SCHOOL OF ENGINEERING

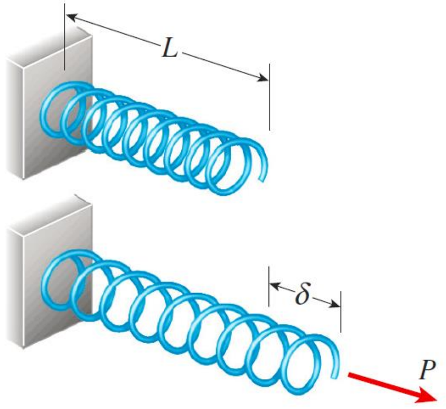
DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

CEE 3211– MECHANICS OF MATERIALS

LECTURE 2 – AXIALLY LOADED MEMBERS

Coiled Springs

- Tension– load acts away; spring elongates
- Compression– load acts towards; spring shortens



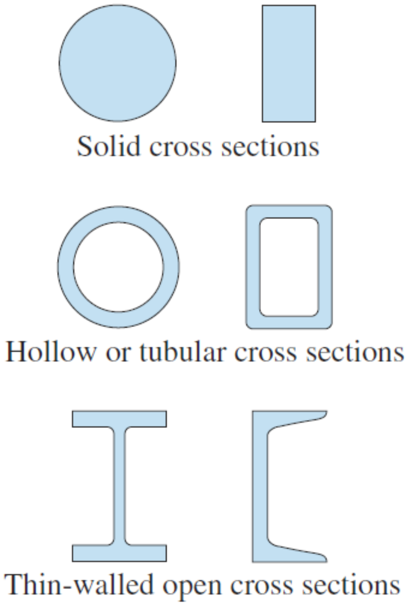
- Linearly Elastic Equations

Load: $P = k\delta$ Elongation: $\delta = fP$

Stiffness: $k = \frac{P}{\delta} = \frac{1}{f}$ Flexibility: $f = \frac{\delta}{P} = \frac{1}{k}$

Prismatic Bars

- Requirement #1: Straight longitudinal axis
- Requirement #2: Constant cross section
  - Geometry can vary (see image)
- Requirement #3: Linearly elastic
- Force-Displacement Relation

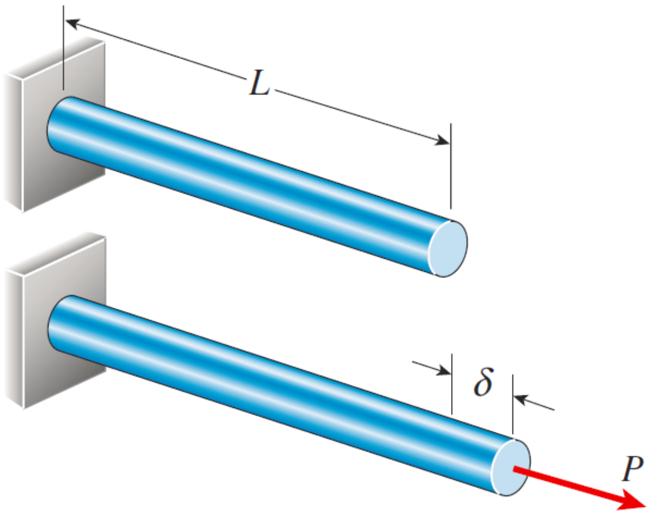


$$\sigma = \frac{P}{A}, \quad \varepsilon = \frac{\delta}{L}$$

Substitute → $\sigma = E\varepsilon$ → Yields → $\delta = \frac{PL}{EA}$

Prismatic Bars (cont.)

- Deformation Sign Convention
  - Elongation: Positive (+)
  - Shortening: Negative (-)



- Stiffness: $k = \dfrac{P}{\delta} \rightarrow k = \dfrac{EA}{L}$

- Flexibility: $f = \dfrac{\delta}{P} \rightarrow f = \dfrac{L}{EA}$

Cables

- Tension only (cannot resist compression)
- δ of cable > δ of prismatic-bar for the same load, material, and cross-section.
- Modulus of elasticity for a cable < modulus of elasticity of the material.
- In analysis, use the cable's effective modulus and not that of the material.

Bars with Intermediate Axial Loads

- Step 1: Identify segments (ex: AB, BC, and CD as 1, 2, and 3).

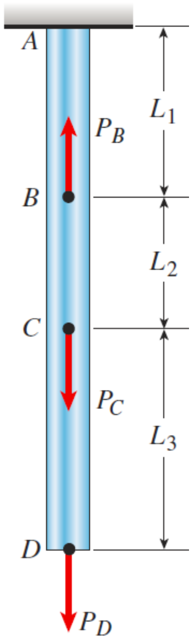
- Step 2: Determine the internal axial forces N_1, N_2, and N_3.
  - Make a sectional cut at each segment and evaluate FBD.

- Step 3: Determine elongation for each segment.

Ex: $\delta_1 = \frac{N_1 L_1}{EA} \quad \delta_2 = \frac{N_2 L_2}{EA} \quad \delta_3 = \frac{N_3 L_3}{EA}$

- Step 4: Add elongation values for overall elongation.

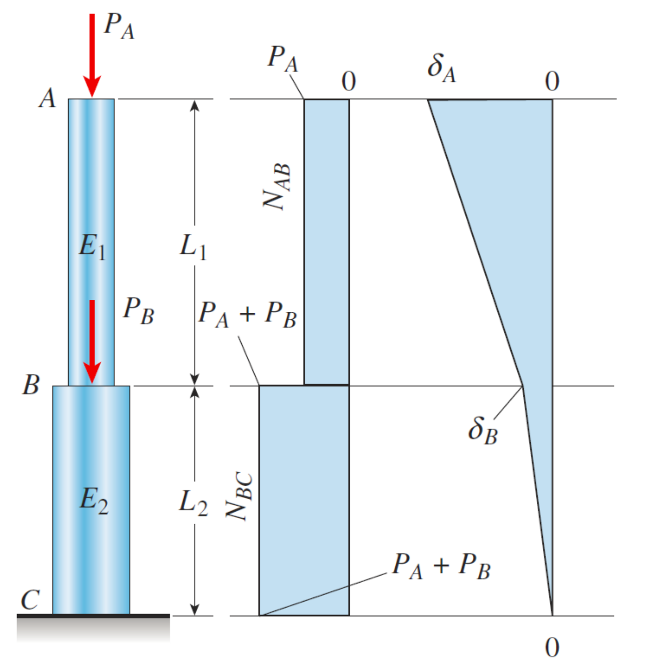
Ex: $\delta_{bar} = \sum_{i=1}^{3} \delta_i = \delta_1 + \delta_2 + \delta_3$



