SCHOOL OF ENGINEERING

DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

CEE 3211– MECHANICS OF MATERIALS

LECTURE 6 & 7– TORSION PART II

What will be covered

1 Determine the shear stresses in a circular shaft due to torsion

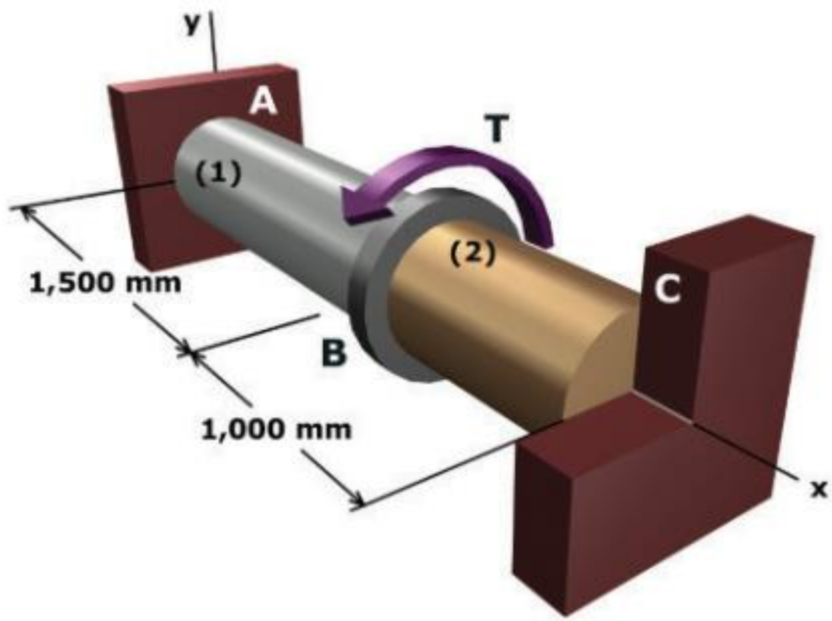
2 Determine the angle of twist

3 Analyze statically indeterminate torque-loaded members

4 Deal with solid non-circular shafts and thin-walled tubes

Statically Indeterminate Torque-Loaded Members

A torsionally loaded shaft will be statically indeterminate if the moment equation of equilibrium, applied about the axis of the shaft, is not adequate to determine the unknown torques acting on the shaft



Statically Indeterminate Torque-Loaded Members

Procedure for analysis:

- Use both equilibrium and compatibility equations

Equilibrium

- Draw a free-body diagram of the shaft in order to identify all the torques that act on it. Then write the equations of moment equilibrium about the axis of the shaft.

Compatibility

- To write the compatibility equation, investigate the way the shaft will twist when subjected to the external loads, and give consideration as to how the supports constrain the shaft when it is twisted.

Statically Indeterminate Torque-Loaded Members (2 of 2)

