Q15

ⓐ $R(t) = (\sec t)\hat{i} + (\tan t)\hat{j} + \frac{4}{3}t\,k, \quad t = \frac{\pi}{6}$

* 1^{st} derivative of $R(t)$ = Velocity ($\vec{v}$)

* 2^{nd} derivative of $R(t)$ = Acceleration ($\vec{a}$)

* Speed = $\overbrace{|\vec{v}|}^{\text{magnitude of velocity}}$

* acceleration scaler (a) = $a = \overbrace{|\vec{a}|}^{\text{magnitude of the acceleration}}$

Q16

Length of the indicated portion of a curve

$$\boxed{L = \int_a^b |R'(t)|\,dt}$$

For Q16 ⓐ $a = 0$, $b = \sqrt{8}$ similarly ⓑ $a = 0$, $b = \pi$

$R(t) = (1+2t)\hat{i} + (1+3t)\hat{j} + (6-6t)k$

For Q16 ⓒ determine upper & lower limit from $\underset{\text{lower}}{(-1,-2,12)}$ to $\underset{\text{upper}}{(1,1,6)}$

Lower limit	Upper limit	
x: $1 + 2t = -1$	x: $1+2t = 1$	- Using x, y or z for the lower limit, it turns out that $t = -1$
y: $1 + 3t = -2$	y: $1+3t = 1$	- using x, y or z for the upper limit it turns out that $t = 0$
z: $6 - 6t = 12$	z: $6-6t = 6$	$\therefore$ $a = -1$, $b = 0$ $\boxed{-1 \le t \le 0}$

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Q17

$R(t) = (12\sin t)\hat{i} - (12\cos t)\hat{j} + 5tk$

at a distance from the origine in the direction opposite to the direction of increasing arc length $\hookrightarrow \underline{t=0}$

1st Find arc length function $a=0$, $b=t$

$$\boxed{S(t) = \int_0^t |R'(u)|\, du}$$

$R'(t) = (12\cos t)\hat{i} + (12\sin t)\hat{j} + 5K$

$|R'(t)| = \sqrt{(12\cos t)^2 + (12\sin t)^2 + 5^2} = \underline{13}$

hence

$$S(t) = \int_0^t 13\, du = \underline{13t}$$

distance t from 13π $\rightarrow t = \pi$

$13t = 13\pi$

Evaluate ~~from~~ for $t=\pi$ in $R(t)$

$R(\pi) = (12\sin(\pi))\hat{i} - (12\cos(\pi))\hat{j} + 5\pi k$

$R(\pi) = 12\hat{j} + 5\pi k$

Evaluate for $t=0$ [origin]

$R(0) = [12\sin(0)]\hat{i} - (12\cos(0))\hat{j} + 5\cdot 0 k$

$R(0) = 0\hat{i} - 12\hat{j} + 0k = (0, -12, 0)$

$\therefore$ point on the curve is $\underline{(0, -12, 0)}$

Scanned with CamScanner

Q17

i) Unit tangent vector T

$$\boxed{T = \frac{R'(t)}{\|R'(t)\|}}$$

(ii) Principal Normal vector

$$\boxed{N(t) = \frac{T'(t)}{\|T'(t)\|}}$$

iii) Binormal vector

$$B = \underbrace{T \times N}$$

Cross product

(iv) Curvature K

$$\boxed{K = \frac{|T'(t)|}{|R'(t)|}}$$

Q18

$a = a_T T + a_N N$ without T and N

a = acceleration

a_T = tangent magnitude

T = Unit tangent vector

a_N = Normal magnitude

N = principal Normal vector

$$a_T = \frac{d}{dt}\left[|R'(t)|\right] = \frac{d}{dt}|V|$$

$$a_N = \sqrt{|a|^2 - (a_T)^2}$$

Remember $a = R''(t)$

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