# THE UNIVERSITY OF ZAMBIA
## Department of Mathematics and Statistics
## MAT2110: Engineering Mathematics I
## Tutorial Sheet 9 (2022/2023)

1. (a) Show that each of the following, where $c_1$ and $c_2$ are constants, are solutions to the differential equation

$$\frac{d^2y}{dx^2}(1-x)+\frac{dy}{dx}x-y=0.$$

i. $y=2e^x$ ii. $y=3x$ iii. $y=c_1e^x+c_2x$

(b) Solve the following ODEs using separation of variable.

i. $\frac{dy}{dx}=\frac{5x}{7y}$

ii. $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$

iii. $\frac{dy}{dx}+\frac{1+y^3}{xy^2(1+x^2)}=0$

iv. $\frac{dy}{dx}=\frac{\cos^2 y}{\sin^2 x}$

v. $\frac{dy}{dx}=ay(1-by)$ where $a$ and $b$ are constants.

2. (a) Show that each of the following ODEs are homogeneous and hence solve.

i. $\frac{dy}{dx} = \frac{x+y}{x}$

ii. $2xy\,dy = (x^2 - y^2)\,dx$

iii. $\frac{dy}{dx} = (x+y)^2$

iv. $\frac{dy}{dx} = \frac{x+2y}{3y-2x}$

v. $\frac{dy}{dx} = -\frac{2x^2+y^2}{2xy+3y^2}$

(b) Solve the following differential equations by means of an integration factor.

i. $\frac{dy}{dx} + y = e^x$

ii. $\frac{dy}{dx} + \frac{1}{3}y = 1$

iii. $x\frac{dy}{dx} + y = x$

iv. $\frac{dy}{dx} - y = \sin x$

v. $\tan x\frac{dy}{dx} + y = \sec x$

3. Show that each of the following differential equations is exact and use that property to find the general solution.

(a) $\frac{1}{x}dy - \frac{y}{x^2}dx = 0$

(b) $2xy\frac{dy}{dx} + y^2 - 2x = 0$

(c) $2(y+1)e^x dx + 2(e^x - 2y)dy = 0$

(d) $(3x^2 + y\cos x)dx + 2(\sin x - 4y^3)dy = 0$

4. Solve the following Bernoulli equations.

(a) $\frac{dy}{dx} + 3x^2yy = x^2y^3$

(b) $\frac{dy}{dx} + \frac{y}{x} = xy^2$

(c) $\frac{dy}{dx} + \frac{y}{x} = y^2$

(d) $\frac{dy}{dx} + \frac{1}{3}y = e^x y^4$

(e) $\frac{dy}{dx} + \frac{2}{x}y = -x^2(\cos x)y^2$

5. Find a particular solution to each of the following ODEs.

(a) $\frac{dy}{dx} + \frac{y}{x} = 0$ given that at $x = 2$, $y = 2$.

(b) $2(x + 2y)\,dx + (y - x)\,dy = 0$ given that when $x = 1$, $y = 0$.

(c) $x^3\frac{dy}{dx} + 2y = e^{\frac{1}{x^2}}$ at $(1, e)$.

(d) $\frac{dy}{dx} + 2xy = xy^2$ at $(0, 1)$.

6. (a) Solve the auxiliary equation for each of the following differential equations.

i. $\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 36y = 0.$

ii. $\frac{d^2y}{dx^2} + 7y = 0.$

iii. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 0.$

iv. $2\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 0.$

v. $\frac{d^2y}{dx^2} - 9y = 0.$

(b) Hence solve each of the differential equations in (a) above.

(c) Solve each of the following ODEs by the method of undetermined coefficients.

i.
$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8.$$

ii.
$$\frac{d^2y}{dx^2} - 4y = 10e^{3x}.$$

iii.
$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}.$$

iv.
$$\frac{d^2y}{dx^2} + 25y = 5x^2 + x.$$

v.
$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x.$$

vi.
$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x},$$
given that $x = 0$, $y = 1$, $\frac{dy}{dx} = -2$.

7. Solve each of the following ODEs by the method of variation of parameters.

(a)
$$\frac{d^2y}{dx^2} + y = \sec x.$$

(b)
$$\frac{d^2y}{dx^2} + y = \sec x \tan x.$$

(c)
$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2e^{2x}.$$

(d)
$$\frac{d^2y}{dx^2} + 4y = \csc 2x.$$

(e)
$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \ln x.$$

(f)
$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = \frac{e^{2x}}{x}.$$