta y$
- The unit load does –ve work = -1 δy'

Qualitative Influence Lines







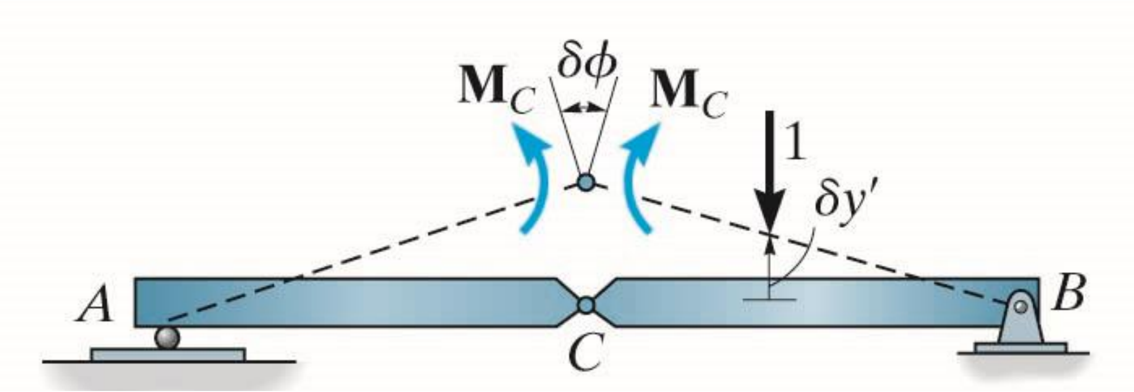
Qualitative Influence Lines

- Since the beam is in equilibrium, the virtual work sums to zero

$$A_y \delta y - 1\delta y' = 0$$

$$\text{If } \delta y = 1, \text{ then} \Rightarrow A_y = \delta y'$$

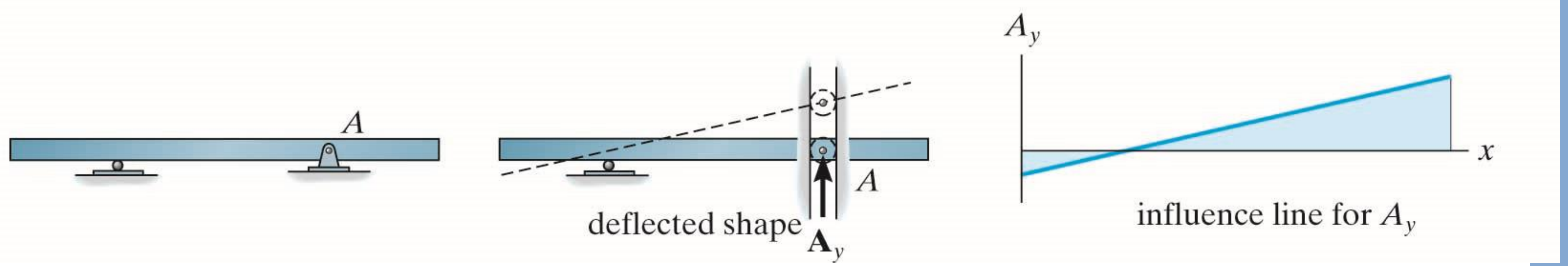
- The value of A_y represents the ordinate of the influence line at the position of the unit load

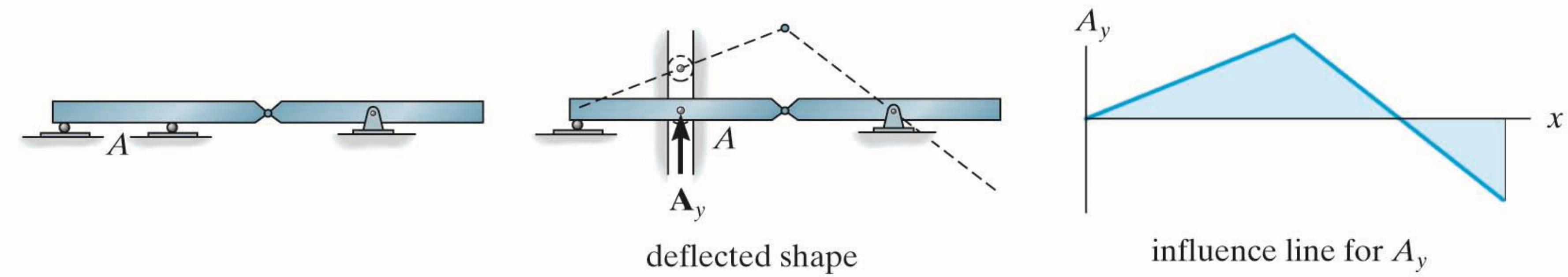


Qualitative Influence Lines

Example 6.9

For each beam sketch the influence line for the vertical reaction at *A*.

