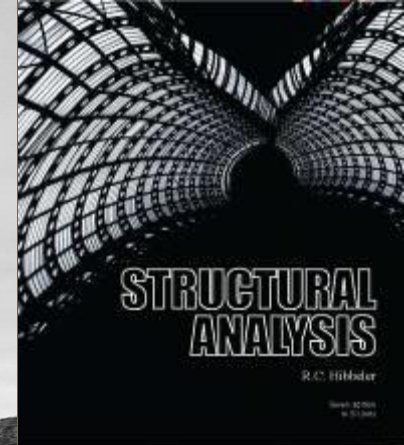




Chapter 8: Deflections

CEE 3222 THEORY OF STRUCTURES

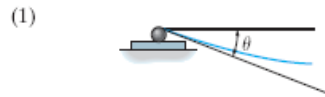
Dr. Lenganji Simwanda, PhD

Deflection- Displacement & Rotation

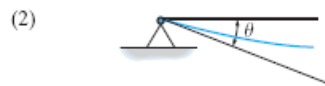
- Deflections may be due to **loads, temperature, fabrication errors or settlement**
- **Linear elastic behavior:** linear stress-strain relationship and small deflection
- **Need to:**
 - ▣ Plot qualitative deflection shapes before/after the analysis
 - ▣ Calculate deflections at any location of a structure

Deflection Diagrams/Shapes

TABLE 8-1



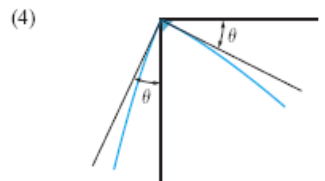
$\Delta = 0$
roller or rocker



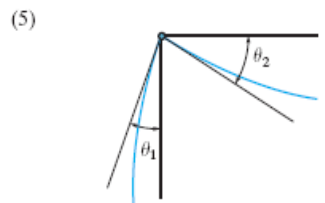
$\Delta = 0$
pin



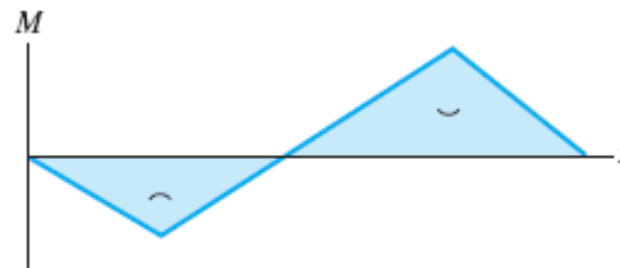
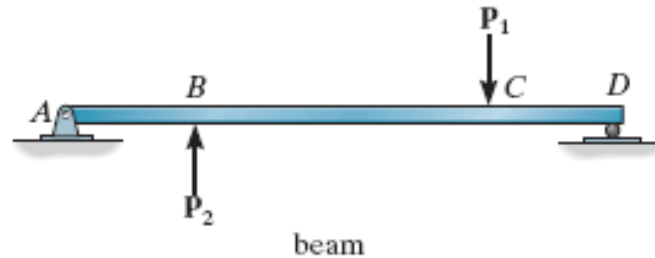
$\Delta = 0$
 $\theta = 0$
fixed support



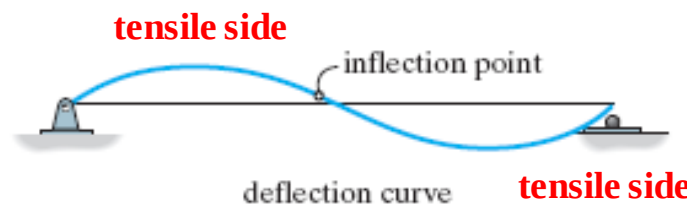
fixed-connected joint



pin-connected joint



moment diagram



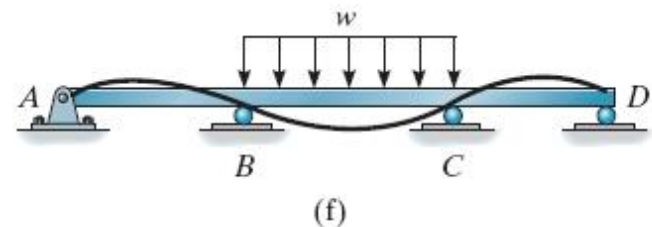
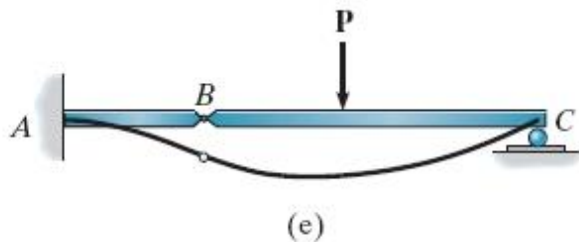
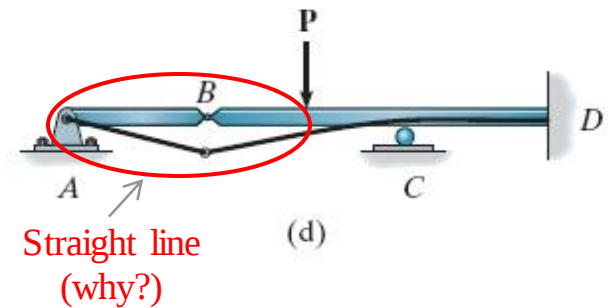
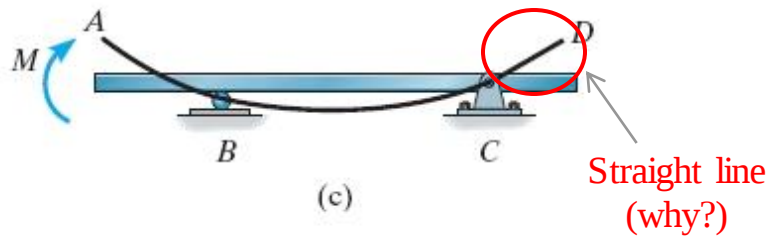
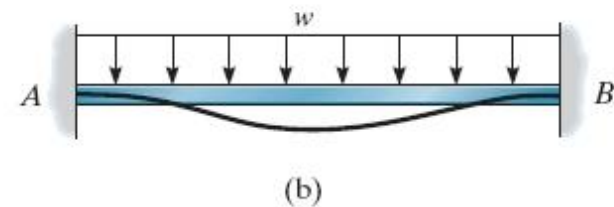
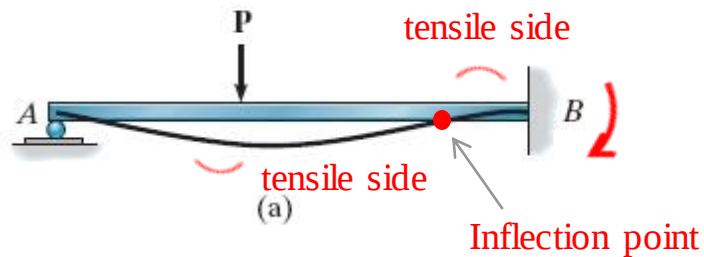
$$\frac{M}{EI} = \text{curvature}$$



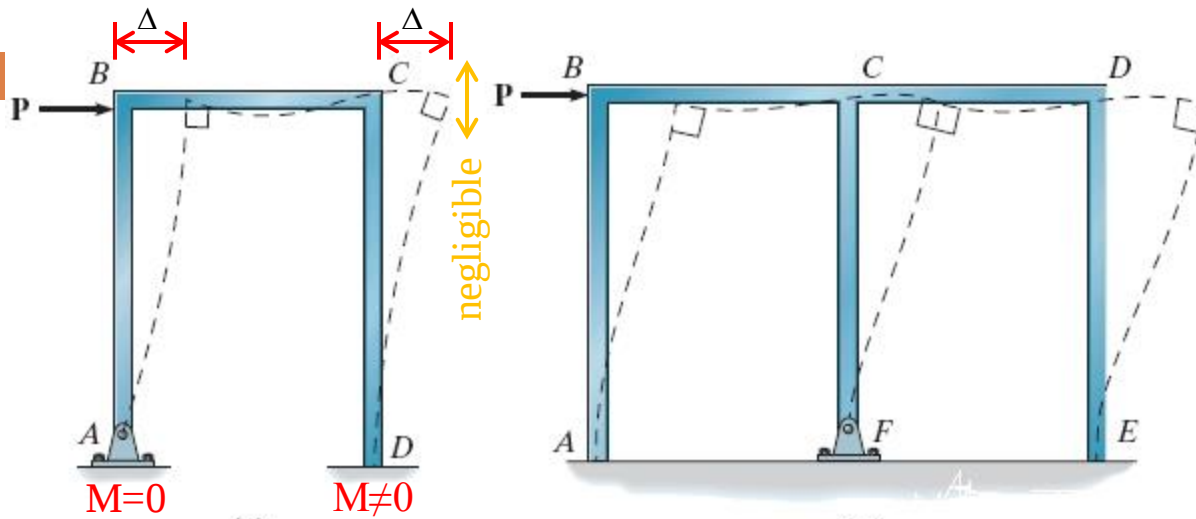
Example 8.1

Draw the deflected shape of each of the beams.

