SCHOOL OF ENGINEERING

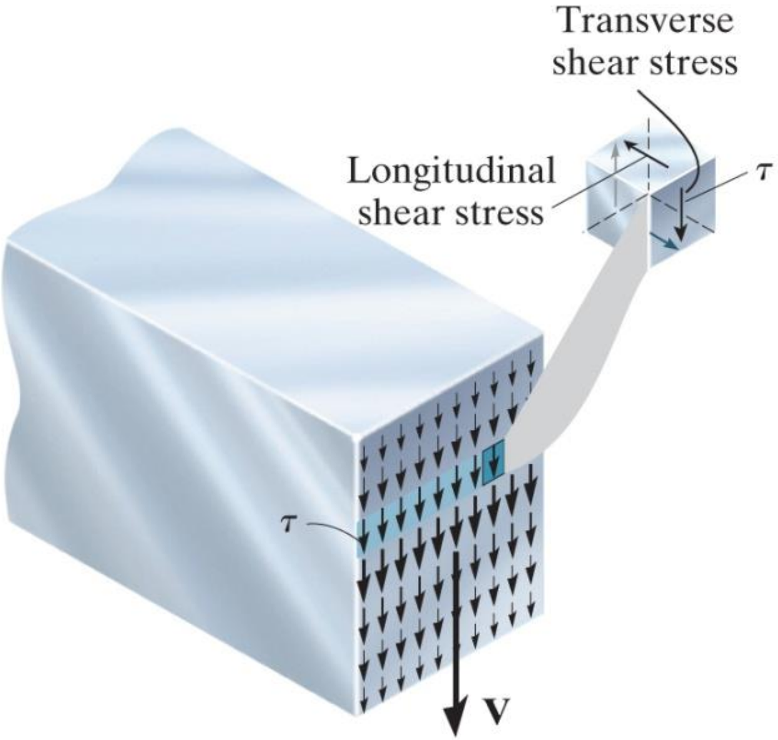
DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

CEE 3211– MECHANICS OF MATERIALS

LECTURE 12 – TRANSVERSE SHEAR

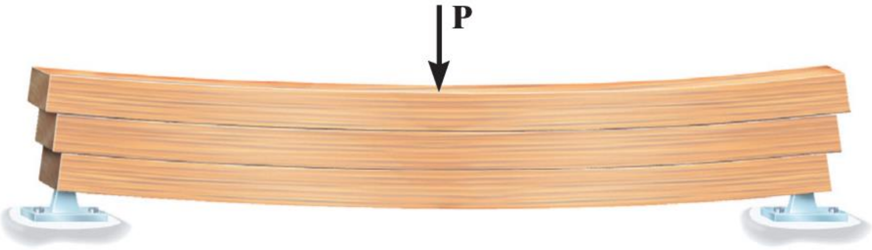
SHEAR IN A STRAIGHT BEAM

- Transverse shear stress always has its associated longitudinal shear stress acting along longitudinal planes of the beam.

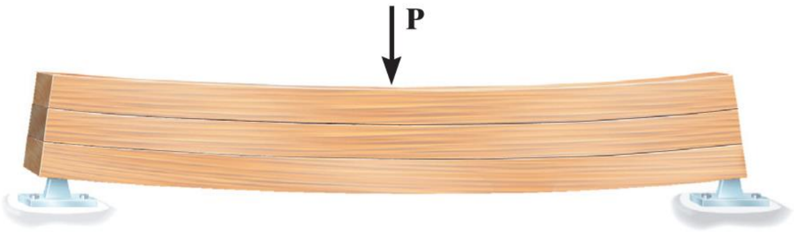


SHEAR IN A STRAIGHT BEAM (cont)

- Effects of Shear Stresses:

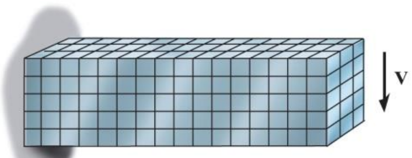


Boards not bonded together
(a)

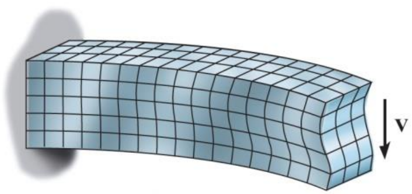


Boards bonded together
(b)