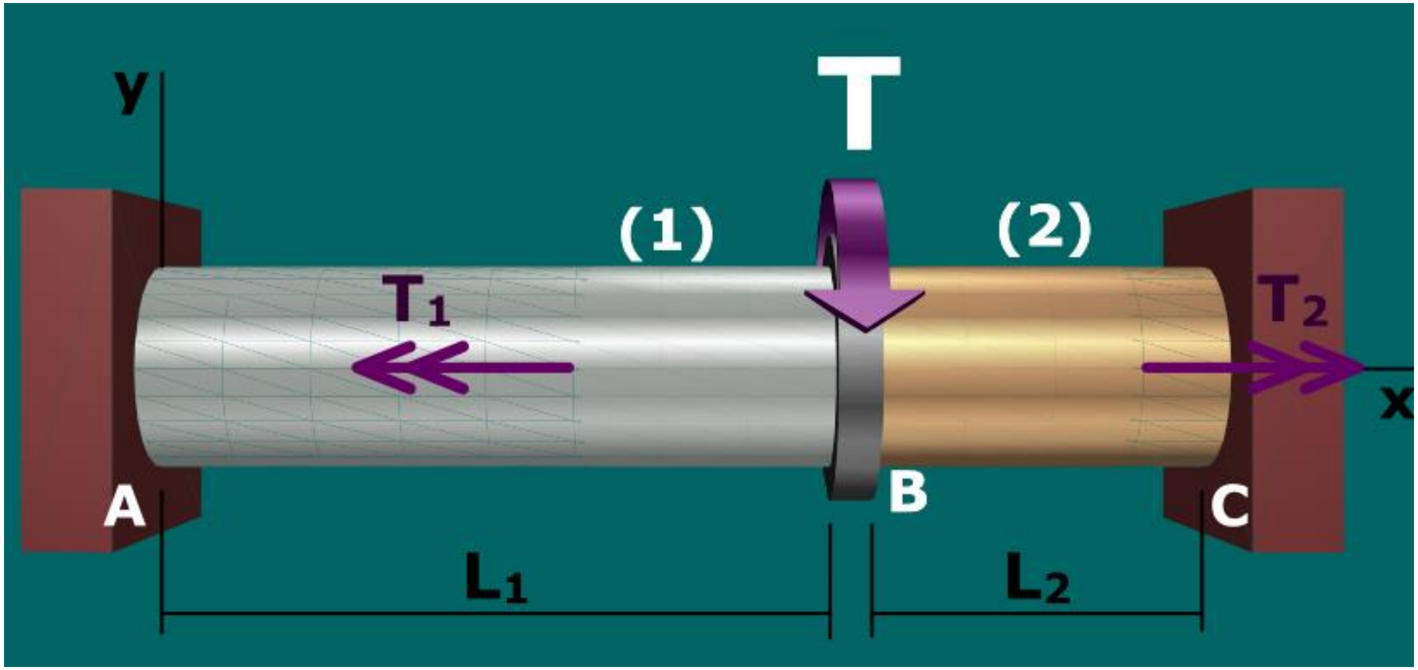
- Express the compatibility condition in terms of the rotational displacements caused by the reactive torques, and then use a torque-displacement relation, such as $\phi = TL / JG$, to relate the unknown torques to the unknown displacements.

- Solve the equilibrium and compatibility equations for the unknown reactive torques.

- If any of the magnitudes have a negative numerical value, it indicates that this torque acts in the opposite sense of direction to that indicated on the free-body diagram.

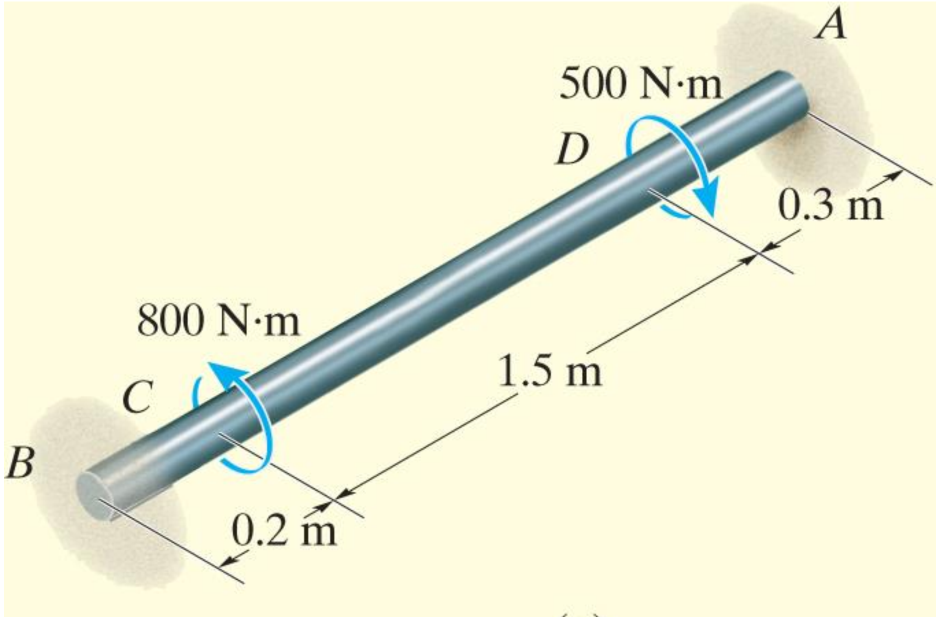
Example 4

A shaft is fixed on both ends (A & C) and a Torque of 500N.m is applied at the location shown. Both segments have a diameter of 20mm. L1=600mm and L2= 400m. Find the reaction Torques at A and B



Example 5 (1 of 3)

The solid steel shaft shown below has a diameter of 20 mm. It is fixed on both ends. If it is subjected to the two torques, determine the reactions at the fixed supports *A* and *B*.



