THE UNIVERSITY OF ZAMBIA

SCHOOL OF ENGINEERING

CIVIL AND ENVIRONMENTAL ENGINEERING

CEE 3222

THEORY OF STRUCTURES

TUTORIAL SHEET 2 SOLUTIONS

(2023)

DR. LENGANJI SIMWANDA

Chapter 4: Internal Loadings Developed in Structural Members

QUESTION 1.1

Determine the axial forces, shears, and bending moments at points A and B of the structure shown.

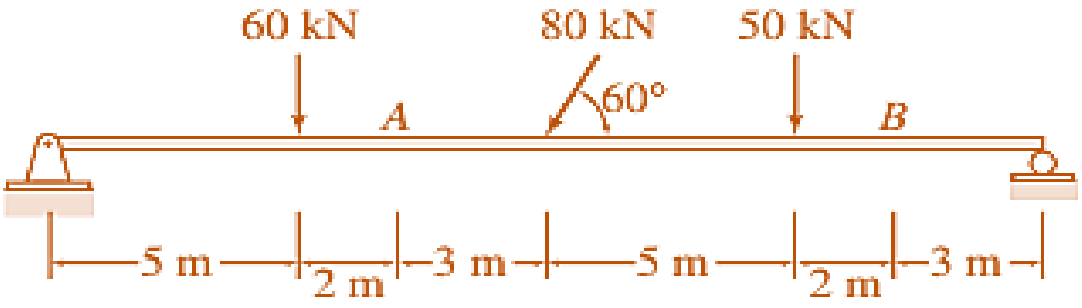
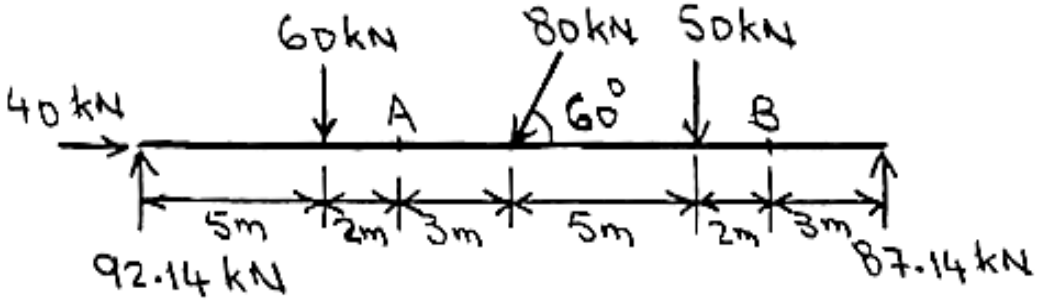


Fig 1.1a



$Q_A = \underline{-40\ kN}$

$S_A = 92.14 - 60 = \underline{32.14\ kN}$

$M_A = 92.14(7) - 60(2) = \underline{524.98\ kN.m}$

$Q_B = \underline{0}$

$S_B = \underline{-87.14\ kN}$

$M_B = 87.14(3) = \underline{261.42\ kN.m}$

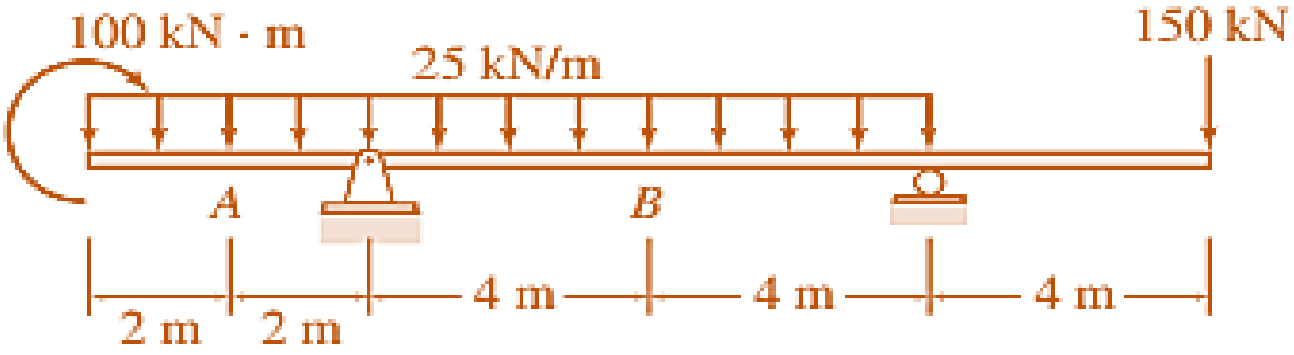
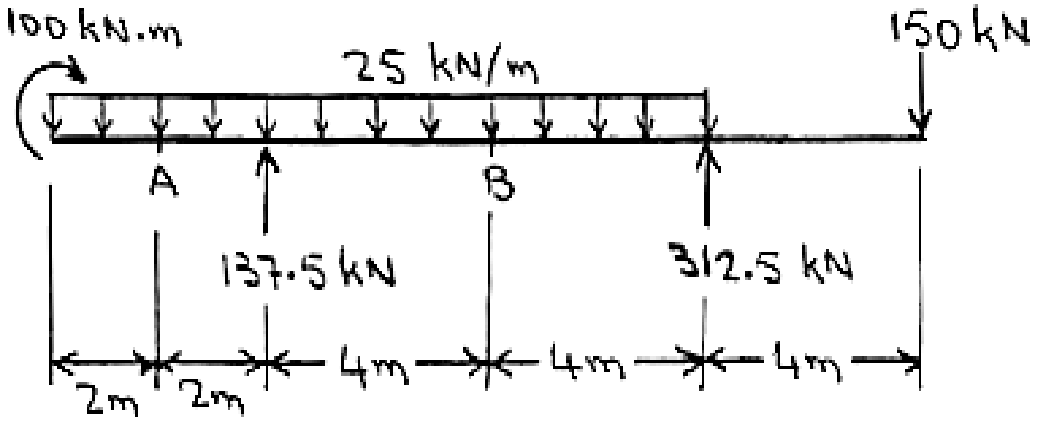