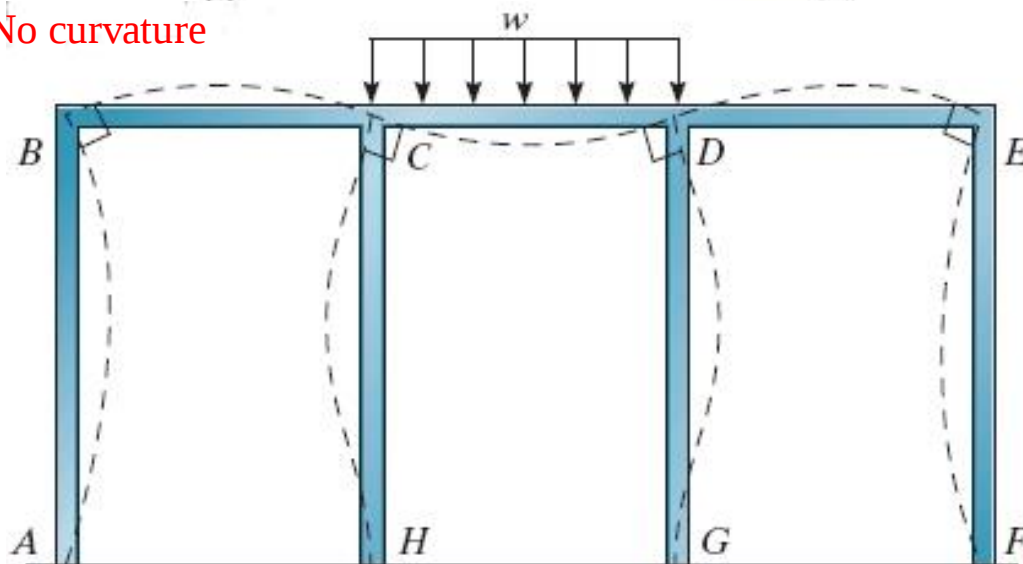
Need to show 1st order (slope) and 2nd order (curvature) information



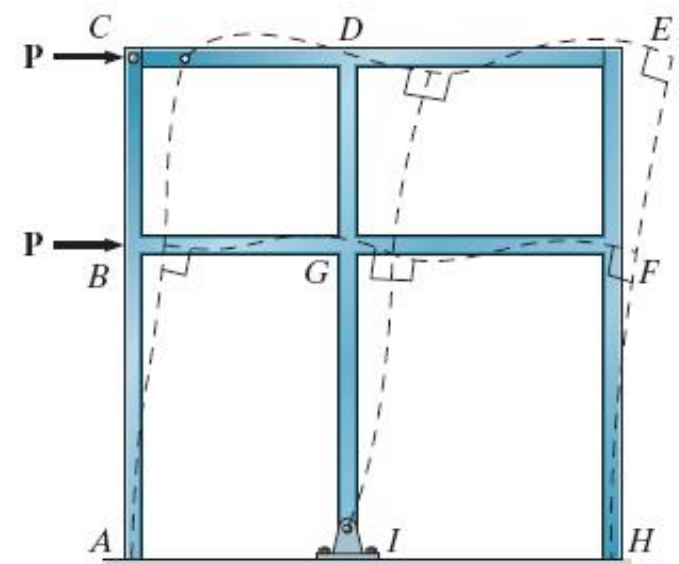
Example 8.2



No curvature



All members are axially inextensible!



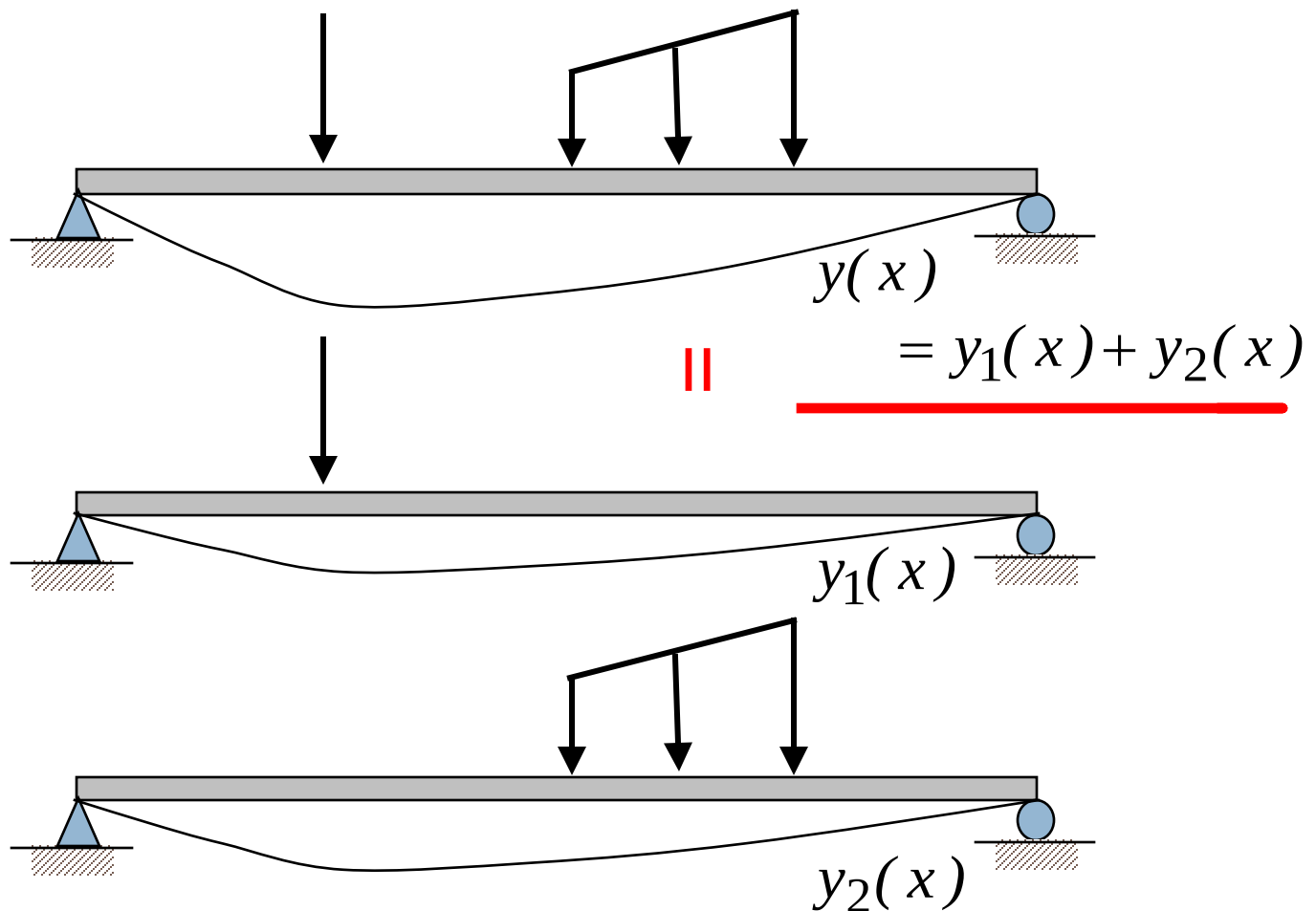
(d)