Bars Consisting of Prismatic Segments

- Analysis follows the same procedure.
- However, EA are no longer constants.
- General Elongation Equation

$$\delta = \sum_{i=1}^{n} \frac{N_i L_i}{E_i A_i}$$



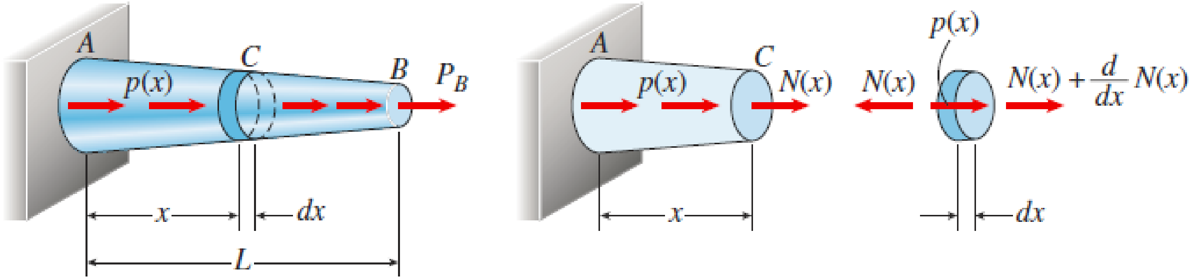
Axial force (AFD)

Axial displacement (ADD)

Bars with Continuously Varying Loads or Dimensions

- Condition #1: Continuously varying cross-sectional area A.
- Condition #2: Continuously varying axial force N.
- Goal: Determine the elongation of a differential element of the bar and integrate over the length of the bar.
- Limitations: Must be linearly elastic with any taper angle being small.

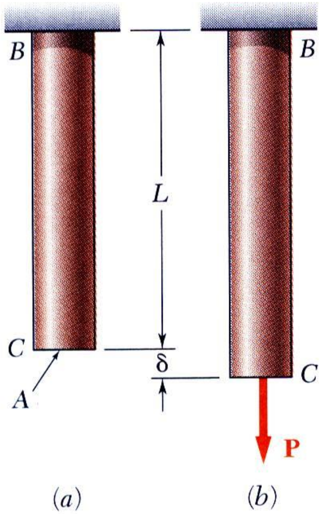
$$\delta = \int_0^L d\delta = \int_0^L \frac{N(x)dx}{EA(x)}$$



Stress & Strain: Axial Loading

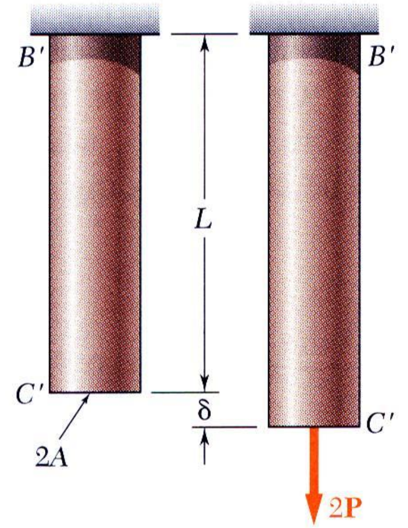
- Suitability of a structure or machine may depend on the deformations in the structure as well as the stresses induced under loading. Statics analyses alone are not sufficient.
- Considering structures as deformable allows determination of member forces and reactions which are statically indeterminate.
- Determination of the stress distribution within a member also requires consideration of deformations in the member.
- Concerned with deformation of a structural member under axial loading.

Normal Strain



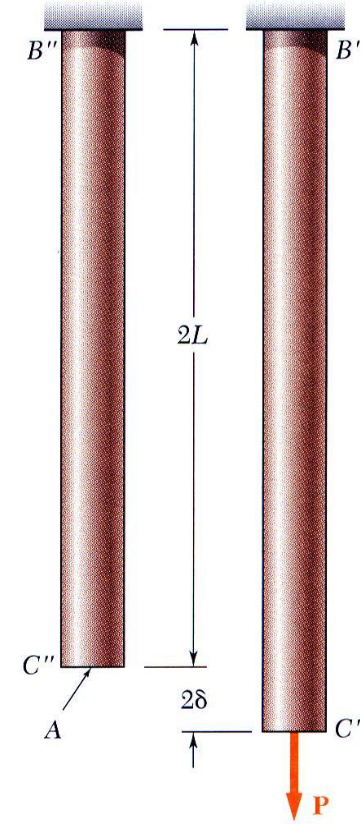
$$\sigma = \frac{P}{A} = \text{stress}$$

