SCHOOL OF ENGINEERING

DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

CEE 3211– MECHANICS OF MATERIALS

LECTURE 5 – TORSION PART I

What will be covered

1 Determine the shear stresses in a circular shaft due to torsion

2 Determine the angle of twist

3 Analyze statically indeterminate torque-loaded members

4 Deal with solid non-circular shafts and thin-walled tubes

INTRODUCTION

- Torque (T) is a moment that tends to twist a member about its longitudinal axis.

- In the design of machinery (and some structures), the problem of transmitting a torque from one plane to a parallel plane is frequently encountered

- The simplest device for accomplishing this function is called a shaft

- The torsion problem is concerned with the determination of stresses in shaft and the deformation of the shaft.

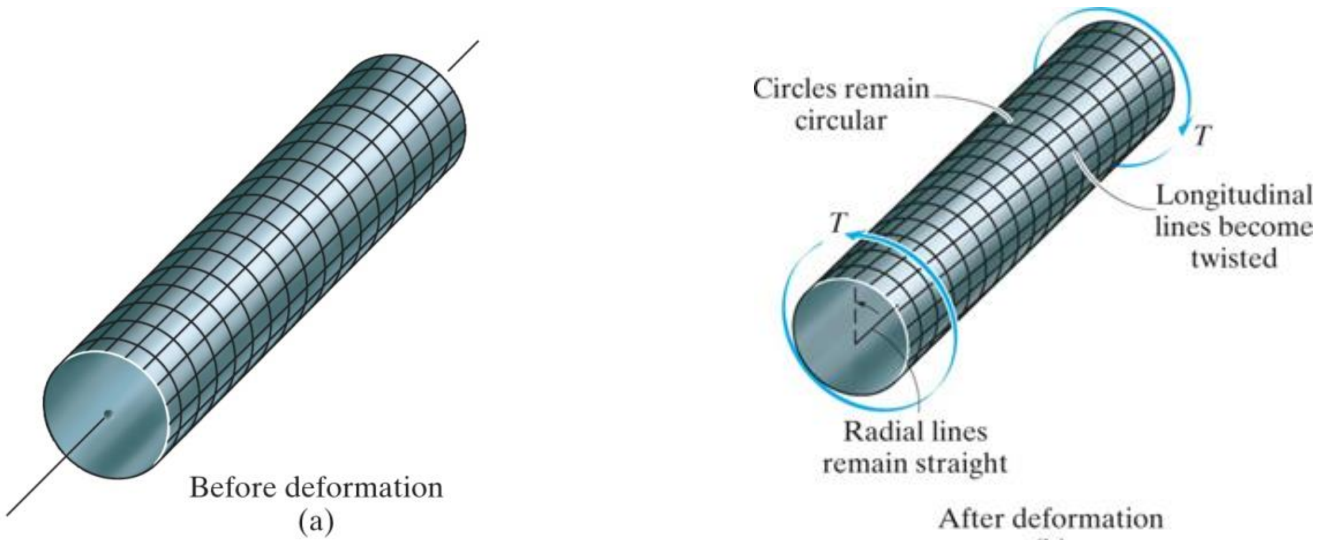
Comparisons between Axial Deformation and Torsion

Axial Deformation	Torsion
Axial Force (F)	Torque (T)
Elongation (e)	Twist angle (ϕ)
Normal stress (σ)	Shear stress (τ)
Extensional strain (ϵ)	Shear strain (γ)
Modulus of elasticity (E)	Shear modulus (G)

Torsion Formula (1 of 4)

- Assumptions:
  - Linear and elastic deformation
  - Plane section remains plane and undistorted