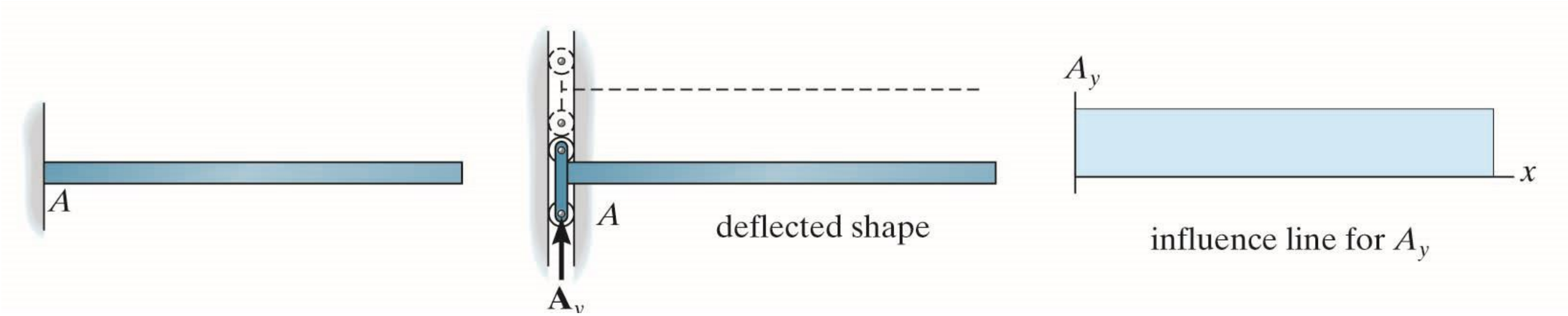




Qualitative Influence Lines

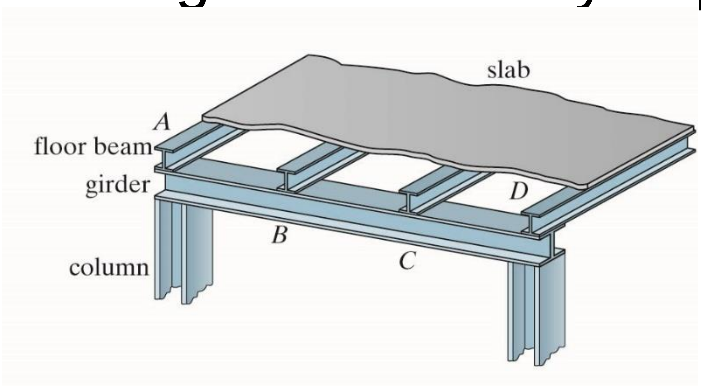
Example 6.9

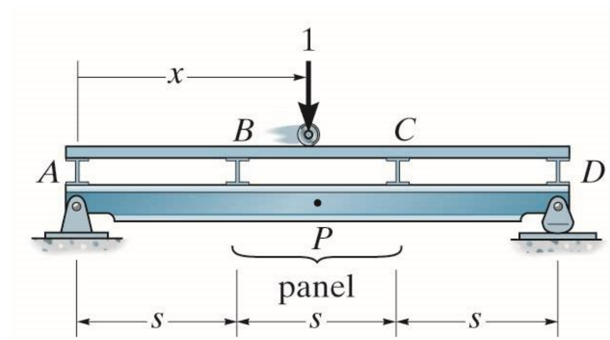
For each beam sketch the influence line for the vertical reaction at *A*.

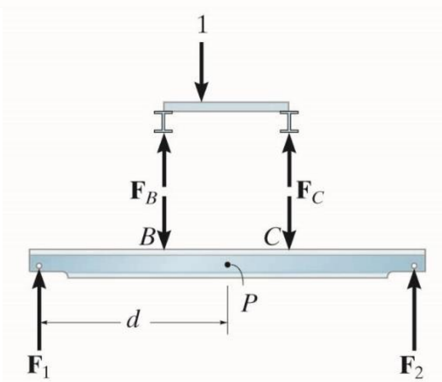


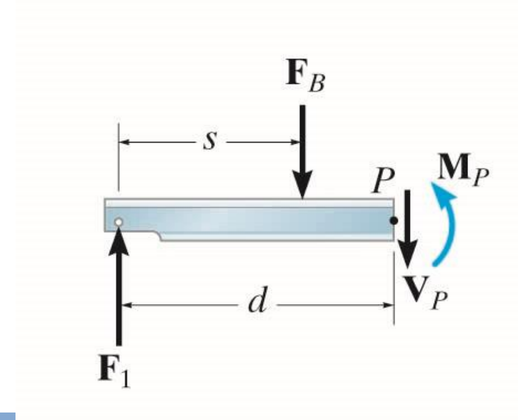
Influence Lines for Floor Girders

- Floor loads are transmitted from slabs to floor beams then to side girders & finally supporting columns









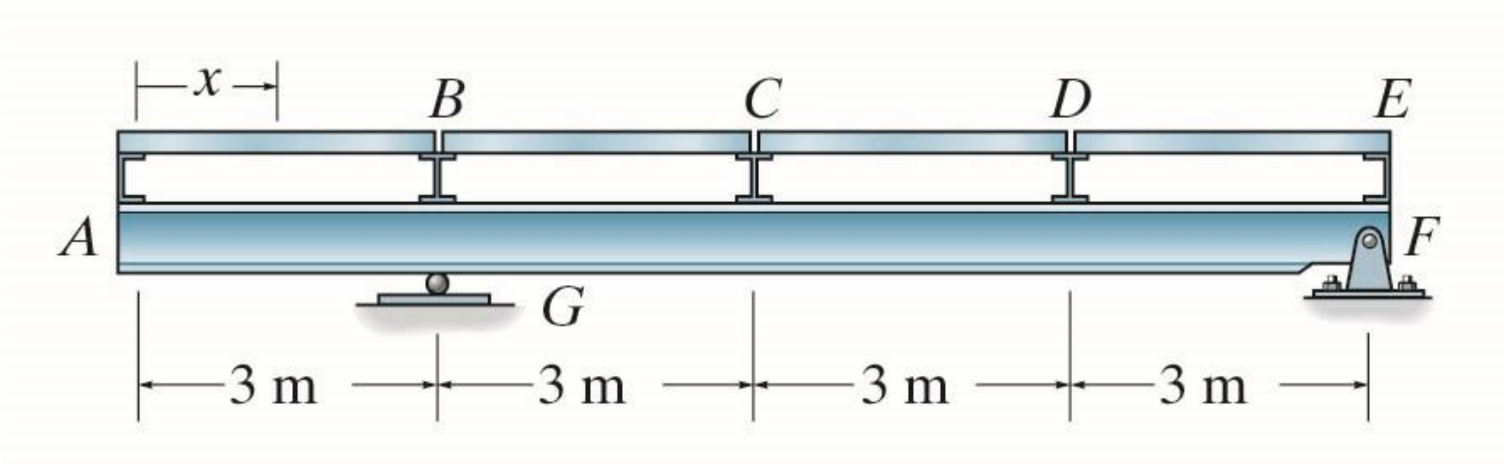
Influence Lines for Floor Girders

- The influence line for a specified point on the girder can be determined using the same statics procedure
- In particular, the value for the internal moment in a girder panel will depend upon where point *P* is chosen for the influence line
- Magnitude of $\mathbf{M}_P$ depends upon the point's location from end of the girder
- Influence lines for shear in floor girders are specified for panels in the girder and not specific points along the girder
- This shear is known as girder shear

Influence Lines for Floor Girders

Example 6.13

Draw the influence line for the shear in panel *CD* of the floor girder.