Fig 1.1b



$Q_A = \underline{0}$

$S_A = -25(2) = \underline{-50\ kN}$

$M_A = 100 - 25(2)(1) = \underline{50\ kN.m}$

$Q_B = \underline{0}$

$S_B = 150 - 312.5 + 25(4) = \underline{-62.5\ kN}$

$M_B = -150(8) + 312.5(4) - 25(4)(2)$

$M_B = \underline{-150\ kN.m}$

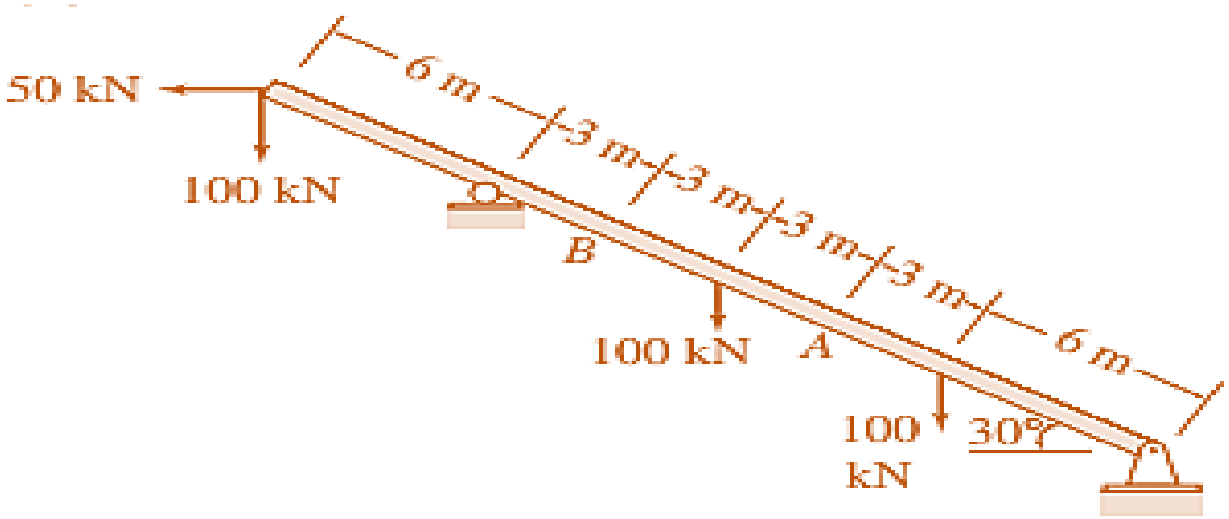
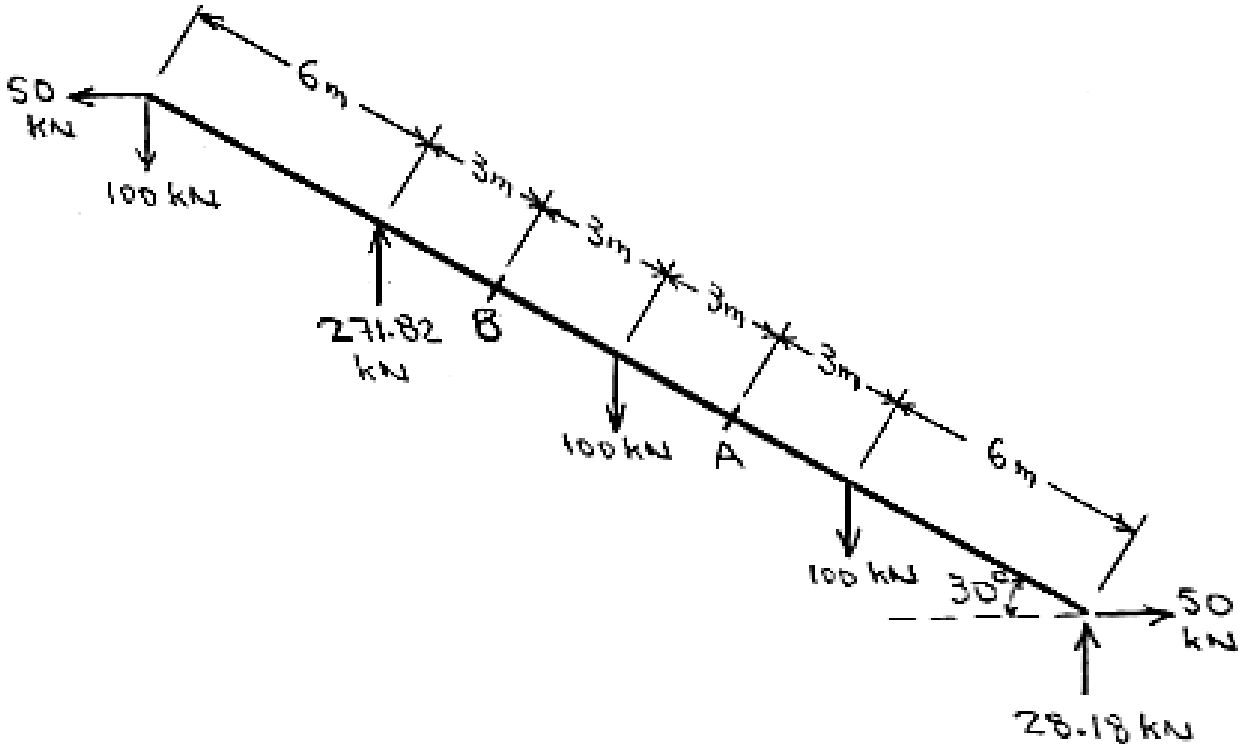