$$\varepsilon = \frac{\delta}{L} = \text{normal strain}$$



$$\sigma = \frac{2P}{2A} = \frac{P}{A}$$

$$\varepsilon = \frac{\delta}{L}$$



$$\sigma = \frac{P}{A}$$

$$\varepsilon = \frac{2\delta}{2L} = \frac{\delta}{L}$$

Stress-Strain Test

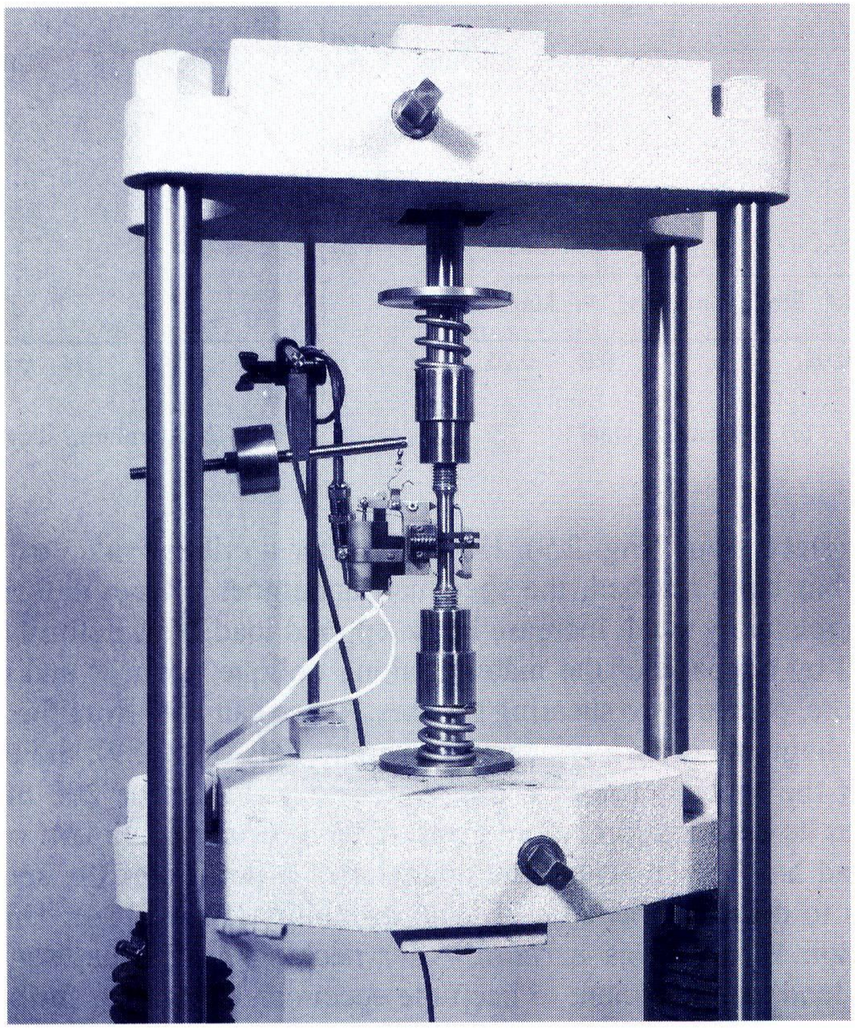


Fig. 2.7 This machine is used to test tensile test specimens, such as those shown in this chapter.

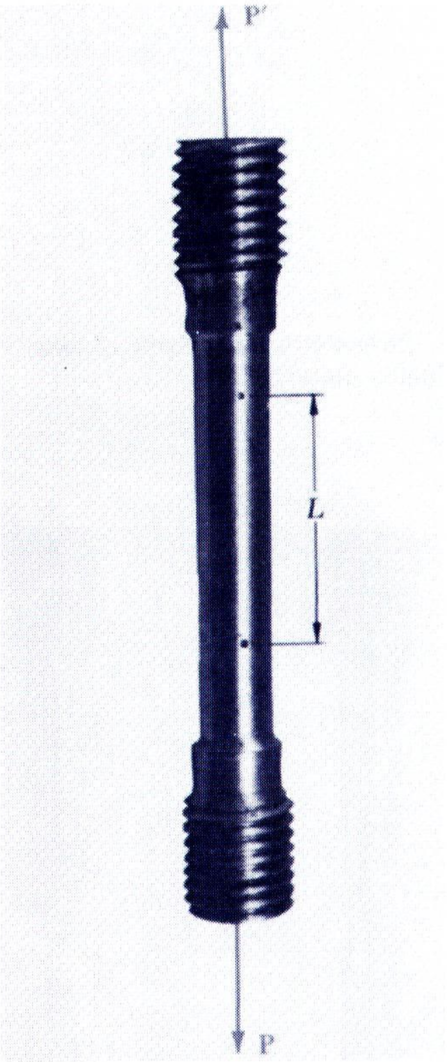
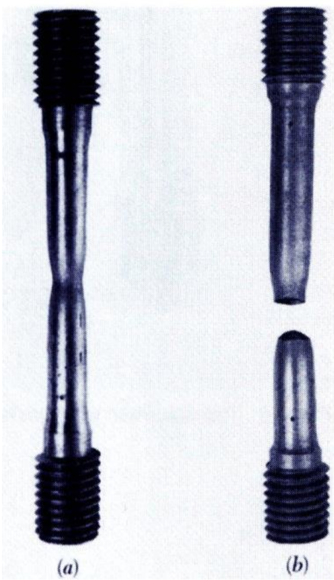
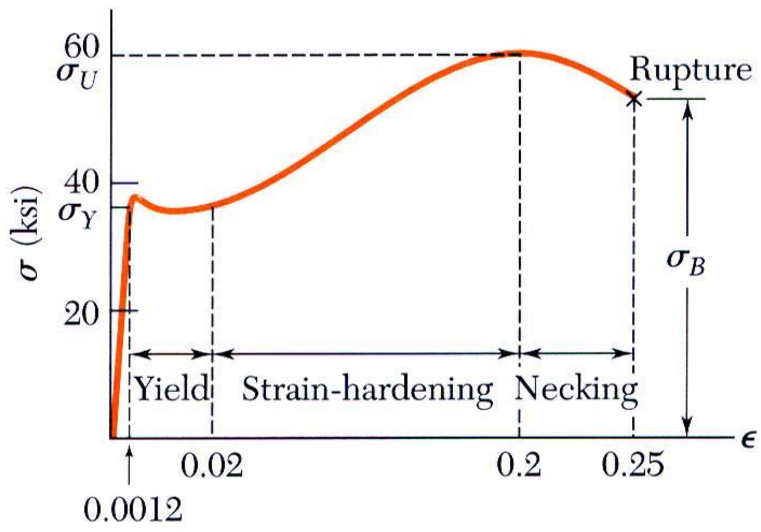


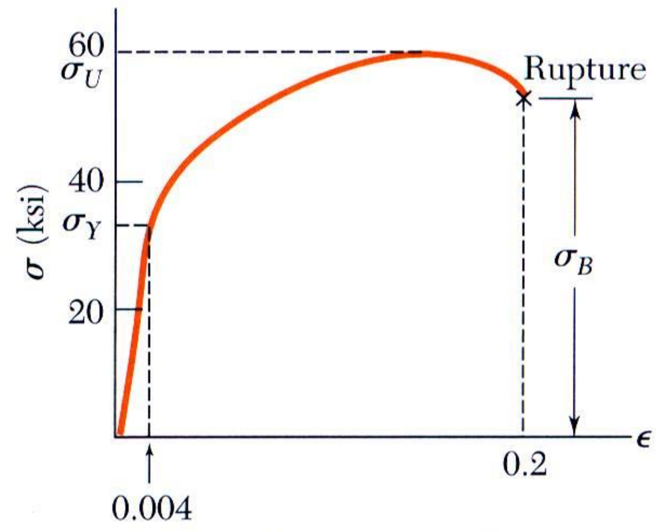
Fig. 2.8 Test specimen with tensile load.