Figure 1

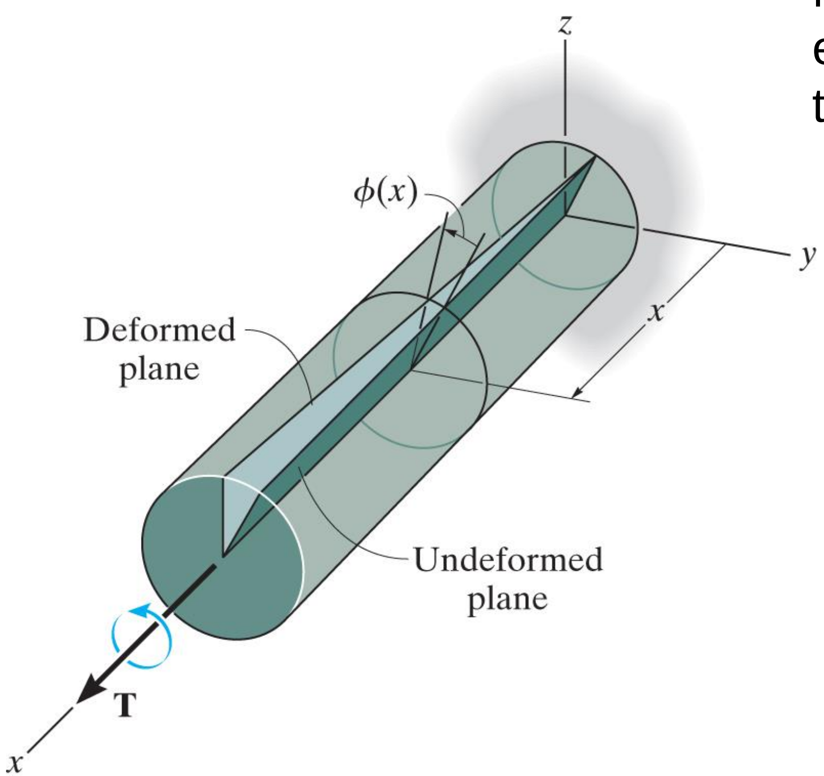


Before deformation
(a)

After deformation
(b)

Torsion Formula (2 of 4)

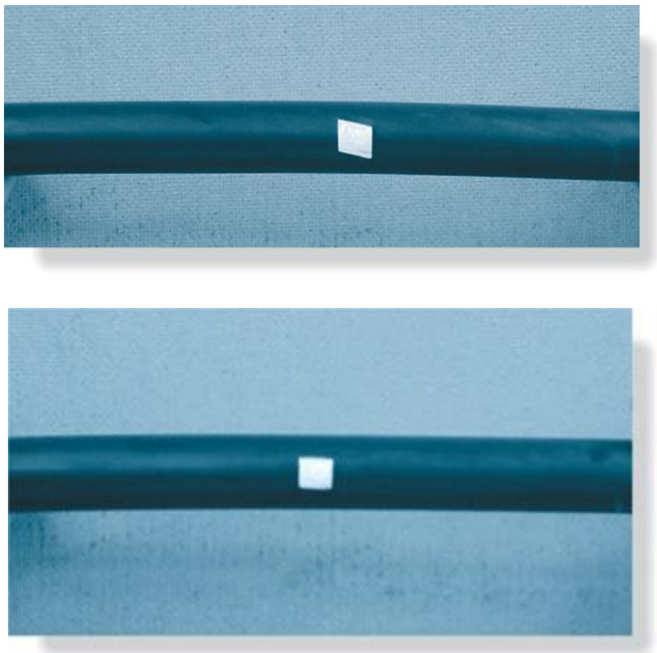
Figure 2



The angle of twist $\phi(x)$ increases as x increases.

Figure 3

Notice the deformation of the rectangular element when this rubber bar is subjected to a torque.



