SCHOOL OF ENGINEERING

DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

CEE 3211– MECHANICS OF MATERIALS

LECTURE 2 – THERMAL STRESS/STRAIN

Twin Towers – New York 11 Sept 2001

GRENFELL TOWER FIRE – LONDON 2017

BEHAVIOUR OF CONCRETE AT ELEVATED TEMPERATURES

Thermal Strain

- Most engineering materials, (when unrestrained) expand when heated and contract when cooled.
- The thermal strain caused by a one-degree (1°) change in temperature is designated by the Greek letter α (alpha) and is known as the coefficient of thermal expansion.
- The strain due to a temperature change of ΔT is

$$\varepsilon_T = \alpha \Delta T$$

- The coefficient of thermal expansion is approximately constant over a considerable range of temperatures.
- Generally, the coefficient increases with an increase in temperature

Thermal Strain

- For an isotropic material, the coefficient applies to all dimensions (all directions)

- An element could be subjected to both thermal and normal stresses; in this case the strain caused by the two loading are calculated independently. Thus total strain becomes

$$\varepsilon_{\text{total}} = \varepsilon_{\sigma} + \varepsilon_{T}$$

Thermal Effects on Axial Deformation

Temperature changes can cause deformation in an element.

An axial member with a length L, the deformation resulting from a temperature change is

$$\delta_T = \varepsilon_T L = \alpha \Delta T L$$

If an axial member is allowed to elongate or contract freely, temperature change creates no stress in a material.

However, substantial stresses can result in an axial member if elongation or contraction is inhibited.

Combined deformation: Force–Temperature–Deformation Relationship

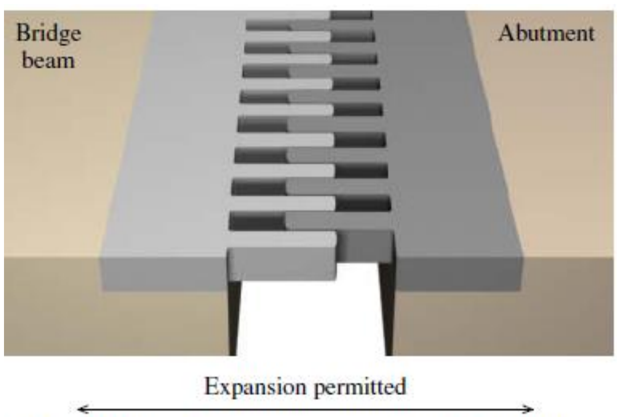
The relationship between internal force and axial deformation developed earlier can be enhanced to include the effects of temperature change:

$$\delta = \frac{PL}{AE} + \alpha \Delta T L$$

The deformation of a statically determinate axial member can be computed from Eqn above since the member is free to elongate or contract in response to a change in temperature.

Example 1

A steel bridge beam has a total length of 150 m. Over the course of a year, the bridge is subjected to temperatures ranging from −40°C to +40°C, and the associated temperature changes cause the beam to expand and contract. Expansion joints between the bridge beam and the supports at the ends of the bridge (called abutments)are installed to allow this change in length to take place without restraint. Determine the change in length that must be accommodated by the expansion joints. Assume that the coefficient of thermal expansion for steel is 11.9×10^{-6}/°C



Typical "finger-type" expansion joint for bridges.

Example 1

SOLUTION

Using the Equation;

$$\varepsilon_T = \alpha \Delta T$$

First, find the change in temperature;

$$\Delta T = 40°\text{C} - (-40°\text{C}) = \mathbf{80°C}$$

$$\varepsilon_T = 11.9 \ \times 10^{-6} 10\ /\ C) \times (80\ °C)$$

$$= \mathbf{0.000952\ m/m}$$

Example 1

SOLUTION

The deformation is:

$$\delta_T = \varepsilon L$$

$$= \left(\mathbf{0.000952}\,\frac{\mathbf{m}}{\mathbf{m}}\right) \times (150\ \mathrm{m})$$

$$= \mathbf{0.1428\ m} = \mathbf{142.8\ mm}$$

Example 2

A hot-rolled steel bar is held between two rigid supports. The bar is stress free at a temperature of 25°C. The bar is then heated uniformly. If the yield strength of the steel is 414 MPa, determine the temperature at which yield first occurs. Take E = 207 GPa; and $\alpha = 11.3\times10^{-6}$/°C