- Warping of cross section



(a) Before deformation



(b) After deformation

Figure: 07_03

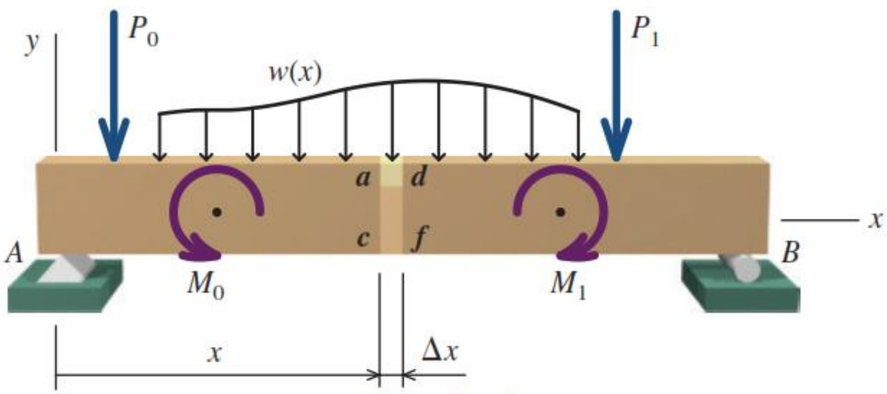
SHEAR IN A STRAIGHT BEAM (cont)

Note:

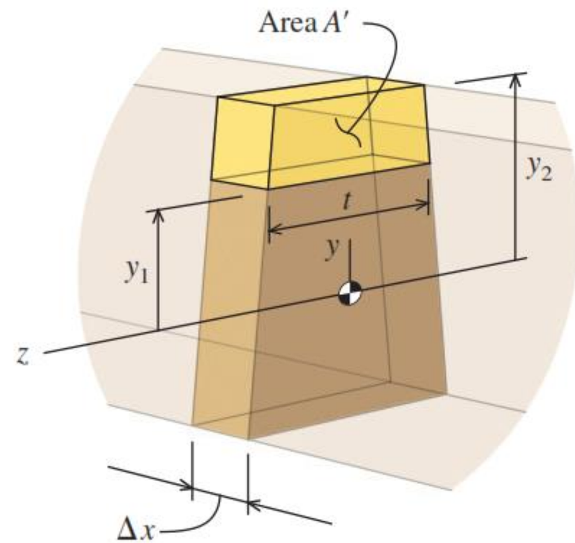
1. Warping" violates the assumptions of "plane section remains plane" in flexure and torsion formulae

2. "Warping" is negligible in "slender beam"