Fig 1.1c



$$Q_A = 50(\cos 30^\circ) - 28.18(\sin 30^\circ) + 100(\sin 30^\circ) = \underline{79.2\ kN}$$

$$S_A = -50(\sin 30^\circ) - 28.18(\cos 30^\circ) + 100(\cos 30^\circ) = \underline{37.2\ kN}$$

$$M_A = 50(\sin 30^\circ)(9) + 28.18(\cos 30^\circ)(9) - 100(\cos 30^\circ)(3)$$

$$= \underline{184.8\ kN.m}$$

$$Q_B = 50(\cos 30^\circ) - 100(\sin 30^\circ) + 271.82(\sin 30^\circ) = \underline{129.2\ kN}$$

$$S_B = -50(\sin 30^\circ) - 100(\cos 30^\circ) + 271.82(\cos 30^\circ) = \underline{123.8\ kN}$$

$$M_B = -50(\sin 30^\circ)(9) - 100(\cos 30^\circ)(9) + 271.82(\cos 30^\circ)(3)$$

$$= \underline{-298.2\ kN.m}$$

QUESTION 1.2

(I) Determine the equations for shear and bending moment for each beam shown using:
   a) Equilibrium (first principle)
   b) The method of intergration

(II) Use the resulting equations to draw the shear and bending moment diagrams.

(III) Use the slope-area method, draw the shear and bending moment diagrams for each of the beams shown below.

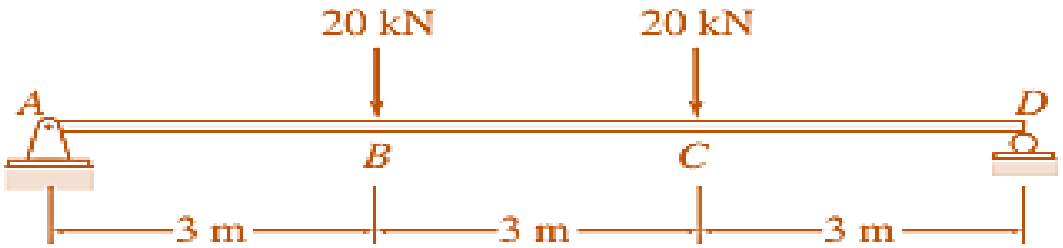
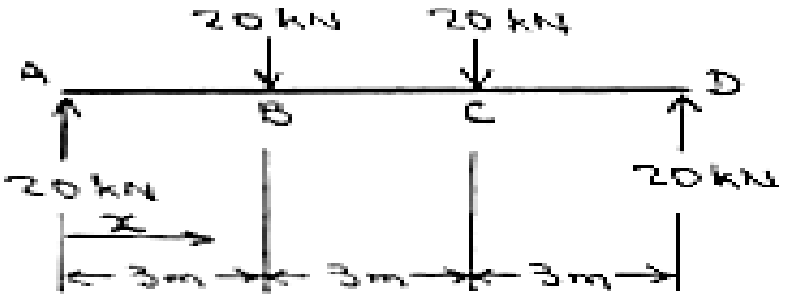


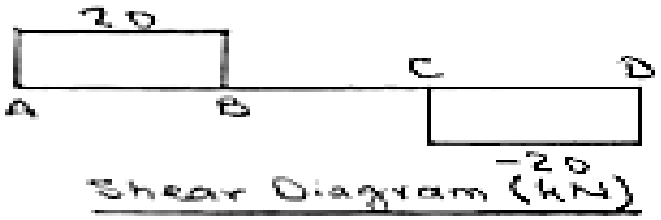
Fig 2.1a



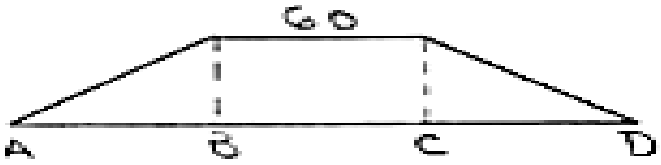
$0 < x < 3m$: $S = 20$, $M = 20x$

$3m < x < 6m$: $S = 20 - 20 = 0$, $M = 20x - 20(x-3) = 60$

$6m < x < 9m$: $S = 20 - 20 - 20 = -20$, $M = 20x - 20(x-3) - 20(x-6) = 180 - 20x$



Shear Diagram (kN)



Bending Moment Diagram (kN.m)