(a)

Example 5 (2 of 3)

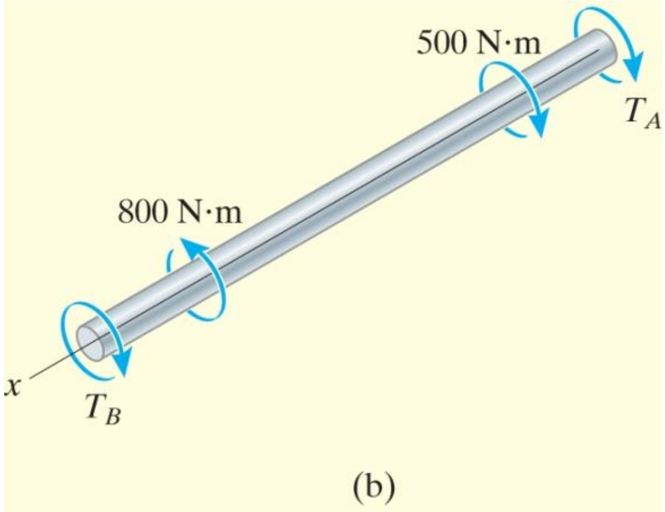
Solutions

- It is seen that the problem is statically indeterminate since there is only **one** available equation of equilibrium and there are 2 unknowns

$$\sum M_x = 0$$
$$-T_B + 800 - 500 - T_A = 0 \qquad (1)$$

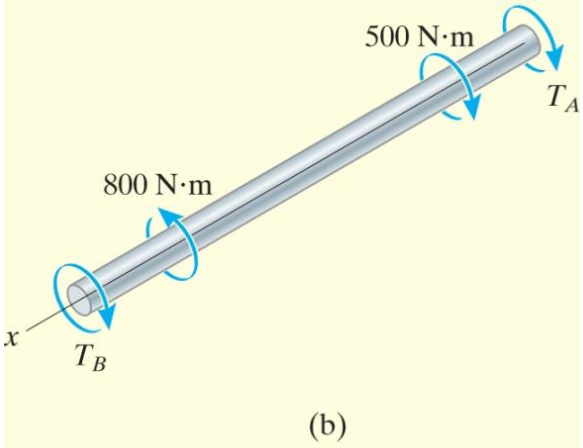
- Since the ends of the shaft are fixed, the angle of twist of one end of the shaft with respect to the other must be zero.

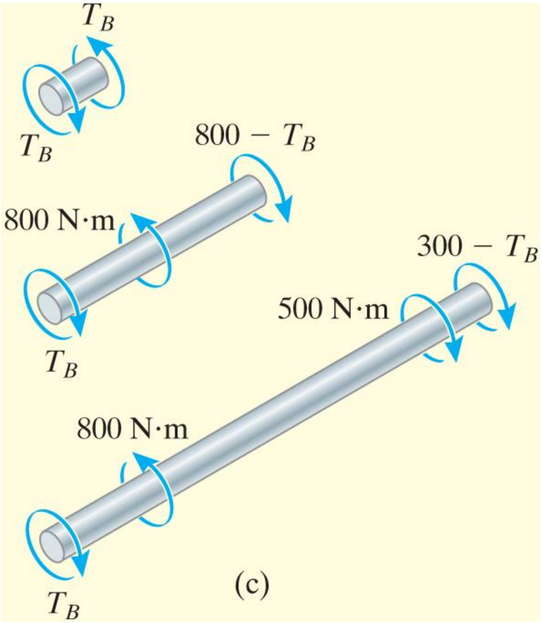
$$\phi_{A/B} = 0$$



Example 5 (3 of 3)