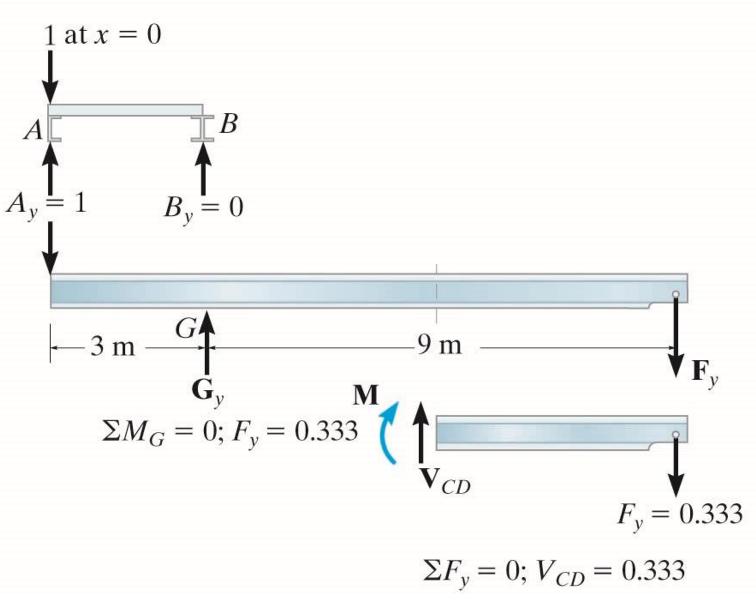


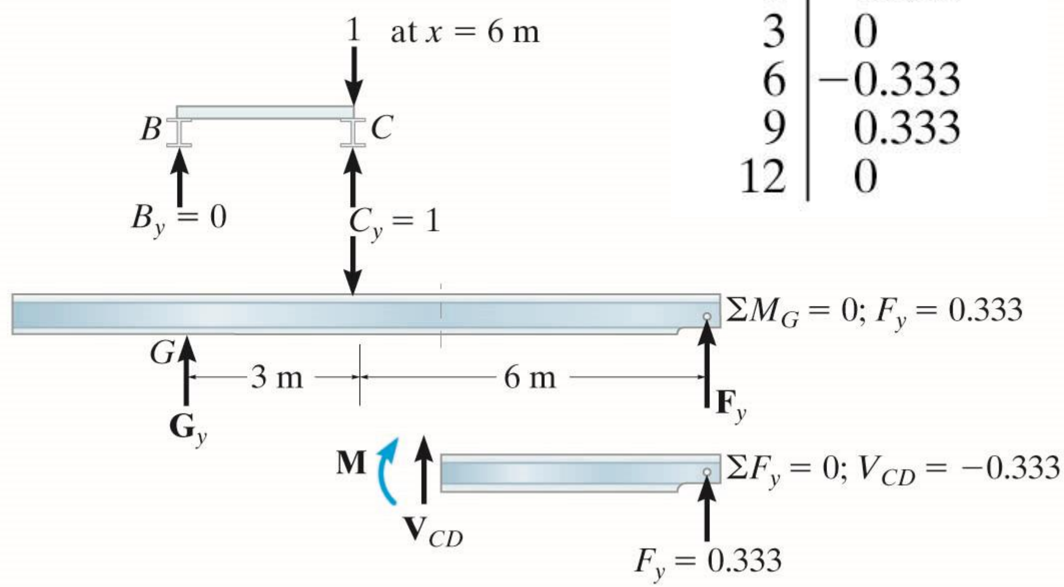
Influence Lines for Floor Girders

Example 6.13 (solution)

The unit load is placed at each floor beam location & the shear in panel *CD* is calculated. Finally a segment of the girder is considered & the internal panel shear V_C calculated.

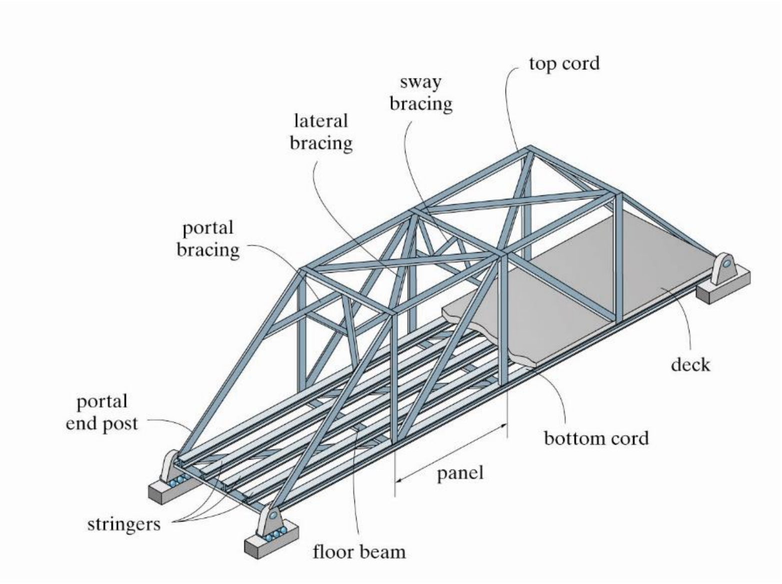
x	V_{CD}
0	0.333
3	0
6	−0.333
9	0.333
12	0