Stress-Strain Diagram: Ductile Materials



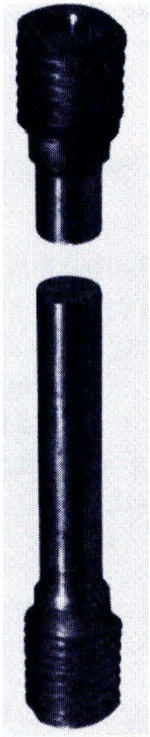


(a) Low-carbon steel



(b) Aluminum alloy

Stress-Strain Diagram: Brittle Materials



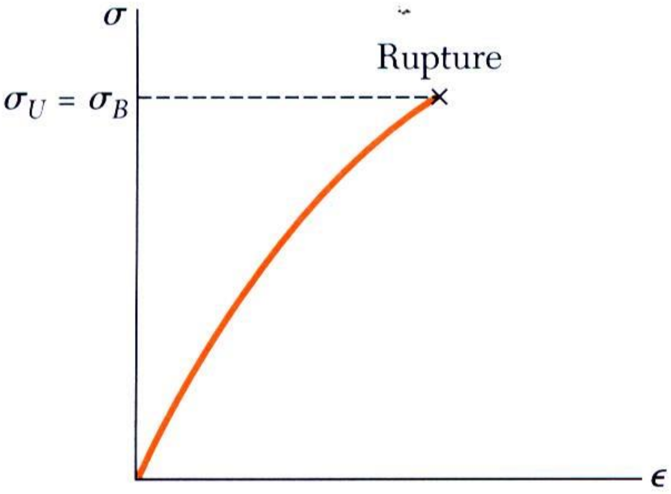
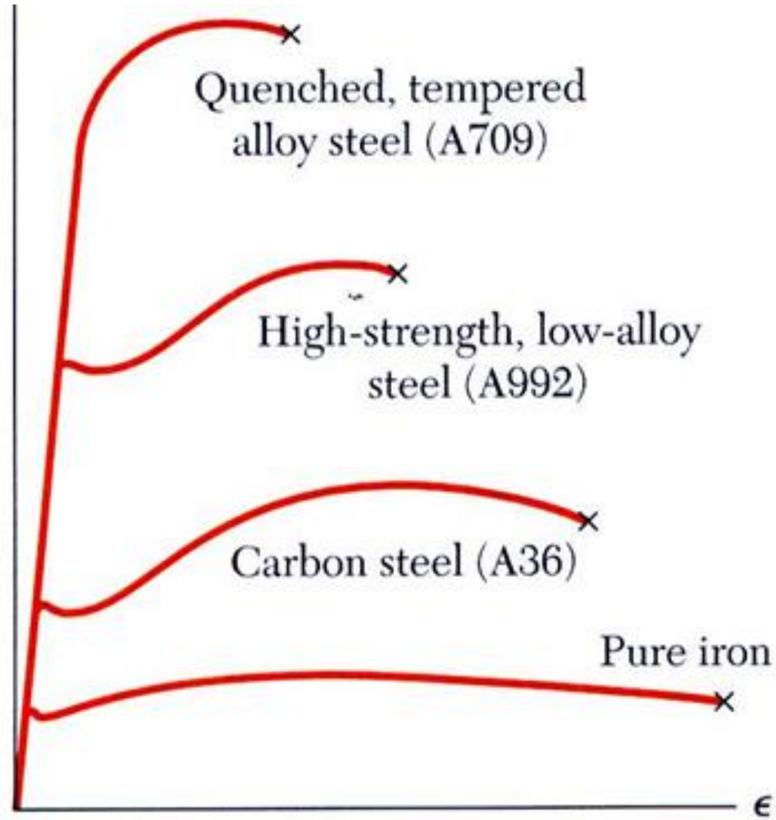


Fig. 2.11 Stress-strain diagram for a typical brittle material.

Hooke's Law: Modulus of Elasticity



Stress-strain diagrams for iron and different grades of steel.