Calculation of Deflections

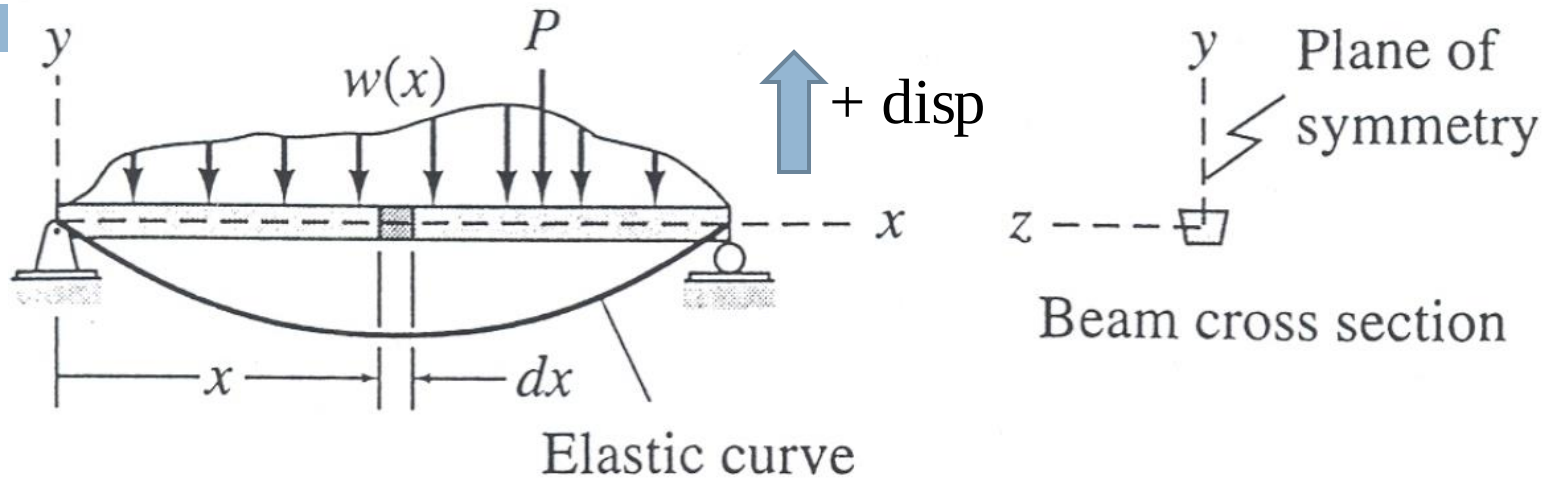
- Direct integration
- Moment-area method
- Conjugate-beam method
- Energy methods (in Chapter 9)

Calculation of Deflections

- Note: Superposition can be used for linear structures



Direct Integration

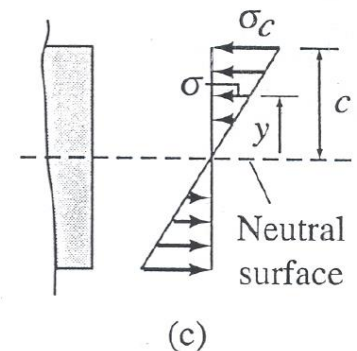
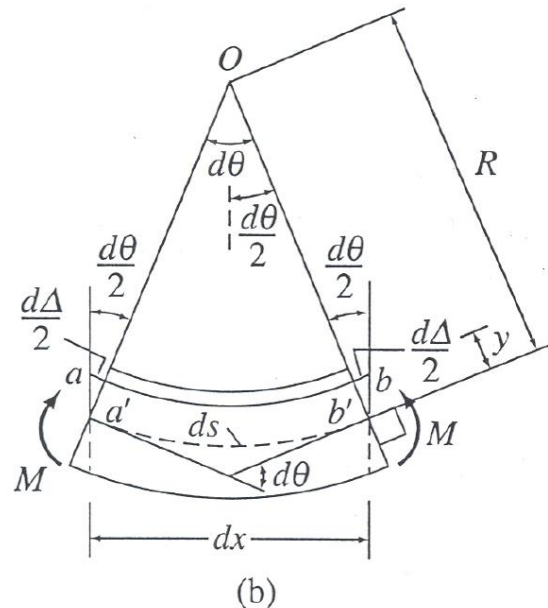


Bernoulli-Euler beam

$$\frac{d^2 y}{dx^2} = \frac{M}{EI} \quad \frac{d\theta}{dx} = \frac{M}{EI}$$

$$y = \iint \frac{M}{EI} dx dx \quad \theta = \int \frac{M}{EI} dx$$

apply boundary conditions

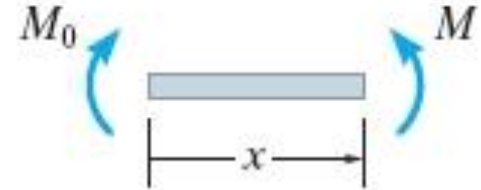


Example 8.3

The cantilevered beam is subjected to a couple moment M_o at its end. Determine the eqn of the elastic curve. EI is constant.



(a)



(b)

$$EI \frac{d^2v}{dx^2} = M_o$$

$$EI \frac{dv}{dx} = M_o x + C_1$$

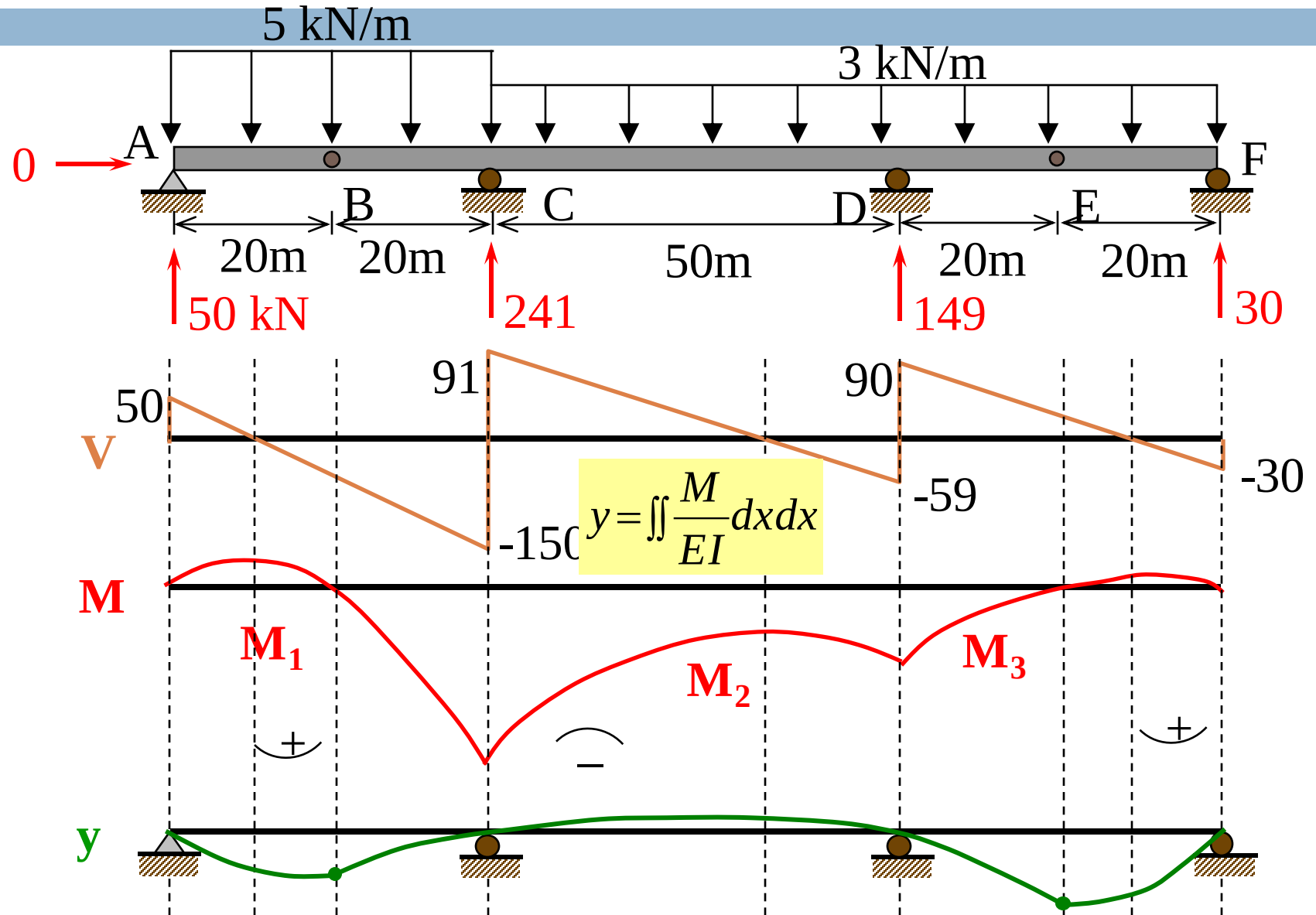
$$EI v = \frac{M_o x^2}{2} + C_1 x + C_2$$

