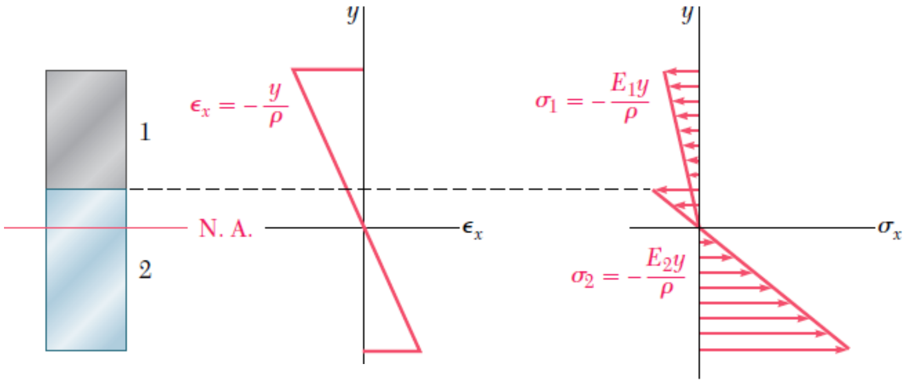
SCHOOL OF ENGINEERING

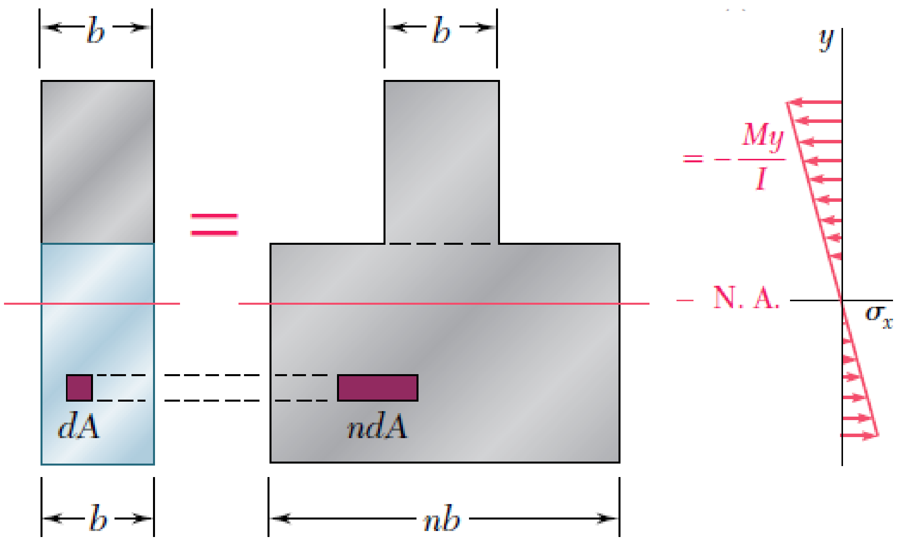
DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

CEE 3211– MECHANICS OF MATERIALS

LECTURE 10 – Bending of Composite Materials

Bending of Composite Materials





$$\sigma_x = -\frac{My}{I}$$

$$\sigma_1 = \sigma_x \qquad \sigma_2 = n\sigma_x$$

- Consider a composite beam formed from two materials with E_1 and E_2. *with* $E_2 > E_1$
- Normal strain varies linearly.

$$\varepsilon_x = -\frac{y}{\rho}$$

- Piecewise linear normal stress variation.

$$\sigma_1 = E_1\varepsilon_x = -\frac{E_1 y}{\rho} \qquad \sigma_2 = E_2\varepsilon_x = -\frac{E_2 y}{\rho}$$

- Neutral axis does not pass through section centroid of composite section.
- Elemental forces on the section are

$$dF_1 = \sigma_1 dA = -\frac{E_1 y}{\rho} dA \quad dF_2 = \sigma_2 dA = -\frac{E_2 y}{\rho} dA$$