Torsion Formula (3 of 4)

- Linear distribution of stress: $\tau = \dfrac{\rho}{c}\tau_{\max}$

- Torsion – shear relationship:

$$T = \int_A \rho(\tau) dA = \int_A \rho\left(\frac{\rho}{c}\right)\tau_{\max} dA$$

$$T = \frac{\tau_{\max}}{c}\int_A \rho^2 dA \qquad \int_A \rho^2 dA = J$$

$$\tau_{\max} = \frac{Tc}{J}$$

Similarly, $\tau = \dfrac{T\rho}{J}$

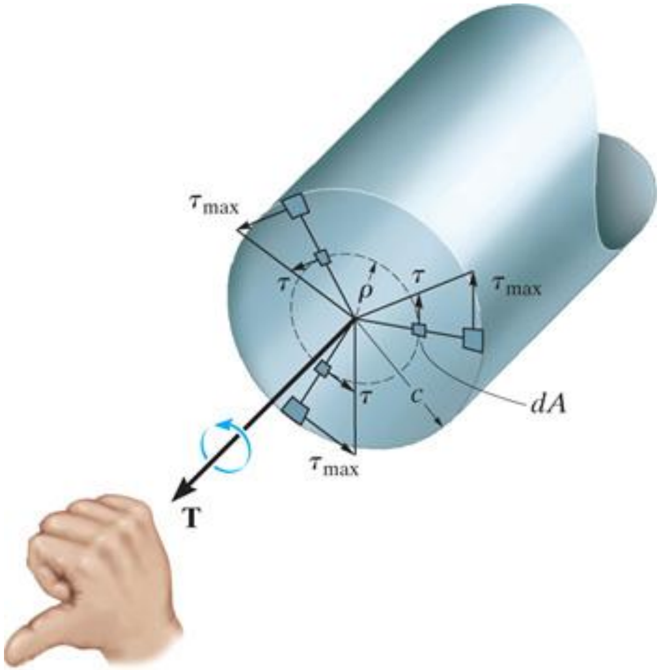
Torsional Formula

$\rho =$ radial distance

$c =$ outer radius of shaft

Figure 4

Shear stress τ varies linearly from Zero (at axis) along radial line of the cross section to τ_{max} at outer surface



Torsion Formula (4 of 4)

- **Polar moment of inertia**
  - For solid shaft: $$J=\int_A \rho^2 dA=\int_0^c \rho^2\left(2\pi\rho d\rho\right)=2\pi\int_0^c \rho^3 d\rho=2\pi\left(\frac{1}{4}\right)\rho^4\Big|_0^c$$
    $$J=\frac{\pi}{2}c^4$$
  - For tubular shaft: $$J=\frac{\pi}{2}\left(c_o^4-c_i^4\right)$$

Figure 5

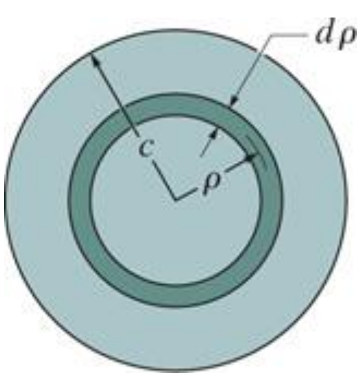
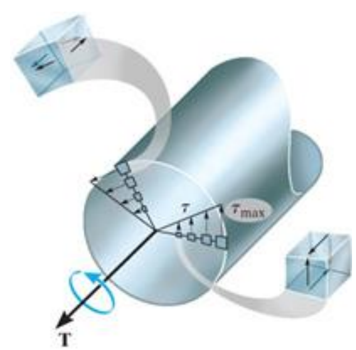
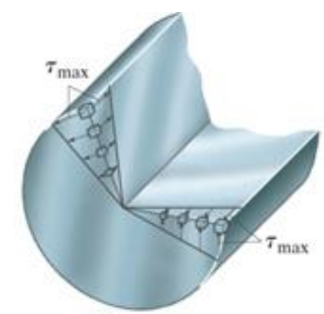


Figure 6a



(a)

