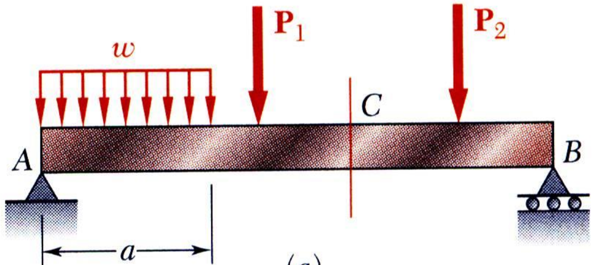
SCHOOL OF ENGINEERING

DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

CEE 3211– MECHANICS OF MATERIALS

LECTURE 7 & 8 – SHEARING FORCE AND BENDING MOMENT IN STRAIGHT BEAMS

Introduction



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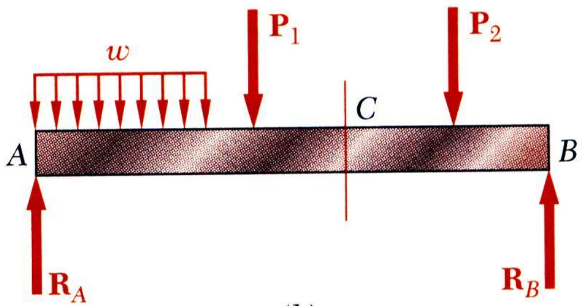

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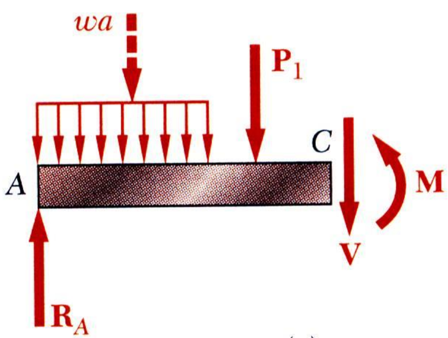


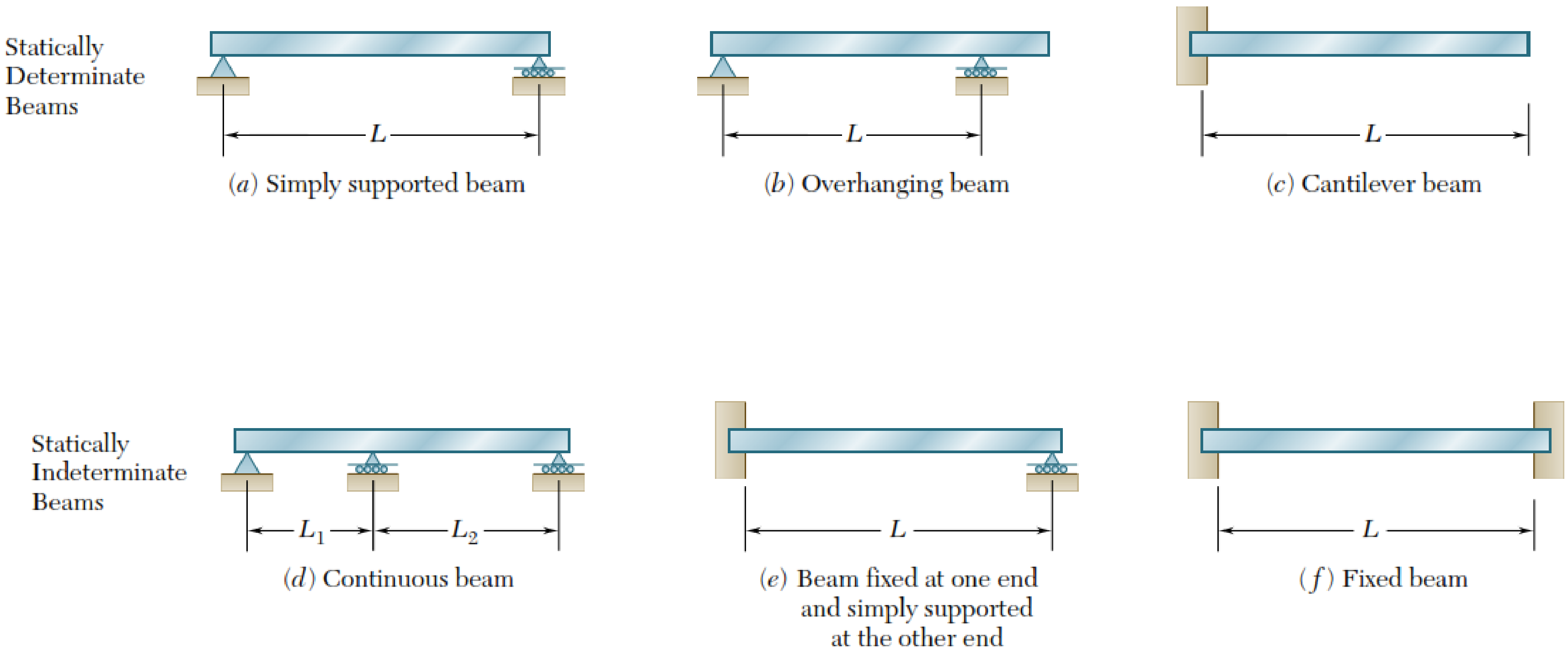
- Objective - Analysis and design of beams
- *Beams* - Members that are slender and support loadings that are applied perpendicular to their longitudinal axis
- Transverse loadings of beams are classified as *concentrated* loads or *distributed* loads
- Applied loads result in internal forces consisting of a shear force (from the shear stress distribution) and a bending couple (from the normal stress distribution)
- Normal stress is often the critical design criteria

$$\sigma_x = -\frac{My}{I} \qquad \sigma_m = \frac{|M|c}{I} = \frac{|M|}{S}$$

Requires determination of the location and magnitude of largest bending moment

Introduction

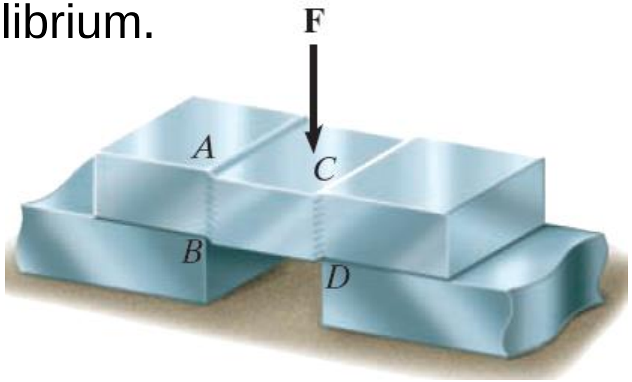
Classification of Beam Supports

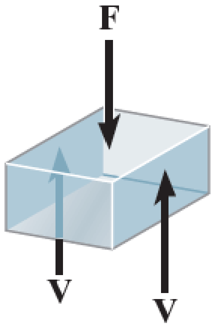


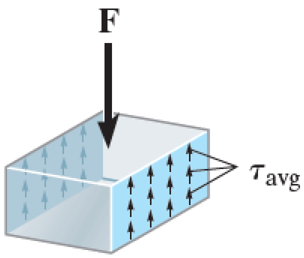
Shear Stress

- ***Shear stress***, τ (tau) is defined as the shear force per unit area.

- If **F** is large enough, it can cause the material of the bar to deform and fail along the planes identified by *AB* and *CD*.
- A free-body diagram of the unsupported centre segment of the bar, indicates that the shear force $V = F/2$ must be applied at each section to hold the segment in equilibrium.







The shear stress is rarely uniform, but the average value can be found in terms of the shear force, V, and the area, A, on which it acts.

$$\tau_{avg} = \frac{V}{A}$$

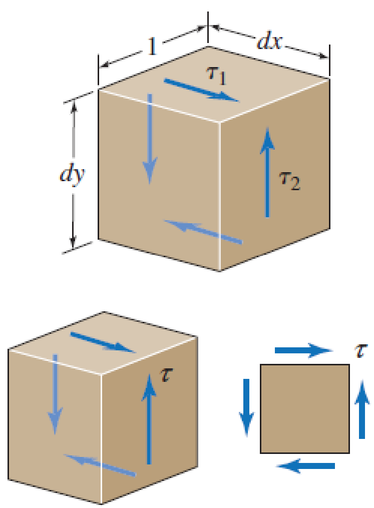
Shear Stress Equilibrium

The shear stress is shown as τ_1 acting on the top and bottom faces of a small element. There are likewise equal and opposite shear stresses, τ_2, on the vertical faces.