Solution

Force-Temperature-Deformation Relationship

The relationship between internal force, temperature change, and deformation of an axial member is:

$$\delta = \frac{FL}{AE} + \alpha \Delta T \, L$$

Since the bar is attached to rigid supports, $\delta = 0$.

$$\frac{FL}{AE} + \alpha \Delta T \, L = 0$$

which can also be expressed in terms of the bar normal stress:

$$\sigma \frac{L}{E} + \alpha \Delta T \, L = 0$$

Temperature at which yield first occurs: Divide this expression by *L* and solve for ΔT corresponding to a 414 MPa yield stress in the steel bar. Note that we are considering a compressive stress because we are told that the bar will be heated.

$$\Delta T = -\frac{\sigma}{\alpha E}$$

$$= -\frac{-414 \text{ MPa}}{\left(11.3\times10^{-6} \,/\, {}^\circ\text{C}\right)\left(207{,}000 \text{ MPa}\right)} = 176.99^\circ\text{C}$$

Initially, the bar is at a temperature of 25°C. Therefore, the temperature at which yield first occurs is:

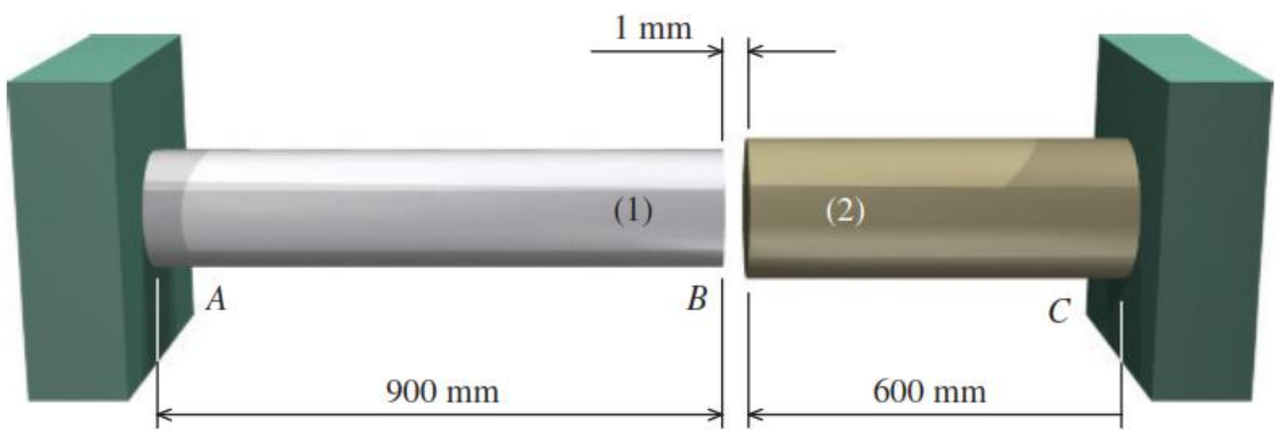
$$T_{\text{final}} = T_{\text{initial}} + \Delta T = 25^\circ\text{C} + 176.99^\circ\text{C} = \boxed{202^\circ\text{C}}$$

Example 3

An aluminum rod (1) [E = 70 GPa; α = 22.5 x 10^{-6}/°C] and a brass rod (2) [E =105 GPa; α = 18.0 x 10^{-6}/°C] are connected to rigid supports, as shown. The cross-sectional areas of rods (1) and (2) are 2,000 mm^2 and 3,000 mm^2, respectively. The temperature of the structure will increase.

(a) Determine the temperature increase that will close the initial 1 mm gap between the two axial members.

(b) Compute the normal stress in each rod if the total temperature increase is 60°C.