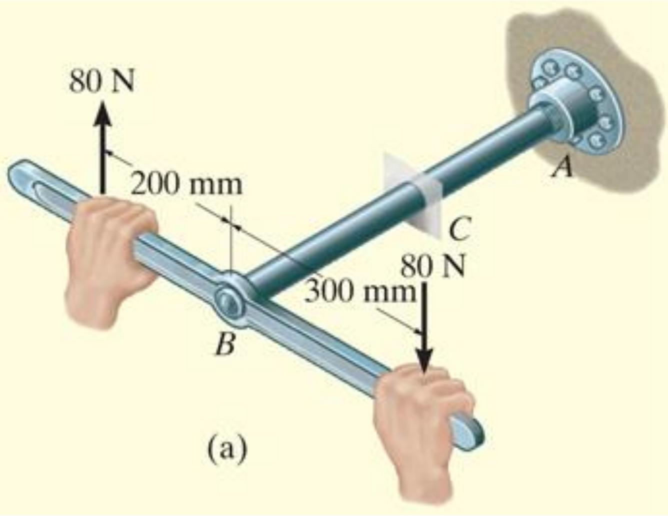
Figure 6b

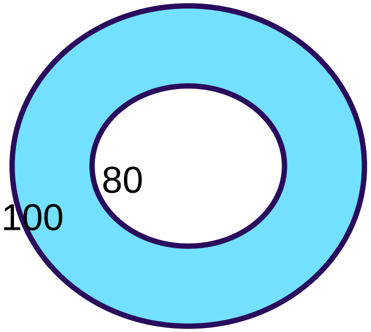


Shear stress varies linearly along each radial line of the cross section.
(b)

Example 1 (1 of 4)

The pipe shown below has an inner diameter of 80 mm and an outer diameter of 100 mm. If its end is tightened against the support at *A* using a torque wrench at *B*, determine the shear stress developed in the material at the inner and outer walls along the central portion of the pipe when the 80-N forces are applied to the wrench.



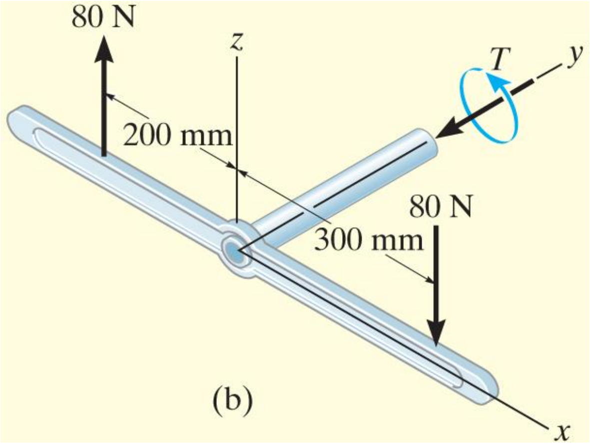


Example 1 (2 of 4)

Solutions

- The only unknown at the section is the internal torque **T**

$$\sum M_y = 0;$$
$$80(0.3) + 80(0.2) - T = 0$$
$$T = 40\ \mathrm{N \cdot m}$$



- The polar moment of inertia for the pipe's cross-sectional area is

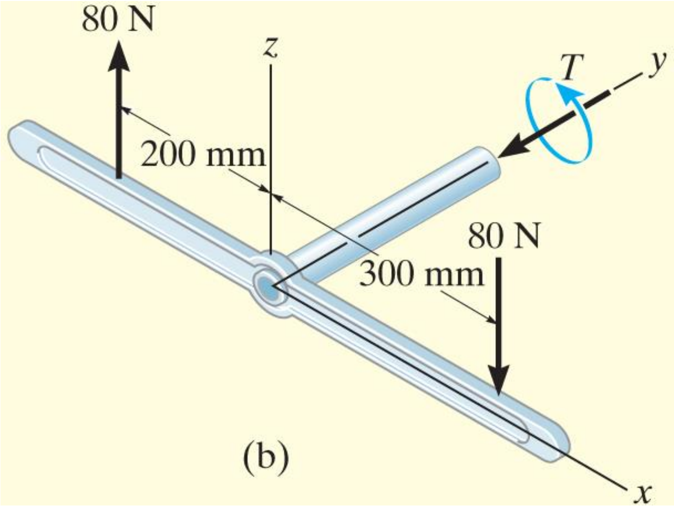
$$J = \frac{\pi}{2}\left[(0.05)^4 - (0.04)^4\right] = 5.796\left(10^{-6}\right)\ \mathrm{m}^4$$