- Using the sign convention established,





$$\frac{-T_B(0.2)}{JG} + \frac{(800 - T_B)(1.5)}{JG} + \frac{(300 - T_B)(0.3)}{JG} = 0$$

$$T_B = 645\ \mathrm{N \cdot m} \quad \text{(Ans)}$$

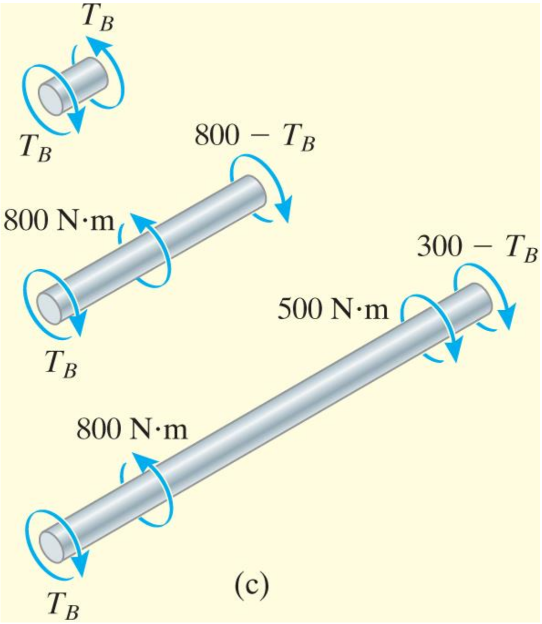
Example 5 (3 of 3)

From (1);

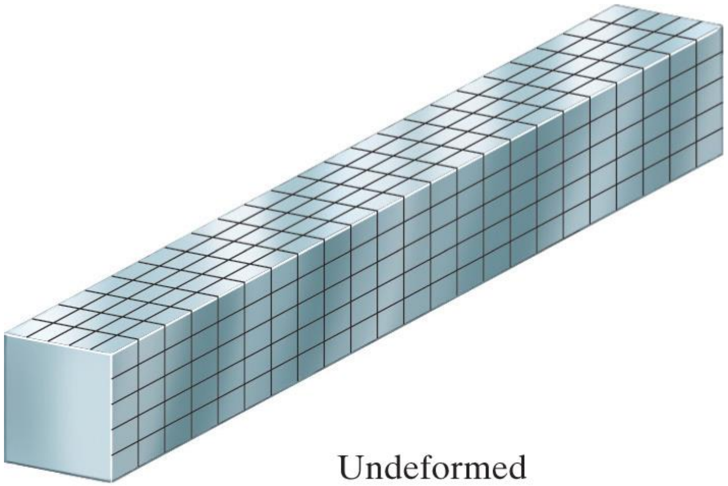
$$-645 + 800 - 500 - T_A = 0$$

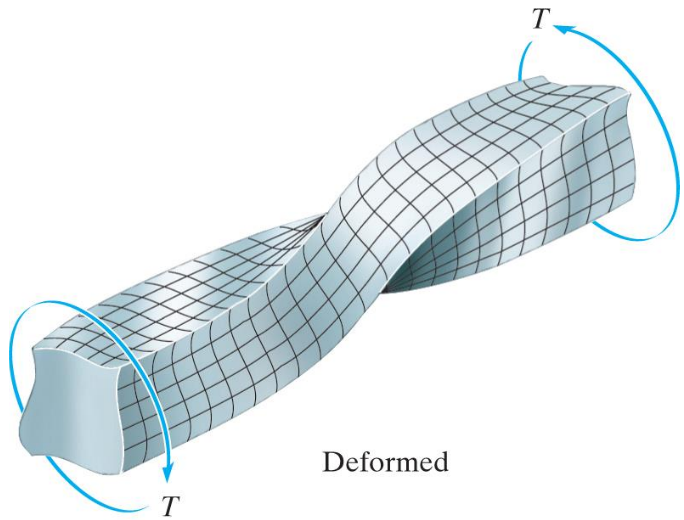
$$\boxed{T_A = -345\ \text{N}\cdot\text{m}}$$