- Below the yield stress

$$\sigma = E\varepsilon$$

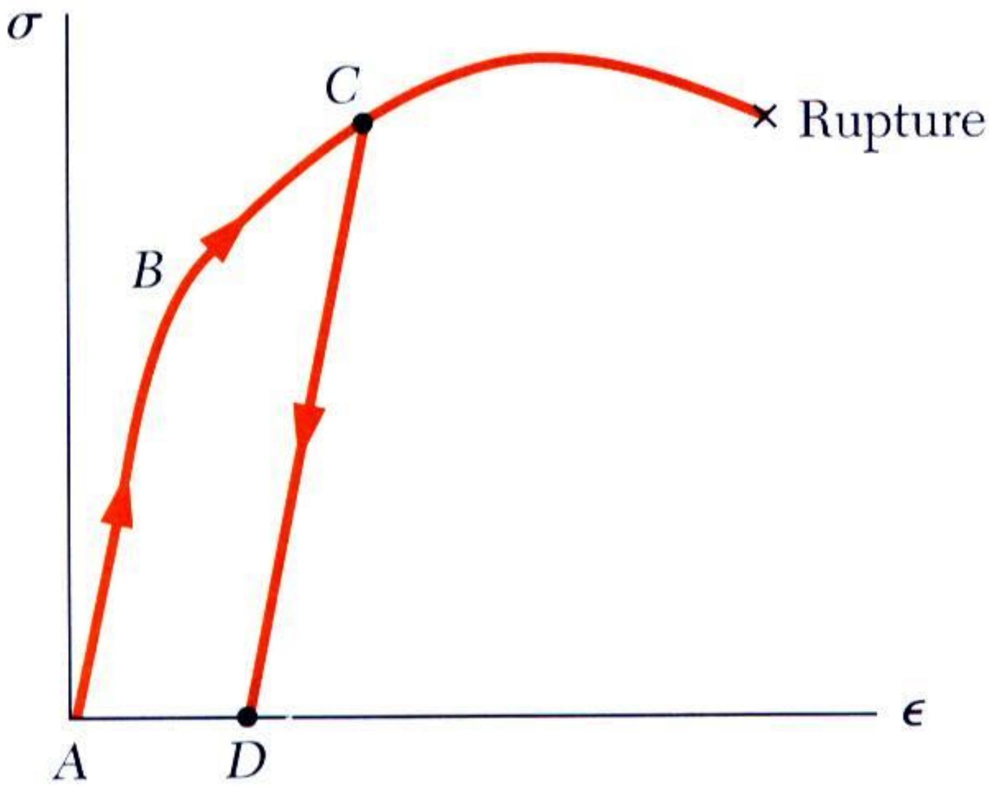
E = Youngs Modulus or Modulus of Elasticity

All the 4 steel grades, possess the same modulus of elasticity; in other words, their "stiffness," or ability to resist a deformation within the linear range, is the same

Therefore, if a high-strength steel is substituted for a lower-strength steel in a given structure, and if all dimensions are kept the same, the structure will have an increased load-carrying capacity, but its stiffness will remain unchanged

Strength is affected by alloying, heat treating, and manufacturing process but stiffness (Modulus of Elasticity) is not.

Elastic vs. Plastic Behavior



- If the strain disappears when the stress is removed, the material is said to behave *elastically*.
- The largest stress for which this occurs is called the *elastic limit*.
- When the strain does not return to zero after the stress is removed, the material is said to behave *plastically*.

Factor of Safety

Structural members or machines must be designed such that the working stresses are less than the ultimate strength of the material.

$$FS = \text{Factor of safety}$$

$$FS = \frac{\sigma_u}{\sigma_{all}} = \frac{\text{ultimate stress}}{\text{allowable stress}}$$

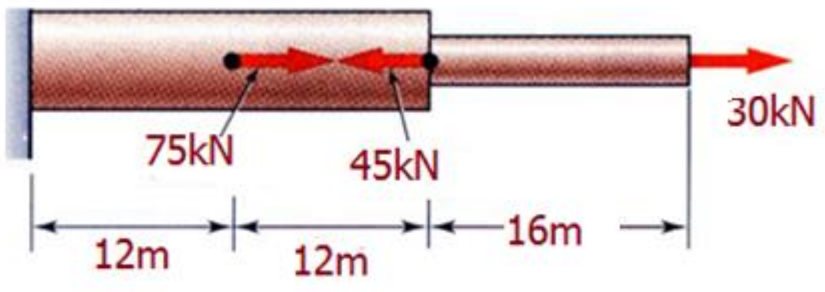
$$FS = \frac{\text{Failure Load}}{\text{allowable Load}}$$

$$FS > 1$$

Factor of safety considerations:

- uncertainty in material properties
- uncertainty of loadings
- uncertainty of analyses
- number of loading cycles
- types of failure
- maintenance requirements and deterioration effects
- importance of member to structures integrity
- risk to life and property
- influence on machine function

Example 1



$$E = 200GPA$$

$$D = 1.07 \text{ m.} \quad d = 0.618 \text{ m.}$$

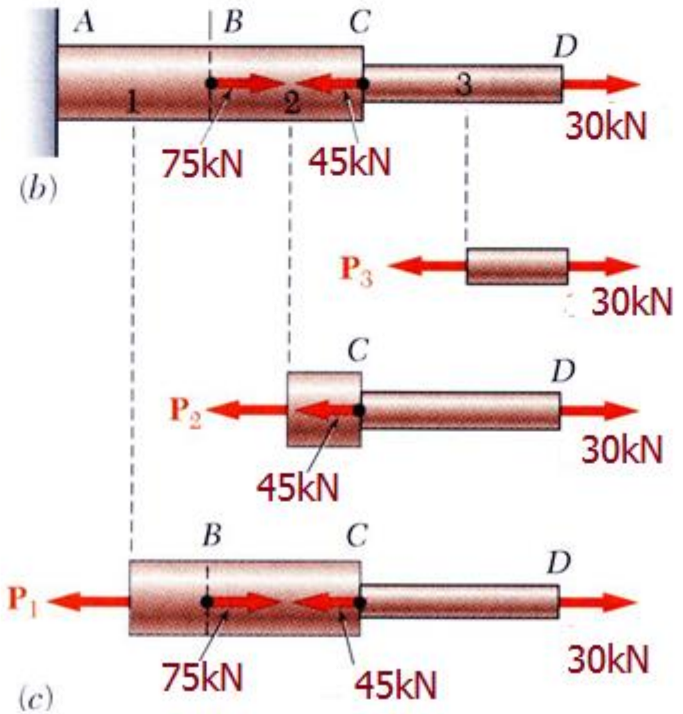
Determine the deformation of the steel rod shown under the given loads.

SOLUTION:

- Divide the rod into components at the load application points.
- Apply a free-body analysis on each component to determine the internal force
- Evaluate the total of the component deflections.