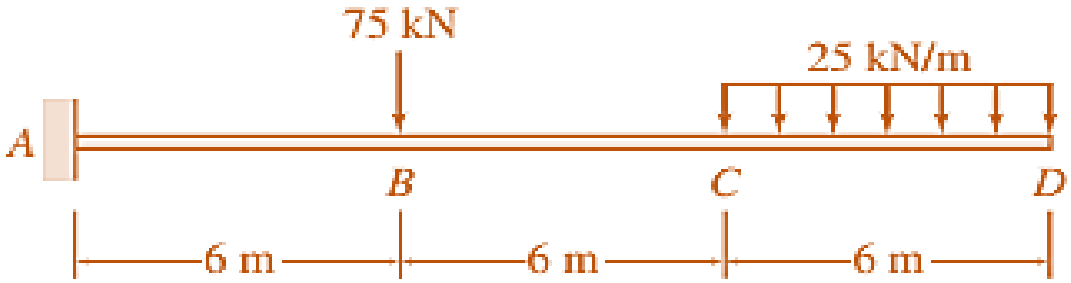
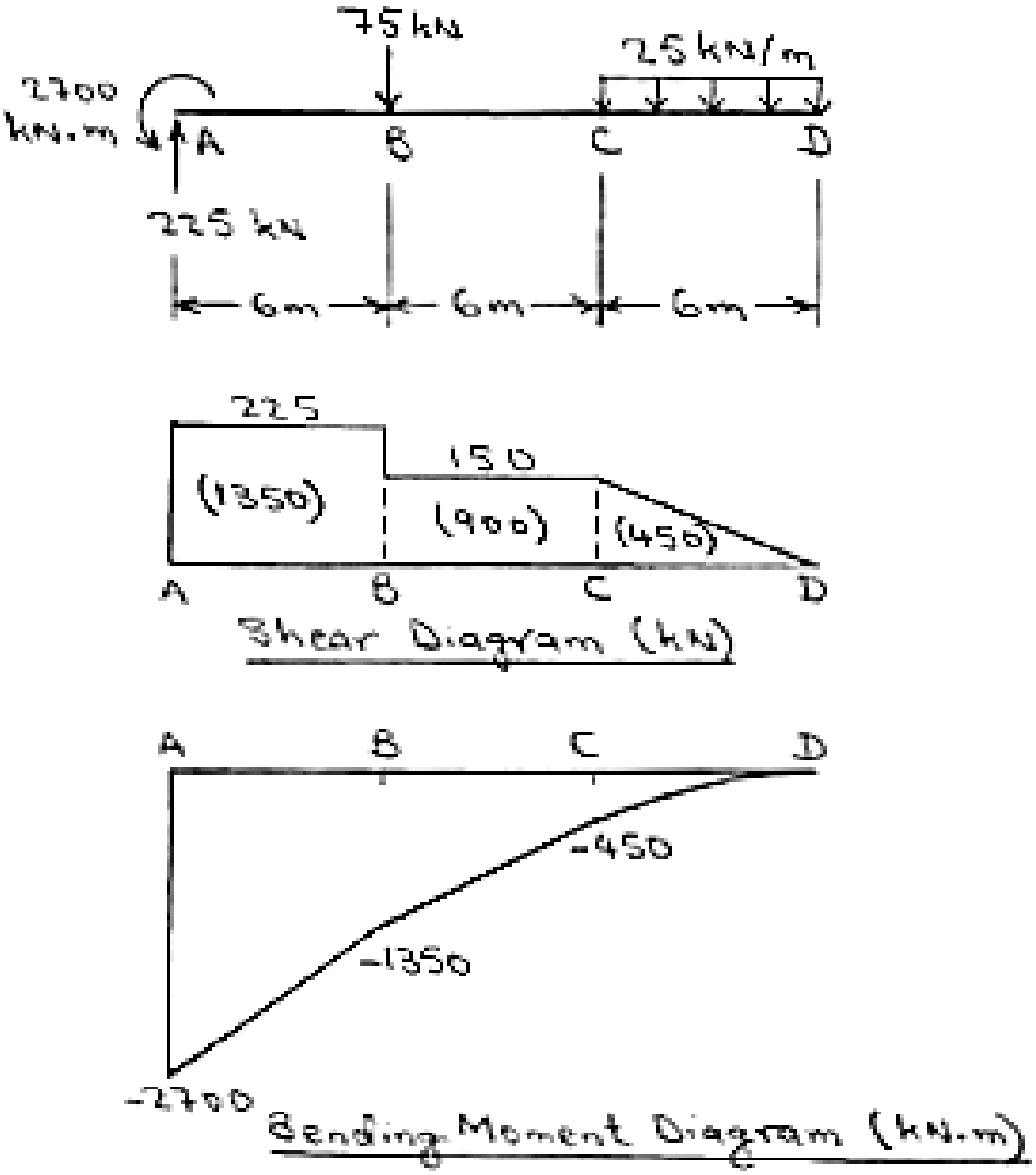

75 kN
25 kN/m
A
B
C
D
6 m
6 m
6 m

Fig 2.1b


75 kN
25 kN/m
2700
kN.m
A
B
C
D
225 kN
6m
6m
6m
225
150
(1350)
(900)
(450)
A
B
C
D
Shear Diagram (kN)
A
B
C
D
-450
-1350
-2700
Bending Moment Diagram (kN.m)