For this element to be in equilibrium, the moment about the center must be zero. If the element is *dx* by *dy* by 1 thick, then

$$\sum M = -\tau_1(dx)(1)(dy) + \tau_2(dx)(1)(dy) = 0$$

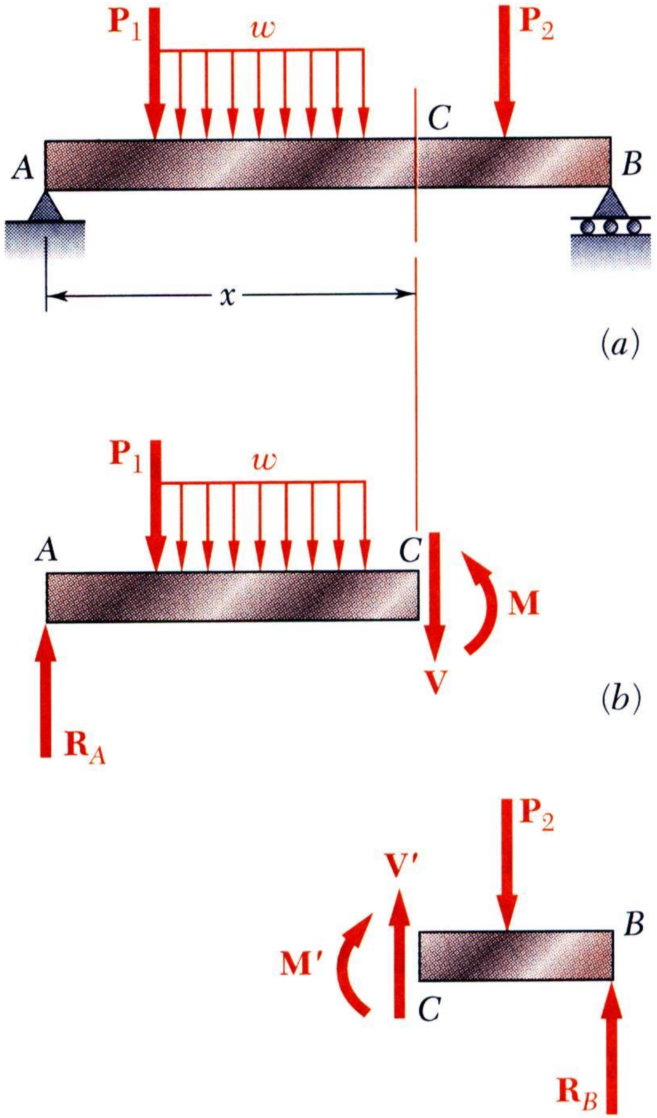
So, the shear stresses must be equal: $\tau_1 = \tau_2$. A single shear stress, τ , describes the shear force per area on horizontal and vertical faces.



All four shear stresses must have equal magnitude and be directed either toward or away from each other at opposite edges of the element,

This is referred to as the ***complementary property of shear***, and the element in this case is subjected to *pure shear*.

Shear and Bending Moment Diagrams

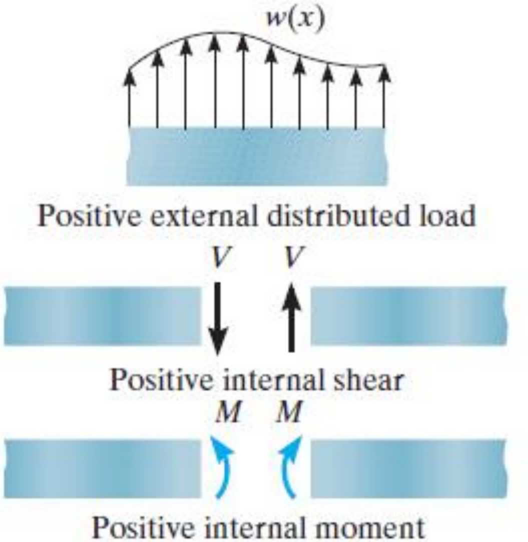


In order to properly design a beam it's necessary to determine the ***maximum*** shear and moment in the beam. One way to do this is to express ***V*** and ***M*** as functions of their arbitrary position ***x*** along the beam's axis. These ***shear*** *and* ***moment functions*** can then be plotted and represented by graphs called ***shear and moment diagrams***.

Shear force and bending couple at a point are determined by passing a section through the beam and applying an equilibrium analysis on the beam portions on either side of the section.

Beam Sign Convention

Before plotting shear and moment diagrams, it is first necessary to establish a ***sign convention*** so as to define "positive" and "negative" values for *V* and *M*.



Beam sign convention

The ***positive directions*** are as follows:

- The ***distributed load*** acts ***upward*** on the beam;
- The internal ***shear force*** causes a ***clockwise*** rotation of the beam segment on which it acts;
- The internal ***moment*** causes ***compression*** in the *top fibers* of the segment such that it bends the segment so that it holds water. Loadings that are opposite to these are considered negative.

SF & BM Diagrams – Procedure for Analysis

The shear and moment diagrams for a beam can be constructed using the following procedure.

Support Reactions.

• Determine all the reactive forces and couple moments acting on the beam, and resolve all the forces into components acting perpendicular and parallel to the beam's axis.

Shear and Moment Functions.

• Specify separate coordinates *x* having an origin at the beam's *left end* and extending to regions of the beam between concentrated forces and/or couple moments, or where there is no discontinuity of distributed loading.

• Section the beam at each distance *x*, and draw the FBD of one of the segments. Be sure **V** and **M** are shown acting in their positive sense

SF & BM Diagrams -Procedure for Analysis

• The shear is obtained by summing forces perpendicular to the beam's axis.

• To eliminate V, the moment is obtained directly by summing moments about the sectioned end of the segment.

Shear and Moment Diagrams.

• Plot the shear diagram (V versus x) and the moment diagram (M versus x). If numerical values of the functions describing V and M are *positive*, the values are plotted above the x axis, whereas negative values are plotted below the axis.

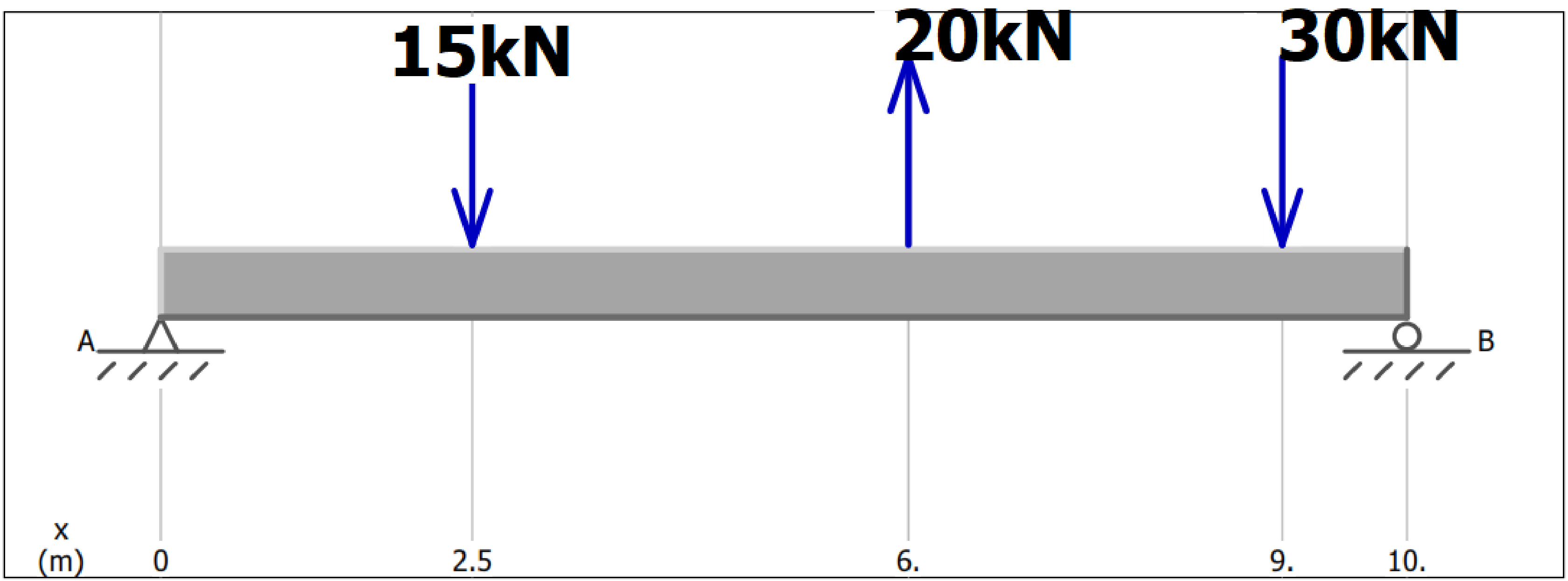
• Generally it is convenient to show the shear and moment diagrams below the free-body diagram of the beam.