SOLUTION:

- Divide the rod into three components:



$L_1 = L_2 = 12$ m. $\quad L_3 = 16$ m

$A_1 = A_2 = 0.9\ \text{m}^2 \quad A_3 = 0.3\ \text{m}^2$

- Apply free-body analysis to each component to determine internal forces,

$$P_1 = 60 \times 10^3\, N$$

$$P_2 = -15 \times 10^3\, N$$

$$P_3 = 30 \times 10^3\, N$$

- Evaluate total deflection,

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i} = \frac{1}{E}\left(\frac{P_1 L_1}{A_1} + \frac{P_2 L_2}{A_2} + \frac{P_3 L_3}{A_3}\right)$$

$$= \frac{1}{200 \times 10^9}\left[\frac{(60 \times 10^3)12}{0.9} + \frac{(-15 \times 10^3)12}{0.9} + \frac{(30 \times 10^3)16}{0.3}\right]$$

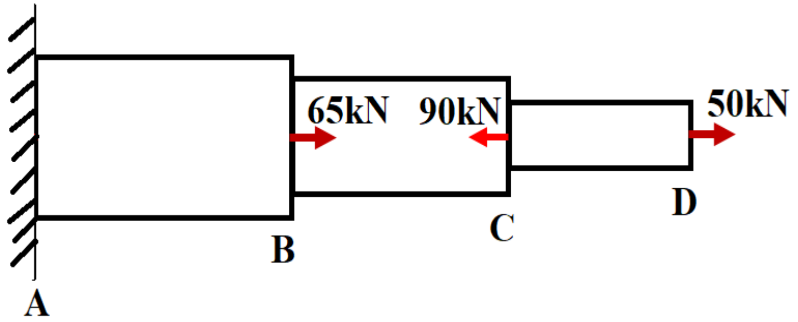
$$= 0.011\text{mm.}$$

$$\delta = 0.011 \text{ mm.}$$

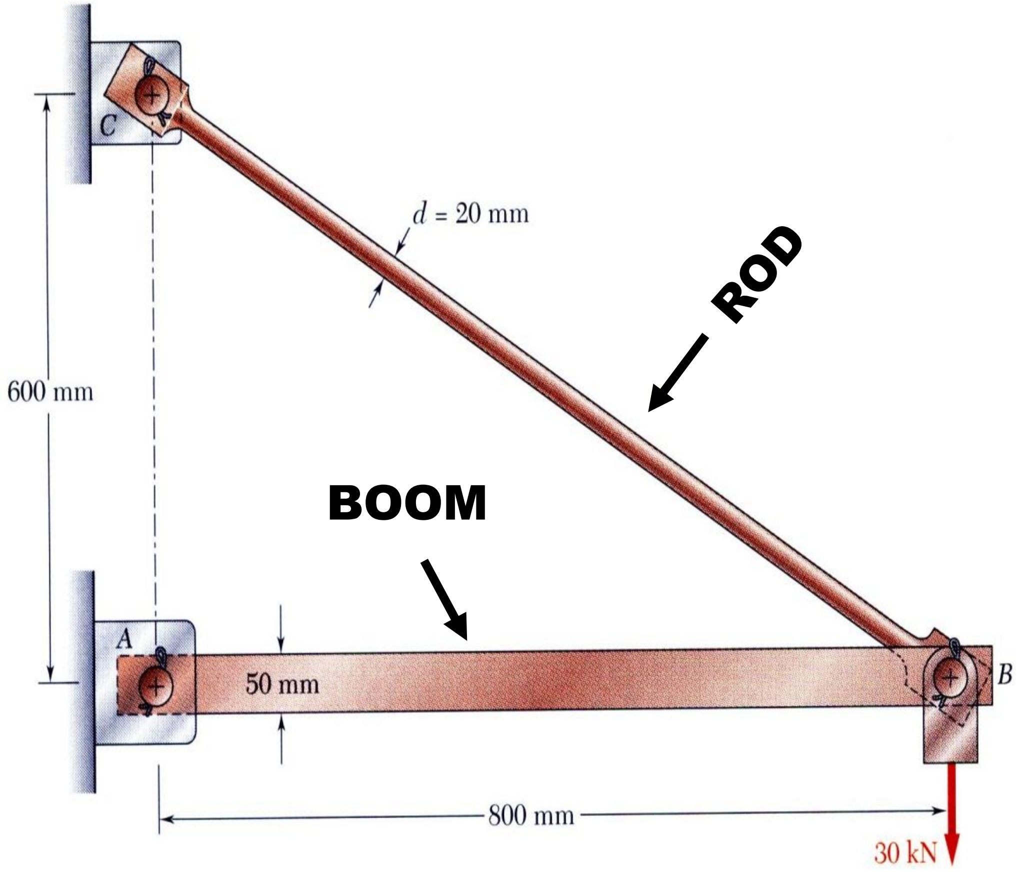
2019 EXAM QUESTION

A composite rod has 3 segments shown in Fig Q1. The diameters for each segment are given as; AB = 120 mm, BC = 90mm and CD = 60 mm. AB is made of A-36 structural Steel (E = 200 GPa), BC is made of Aluminium (E = 68.9 GPa) while CD is made of Brass-C83400 (E = 101 GPa). The lengths of the segments are as follows: AB = 30 cm, BC = 35 cm and CD = 45 cm. Determine the following:

- The reaction force at support A **[2 marks]**
- The normal stress induced in each segment **[10 Marks]**
- The total elongation **[8 Marks]**