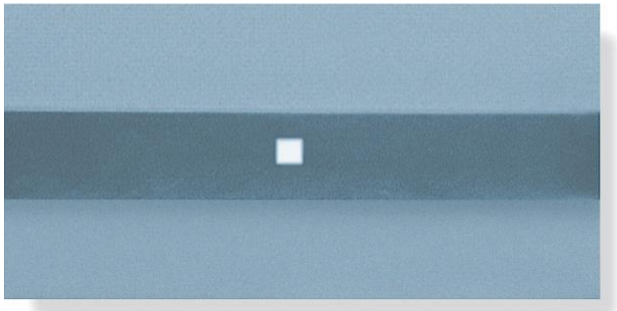
- The negative sign indicates that acts in the opposite direction of that shown in Figure b.

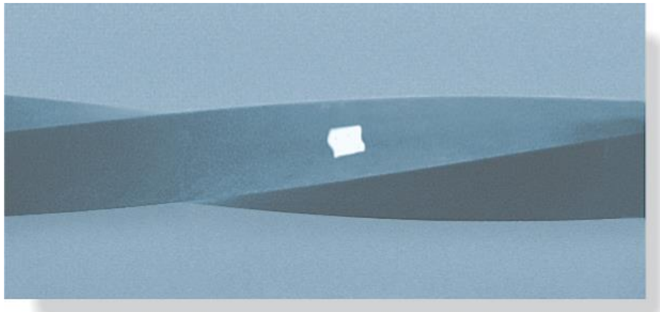


Solid Non-Circular Shafts





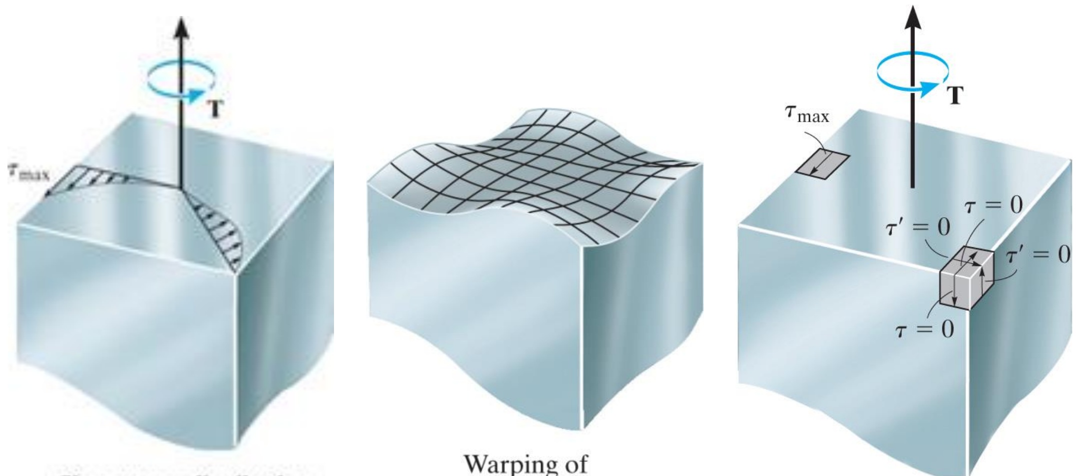




Solid Non-Circular Shafts

Table-1

Shape of cross section	τ_{max}	ϕ
Square	$\dfrac{4.81\,T}{a^3}$	$\dfrac{7.10\,TL}{a^4G}$
Equilateral triangle	$\dfrac{20\,T}{a^3}$	$\dfrac{46\,TL}{a^4G}$
Ellipse	$\dfrac{2\,T}{\pi ab^2}$	$\dfrac{(a^2+b^2)TL}{\pi a^3b^3G}$



Shear stress distribution along two radial lines

(a)