- Define a transformed section such that

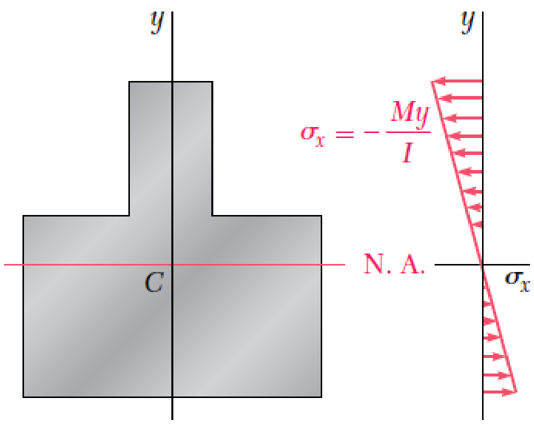
$$dF_2 = -\frac{(nE_1)y}{\rho} dA = -\frac{E_1 y}{\rho}(n\,dA) \qquad n = \frac{E_2}{E_1}$$

Bending of Composite Materials

- In other words, the resistance to bending of the bar would remain the same if both portions were made of the first material, provided that the width of each element of the lower portion were multiplied by the factor *n*.

- Note that this widening (if $n > 1$), or narrowing (if $n < 1$), must be effected *in a direction parallel to the neutral axis of the section*, since it is essential that the distance *y* of each element from the neutral axis remain the same

- The new cross section obtained in this way is called the *transformed section* of the member



The neutral axis will be drawn *through the centroid of the transformed section* and the stress σ_x at any point of the corresponding fictitious homogeneous member will be obtained from

$$\sigma_x = -\frac{My}{I}$$

Where: *y* is distance from the neutral surface

I the moment of inertia of the transformed section with respect to its centroidal axis

Bending of Composite Materials

- To obtain the stress σ_1 at a point located in the upper portion of the cross section of the original composite bar, we simply compute the stress σ_x at the corresponding point of the transformed section.

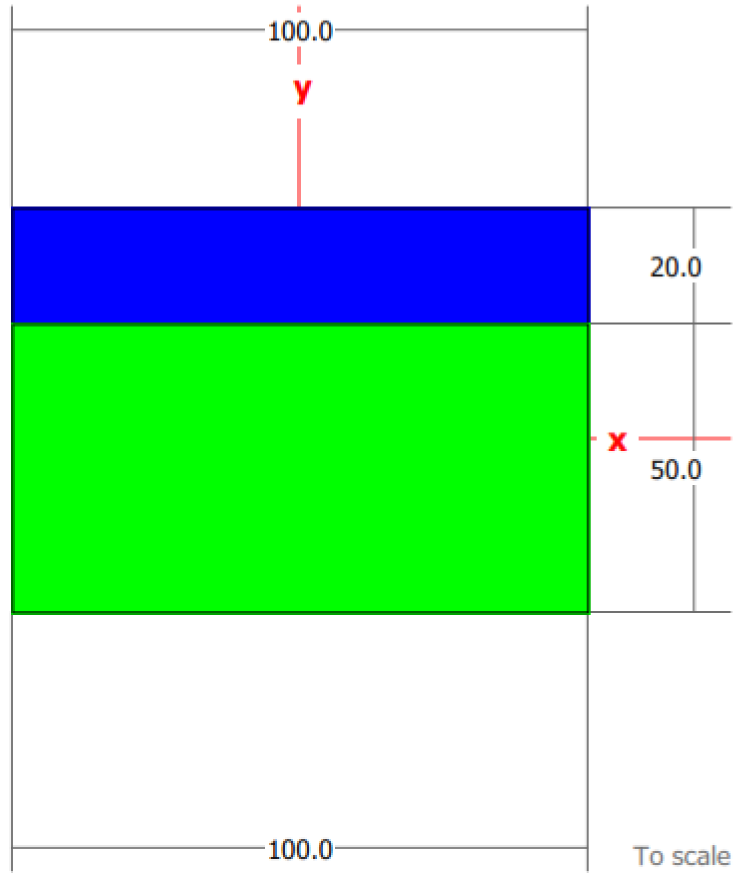
- However, to obtain the stress σ_2 at a point in the lower portion of the cross section, we must *multiply by n* the stress σ_x computed at the corresponding point of the transformed section.