Shear Force and Bending Moment Diagram

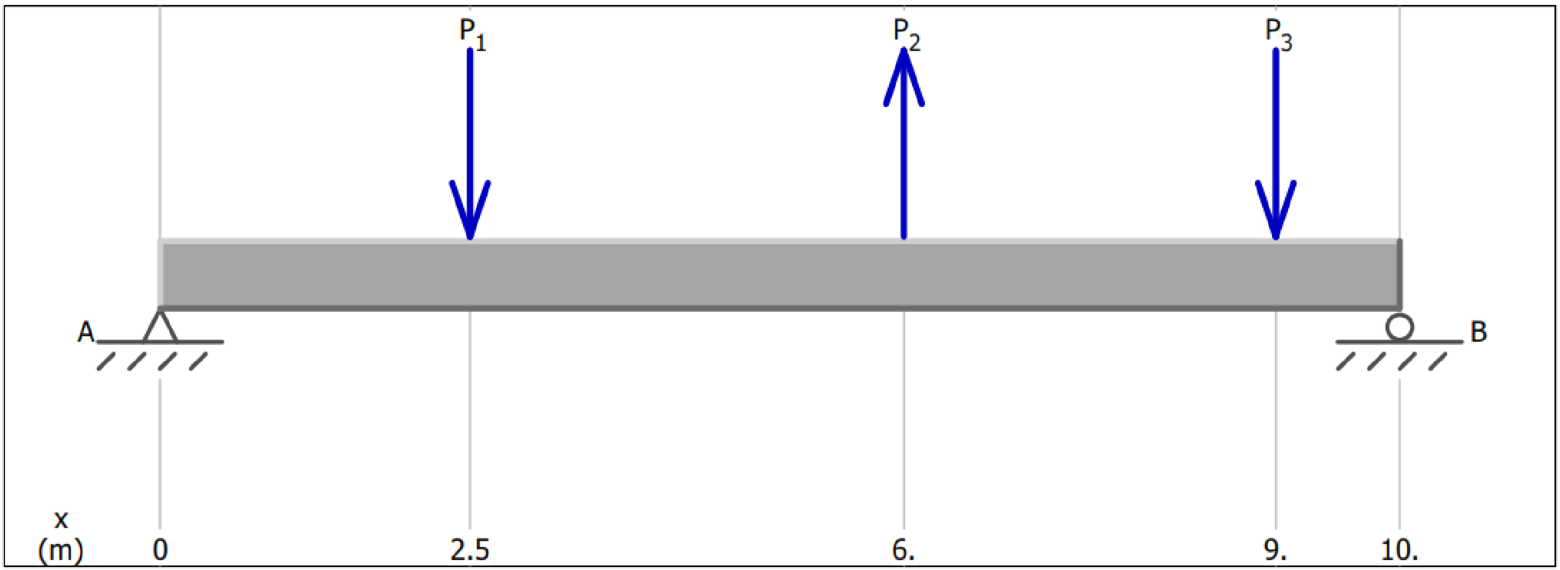
GRAPHIC METHOD

Example 1- Pointed Loads

Draw the Shear Force and Bending Moment Diagram



Example 1- Pointed Loads

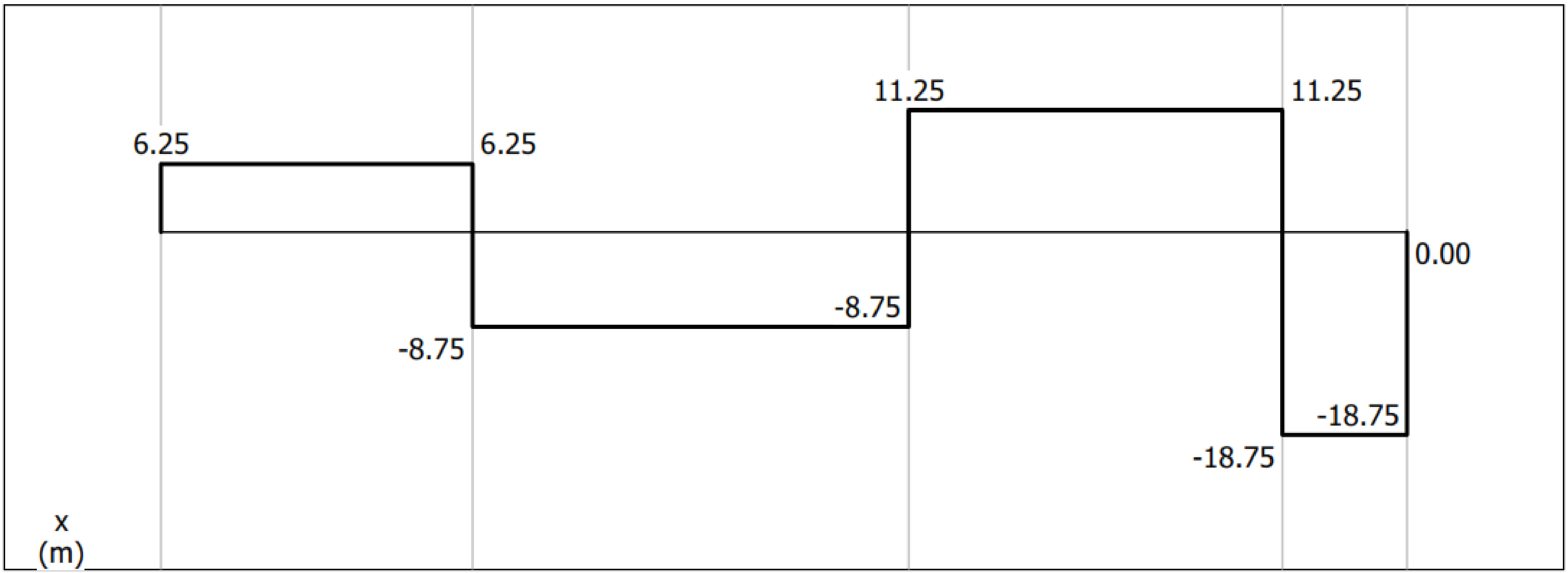


Load Diagram

P_1 = 15.0 kN (down)
P_2 = 20.0 kN (up)
P_3 = 30.0 kN (down)

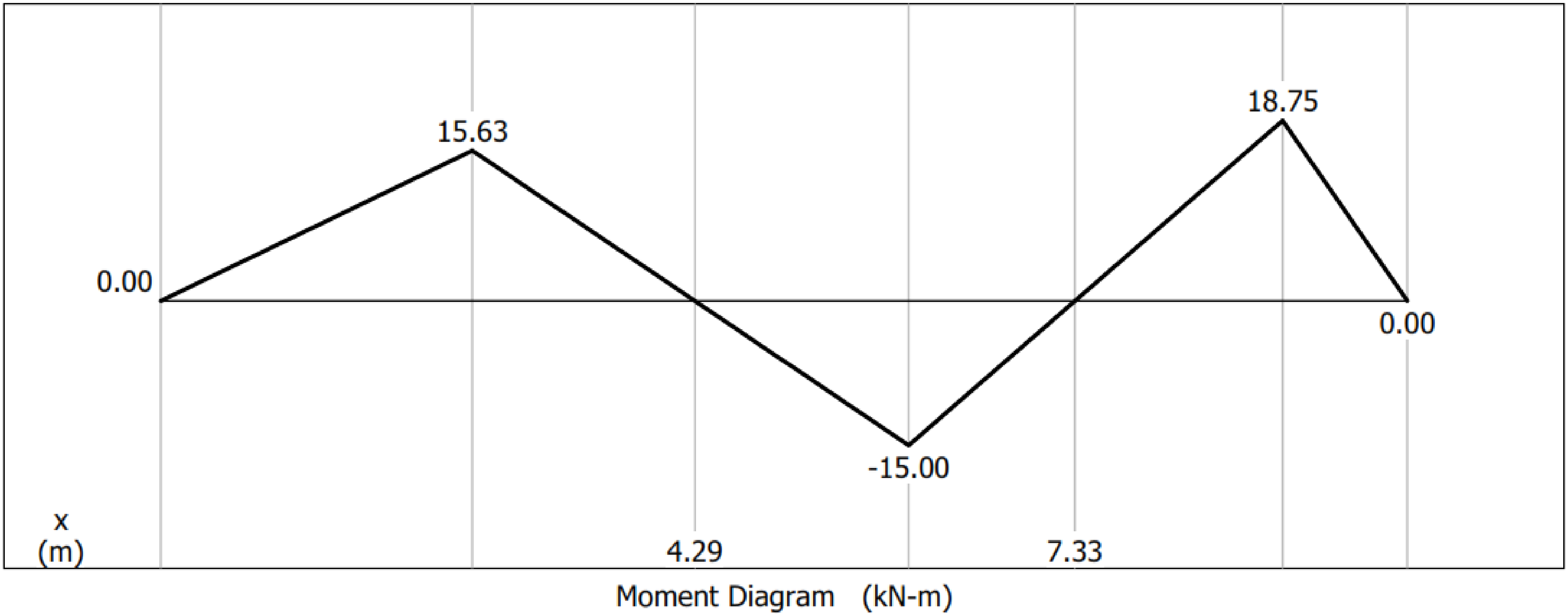
A_y = 6.25 kN (up)
B_y = 18.75 kN (up)