Influence Lines for Trusses

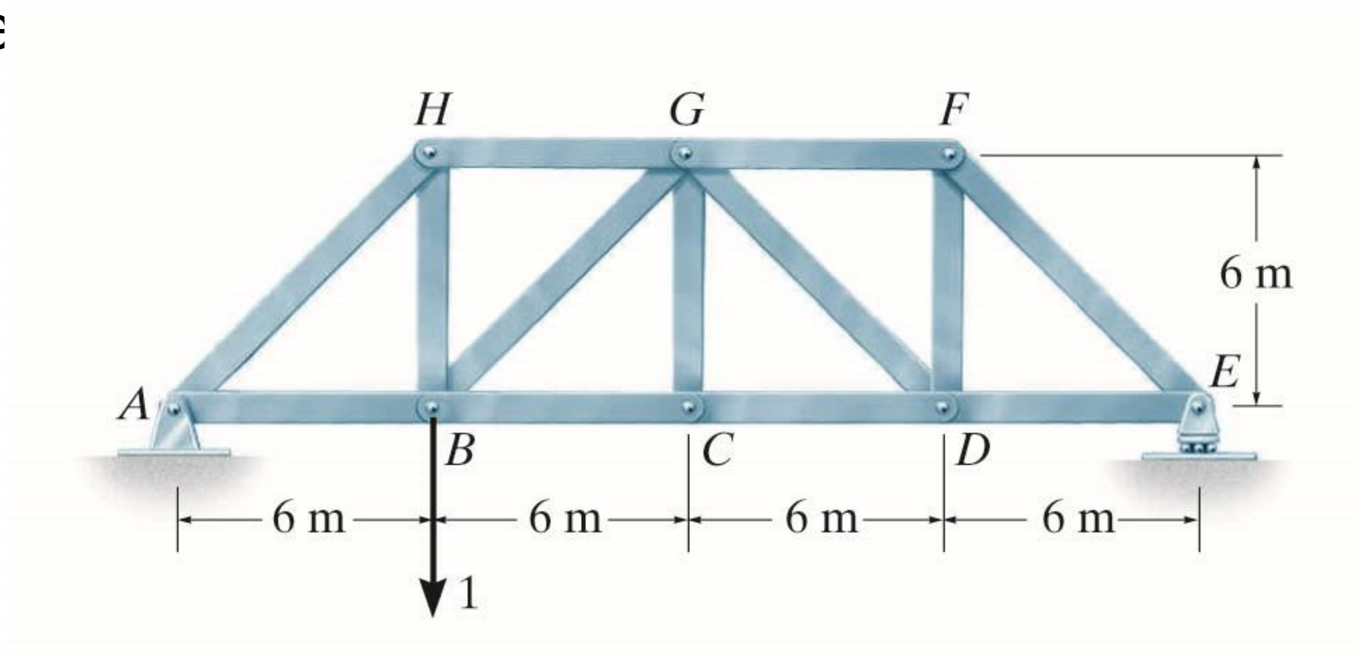
- The loading on the bridge deck is transmitted to stringers which in turn transmit the loading to floor beams and then to joints along the bottom cord
- We can obtain the ordinate values of the influence line for a member by loading each joint along the deck with a unit load and then use the method
  of joints or method of sections to calculate the force in the member



Influence Lines for Trusses

Example 6.15

Draw the influence line for the force in member *GB* of the bridge



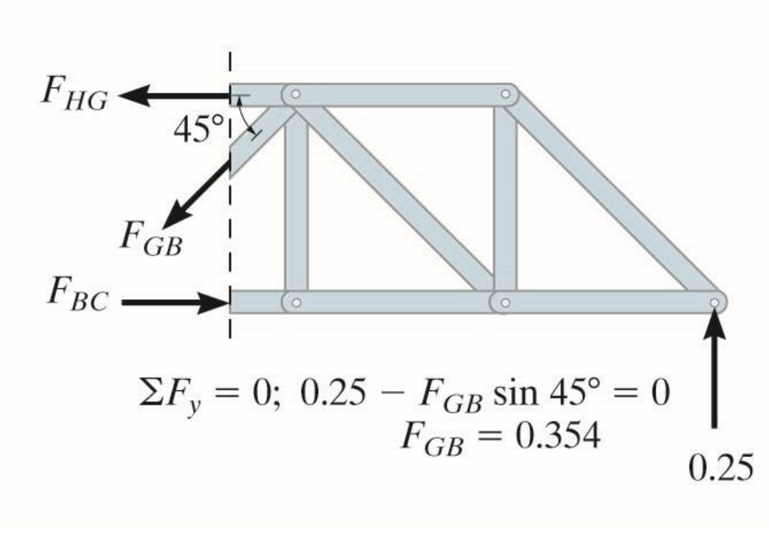
Influence Lines for Trusses

Example 6.15 (solution)

Each successive joint at the bottom cord is loaded with a unit load and the force in member *GB* is calculated using the method of sections.

Since the influence line extends over the entire span of truss, member *GB* is referred to as a primary member.

x	F_{GB}
0	0
6	0.354
12	−0.707
18	−0.354
24	0

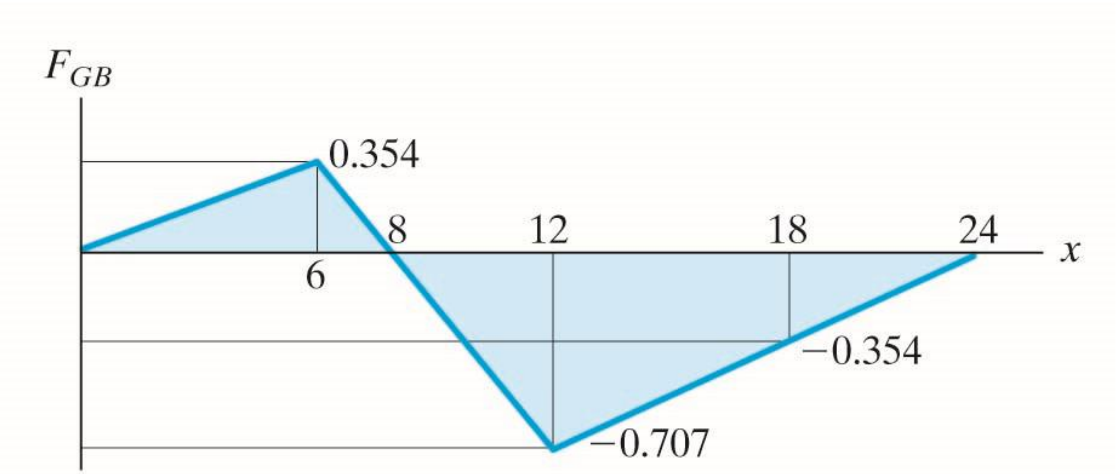


Influence Lines for Trusses

Example 6.15 (solution)

This means that *GB* is subjected to a force regardless of where the bridge deck is loaded.