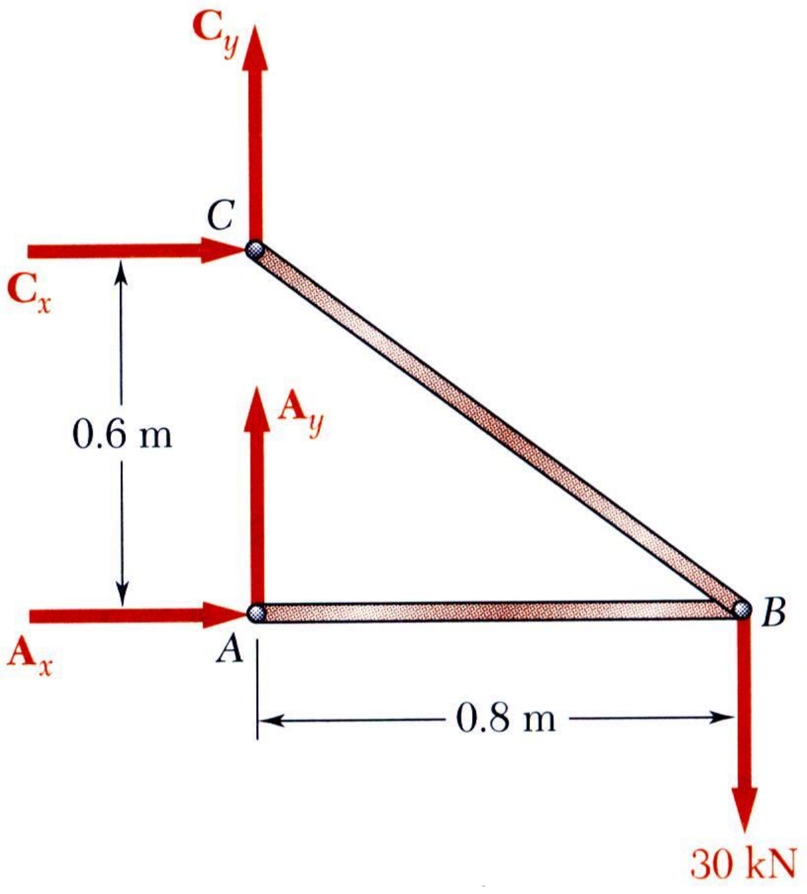


Review of Statics



- The structure is designed to support a 30 kN load
- The structure consists of a boom and rod joined by pins (zero moment connections) at the junctions and supports
- Perform a static analysis to determine the internal force in each structural member and the reaction forces at the supports

Structure Free-Body Diagram



- Structure is detached from supports and the loads and reaction forces are indicated

- Conditions for static equilibrium:

$$\sum M_C = 0 = A_x(0.6\,\mathrm{m}) - (30\,\mathrm{kN})(0.8\,\mathrm{m})$$

$$A_x = 40\,\mathrm{kN} \longrightarrow \text{Eqn. (a)}$$

$$\sum F_x = 0 = A_x + C_x$$

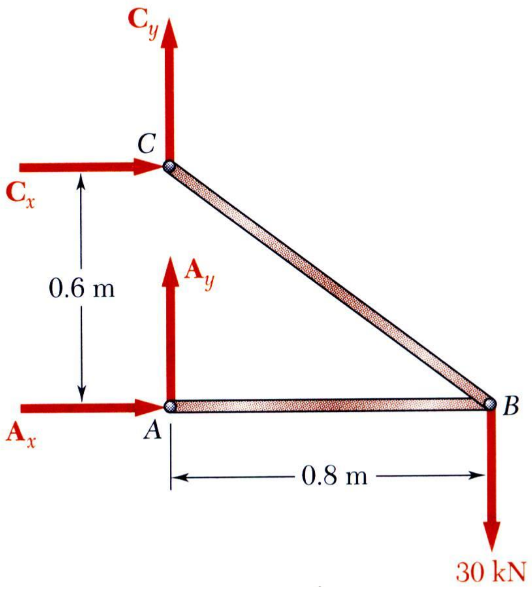
$$C_x = -A_x = -40\,\mathrm{kN} \longrightarrow \text{Eqn. (b)}$$

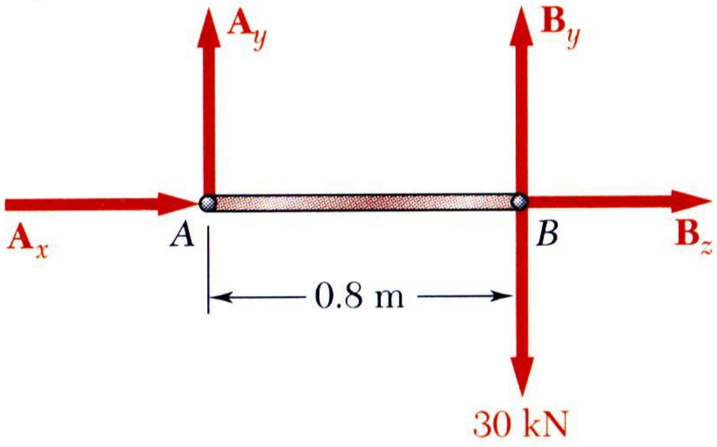
$$\sum F_y = 0 = A_y + C_y - 30\,\mathrm{kN} = 0$$

$$A_y + C_y = 30\,\mathrm{kN} \longrightarrow \text{Eqn. (c)}$$

- A_y and C_y can not be determined from these equations

Component Free-Body Diagram





- In addition to the complete structure, each component must satisfy the conditions for static equilibrium

- Consider a free-body diagram for the boom:

$$\sum M_B = 0 = -A_y(0.8\,\text{m})$$

$$A_y = 0$$

substitute into the structure equilibrium equation i.e Eqn (c).

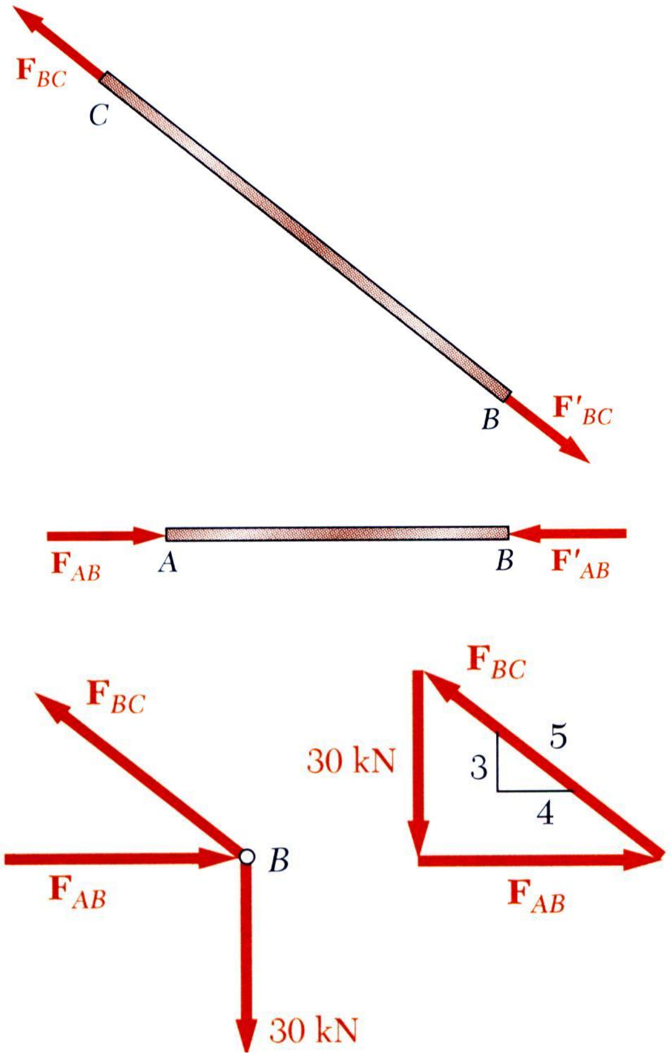
$$C_y = 30\,\text{kN}$$

- Results:

$$A = 40\,\text{kN} \rightarrow \quad C_x = 40\,\text{kN} \leftarrow \quad C_y = 30\,\text{kN} \uparrow$$