Warping of cross-sectional area

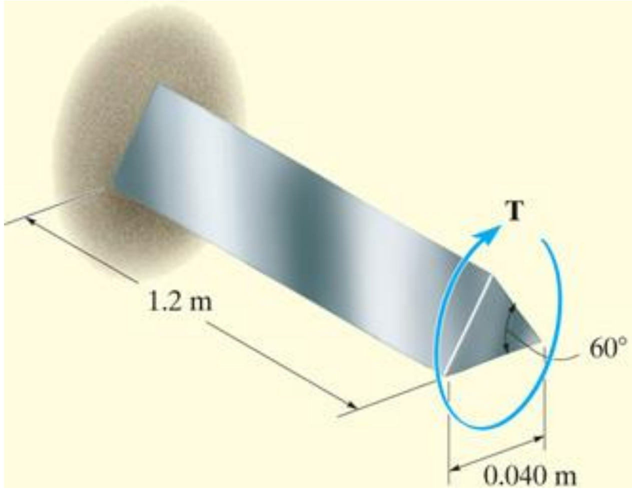
(b)

(c)

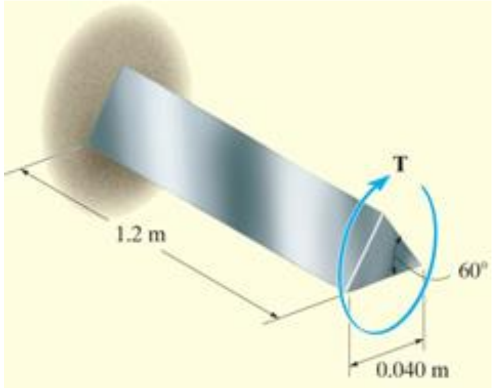
Example 6 (1 of 3)

The 6061-T6 aluminum shaft shown in Figure below has a cross-sectional area in the shape of an equilateral triangle.

1. Determine the largest torque *T* that can be applied to the end of the shaft if the allowable shear stress is τ_{allow} = 56 MPa and the angle of twist at its end is restricted to $\emptyset_{allow} = 0.02$ rad.
2. How much torque can be applied to a shaft of circular cross section made from the same amount of material? Take G_{al} as 26 GPa



Example 6 (2 of 3)



Solutions

- By inspection, the resultant internal torque at any cross section along the shaft's axis is also **T**.

$$\tau_{allow} = \frac{20T}{a^3}; \qquad 56 = \frac{20T}{40^3} \Rightarrow T = \mathbf{1779.2\ N.m}$$

$$\phi_{allow} = \frac{46T}{a^4 G_{al}}; \qquad 0.02 = \frac{46T(1.2)(10^3)}{40^4[26(10^3)]} \Rightarrow T = \mathbf{24.12\ N.m}$$

- By comparison, the torque is limited due to the angle of twist.

$$T = \mathbf{24.12\ N.m} \quad \textbf{[ANS]}$$

Example 6 (3 of 3)

Solutions

- For circular cross section, we have

$$A_{circle} = A_{triangle}; \qquad \pi c^2 = \frac{1}{2}(40)(40\sin 60^\circ) \Rightarrow c = 14.85 \text{ mm}$$

- The limitations of stress and angle of twist then require

$$\tau_{allow} = \frac{Tc}{J}; \qquad 56 = \frac{T(14.85)}{(\pi/2)(14.85)^4} \Rightarrow T = 288.06 \text{ N.m}$$

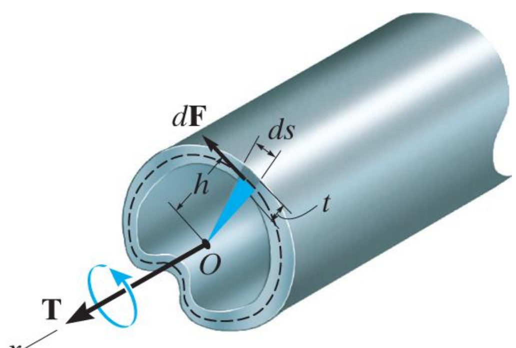
$$\phi_{allow} = \frac{TL}{JG_{al}}; \qquad 0.02 = \frac{T(1.2)(10^3)}{(\pi/2)(14.85)^4[26(10^3)]} \Rightarrow T = 33.10 \text{ N.m}$$

  - Again, the angle of twist limits the applied torque.