Example 1 (3 of 4)

- For any point lying on the outside surface of the pipe,

$$\rho = c_0 = 0.05 \text{ m}$$

$$\tau_0 = \frac{Tc_0}{J} = \frac{40(0.05)}{5.796(10^{-6})} = 0.345 \text{ MPa} \quad \text{(Ans)}$$



(b)

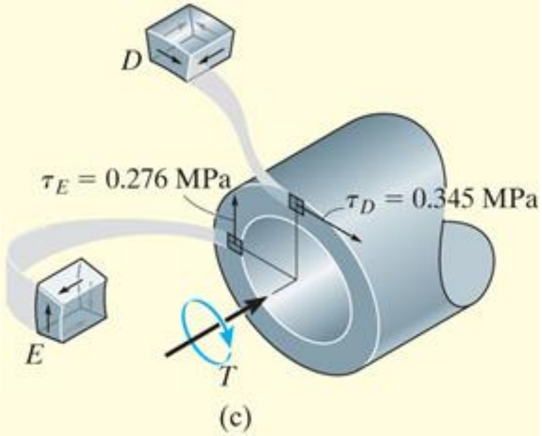
Example 1 (4 of 4)

- And for any point located on the inside surface, $\rho = c_i = 0.04$ m

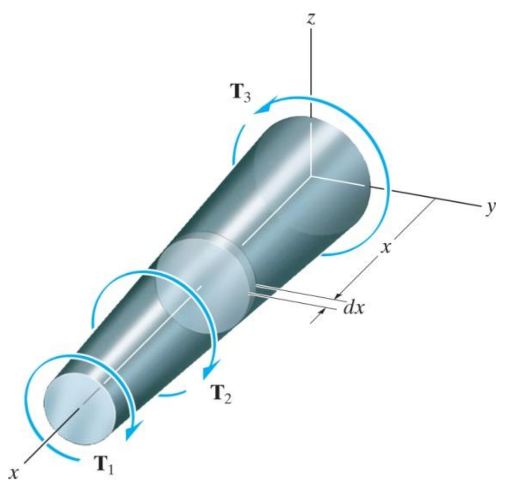
$$\tau_i = \frac{Tc_i}{J} = \frac{40(0.04)}{5.796(10^{-6})} = 0.276 \text{ MPa} \quad \text{(Ans)}$$

- The resultant internal torque is equal but opposite.

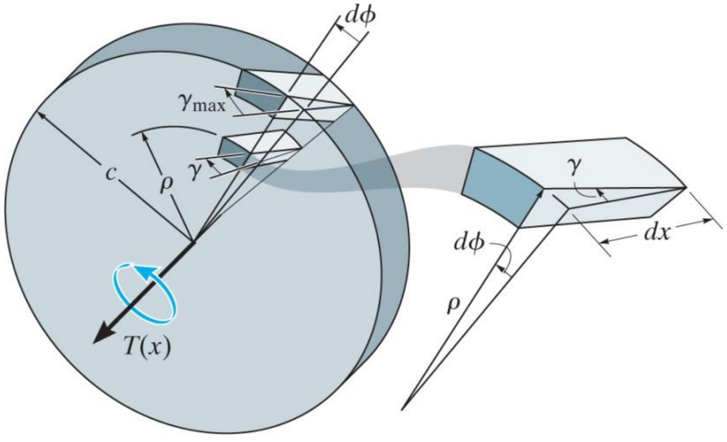
Figure 7



Angle of Twist



(a)