Example 1- Pointed Loads

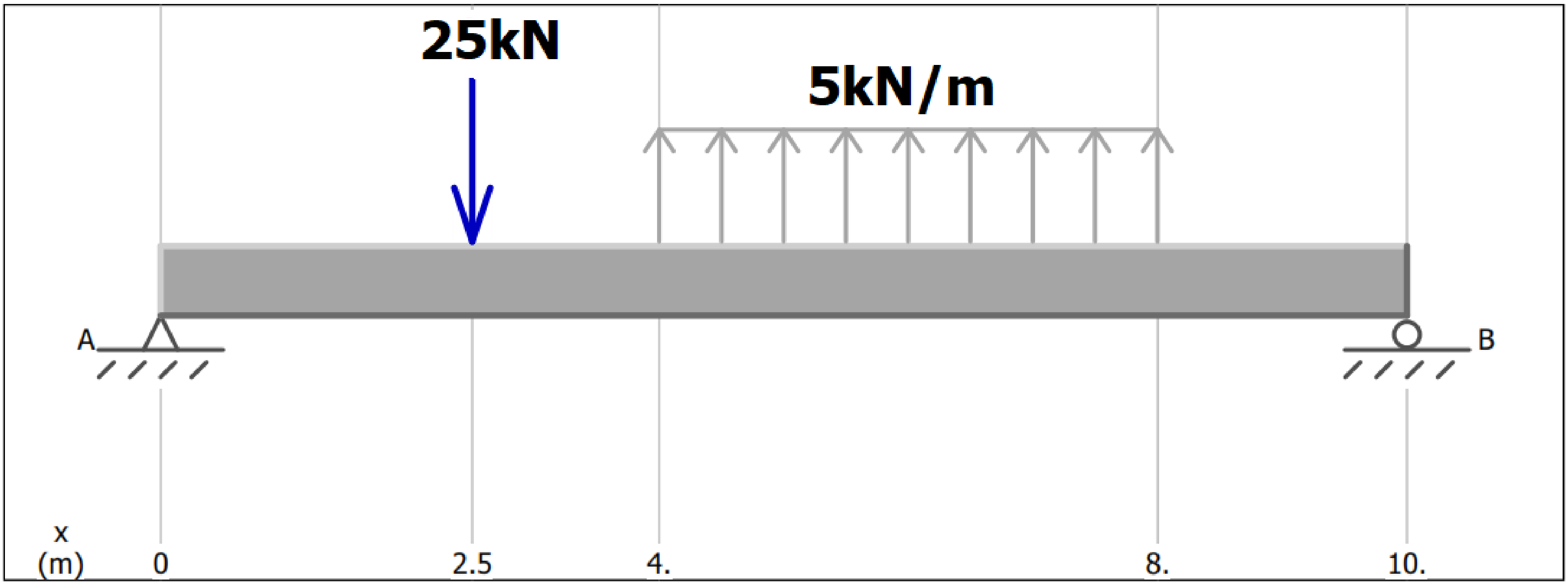


Shear Diagram (kN)