- The same elementary force *dF*2 is applied to an element of area *n dA* of the transformed section and to an element of area *dA* of the original section.

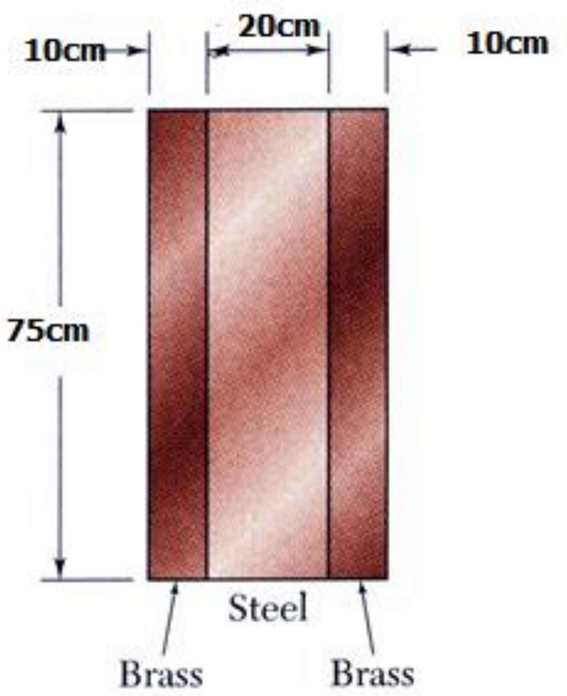
- Thus, the stress σ_2 at a point of the original section must be *n* times larger than the stress at the corresponding point of the transformed section.

Example 1

Bar is made from bonded pieces of A992 structural steel (E_s = 200GPa) and Gray Cast Iron-A48 (E_I = 84GPa). Determine the maximum stress in the steel and cast Iron when a moment of 25 kN.m is applied.



Example 2

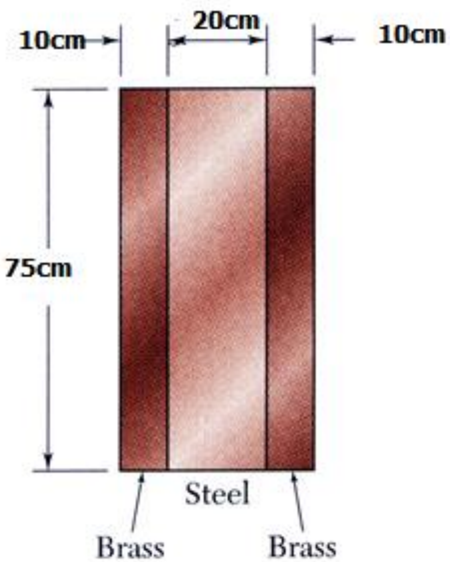


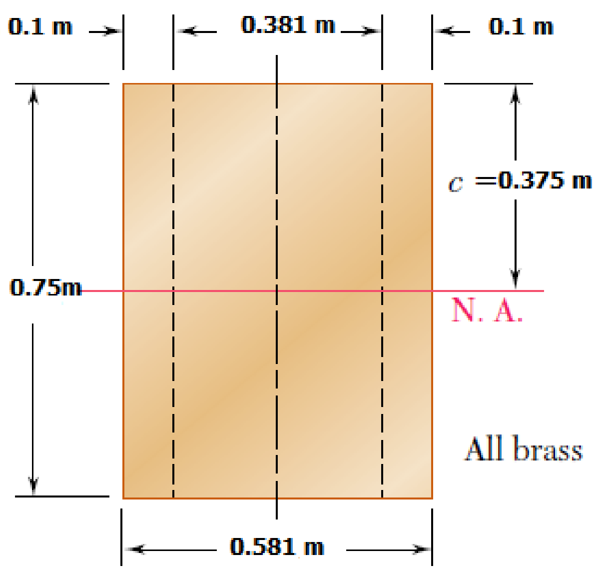
Bar is made from bonded pieces of steel (E_s = 200GPa) and brass (E_b = 105GPa). Determine the maximum stress in the steel and brass when a moment of 40 N.m is applied.