(b)

$$d\phi = \gamma \frac{dx}{\rho}$$

$$d\phi = \frac{T(x)}{J(x)G} dx$$

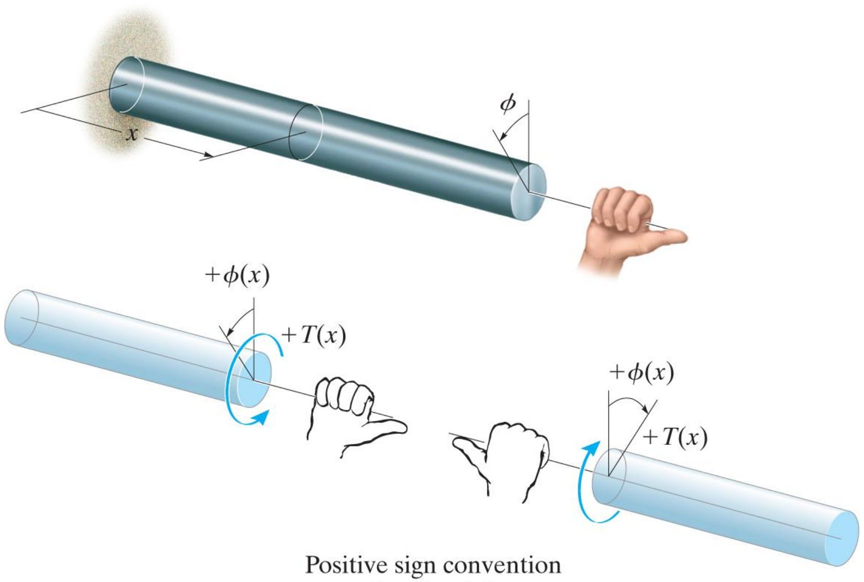
- For constant torque and cross-sectional area:

$$\boxed{\phi = \frac{TL}{JG}}$$

Angle of Twist (2 of 2)

- Sign convention for both torque and angle of twist
  - positive if (right hand) thumb directs outward from the shaft

Figure 8



Positive sign convention for T and ϕ.

Example 2

A rod specimen of ductile cast iron was tested in a torsion-testing machine. The rod diameter was 22 mm, and the rod length was 300 mm. When the applied torque reached 271.4 N·m, a shear strain of 2,015 micro-radians was measured in the specimen.

What was the angle of twist in the specimen?

Example 2 - SOLUTION

The polar moment of inertia for the specimen is

$$J = \frac{\pi}{2}(11\ \text{mm})^4 = 22{,}998\ \text{mm}^4$$

The shear stress in the specimen at the specified torque is:

$$\tau = \frac{Tc}{J} = \frac{(271.4\ \text{N}\cdot\text{m})(11\ \text{mm})(1{,}000\ \text{mm/m})}{22{,}998\ \text{mm}^4}$$

$$= 129.81\ \text{Mpa}$$

Example 2 - SOLUTION

From Hooke's law, calculate the shear modulus from the specified shear strain and the calculated shear stress:

$$\tau = G\gamma \qquad \therefore G = \frac{\tau}{\gamma} = \frac{129.81\ \text{N/mm}^2}{2{,}015 \times 10^{-6}\ \text{rad}} = 64{,}422.4\ \text{N/mm}^2$$

The magnitude of the angle of twist in a 300 mm length of the specimen is

$$\phi = \frac{TL}{JG} = \frac{(271.4\ \text{N}\cdot\text{m})(300\ \text{mm})(1{,}000\ \text{mm/m})}{(22{,}998\ \text{mm}^4)(64{,}422.4\ \text{N/mm}^2)}$$

$$= \boxed{0.0550\ \text{rad}} = \boxed{3.15°}$$