at $x = 0$
 $dv/dx = 0$ & $v = 0$
 $C_1 = C_2 = 0.$

$$\theta = \frac{M_o x}{EI}; \quad v = \frac{M_o x^2}{2EI}$$

Max $\theta_A = \frac{M_o L}{EI}; \quad v_A = \frac{M_o L^2}{2EI}$

Direct Integration



Moment-Area Theorems

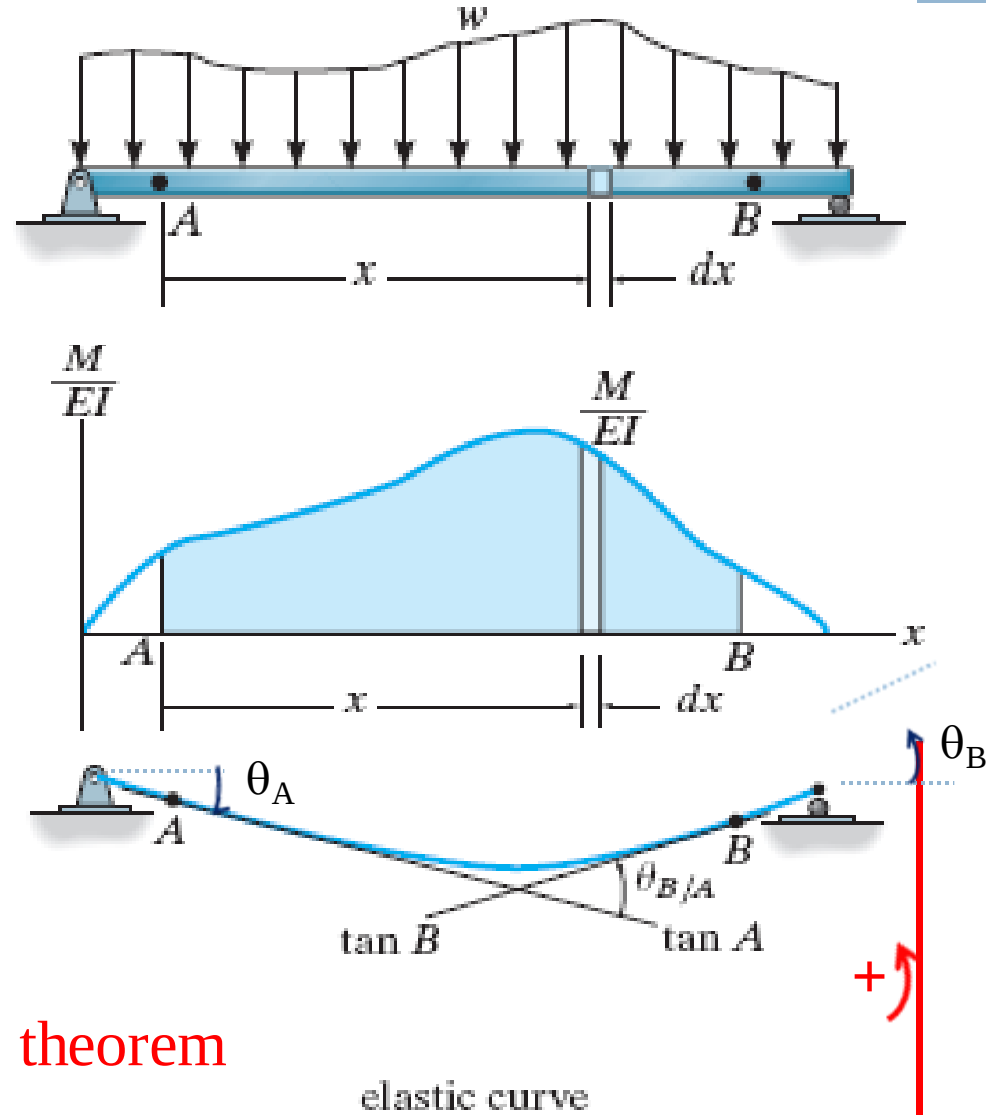
$$\frac{d^2 y}{dx^2} = \frac{M}{EI} \quad \frac{d\theta}{dx} = \frac{M}{EI}$$

$$d\theta = \left(\frac{M}{EI} \right) dx$$

$$\theta_B - \theta_A = \int_A^B \frac{M}{EI} dx$$

$\theta_{B/A}$

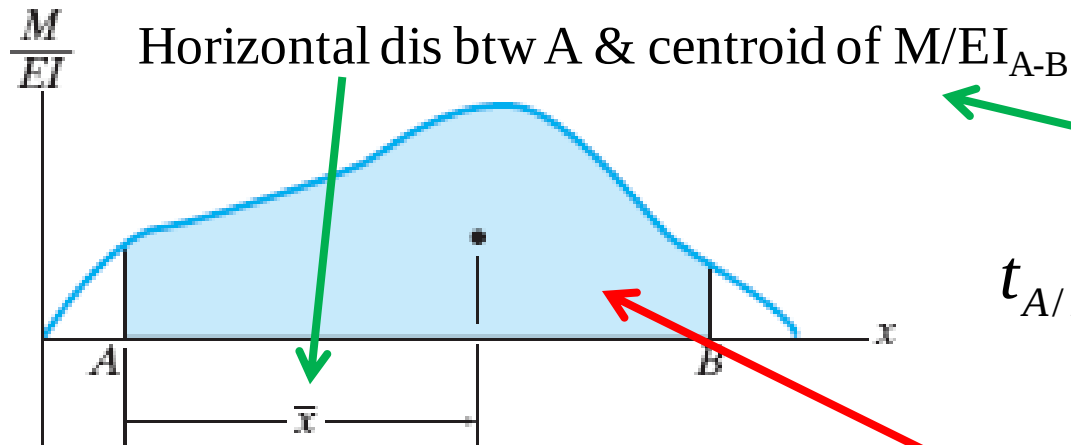
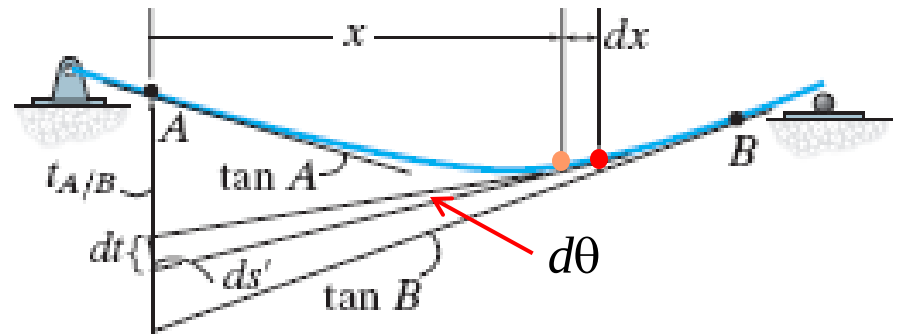
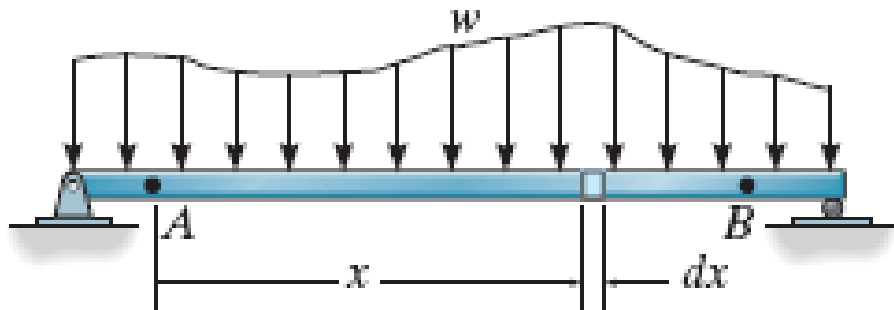
1st moment-area theorem