The point of zero force is determined by similar triangles.



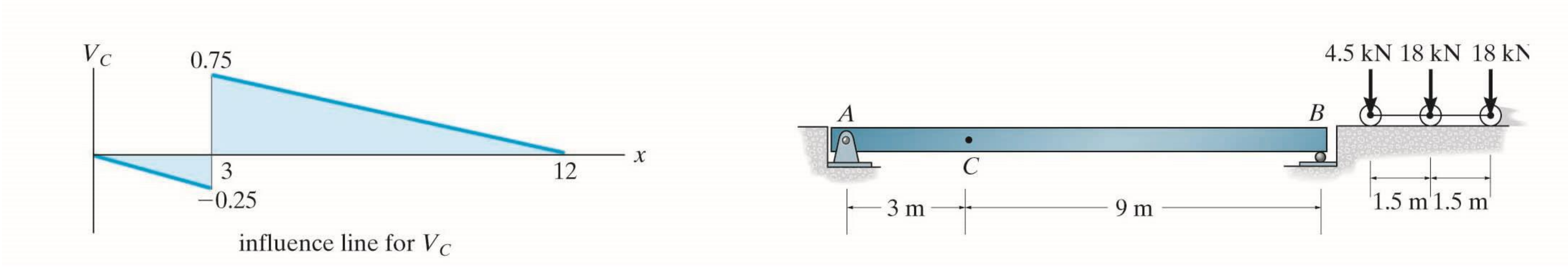
influence line for F_{GB}

Maximum Influence at a Point due to a Seri of Concentrated Loads

- The max effect caused by a live concentrated force is determined by multiplying the peak ordinate of the influence line by the magnitude of the force
- In some cases, e.g. wheel loadings, several concentrated loadings must be placed on structure
- Trial-and-error procedure can be used or a method that is based on the change in function that takes place as the load is moved

Maximum Influence at a Point due to a Series of Concentrated Loads

- Shear
  - Consider the simply supported beam with associated influence line for shear at point *C*
  - The max +ve shear at *C* is to be determined due to the series of concentrated loads moving from right to left
  - Critical loading occurs when one of the loads is placed just to the right of *C*



9

Maximum Influence at a Point due to a Series of Concentrated Loads

- Shear
  - By trial & error, each of three possible cases can therefore be investigated

$$\text{Case 1}: (V_C)_1 = 4.5(0.75) + 18(0.625) + 18(0.5) = 23.63kN$$

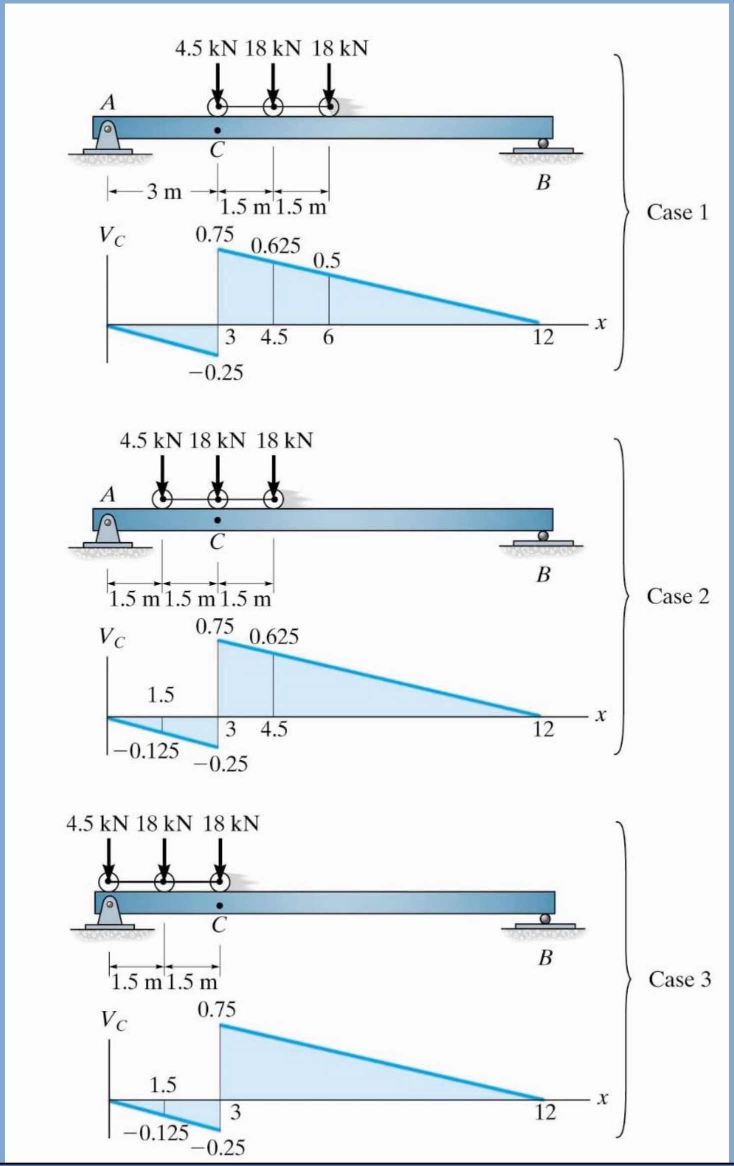
$$\text{Case 2}: (V_C)_2 = 4.5(-0.125) + 18(0.75) + 18(0.625) = 24.19kN$$