SOLUTION:

- Transform the bar to an equivalent cross section made entirely of brass
- Evaluate the cross sectional properties of the transformed section
- Calculate the maximum stress in the transformed section. This is the correct maximum stress for the brass pieces of the bar.
- Determine the maximum stress in the steel portion of the bar by multiplying the maximum stress for the transformed section by the ratio of the moduli of elasticity.

Example 2





SOLUTION:

- Transform the bar to an equivalent cross section made entirely of brass.

$$n = \frac{E_s}{E_b} = \frac{200GPa}{105GPa} = 1.905$$

$$b_T = 0.1\text{ m} + 1.905 \times 0.20\text{ m} + 0.1\text{ m} = 0.581\text{ m}$$

Evaluate the transformed cross sectional properties
Central portion of brass = (0.2m)(1.905) = 0.381m

$$I = \tfrac{1}{12} b_T h^3 = \tfrac{1}{12}(0.581\text{ m.})(0.75\text{ m})^3$$
$$= 0.0204\text{ m}^4$$

- Calculate the maximum stresses

$$\sigma_m = \frac{Mc}{I} = \frac{(40\text{ N}\cdot\text{m})(0.375\text{ m})}{0.0204\text{ m}^4} = 735.3\text{Pa}$$

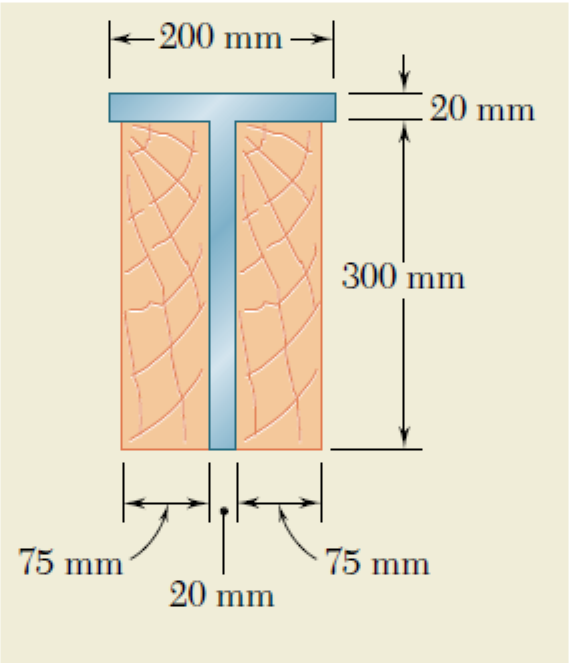
$$(\sigma_b)_{\max} = \sigma_m$$
$$(\sigma_s)_{\max} = n\sigma_m = 1.905 \times 735.3\text{ Pa}$$

$$(\sigma_b)_{\max} = 735.3\text{Pa}$$
$$(\sigma_s)_{\max} = 1400\text{ Pa}$$

Example 3

Two steel plates have been welded together to form a beam in the shape of a T that has been strengthened by securely bolting to it the two oak timbers shown. The modulus of elasticity is 12.5 GPa for the wood and 200 GPa for the steel. Knowing that a bending moment M = 50 kN .m is applied to the composite beam, determine (*a*) the maximum stress in the wood, (*b*) the stress in the steel along the top edge.