Moment-Area Theorems

□ 2nd moment-area theorem

$$\frac{d\theta}{dx} = \frac{M}{EI}$$

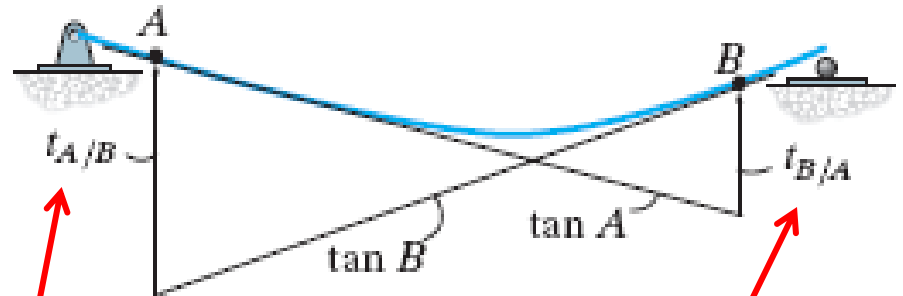
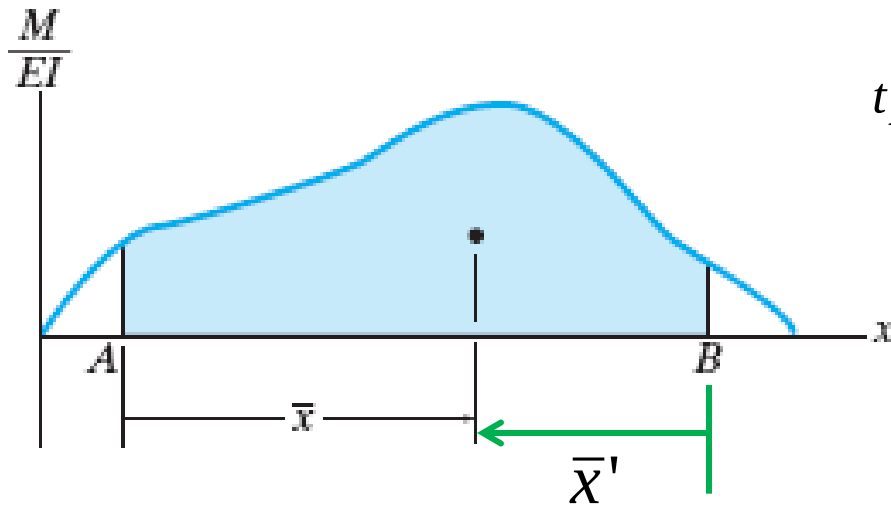
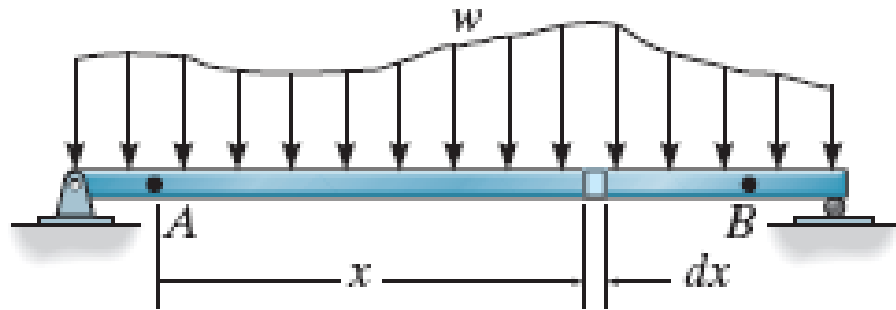