$$\text{Case 3}: (V_C)_3 = 4.5(0) + 18(-0.125) + 18(0.75) = 11.25kN$$

Maximum Influence at a Point due to a Series of Concentrated Loads

- Shear
  - Case 2 yields the largest value for V_C and therefore rep the critical loading
  - Investigation of Case 3 is unnecessary since by inspection such an arrangement of loads would yield $(V_C)_3 < (V_C)_2$
  - Trial-and-error can be tedious at times
  - The critical position of the loads can be determined in a more direct manner by finding ΔV which occurs when the loads are moved from Case 1 to 2, then from Case 2 to 3
  - As long as computed ΔV is +ve, the new position will yield a larger shear $$\Delta V = Ps(x_2 - x_1)$$
  - Each movement is investigated until a –ve ΔV is computed

Maximum Influence at a Point due to a Series of Concentrated Loads

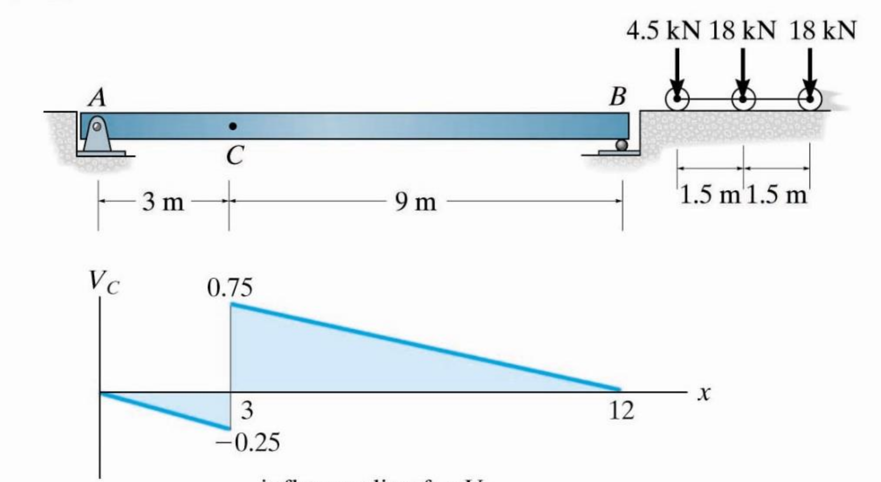


Maximum Influence at a Point due to a Seri of Concentrated Loads

- Shear
  - If the load moves past a point where there is a discontinuity in the influence line, the change in shear is:

$$\Delta V = P(y_2 - y_1)$$

  - Use of above eqn will be illustrated with
  - reference to the beam, loading influence line for V_c shown



influence line for V_C

$$\text{slope}, s = 0.75/(12-3) = 0.75/(9) = 0.0833$$

$$\text{jump at C} = 0.75 + 0.25 = 1$$

Maximum Influence at a Point due to a Series of Concentrated Loads

- Shear
  - Consider the loads moving 1.5 m
  - When this occurs, the 4.5 kN load jumps down (-1) & all the loads move up the slope of the influence line
  - This causes a change of shear

$$\Delta V_{1-2} = 4.5(-1) + [4.5 + 18 + 18](0.0833)(1.5) = +0.563\,\text{kN}$$

  - Since ΔV_{1-2} is +ve, Case 2 will yield a larger value for V_C than case 1

$$\Delta V_{2-3} = 18(-1) + [4.5 + 18 + 18](0.0833)(1.5) = -12.94\,\text{kN}$$

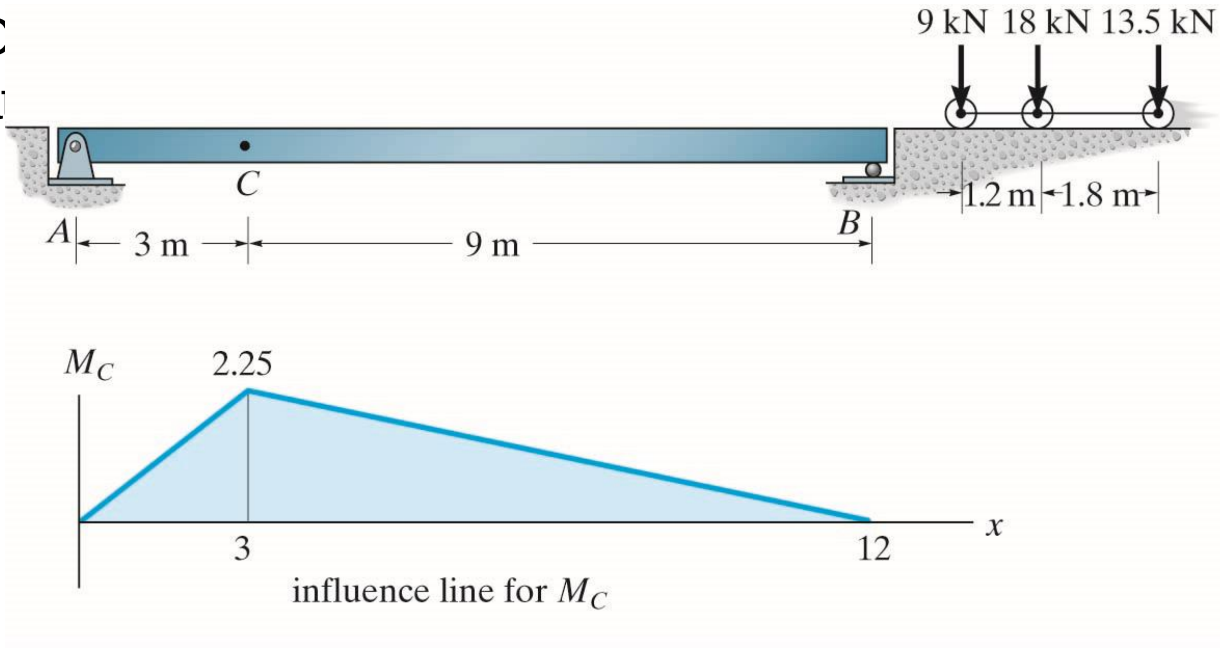
  - Since ΔV_{2-3} is -ve, Case 2 is the position of the critical loading