SOLUTION



Transformed Section

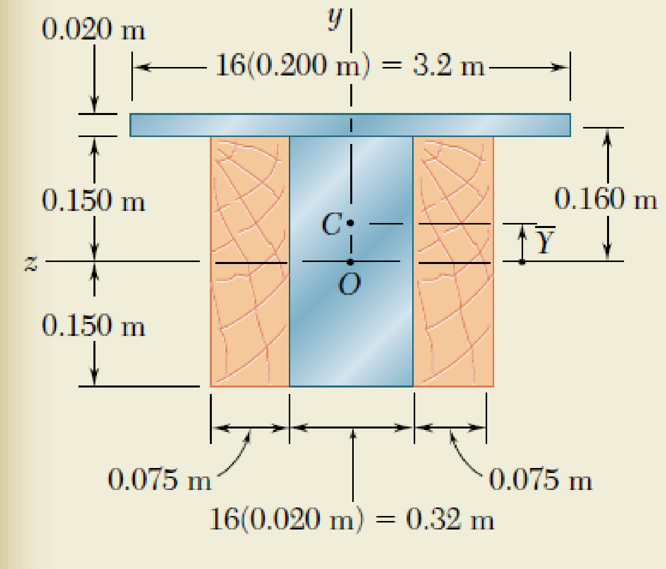
$$n = \frac{E_s}{E_w} = \frac{200GPa}{12.5GPa} = 16$$

Multiply horizontal dimensions of the steel portion of the section by n = 16, we obtain a transformed section made entirely of wood

Example - 3

Neutral Axis. The neutral axis passes through the centroid of the transformed section. Since the section consists of two rectangles, we have

$b_{t(bottom)} = 0.47\text{m}$; $b_{t(top)} = 3.2\text{m}$



$$\bar{Y} = \frac{\Sigma\bar{y}A}{\Sigma A} = \frac{(0.160\text{m})(3.2\text{m x } 0.020) + 0}{3.2\text{m x } 0.02\text{ m} + 0.470\text{ m x } 0.300\text{ m}}$$

$$= 0.050\text{m}$$

Centroidal Moment of Inertia. Using the parallel-axis theorem:

$$I = \tfrac{1}{12}(0.470)(0.300)^3 + (0.470 \times 0.300)(0.050)^2 + \tfrac{1}{12}(3.2)(0.020)^3 + (3.2 \times 0.020)(0.160 - 0.050)^2$$

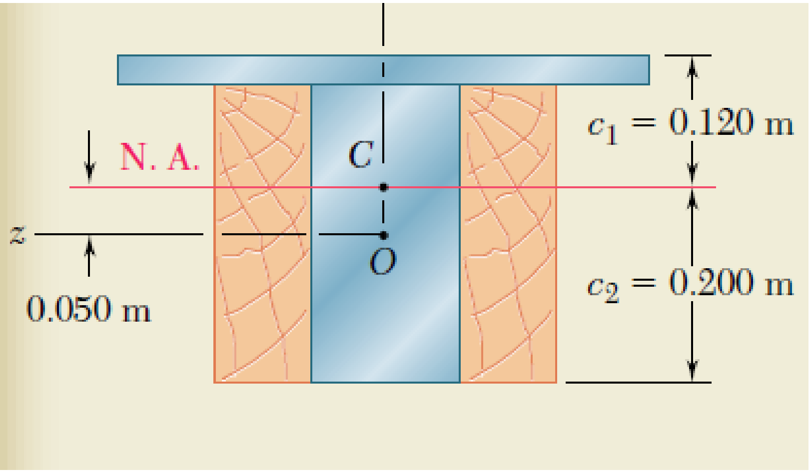
$$I = 2.19 \times 10^{-3}\ \text{m}^4$$

a. Maximum Stress in Wood. The wood farthest from the neutral axis is located along the bottom edge, where $c_2 = 0.200$ m.

$$\sigma_w = \frac{Mc_2}{I} = \frac{(50 \times 10^3\ \text{N} \cdot \text{m})(0.200\ \text{m})}{2.19 \times 10^{-3}\ \text{m}^4}$$

$$\sigma_w = 4.57\ \text{MPa}$$

Example 3



b. Stress in Steel. Along the top edge $c_1 = 0.120$ m. From the transformed section we obtain an equivalent stress in wood, which must be multiplied by n to obtain the stress in steel.

$$\sigma_s = n\frac{Mc_1}{I} = (16)\frac{(50 \times 10^3 \text{ N} \cdot \text{m})(0.120 \text{ m})}{2.19 \times 10^{-3} \text{ m}^4}$$

$$\sigma_s = 43.8 \text{ MPa}$$