$$dt = x d\theta = x \frac{M}{EI} dx$$

$$t_{A/B} = \int_A^B x \frac{M}{EI} dx = \bar{x} \int_A^B \frac{M}{EI} dx$$

M/EI area btw A & B

Moment-Area Theorems

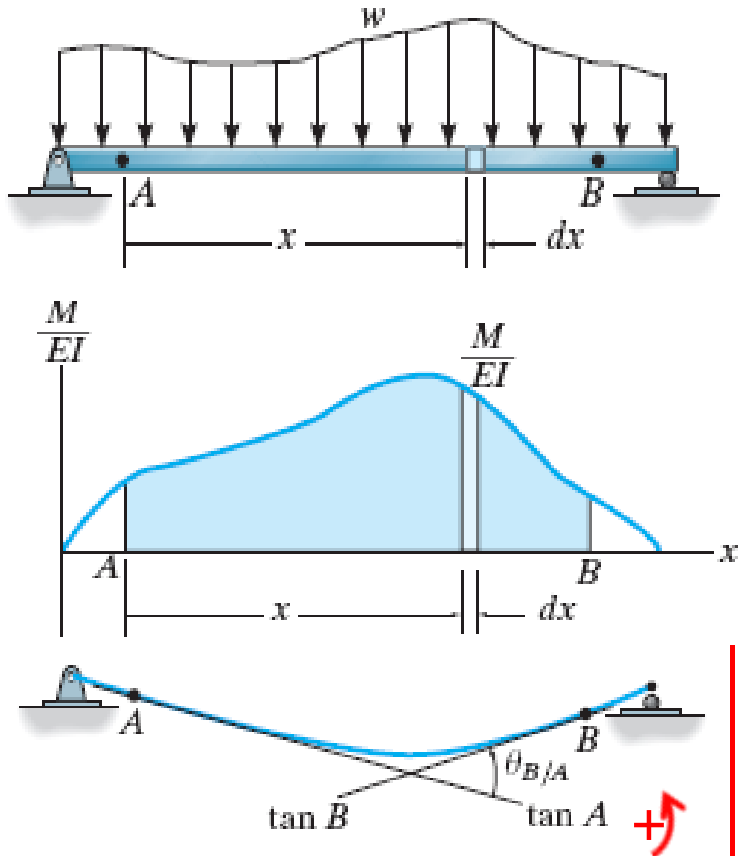


$$t_{A/B} = \int_A^B x \frac{M}{EI} dx = \bar{x} \int_A^B \frac{M}{EI} dx$$

$$t_{B/A} = \int_B^A x \frac{M}{EI} dx = \bar{x}' \int_B^A \frac{M}{EI} dx$$

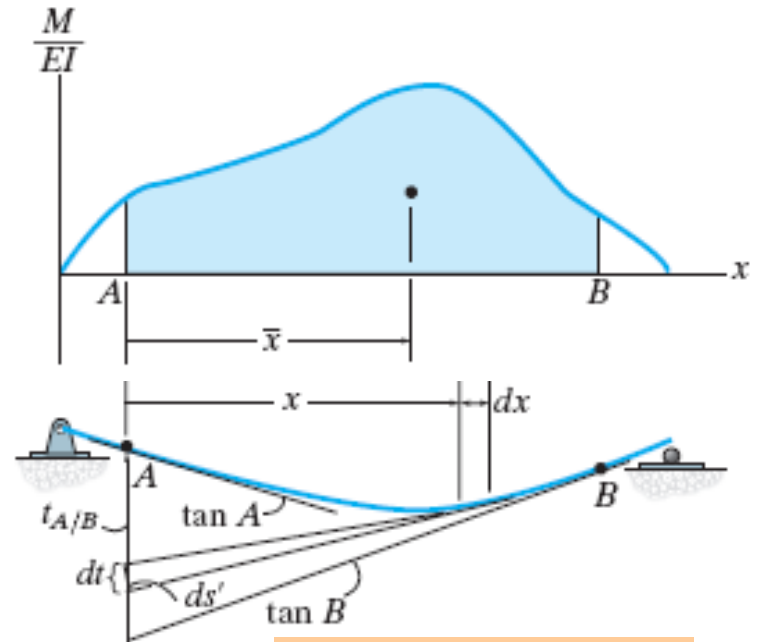
Moment-Area Theorems

1st moment-area theorem



$$\theta_{B/A} = \int_A^B \frac{M}{EI} dx$$

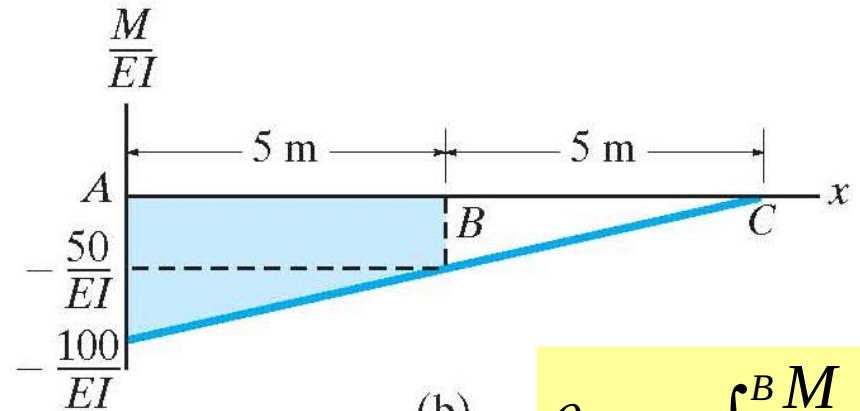
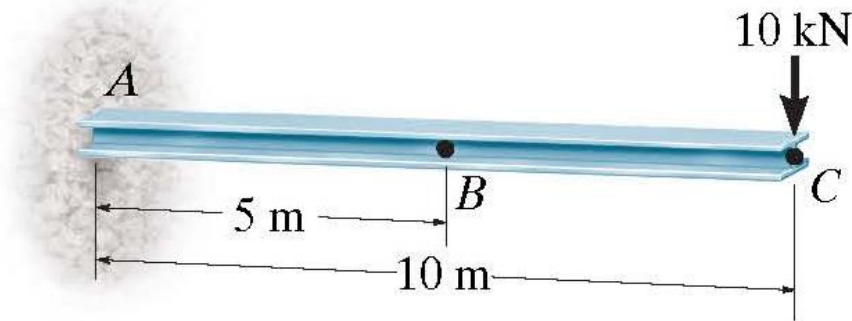
2nd moment-area theorem



$$t_{A/B} = \bar{x} \int_A^B \frac{M}{EI} dx$$

Example 8.5

Determine the slope at points B & C of the beam. Take $E = 200\text{GPa}$, $I = 360(10^6)\text{mm}^4$



(b)

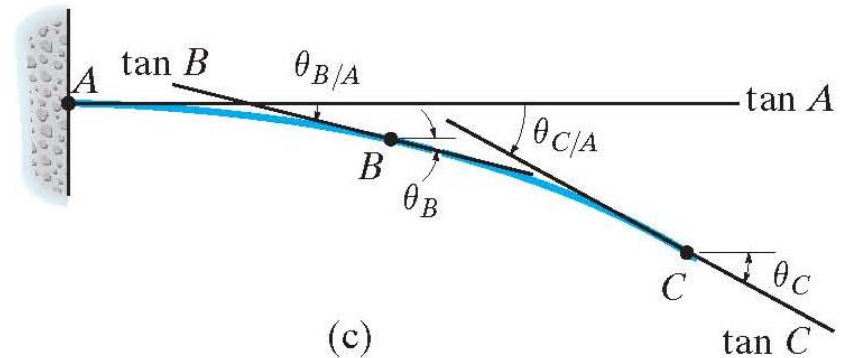
$$\theta_{B/A} = \int_A^B \frac{M}{EI} dx$$

$$\theta_B = \theta_{B/A}; \quad \theta_C = \theta_{C/A}$$

$$\theta_B = \theta_{B/A}$$

$$\begin{aligned} &= -\left(\frac{50\text{kNm}}{EI}\right)(5\text{m}) - \frac{1}{2}\left(\frac{100\text{kNm}}{EI} - \frac{50\text{kNm}}{EI}\right)(5\text{m}) \\ &= -\frac{375\text{kNm}^2}{EI} = -0.00521\text{rad} \end{aligned}$$

$$\theta_C = \theta_{C/A} = -0.00694\text{ rad}$$



(c)

Conjugate-Beam Method

□ Mathematical analogy

$\frac{M}{EI}$ -slope-deflection	Load-shear-moment
$\frac{d\theta}{dx} = \frac{M}{EI}$	$\frac{dV}{dx} = w$
$\frac{dy}{dx} = \theta$	$\frac{dM}{dx} = V$
$\frac{d^2y}{dx^2} = \frac{M}{EI}$	$\frac{d^2M}{dx^2} = w$

Conjugate-Beam Method

□ Mathematical equivalence

$\frac{M}{EI}$ -slope-deflection	Load-shear-moment	
$\frac{M}{EI}$		
θ		
y		
Actual beam	Conjugate beam	

$\frac{M}{EI}$ -slope-deflection	Load-shear-moment
$\frac{d\theta}{dx} = \frac{M}{EI}$	$\frac{dV}{dx} = w$
$\frac{dy}{dx} = \theta$	$\frac{dM}{dx} = V$
$\frac{d^2y}{dx^2} = \frac{M}{EI}$	$\frac{d^2M}{dx^2} = w$

Conjugate-Beam Method

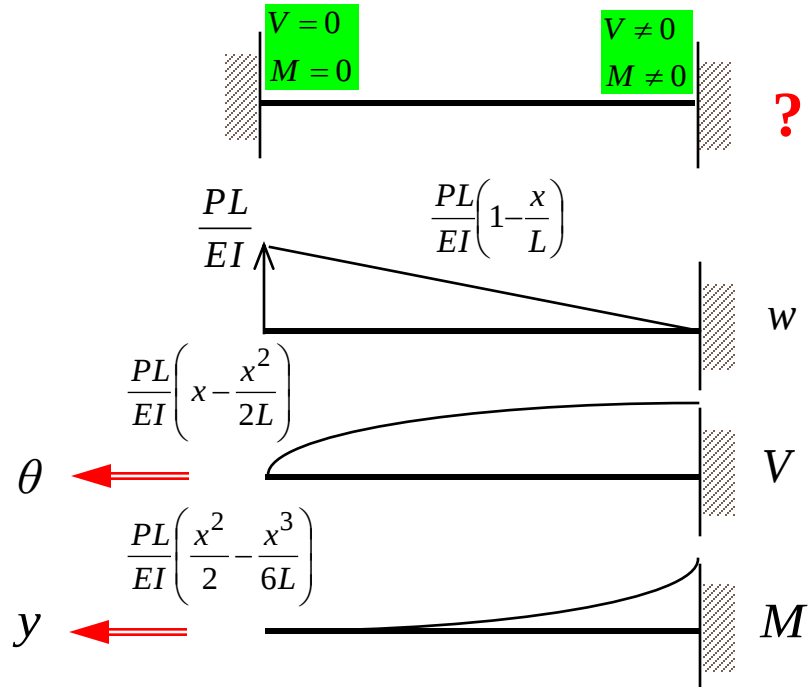
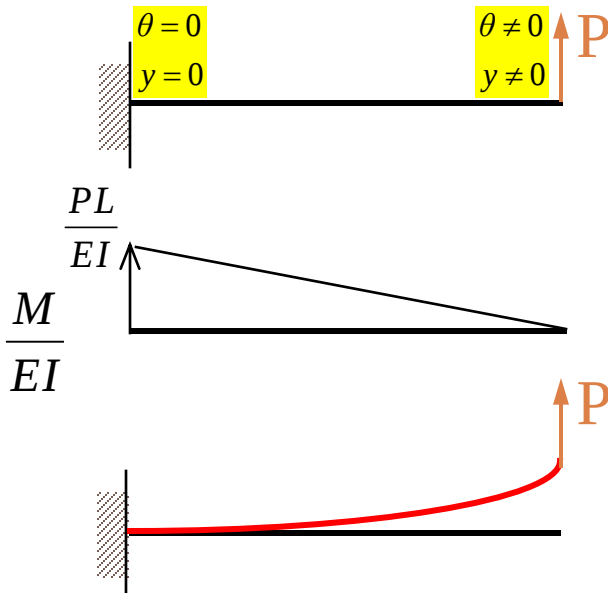
$\frac{M}{EI}$ -slope-deflection	Load-shear-moment
$\frac{M}{EI}$	w
θ	V
y	M