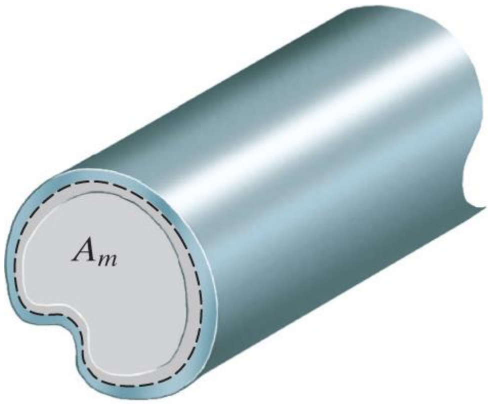
    - $\boldsymbol{T = 33.1}$ **N.m**

Thin Wall Tubes Having Closed Sections

- ***The shear flow measures the force per unit length along the cross section*.**
- The average shear stress can be related to the torque T by considering the torque produced by this shear stress about a selected point O within the tube's boundary



(e)



(f)

Thin Wall Tubes Having Closed Sections

- Average shear stress

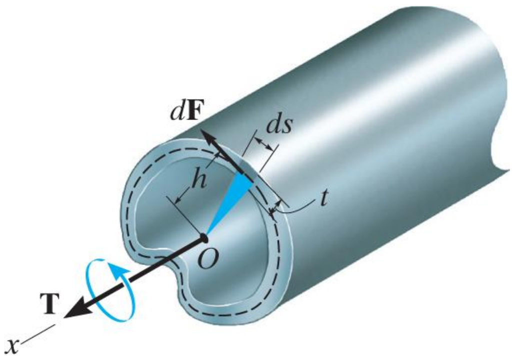
$$\tau_{avg} = \frac{T}{2tA_m}$$

- Shear flow

$$q = \frac{T}{2A_m}$$

- Angle of twist

$$\phi = \frac{TL}{4A_m^2 G} \oint \frac{ds}{t}$$



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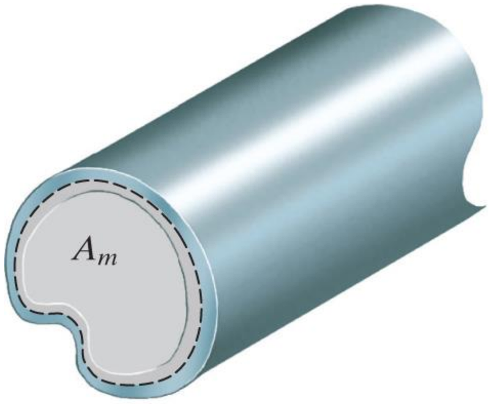
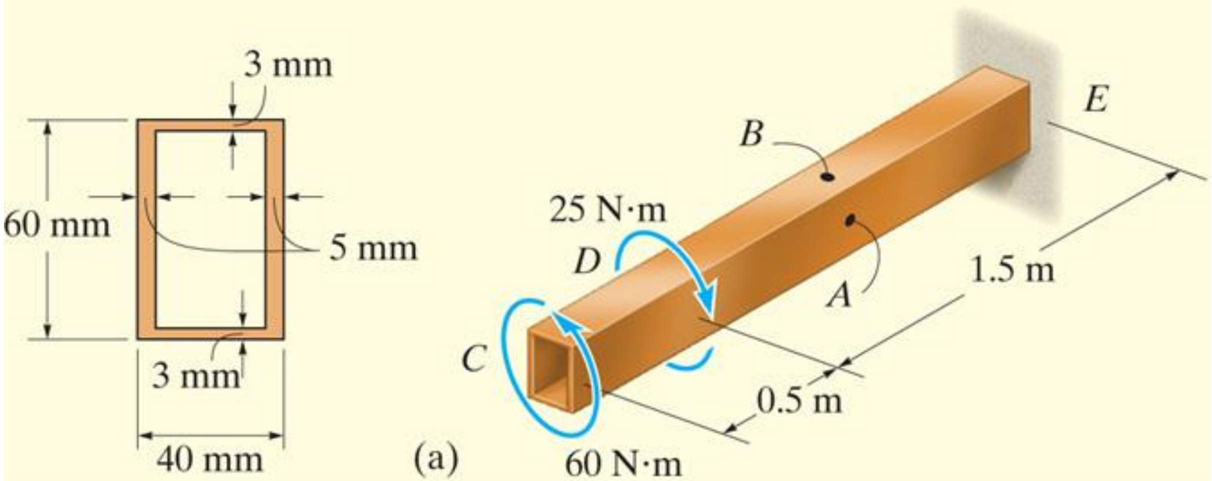