Maximum Influence at a Point due to a Seri of Concentrated Loads

- Moment

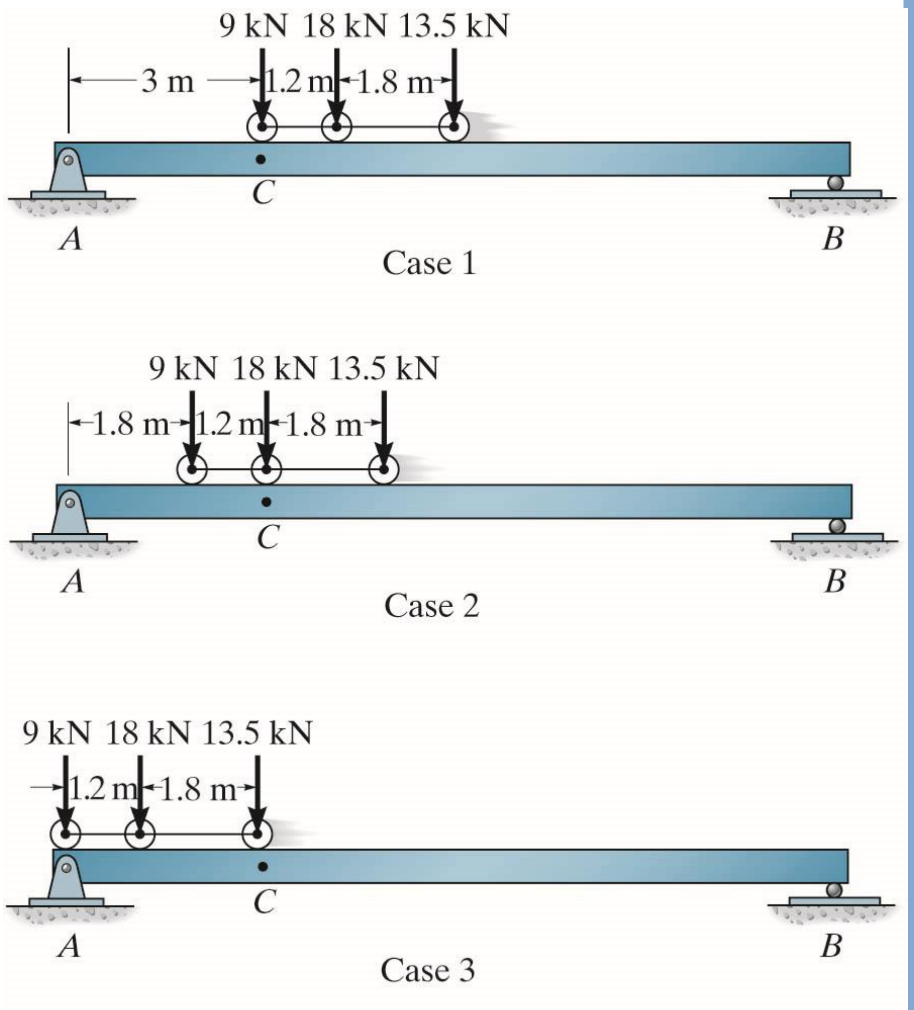
$$\Delta M = Ps(x_2 - x_1)$$

- C … oment
  a…



Maximum Influence at a Point due to a Seri of Concentrated Loads

- Moment
  - When the loads of Case 1 are moved to Case 2, it is observed that the 9 kN load decreases $\Delta M_{1\text{-}2}$
  - Likewise, the 18 kN and 13.5 kN forces cause an increase of $\Delta M_{1\text{-}2}$



Maximum Influence at a Point due to a Series of Concentrated Loads

- Moment

$$\Delta M_{1-2} = 9\left(\frac{2.25}{3}\right)(1.2) + (18+13.5)\left(\frac{2.25}{12-3}\right)(1.2) = 1.35\,\text{kN}\bullet\text{m}$$

- Since $\Delta M_{1\text{-}2}$ is +ve, we compute for loads moved from Cases 2 to 3

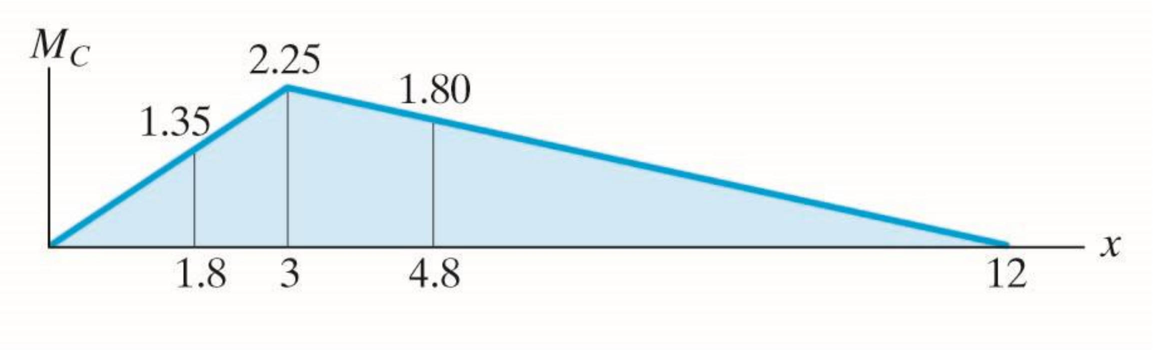
$$\Delta M_{2-3} = -(9+18)\left(\frac{2.25}{3}\right)(1.8) + 13.5\left(\frac{2.25}{12-3}\right)(1.8) = -30.38\,\text{kN}\bullet\text{m}$$