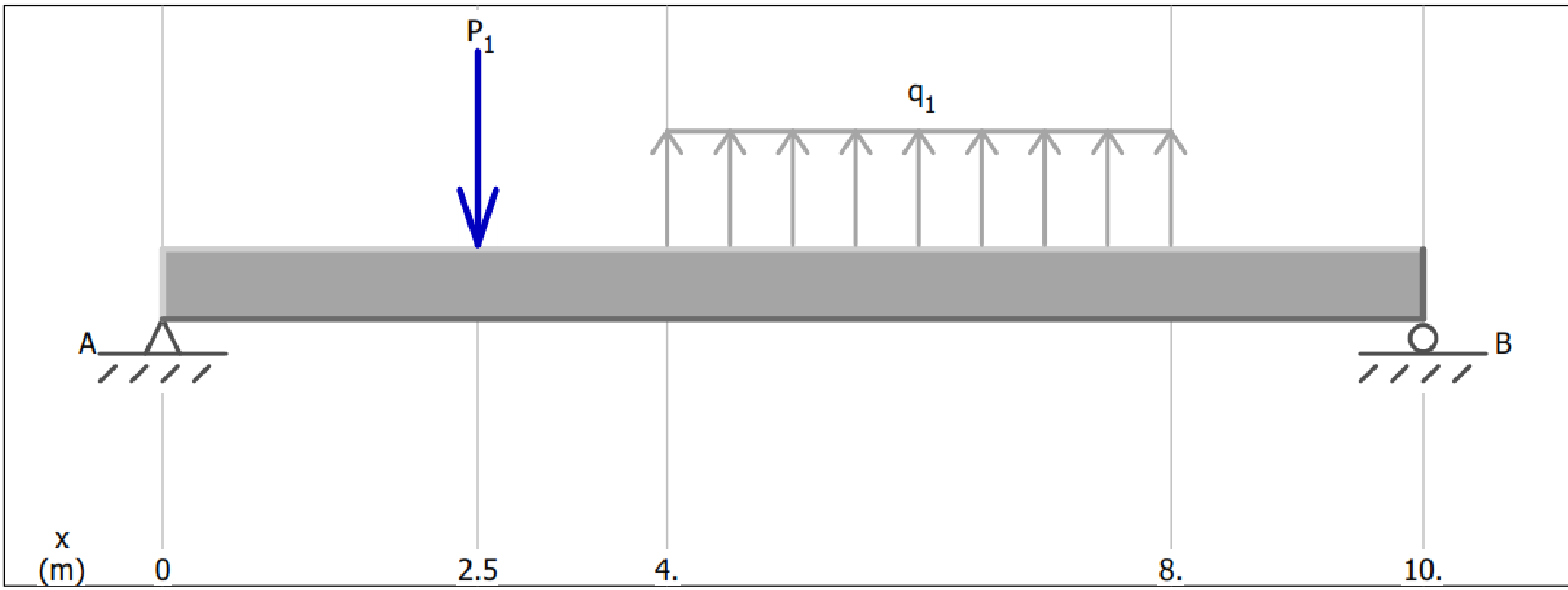
Example 1- Pointed Loads



Example 2- Uniformly Distributed Loads



Example 2- Uniformly Distributed Loads

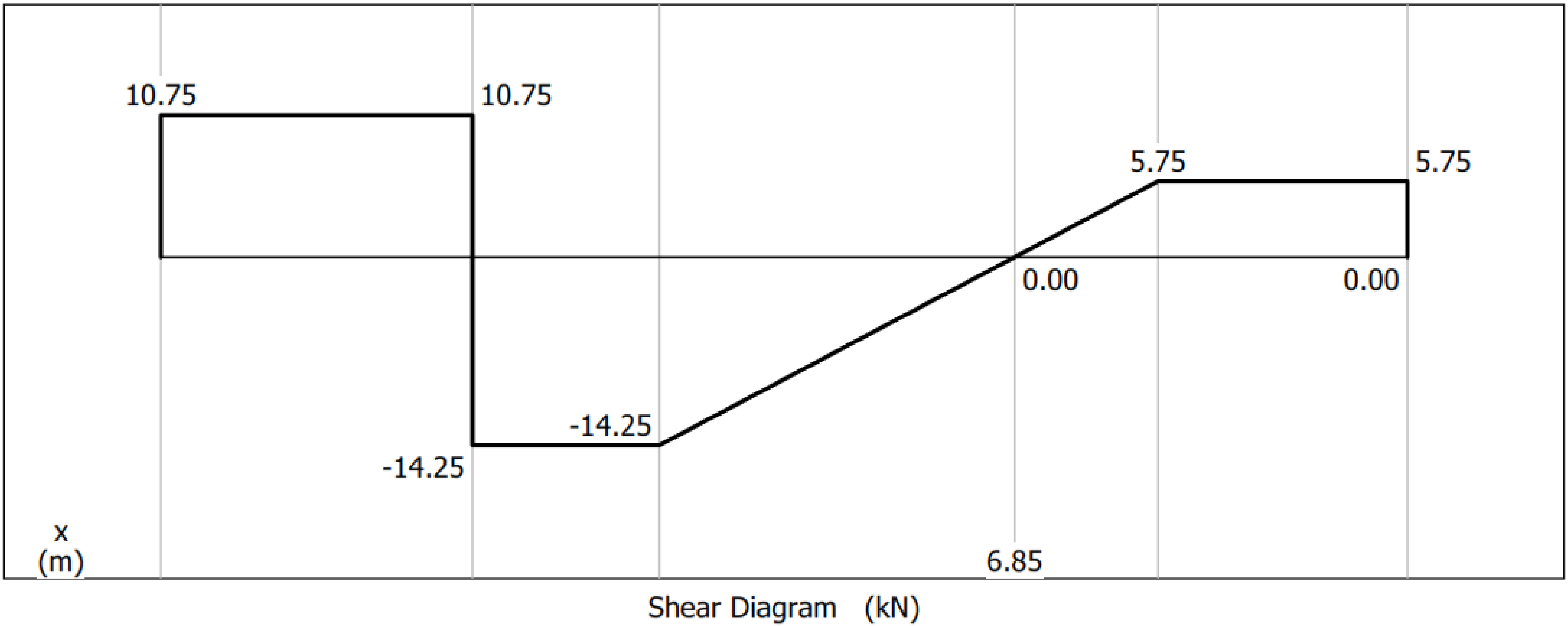


Load Diagram

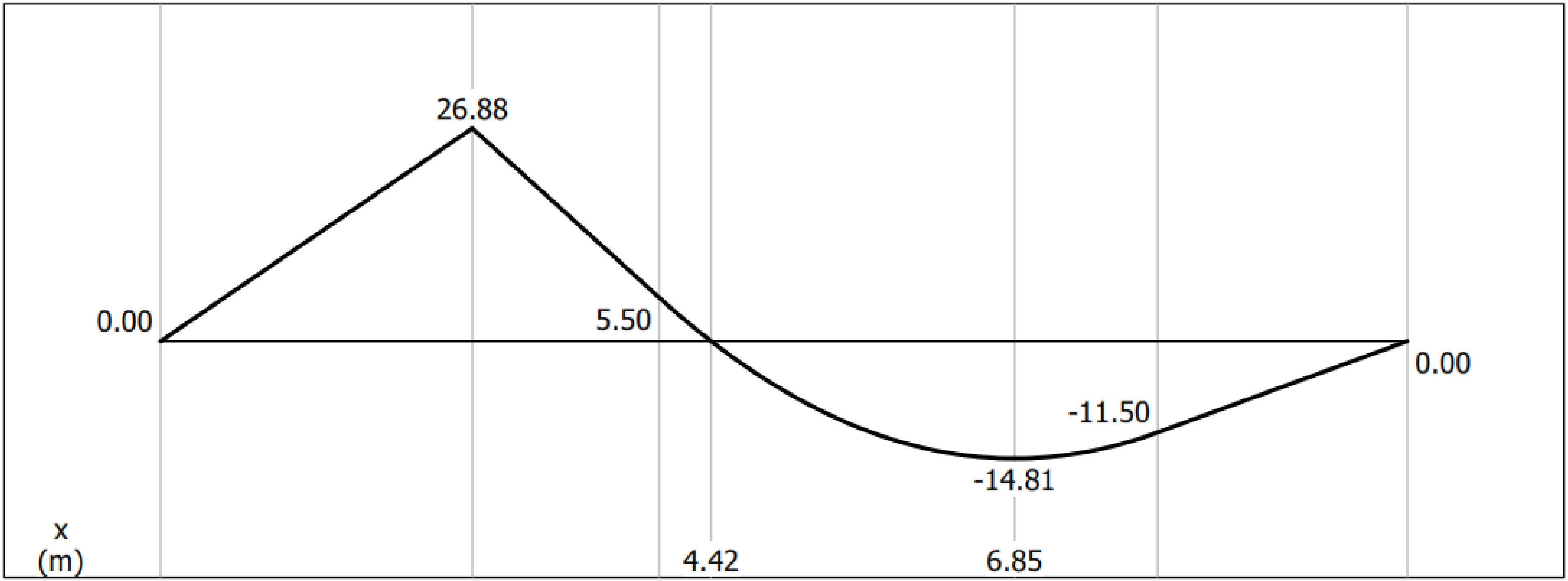
P_1 = 25.0 kN (down)
q_1 = 5.0 to 5.0 kN/m (up)

A_y = 10.75 kN (up)
B_y = 5.75 kN (down)

Example 2- Uniformly Distributed Loads



Example 2- Uniformly Distributed Loads

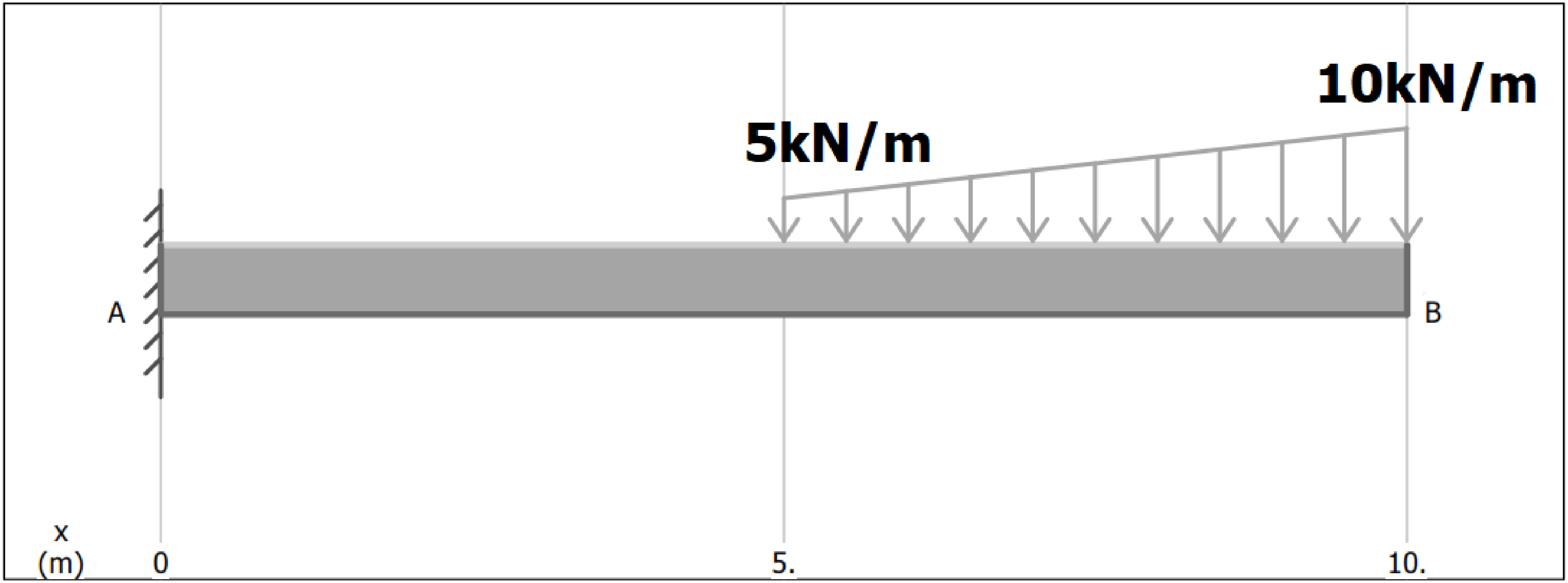


Moment Diagram (kN-m)

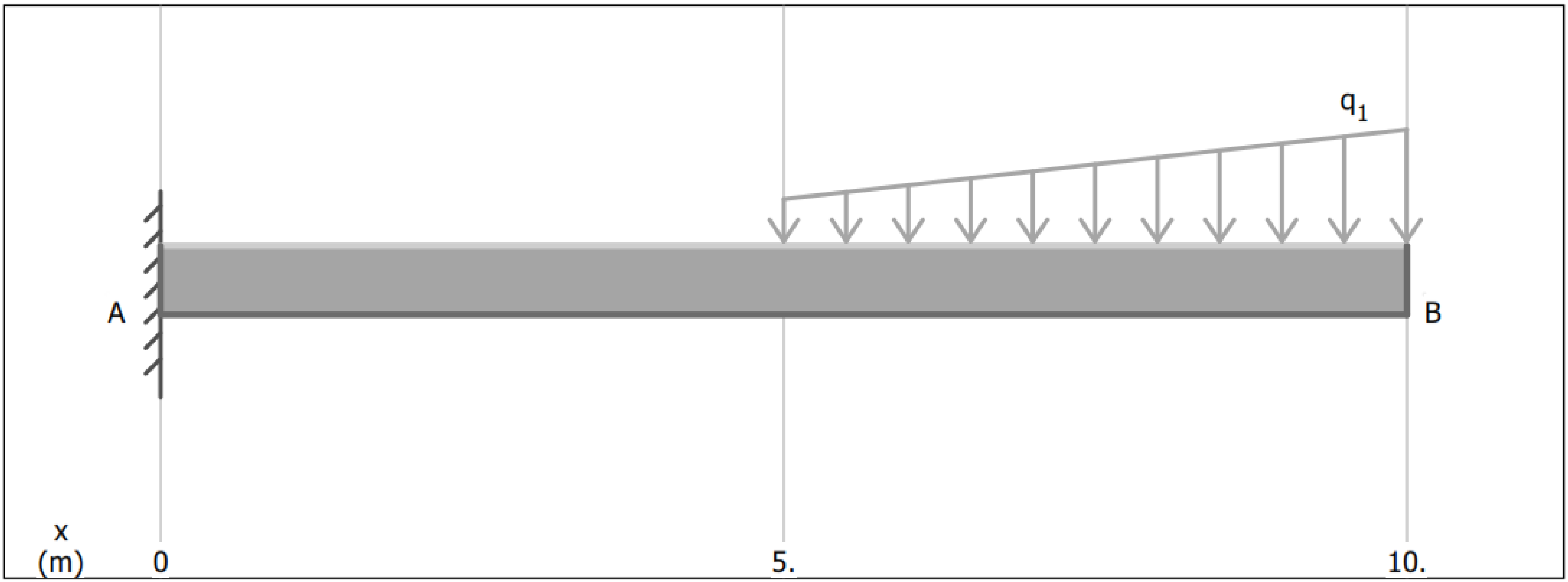
Shear Force and Bending Moment Diagram with a Moment