Example 3

(a) The axial elongation in the two rods due solely to a temperature increase can be expressed as

$$\delta_{1,T} = \alpha_1 \Delta T L_1 \qquad \text{and} \qquad \delta_{2,T} = \alpha_2 \Delta T L_2$$

If the two rods are to touch at B, the sum of the elongations in the rods must equal 1 mm:

$$\delta_{1,T} + \delta_{2,T} = \alpha_1 \Delta T L_1 + \alpha_2 \Delta T L_2 = 1 \text{ mm}$$

We solve this equation for ΔT:

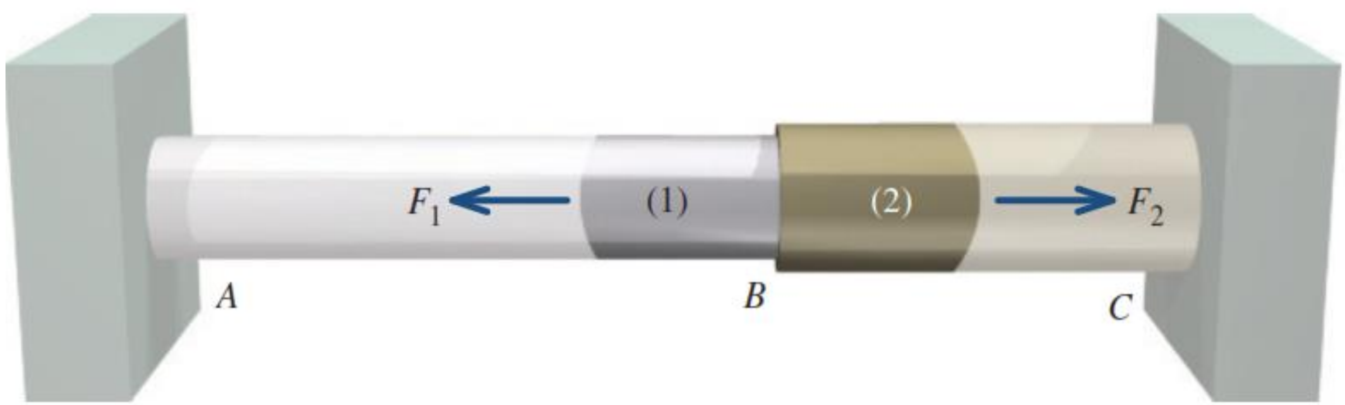
$$(22.5 \times 10^{-6}/^\circ\text{C})\Delta T(900 \text{ mm}) + (18.0 \times 10^{-6}/^\circ\text{C})\Delta T(600 \text{ mm}) = 1 \text{ mm}$$

$$\therefore \Delta T = 32.2^\circ\text{C}$$ **Ans.**

Example 3

(b) Given that a temperature increase of 32.2°C closes the 1 mm gap, a larger temperature increase (i.e., 60°C in this instance) will cause the aluminum and brass rods to compress each other, since the rods are prevented from expanding freely by the supports at *A* and *C*.

Consider a free-body diagram (FBD) of joint *B* after the aluminum and brass rods have come into contact. The sum of forces in the horizontal direction consists exclusively of the internal member forces.