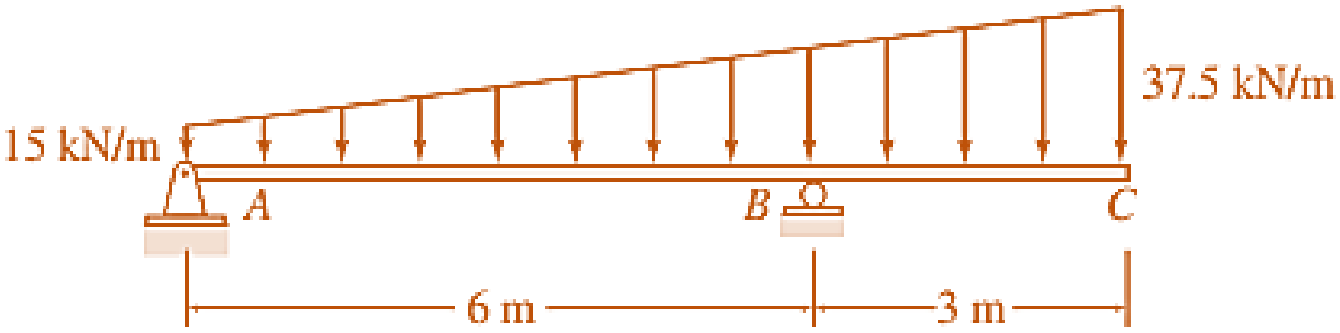
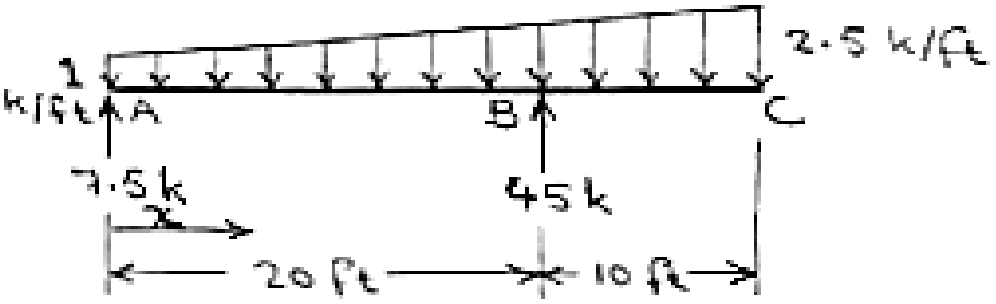


Fig 2.1c

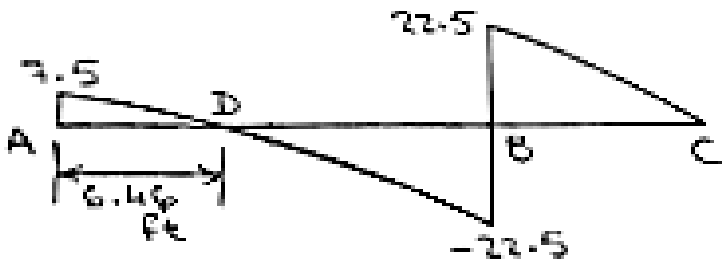


$0 < x < 20'$:

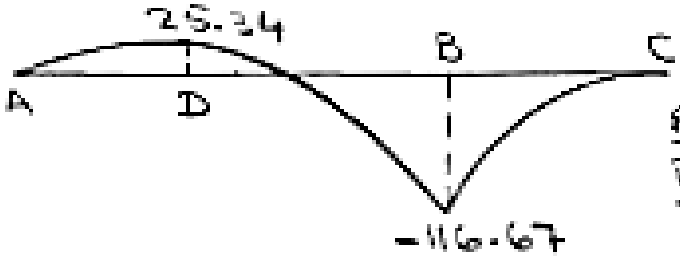
$$S = 7.5 - 1(x) - \frac{1}{2}\left(\frac{x}{20}\right)x$$
$$= -\frac{x^2}{40} - x + 7.5$$
$$M = 7.5x - 1\left(\frac{x^2}{2}\right) - \frac{1}{2}\left(\frac{x}{20}\right)x\left(\frac{x}{3}\right)$$
$$= -\frac{x^3}{120} - \frac{x^2}{2} + 7.5x$$

$20' < x < 30'$:

$$S = 7.5 - 1(x) - \frac{1}{2}\left(\frac{x}{20}\right)x + 45$$
$$= -\frac{x^2}{40} - x + 52.5$$
$$M = 7.5x - 1\left(\frac{x^2}{2}\right) - \frac{x^3}{120} + 45(x-20)$$
$$= -\frac{x^3}{120} - \frac{x^2}{2} + 52.5x - 900$$



Shear Diagram (k)



Bending Moment Diagram (k-ft)

QUESTION 1.3

Draw the shear and bending moment diagrams for each of the frames shown below.