Actual beam

Actual beam

Conjugate beam

Conjugate beam



Conjugate-Beam Method

$\frac{M}{EI}$ -slope-deflection	Load-shear-moment
$\frac{M}{EI}$	w
θ	V
Δ or y	M

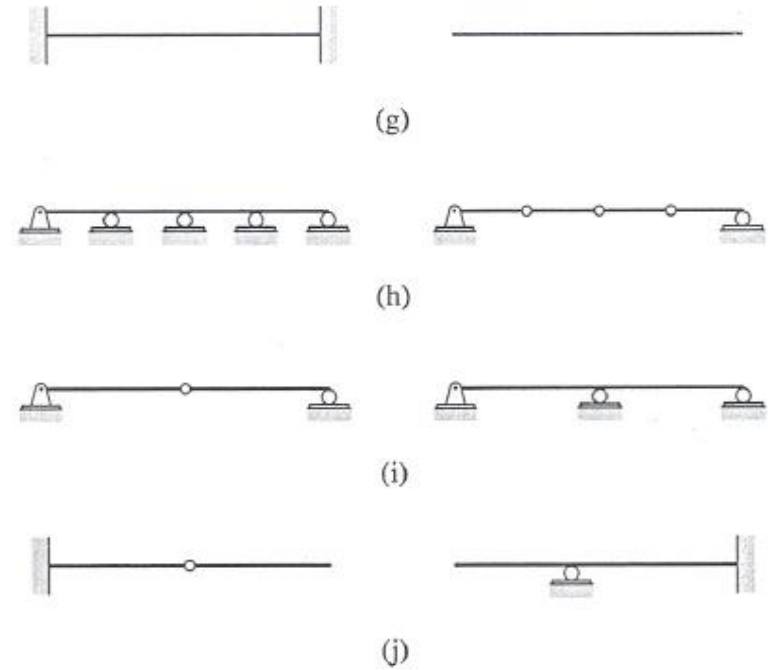
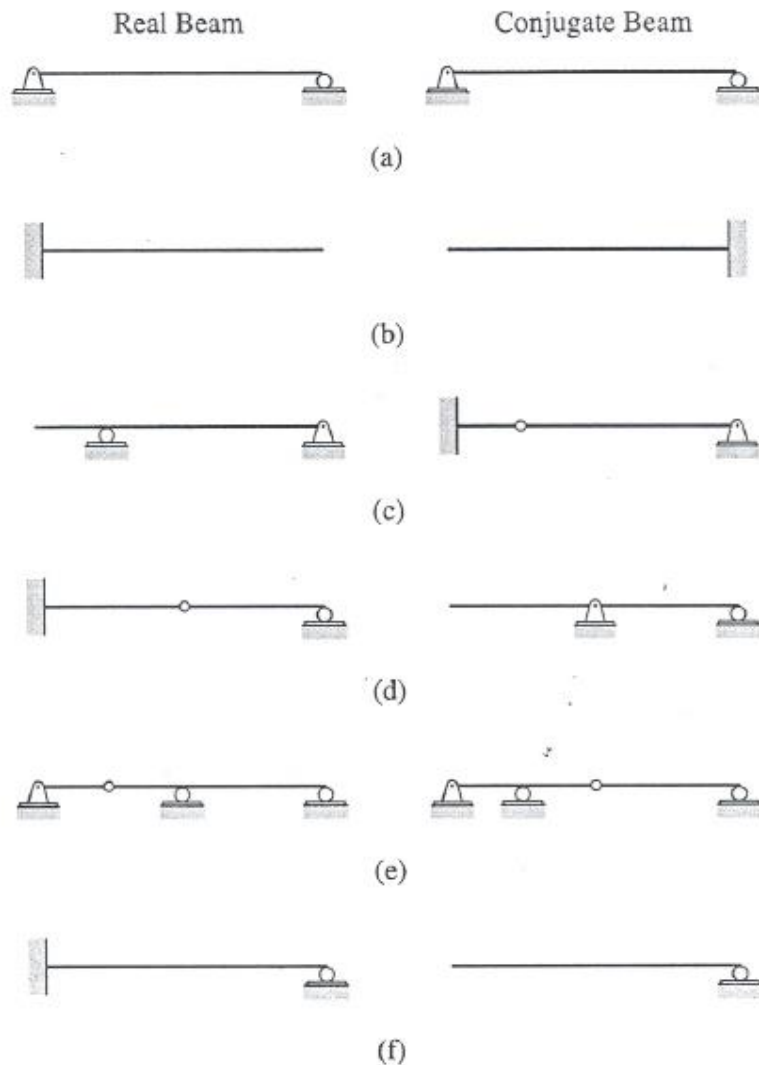
Actual beam

Conjugate beam

TABLE 8-2

Real Beam		Conjugate Beam
1)	θ $\Delta = 0$  pin	V $M = 0$  pin
2)	θ $\Delta = 0$  roller	V $M = 0$  roller
3)	$\theta = 0$ $\Delta = 0$  fixed	$V = 0$ $M = 0$  free
4)	θ Δ  free	V M  fixed
5)	θ $\Delta = 0$  internal pin	V $M = 0$  hinge
6)	θ $\Delta = 0$  internal roller	V $M = 0$  hinge
7)	θ Δ  hinge	V M  internal roller

Conjugate-Beam Method



Conjugate beams are **mathematical/imaginative** beams

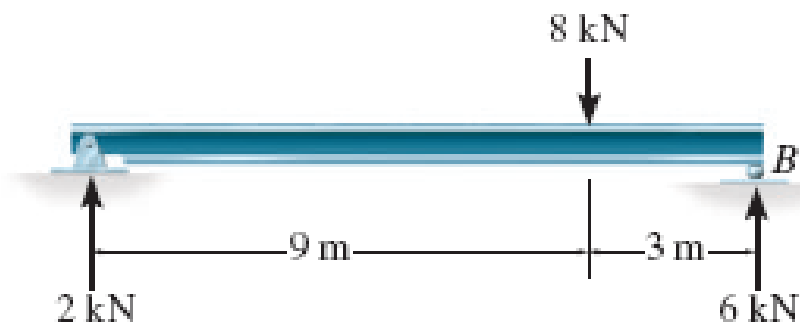
No need to worry about their stability

Just use EQ concept to obtain V and M from w

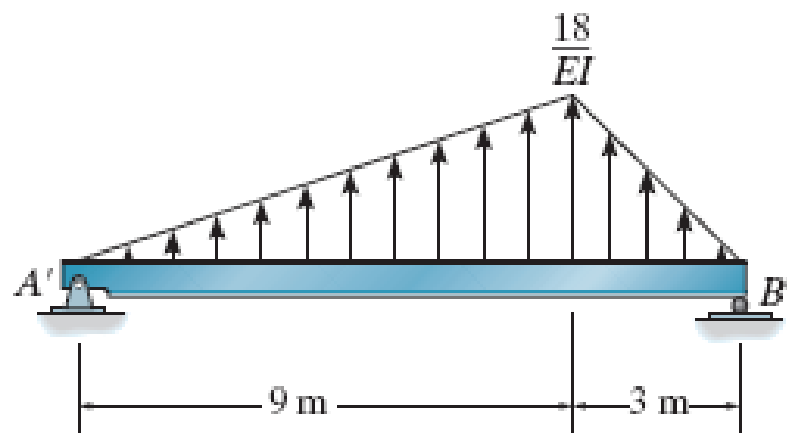
Example 8.5

Determine the max deflection of the steel beam. The reactions have been computed.