Reaction forces are directed along boom and rod

Internal Forces (Method of Joints)



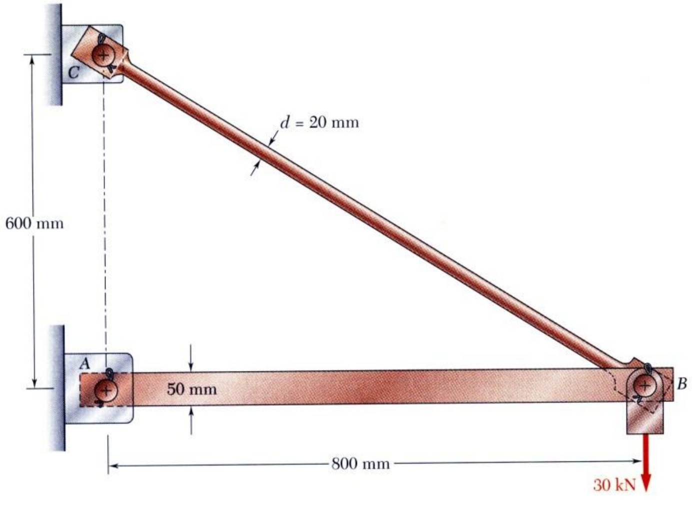
- The boom and rod are 2-force members, i.e., the members are subjected to only two forces which are applied at member ends
- For equilibrium, the forces must be parallel to to an axis between the force application points, equal in magnitude, and in opposite directions
- Joints must satisfy the conditions for static equilibrium which may be expressed in the form of a force triangle:

$$\sum \vec{F}_B = 0$$

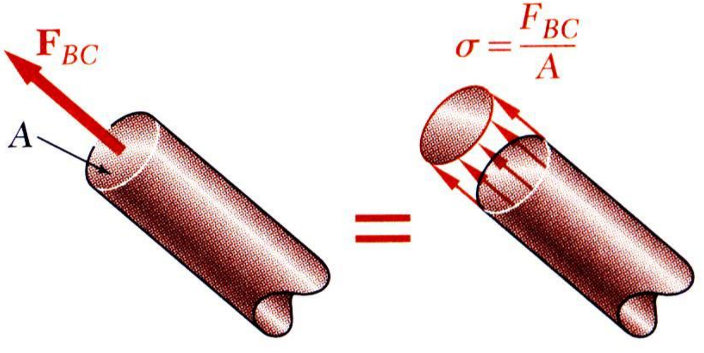
$$\frac{F_{AB}}{4} = \frac{F_{BC}}{5} = \frac{30\,\text{kN}}{3}$$

$$F_{AB} = 40\,\text{kN} \qquad F_{BC} = 50\,\text{kN}$$

Stress Analysis



d_{BC} = 20 mm



Can the structure safely support the 30 kN load?

- From a statics analysis

  F_{AB} = 40 kN (compression)
  F_{BC} = 50 kN (tension)

- At any section through member BC, the internal force is 50 kN with a force intensity or <u>stress</u> of

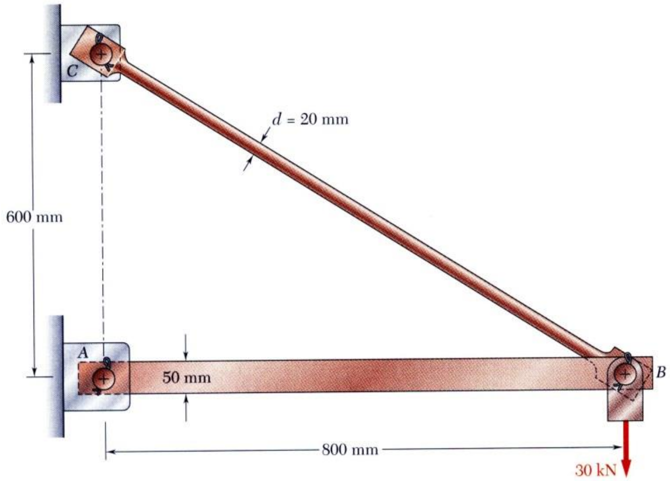
$$\sigma_{BC} = \frac{P}{A} = \frac{50 \times 10^3 \, \text{N}}{314 \times 10^{-6} \, \text{m}^2} = 159 \text{ MPa}$$

- From the material properties for steel, the allowable stress is

$$\sigma_{\text{all}} = 165 \text{ MPa}$$

- Conclusion: the strength of member *BC* is adequate

Design



- Design of new structures requires selection of appropriate materials and component dimensions to meet performance requirements

- For reasons based on cost, weight, availability, etc., the choice is made to construct the rod from aluminum (σ_{all}= 100 MPa). What is an appropriate choice for the rod diameter?

$$\sigma_{all} = \frac{P}{A} \qquad A = \frac{P}{\sigma_{all}} = \frac{50\times10^{3}\,\mathrm{N}}{100\times10^{6}\,\mathrm{Pa}} = 500\times10^{-6}\,\mathrm{m}^2$$

$$A = \pi\frac{d^2}{4}$$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4\left(500\times10^{-6}\,\mathrm{m}^2\right)}{\pi}} = 2.52\times10^{-2}\,\mathrm{m} = 25.2\,\mathrm{mm}$$

- An aluminum rod 26 mm or more in diameter is adequate