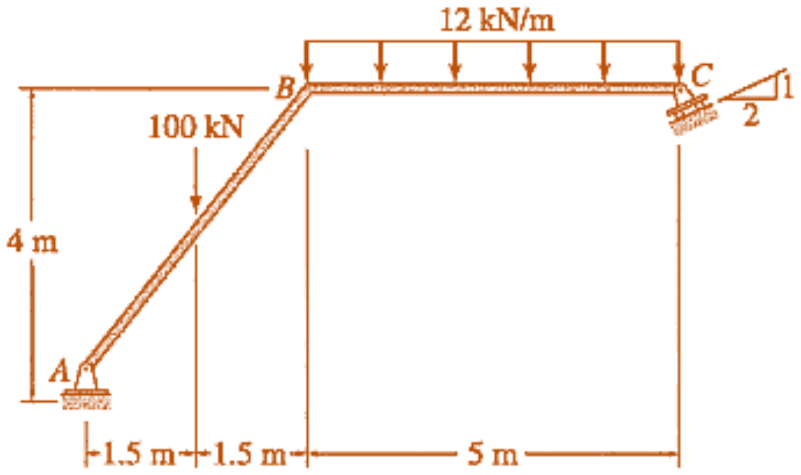
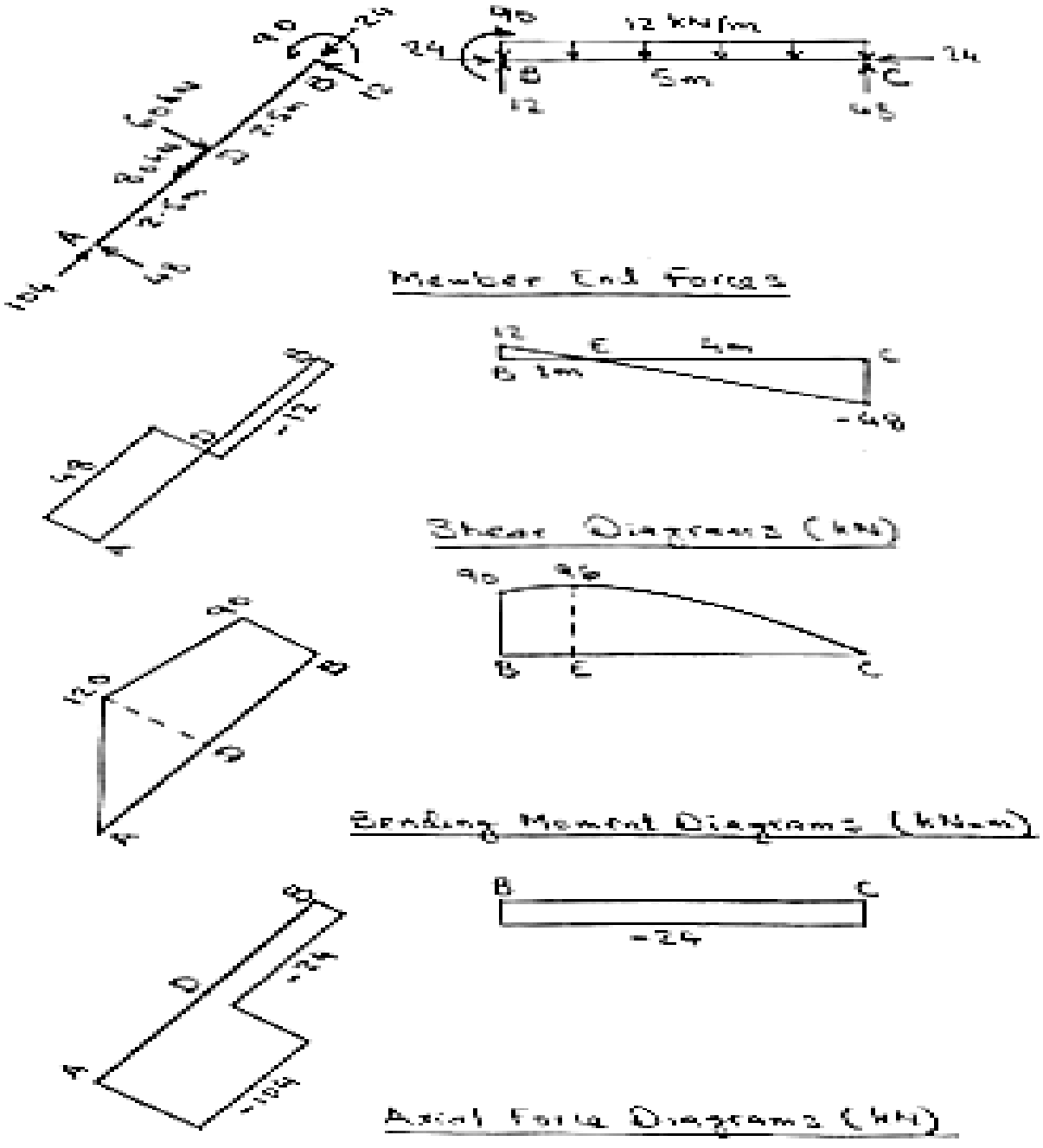