(f)

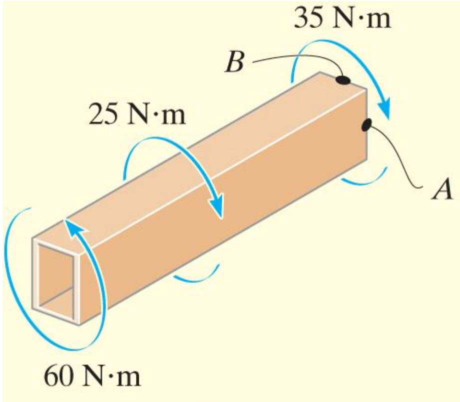
Example 6 (4 of 6)

The tube is made of C86100 bronze and has a rectangular cross section as shown in Figure a. If it is subjected to the two torques, determine the average shear stress in the tube at points *A* and *B*. Also, what is the angle of twist of end *C*? The tube is fixed at *E*. Take G = 38 GPa



Example 7

Solutions



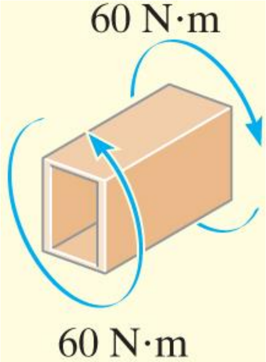
(b)

- As shown in Figure d, the mean area is

$$A_m = (0.035)(0.057) = 0.002 \text{ m}^2$$

- Applying average shear stress eqn for point A,

$$\tau_A = \frac{T}{2tA_m} = \frac{35}{2(0.005)(0.002)} = 1.75 \text{ MPa} \quad \text{(Ans)}$$



(c)

- And for point B,

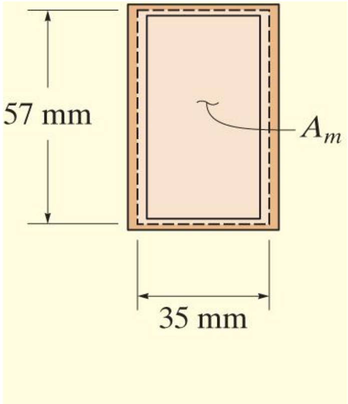
$$\tau_B = \frac{T}{2tA_m} = \frac{35}{2(0.003)(0.002)} = 2.92 \text{ MPa} \quad \text{(Ans)}$$

Example 7

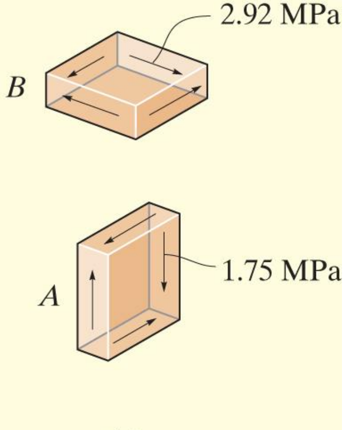
Solutions

- From the free-body diagrams in Figure b and c, the internal torques in regions *DE* and *CD* are 35 Nm and 60 Nm respectively.
- Angle of twist is

$$\phi = \frac{TL}{4A_m^2 G}\oint \frac{ds}{t} = \frac{60(0.5)}{4(0.002)^2[38(10^9)]}\left[2\left(\frac{57}{5}\right) + 2\left(\frac{35}{3}\right)\right]$$
$$+ \frac{35(1.5)}{4(0.002)^2[38(10^9)]}\left[2\left(\frac{57}{5}\right) + 2\left(\frac{35}{3}\right)\right]$$
$$= 6.29(10^{-3})\text{ rad} \quad (\text{Ans})$$



(d)



(e)