Take $E = 200\text{GPa}$, $I = 60(10^6)\text{mm}^4$



real beam



conjugate beam

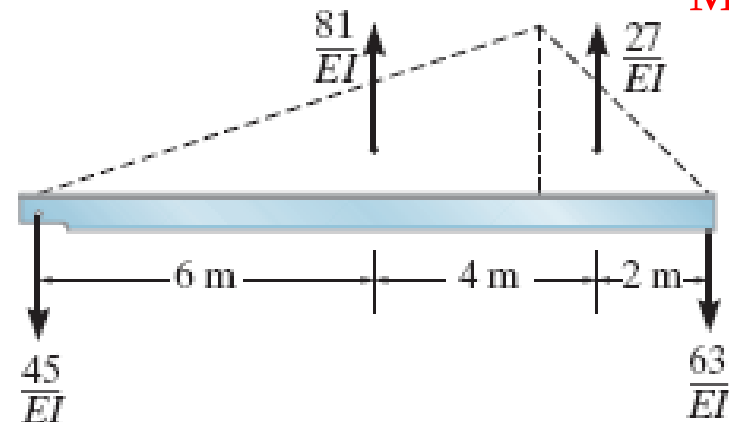
Max disp

$\frac{M}{EI}$ -slope-deflection	Load-shear-moment
$\frac{M}{EI}$	w
θ	V
y	M

Actual beam

Conjugate beam

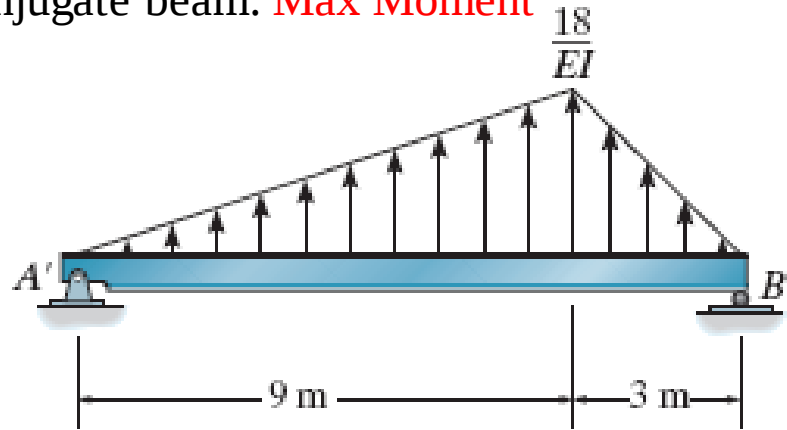
Max M



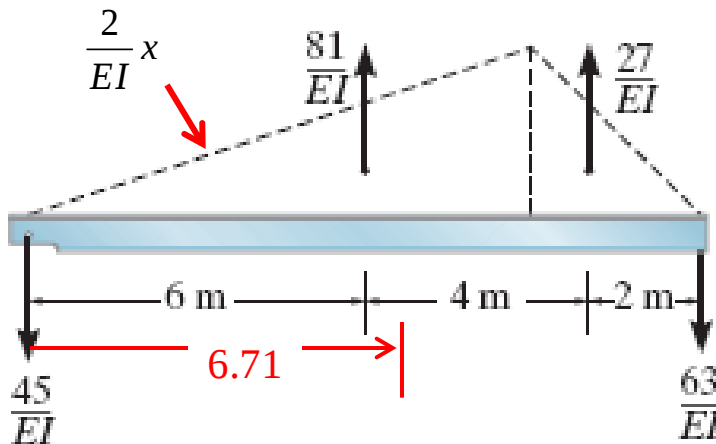
external reactions

Solution

Conjugate beam: **Max Moment**



conjugate beam



external reactions

Note: $\frac{dM}{dx} = V = 0$

when $M = M_{\max}$

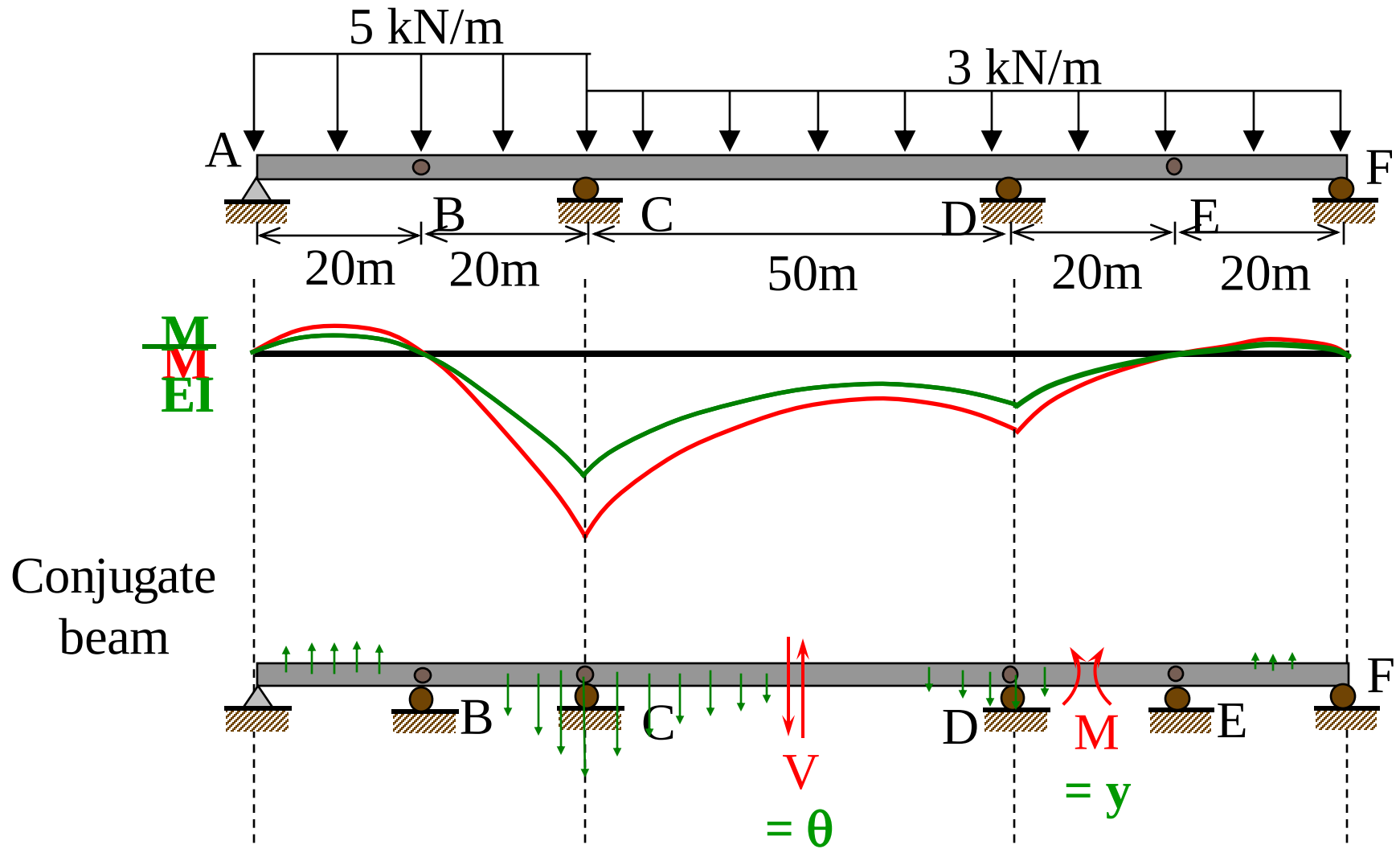
$$-\frac{45}{EI} + \frac{1}{2} \left(\frac{2x}{EI} \right) x = 0$$

$$x = 6.71m \quad (0 \leq x \leq 9m) \quad OK$$

$$\frac{45}{EI}(6.71) - \left[\frac{1}{2} \left(\frac{2(6.71)}{EI} \right) 6.71 \right] \frac{1}{3}(6.71) + M' = 0$$

$$\begin{aligned} \Delta_{\max} = M' &= -\frac{201.2 \text{ kNm}^3}{EI} \\ &= \frac{-201.2 \text{ kNm}^3}{[200(10^6) \text{ kN/m}^2][60(10^6) \text{ mm}^4 (1\text{m}^4 / (10^3)^4 \text{ mm}^4)]} \\ &= -0.0168 \text{ m} = -16.8 \text{ mm} \end{aligned}$$

Conjugate-Beam



Summary

- Can you plot a qualitative deflection shape for a structure under loads (need to show 1st & 2nd order information)?
- Can you calculate structural deformation (displacement & rotation) using
 - ▣ Direct integration?
 - ▣ Moment-area principle?
 - ▣ Conjugate-beam method?