Fig 3.1a



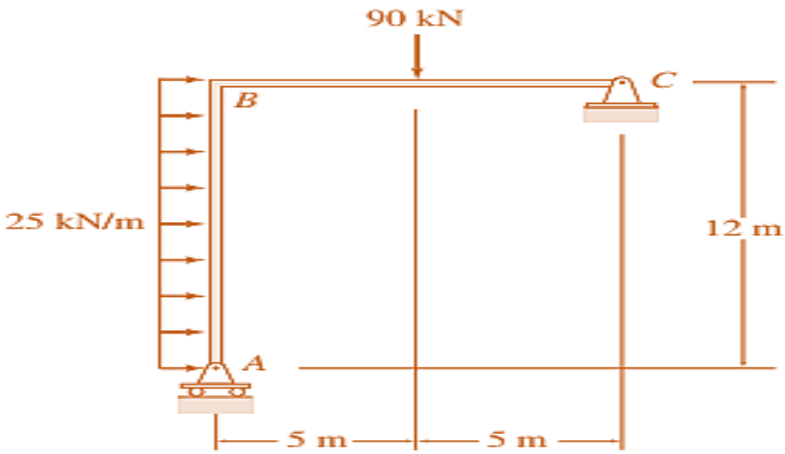
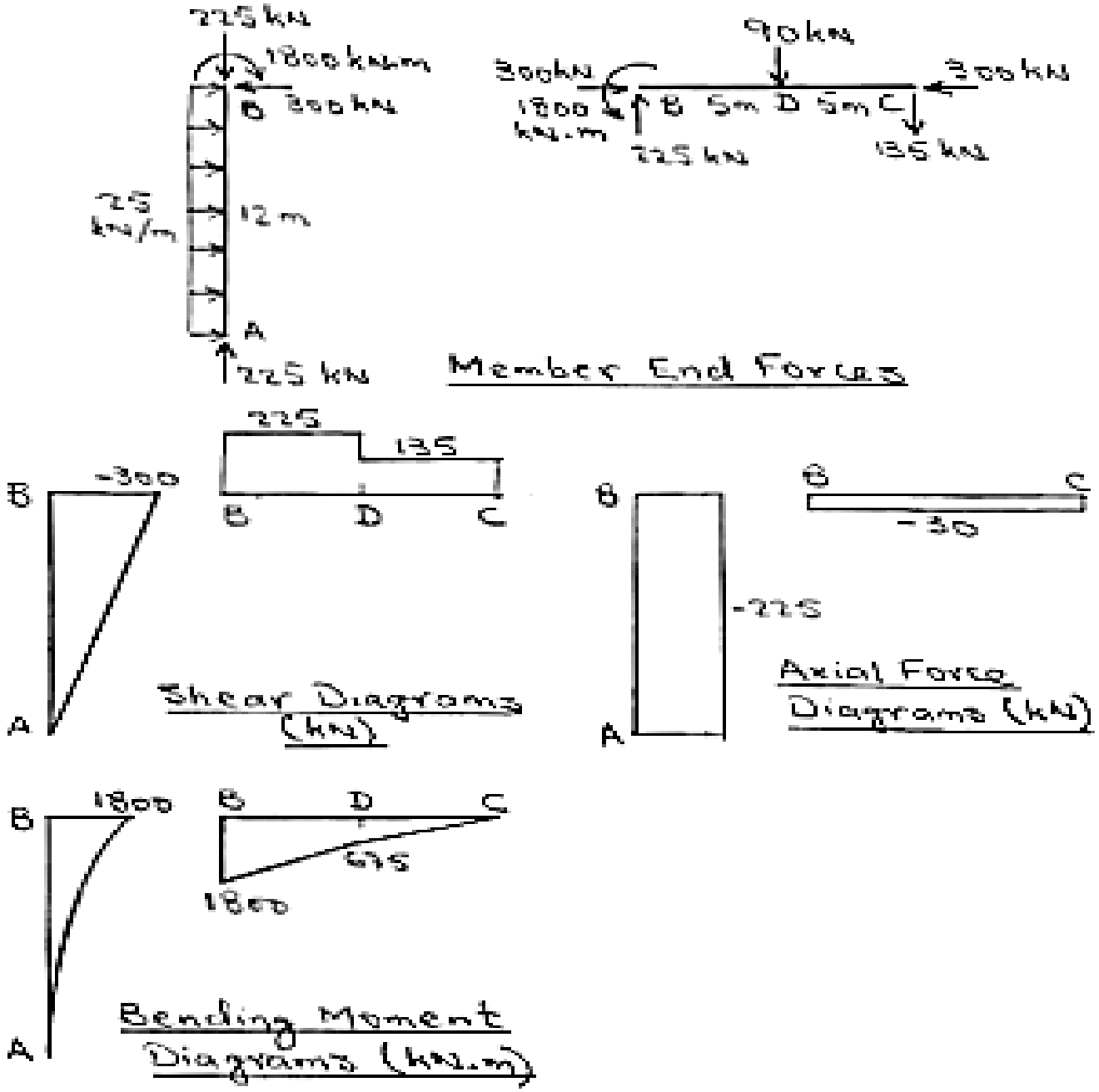

90 kN
B
C
25 kN/m
12 m
A
5 m
5 m

Fig 3.1b


225 kN
1800 kN.m
300 kN
B
25 kN/m
12 m
A
225 kN
Member End Forces
225
135
B
D
C
-300
A
Shear Diagrams (kN)
B
C
-30
-225
A
Axial Force Diagrams (kN)
1800
B
D
C
675
1800
A
Bending Moment Diagrams (kN.m)