- $\Delta M_{1\text{-}2}$ -ve, the greatest moment at C will occur when the beam is loaded as shown in Case 2

Maximum Influence at a Point due to a Series of Concentrated Loads

- Moment
  - The max moment at *C* is therefore,

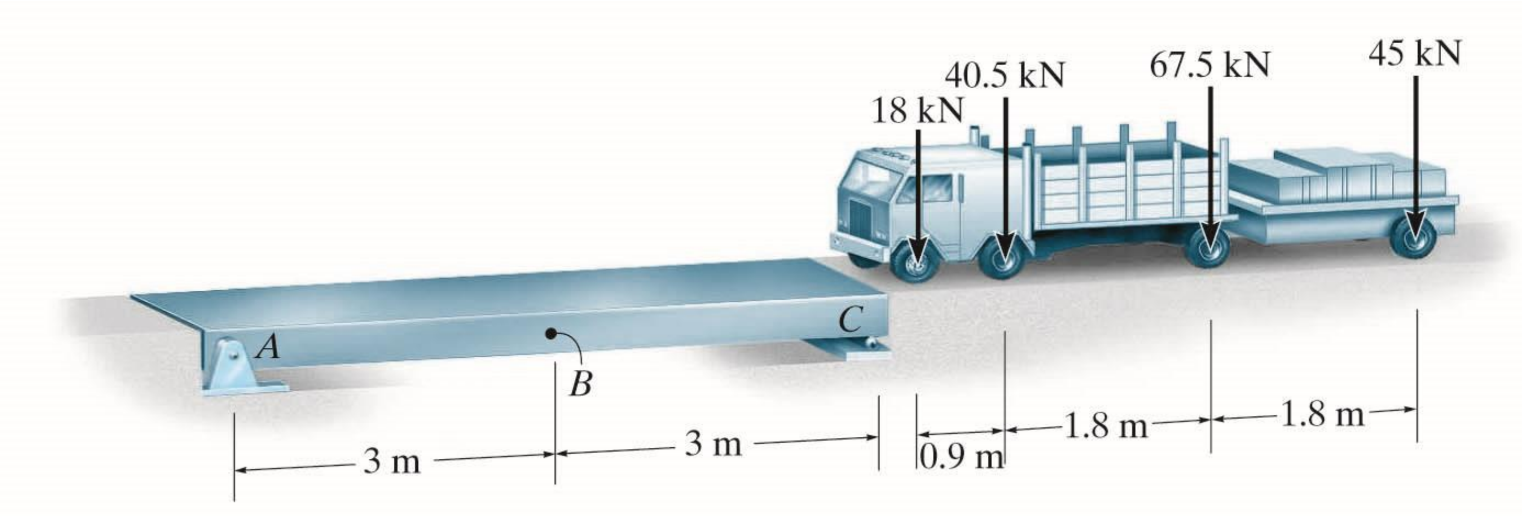
$$(M_C)_{\max} = 9(1.35) + 18(2.25) + 13.5(18) = 77.0\,\text{kN} \bullet \text{m}$$



Maximum Influence at a Point due to a Seri of Concentrated Loads

Example 6.18

Determine the maximum positive shear created at point *B* in the beam due to the wheel loads of the moving truck.



•4
9

Maximum Influence at a Point due to a Series of Concentrated Loads

Example 6.18 (solution)

➔ 0.9 m movement of the 18 kN load

Imagine that the 18 kN load acts just to the right of point *B* so that we obtain its max +ve influence.

Beam segment *BC* is 3 m long, the 4.5 kN load is not as yet on the beam.

When the truck moves 0.9 m to the left, the 18 kN load jumps downward on the influence line 1 unit.

Maximum Influence at a Point due to a Seri of Concentrated Loads

Example 6.18 (solution)

Although the 45 kN load also moves forward 0.9 m, it is still not on the beam. Thus,

$$\Delta V_B = 18(-1) + (18 + 40.5 + 67.5)\left(\frac{0.5}{3}\right)0.9 = +0.9\,\text{kN}$$

➔ 1.8 m movement of the 40.5 kN load

When the 40.5 kN load acts just to the right of *B* & the truck moves 1.8 m to the left, we have

$$\Delta V_B = 40.5(-1) + (18 + 40.5 + 67.5)\left(\frac{0.5}{3}\right)(1.8) + 4.5\left(\frac{0.5}{3}\right)(1.2)$$

$$= +6.3\,\text{kN}$$

Maximum Influence at a Point due to a Series of Concentrated Loads

Example 6.18 (solution)

➔ 1.8 m movement of the 67.5 kN load

If the 67.5 kN load is positioned just to the right of *B* & then the truck moves 1.8 m to the left, the 18 kN load moves only 0.3 m until it is off the beam.

$$\Delta V_B = 67.5(-1) + 18\left(\frac{0.5}{3}\right)(0.3) + 40.5\left(\frac{0.5}{3}\right)(1.2) + (67.5+45)\left(\frac{0.5}{3}\right)(1.8)$$

$$= -24.8\,\text{kN}$$

Likewise, the 40.5 kN load moves only 1.2 m until it is off the beam

Maximum Influence at a Point due to a Series of Concentrated Loads

Example 6.18 (solution)

➔ 1.8 m movement of the 67.5 kN load (cont'd)

Since ΔV_B is -ve, the correct position of the loads occur when 67.5 kN is just to the right of *B*.

$$(V_B)_{\max} = 18(-0.05) + 40.5(-0.2) + 67.5(0.5) + 45(0.2)$$
$$= 33.8\,\text{kN}$$

In practice, one also has to consider motion of the truck from left to right & then choose the max value between these 2 situations.

Reference

Hibbeler, R.C (2009) *Mechanics of Materials*, Pearson, Malaysia.

ACKNOWLEDGEMENT

All the pictures and examples are referred from Pearson Education South Asia Pte Ltd and Hibbeler's textbook for the education purposes.