GRAPHIC METHOD

Example 3- Uniformly Distributed Loads with a concentrated moment



Example 3- Uniformly Distributed Loads with a concentrated moment

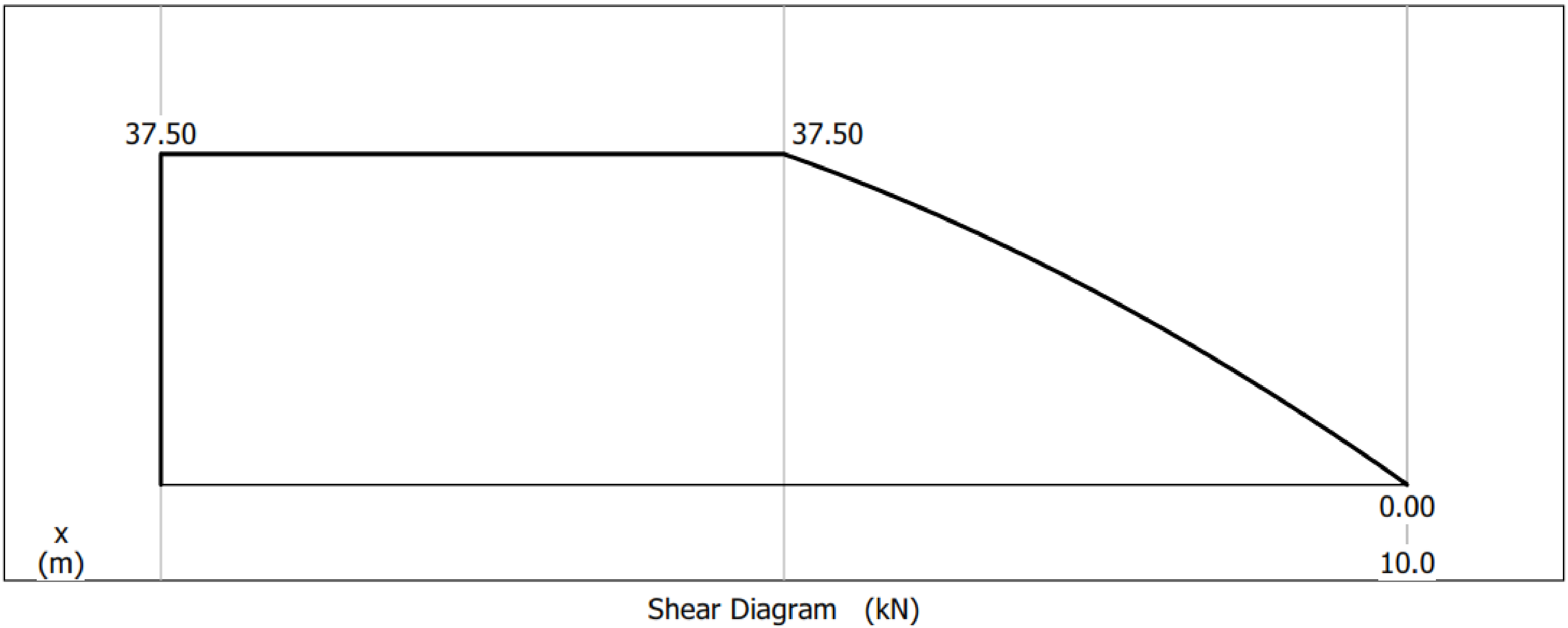


Load Diagram

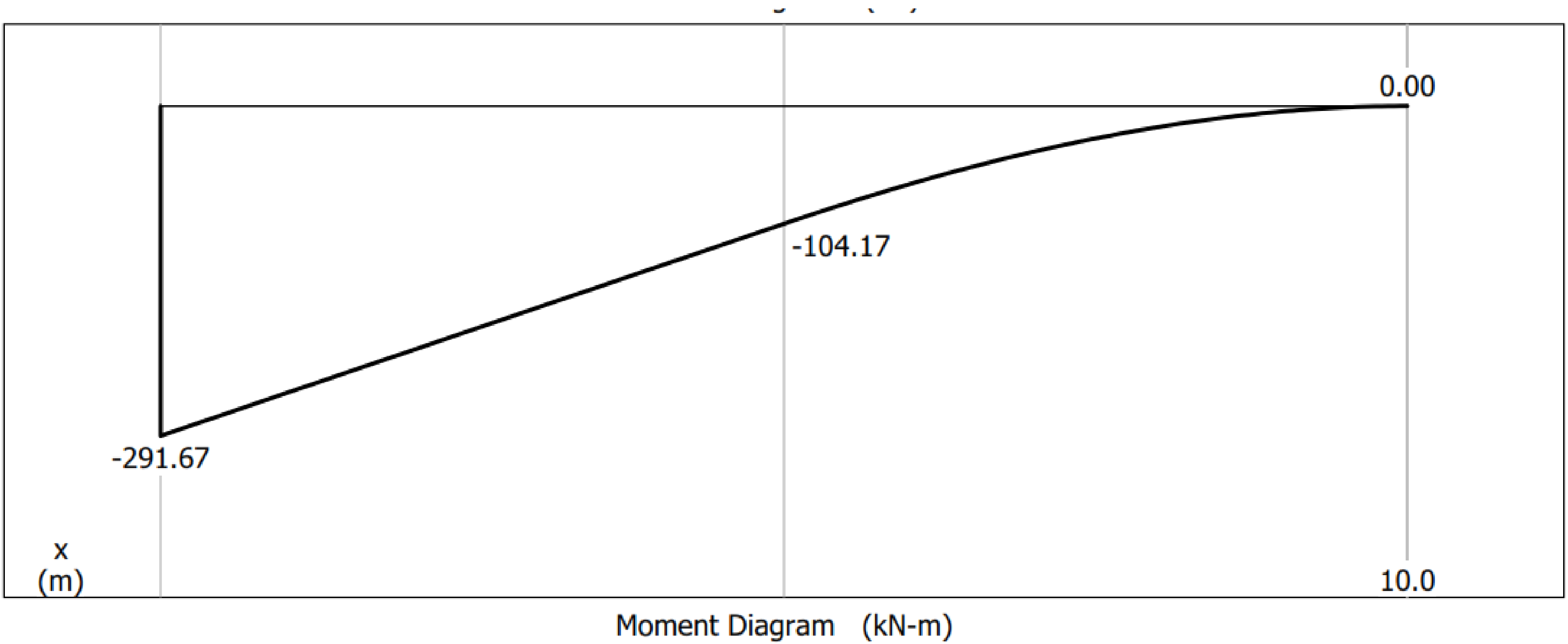
q_1 = 5.0 to 10.0 kN/m (down)

A_y = 37.50 kN (up)
$M_o$ment A = 291.67 kN-m (ccw)

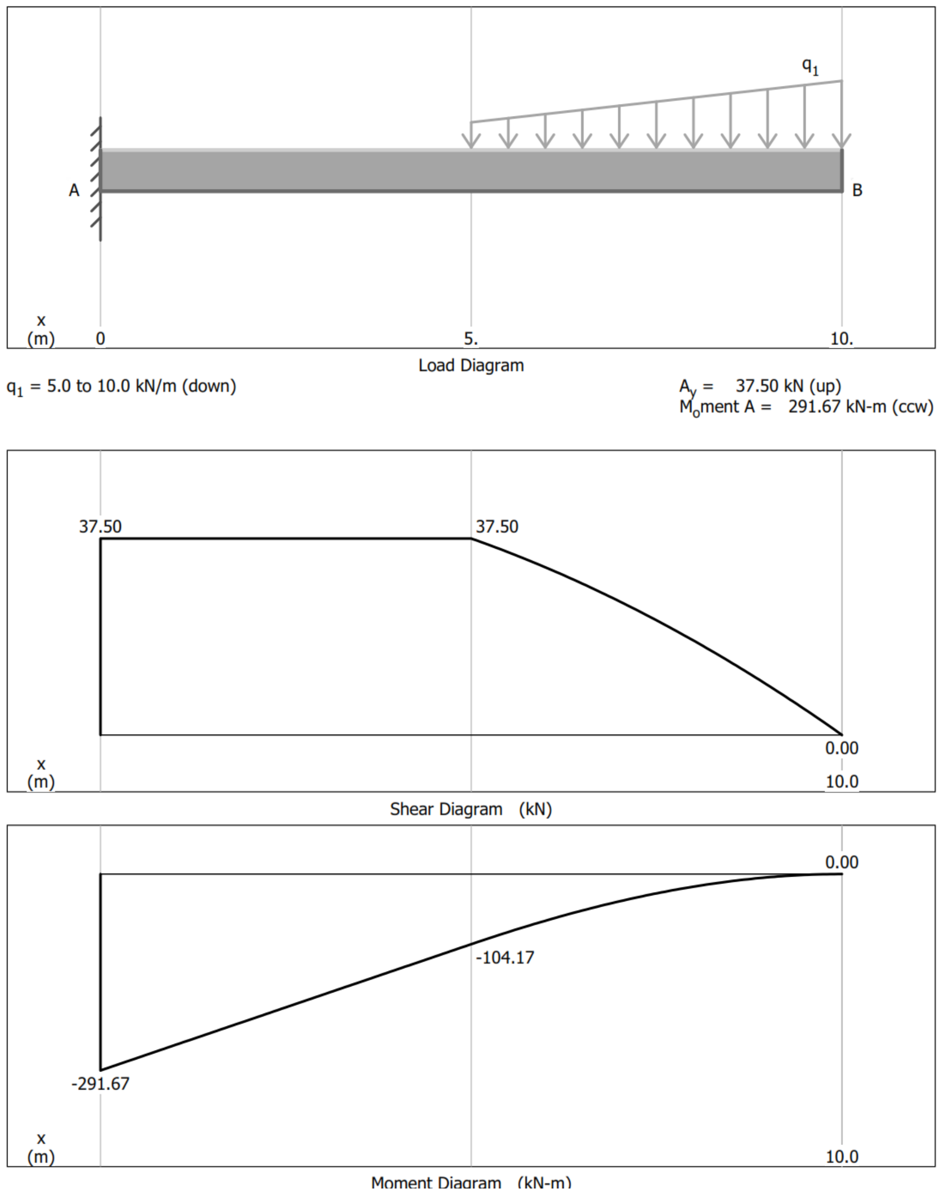
Example 3- Uniformly Distributed Loads



Example 3- Uniformly Distributed Loads



Example 3- Uniformly Distributed Loads



Example 3- Uniformly Distributed Loads

REGION BETWEEN 0 – 5m

- The change in moment between two points on the beam equals the area under the shear curve between the same two points.
- The area under the shear curve between points x = 0.00 m and x = 5.00 m is 187.500 kN-m. The moment at x = 0.00 m is -291.67 kN-m.
- Adding the area under the shear curve (187.500 kN-m) to -291.67 kN-m gives a bending moment of -104.17 kN-m at x = 5.00 m.
- The moment curve is **linear** (i.e., 1st order curve) in this region.