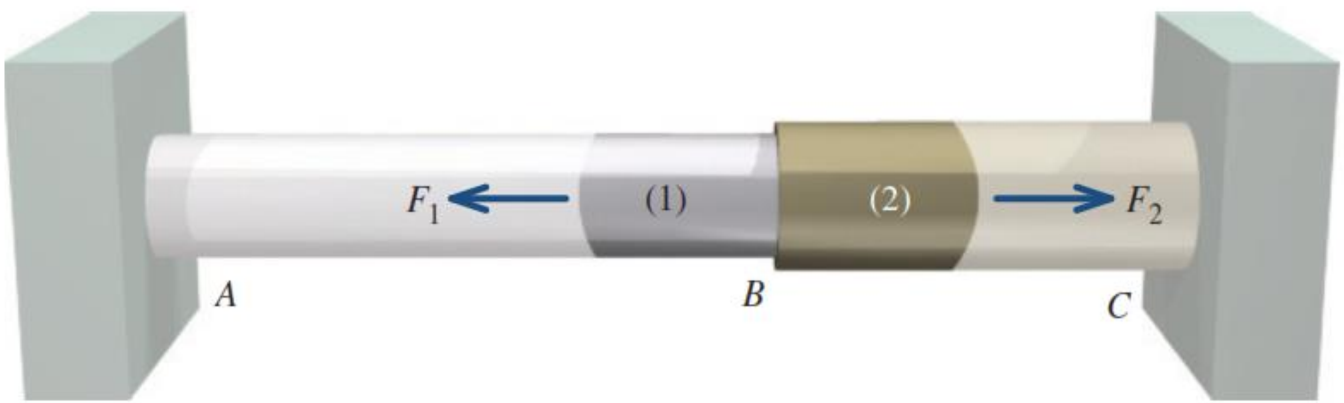


Example 3

(b) Given that a temperature increase of 32.2°C closes the 1 mm gap, a larger temperature increase (i.e., 60°C in this instance) will cause the aluminum and brass rods to compress each other, since the rods are prevented from expanding freely by the supports at A and C.

Step 1 — Equilibrium Equations:

Consider a free-body diagram (FBD) of joint B after the aluminum and brass rods have come into contact. The sum of forces in the horizontal direction consists exclusively of the internal member forces.



Example 3

$$\sum F_x = F_2 - F_1 = 0 \qquad \therefore F_1 = F_2$$

Step 2 — Geometry of Deformation

Since the compound axial member is attached to rigid supports at *A* and *C*, the overall elongation of the structure can be no more than 1 mm. In other words,

$$\delta_1 + \delta_2 = 1 \text{ mm} \qquad \text{(a)}$$

Step 3 — Force–Temperature–Deformation Relationships

$$\boxed{\delta_1 = \frac{P_1 L_1}{A_1 E_1} + \alpha_1 \Delta T L_1} \quad \& \quad \boxed{\delta_2 = \frac{P_2 L_2}{A_2 E_2} + \alpha_2 \Delta T L_2} \qquad \text{(b)}$$

Example 3

Step 4 — compatibility Equation:

Substitute Equations (b) into Equation (a) to obtain the compatibility equation

$$\frac{P_1 L_1}{A_1 E_1} + \alpha_1 \Delta T L_1 + \frac{P_2 L_2}{A_2 E_2} + \alpha_2 \Delta T L_2 = 1 \text{ mm} \qquad \text{(c)}$$

Step 5 — Solve the Equations:

Substitute $P_2 = P_1$ (from the equilibrium equation) into Equation (c), and solve for the internal force P_1:

$$P_1 \left[\frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2} \right] = 1\, mm - \alpha_1 \Delta T L_1 - \alpha_2 \Delta T L_2 \qquad \text{(d)}$$

Example 3

$$P_1\left[\frac{900\text{ mm}}{2000\text{ mm}^2\times 70000\text{ N/mm}^2}+\frac{600\text{ mm}}{3000\text{ mm}^2\times 1050000\text{ N/mm}^2}\right.$$
$$\left.= 1\text{mm} - \left(22.5\times\frac{10^{-6}}{°\text{C}}\right)(60°\text{C})(900\text{ mm}) - (18\times\frac{10^{-6}}{°\text{C}})(60°\text{C})(600\text{ mm})\right]$$

(e)

$P_1 = -103{,}350\ N =$ -103.6 kN

The normal stress in rod (1)

$$\sigma_1 = \frac{P_1}{A_1} = \frac{-103{,}560}{2{,}000} = -51.8\text{ MPa}\ \ (\text{Compression})$$

Example 3

The normal stress in rod (2)

$$\sigma_2 = \frac{P_2}{A_2} = \frac{-103{,}560}{3{,}000} = -34.5\ \mathrm{MPa}\ \ (\mathrm{Compression})$$