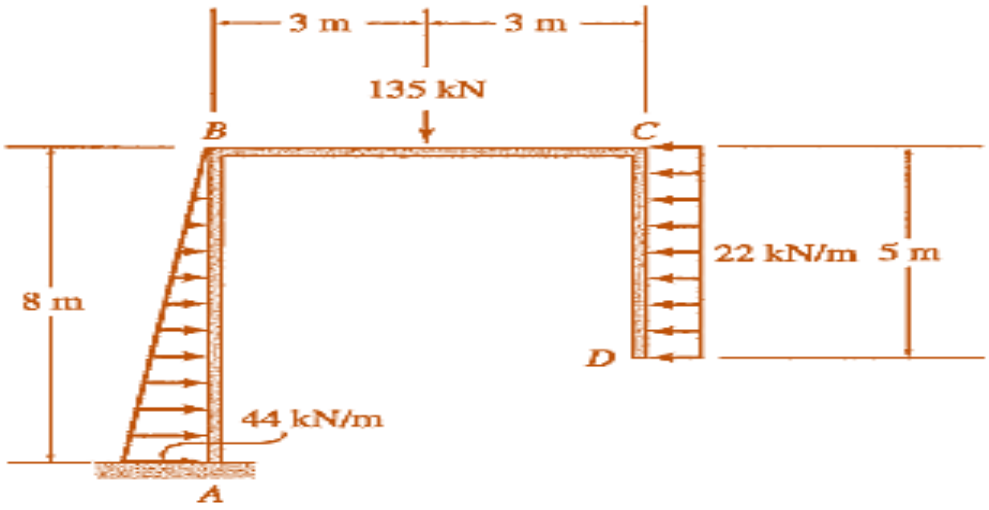
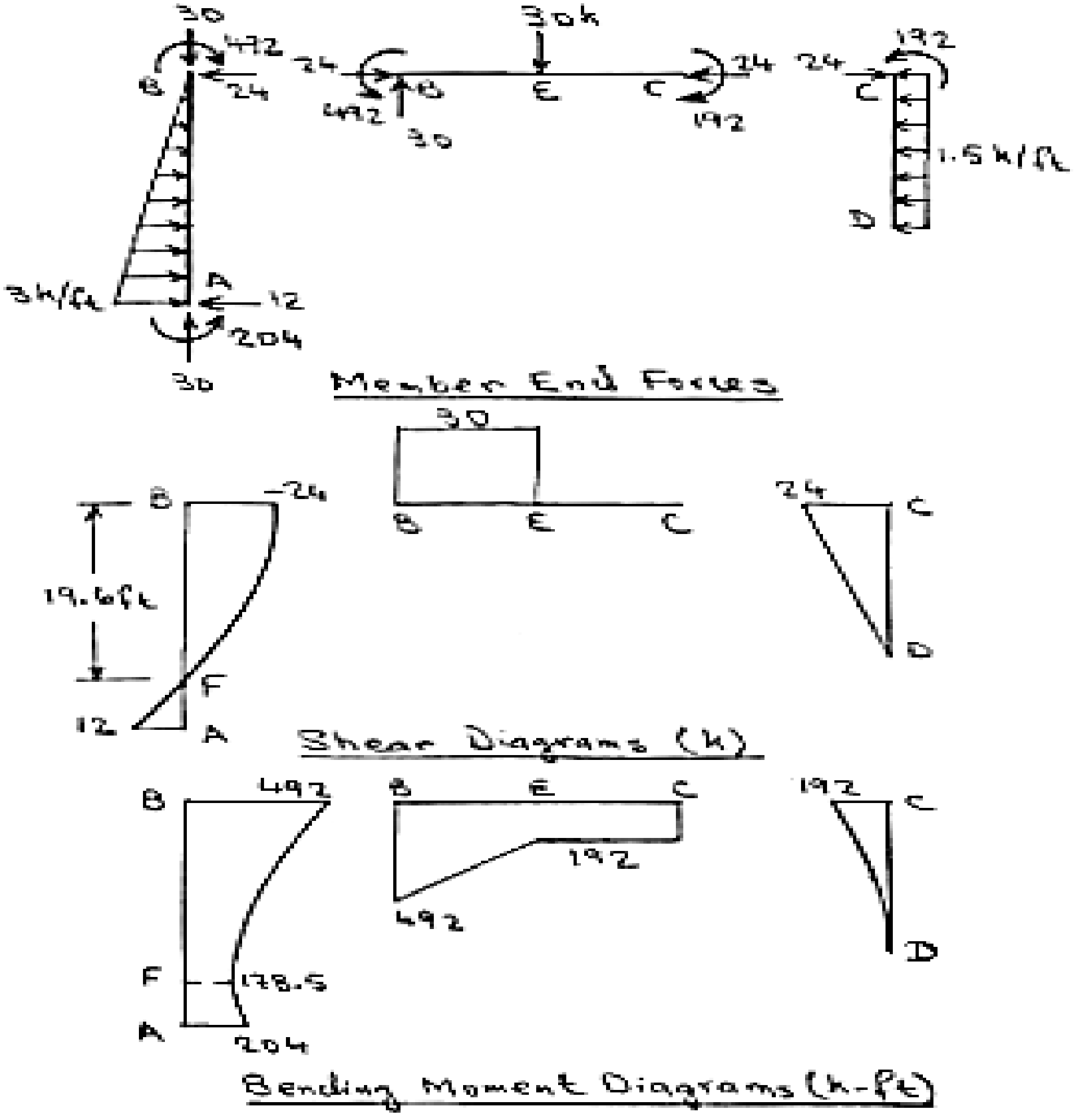

3 m
3 m
135 kN
B
C
8 m
22 kN/m
5 m
D
44 kN/m
A

Fig 3.1c


30
30k
192
492
24
B
E
C
D
1.5 k/ft
3k/ft
A
12
204
Member End Forces
19.6ft
F
Shear Diagrams (k)
178.5
Bending Moment Diagrams (k-ft)