Example 3- Uniformly Distributed Loads

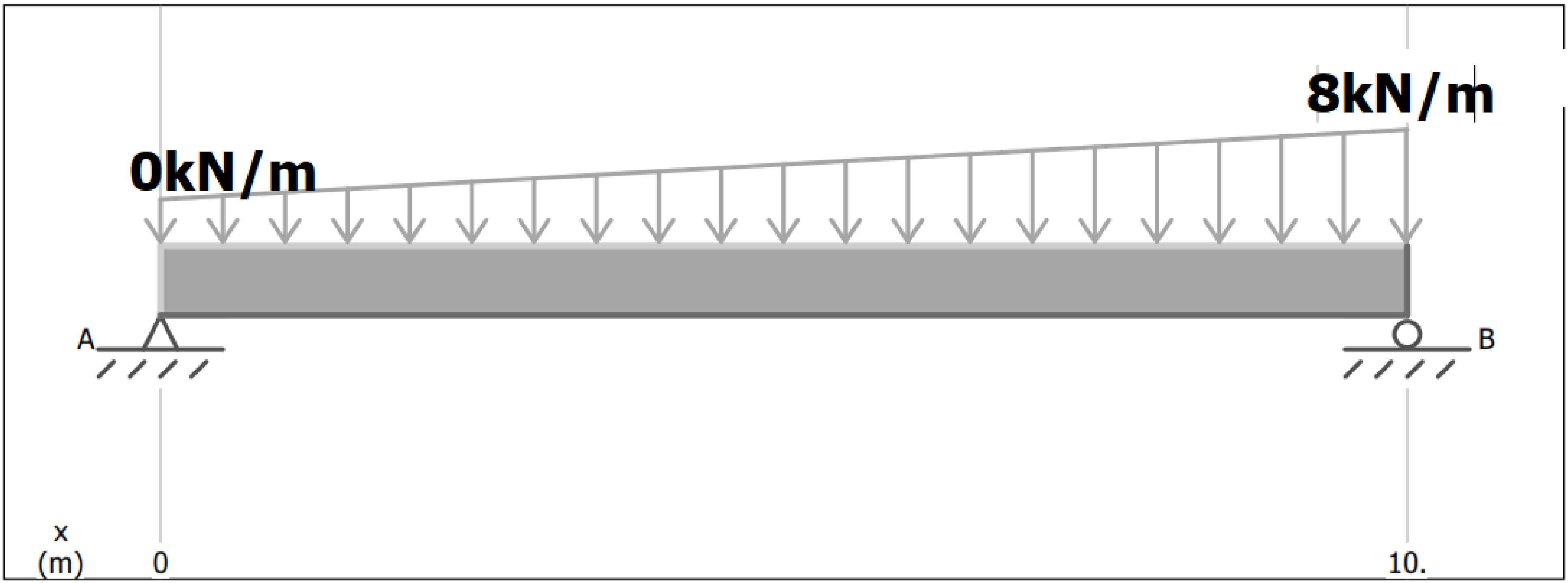
REGION BETWEEN 5 – 10 m

- The change in moment between two points on the beam equals the negative of the area under the shear curve between the same two points.
- The area under the shear curve between points x = 5.00 m and x = 10.00 m is 104.167 kN-m.
- The moment at x = 5.00 m is 104.17 kN-m.
- Adding the negative of the area under the shear curve (104.167 kN-m) to 104.17 kN-m gives a bending moment of 0.00 kN-m at x = 10.00 m.
- In this region, the moment curve is **cubic** (i.e., 3rd order curve), starting with a relatively large negative slope and growing increasingly flatter.

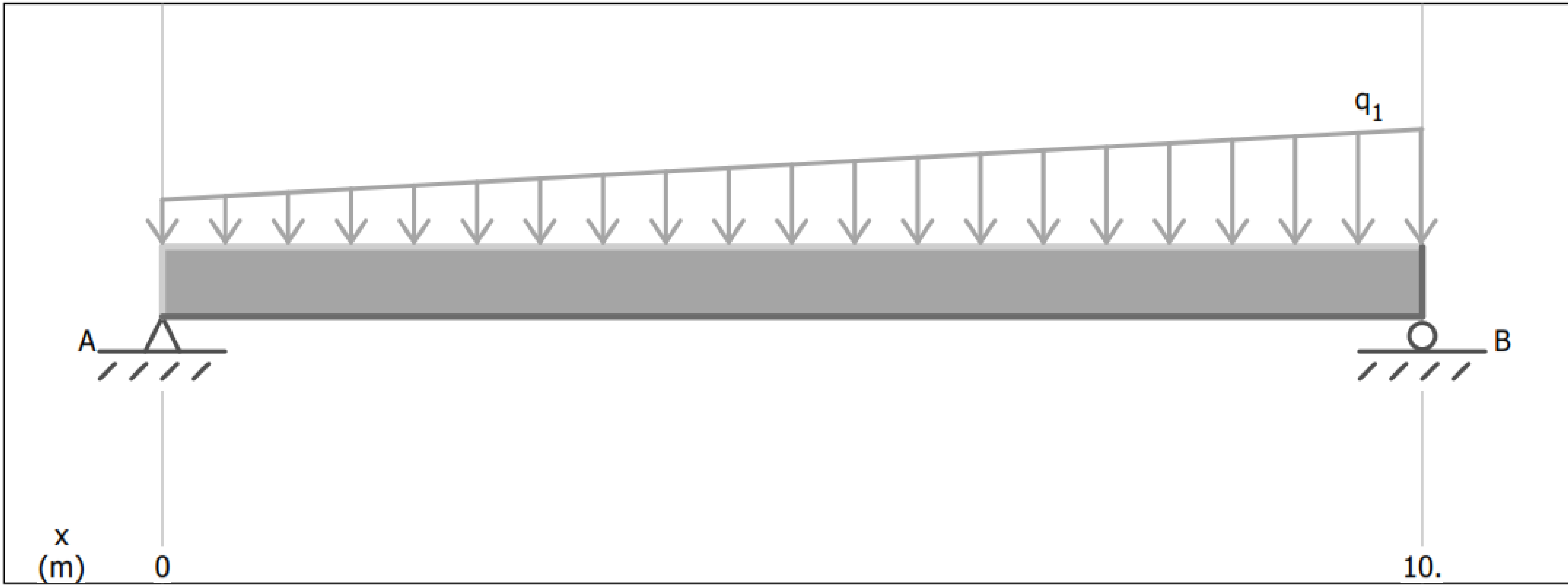
Shear Force and Bending Moment Diagram

EQUATION METHOD

Example 4- Equation Method



Example 4- Equation Method

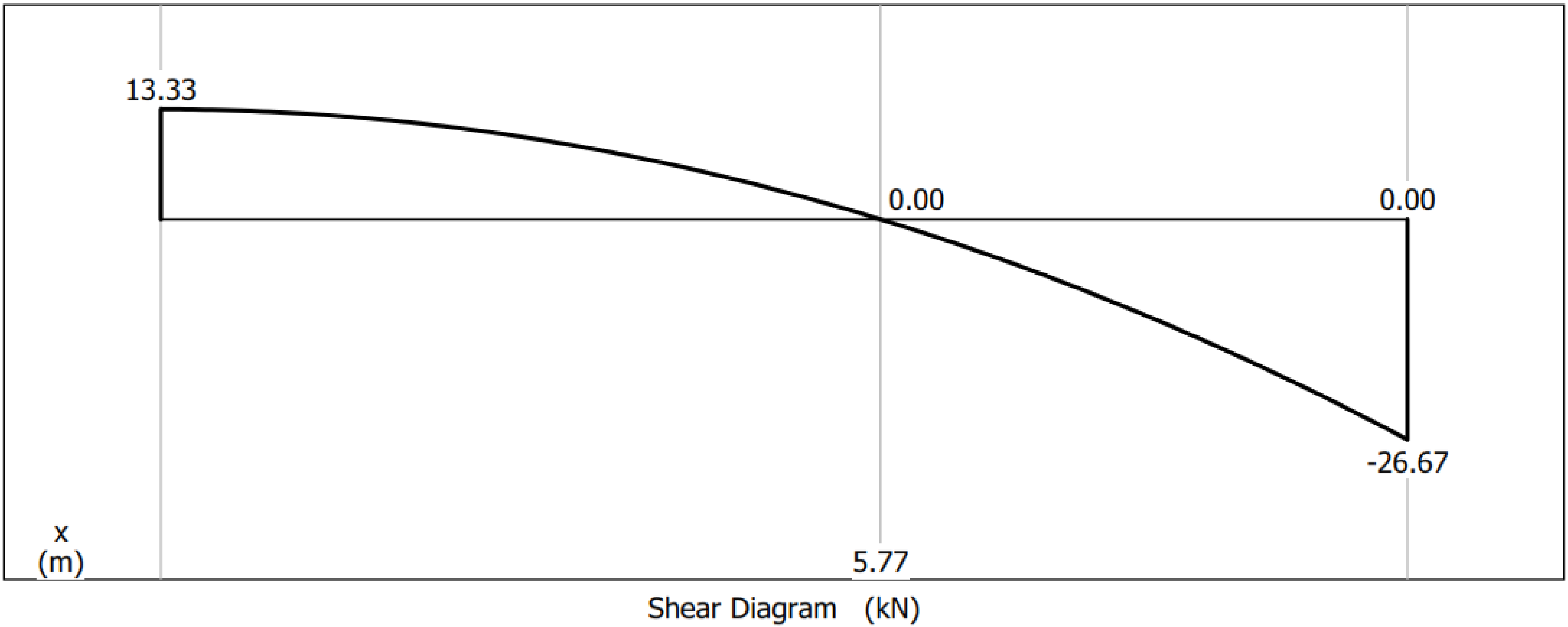


Load Diagram

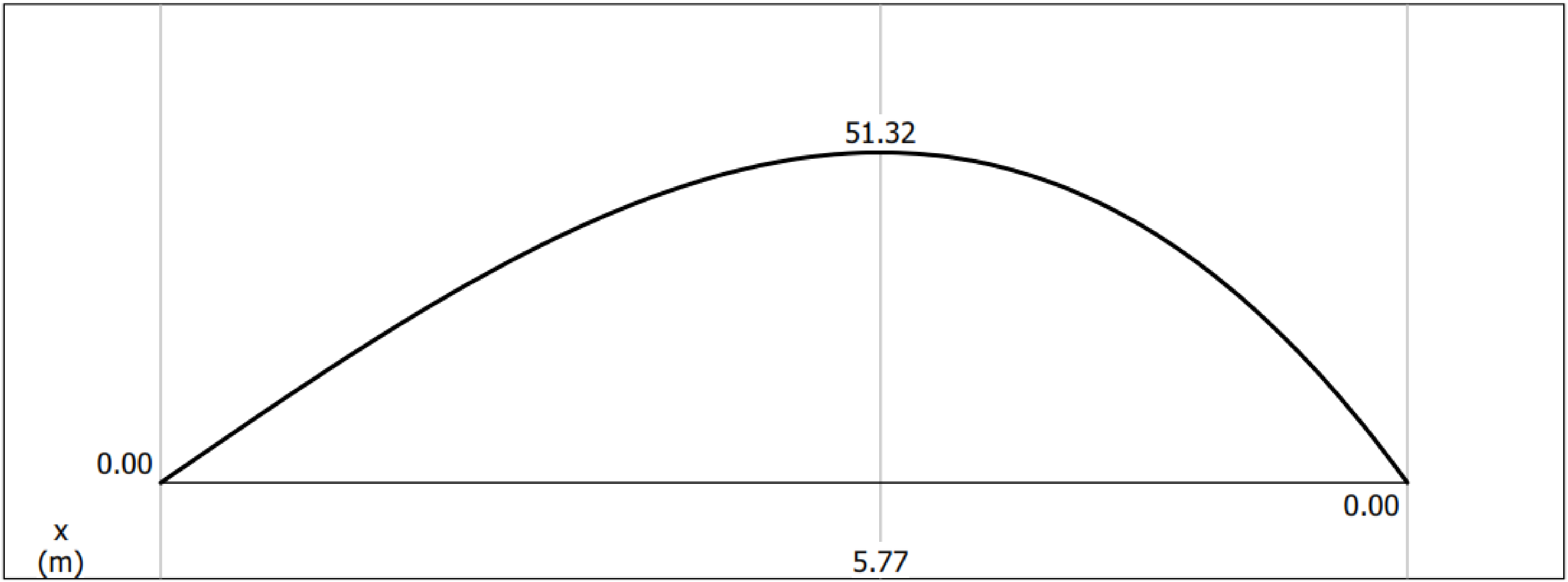
q_1 = 0.0 to 8.0 kN/m (down)

A_y = 13.33 kN (up)
B_y = 26.67 kN (up)

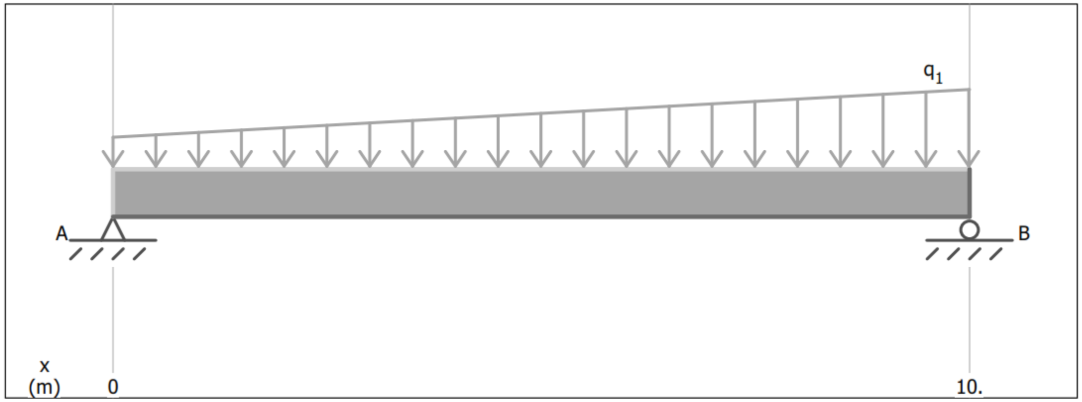
Example 4- Equation Method



Example 4- Equation Method



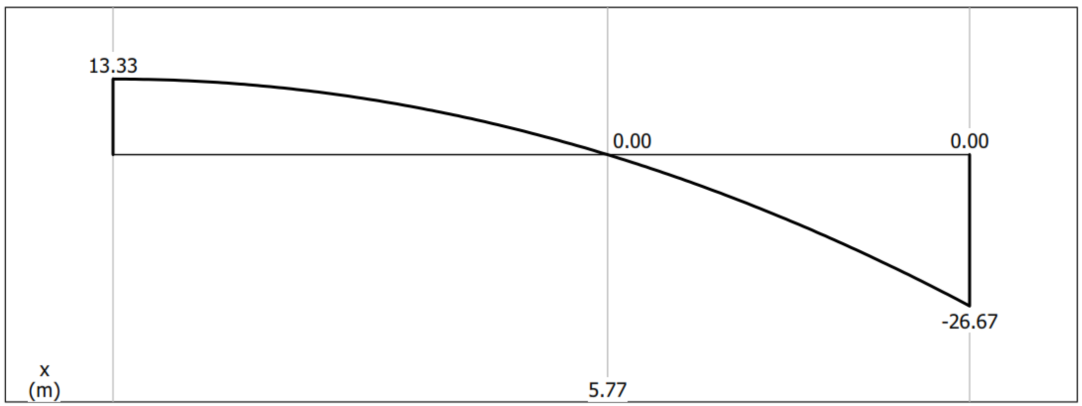
Moment Diagram (kN-m)



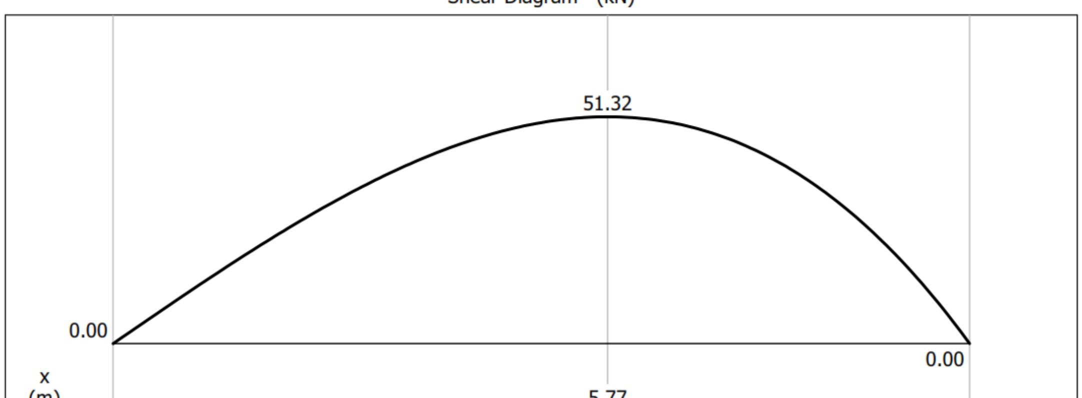
Load Diagram

q_1 = 0.0 to 8.0 kN/m (down)

A_y = 13.33 kN (up)
B_y = 26.67 kN (up)



Shear Diagram (kN)



Moment Diagram (kN-m)

Example 4- Equation Method

- The change in moment between two points on the beam equals the area under the shear curve between the same two points.
- The area under the shear curve between points x = 0.00 m and x = 5.774 m is 51.320 kN-m.

- The moment at x = 0.00 m is 0.00 kN-m.
- Adding the area under the shear curve (51.320 kN-m) to 0.00 kN-m gives a bending moment of 51.32 kN-m at x = 5.774 m.
- In this region, the moment curve is **cubic** (i.e., 3rd order curve), starting with a relatively large positive slope and growing increasingly flatter

Example 4- Equation Method

- The change in moment between two points on the beam equals the area under the shear curve between the same two points.
- The area under the shear curve between points x = 5.774 m and x = 10.00 m is -51.320 kN-m. The moment at x = 5.774 m is 51.32 kN-m.
- Adding the area under the shear curve (-51.320 kN-m) to 51.32 kN-m gives a bending moment of 0.00 kN-m at x = 10.00 m.
- In this region, the moment curve is **cubic** (i.e., 3rd order curve), starting with a flat slope and growing increasingly steeper.