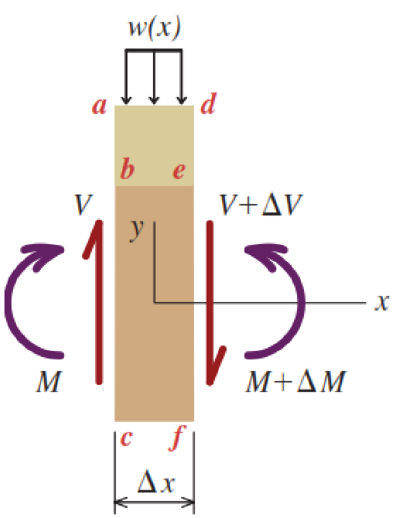
SHEAR FORMULA



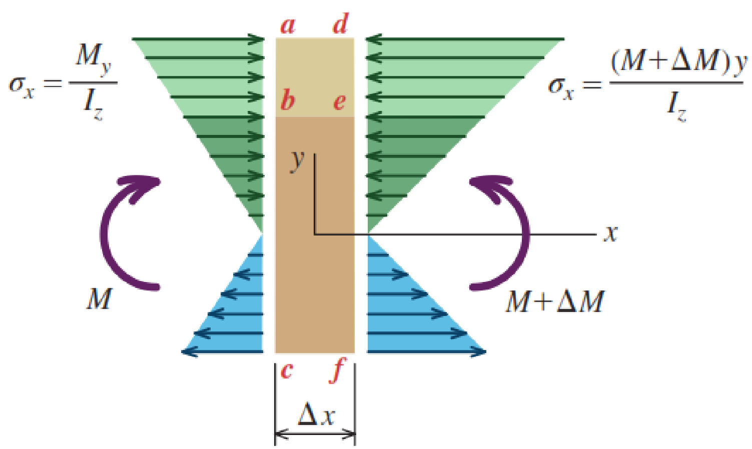
(a) Beam loading



(b) Cross section



(a) Free-body diagram



(b) Bending stresses due to internal bending moments

$$\tau = \frac{VQ}{It}$$

$$\text{where} Q = \int_{A'} y\,dA = \bar{y}'A'$$

Shear Formula

- In deriving a formula for shear stress for a rectangular beam, one must select an elemental area of the cross section at some distance *y* from the neutral axis and recognize that the force acting upon this element is σ dA.
- Substituting in the flexure formula for the normal stress and integrating along the entire cross section of the beam, along with force analysis, yields the following:

$$\tau = \frac{VQ}{Ib}$$

Where:

V – Shear Force

Q – The First Moment

I – Moment of Inertia

b – Width of the location

Shear Formula

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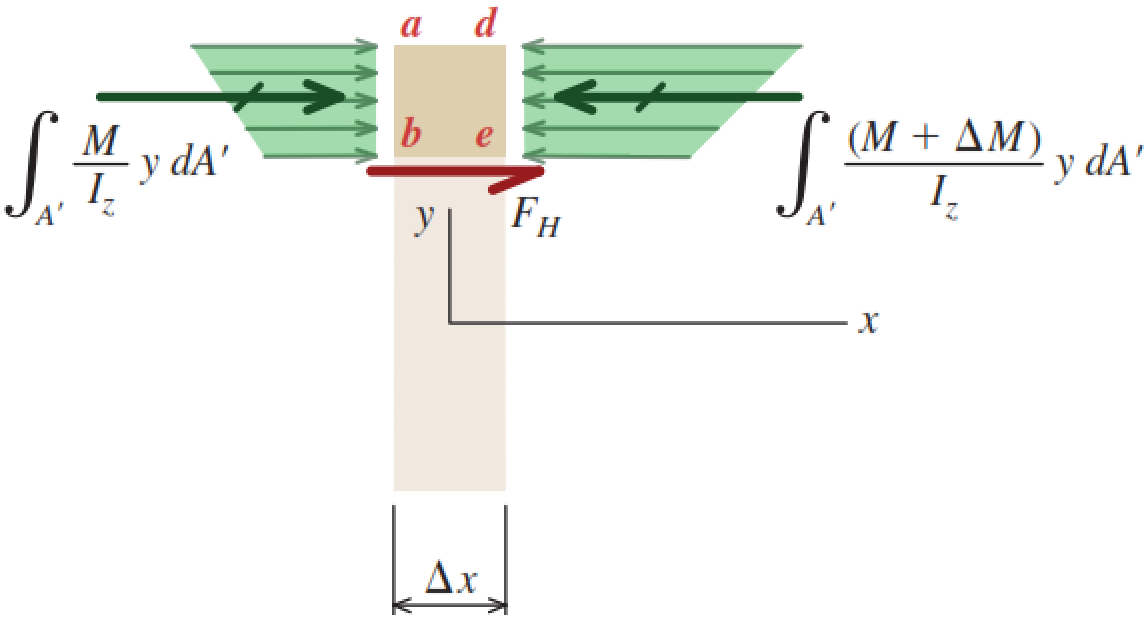
SHEAR FORMULA



SHEAR FORMULA

What is Q?

Q is a mathematical abstraction termed a first moment of area. Recall that a first-moment-of-area term appears as the numerator in the definition of a centroid

$$\boxed{\bar{y} = \frac{\int_A y\, dA}{\int_A dA}}$$

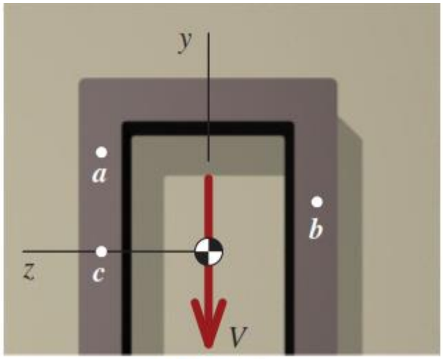
Q is the first moment of area of only portion *A'* of the total cross-sectional area *A*. Equation above can be rewritten in terms of *A'* instead of the total area *A* and multiplied by the denominator of the right side (in terms of *A'*) to give a useful formulation for *Q*:

SHEAR FORMULA

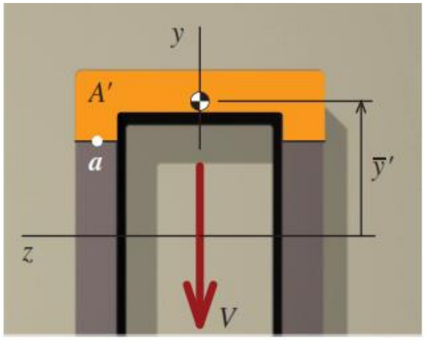
$$Q = \int_{A'} y \, dA' = \bar{y}' \int_{A'} dA' = \bar{y}' A'$$

Here, $\bar{y}'$ is the distance *from the neutral axis* of the cross section to the centroid of area A'.

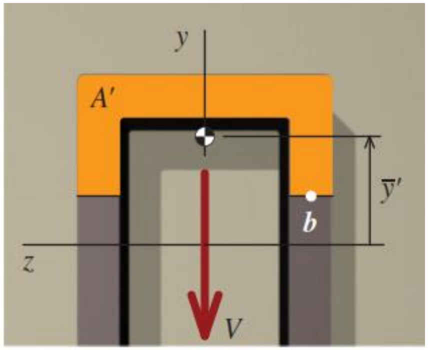
To determine Q at point a in Fig (a), the cross-sectional area is subdivided at a by slicing parallel to the neutral axis (which is perpendicular to the direction of the internal shear



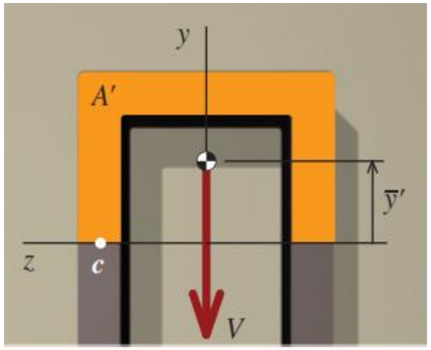
(*a*) Box shape



(*b*) Area A' for calculating Q at point a

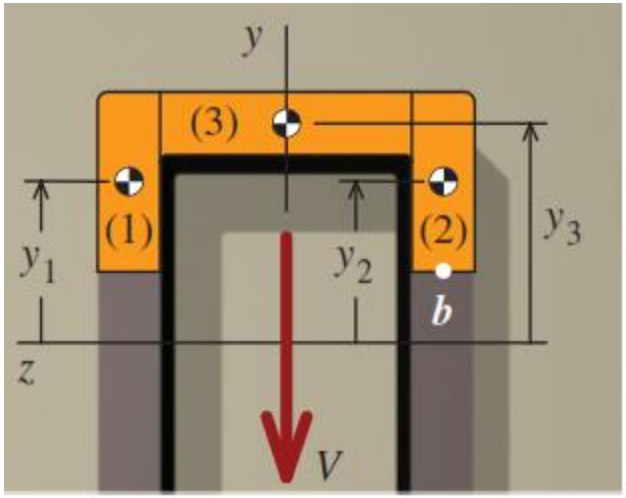


(*c*) Area A' for calculating Q at point b



(*d*) Area A' for calculating Q at point c

SHEAR FORMULA



(*e*) Calculation process

Let us consider the calculation of Q at point b (Figure c) in more detail. The area A' can be divided into three rectangular areas (e) so that $Aa' = A_1 + A_2 + A_3$. The centroid location of the highlighted area with respect to the neutral axis is

$$\bar{y}' = \frac{y_1 A_1 + y_2 A_2 + y_3 A_3}{A_1 + A_2 + A_3}$$

$$Q = y'A' = \frac{y_1 A_1 + y_2 A_2 + y_3 A_3}{A_1 + A_2 + A_3}(A_1 + A_2 + A_3) = y_1 A_1 + y_2 A_2 + y_3 A_3$$

SHEAR FORMULA

This result suggests a more direct calculation procedure that is often expedient. For cross sections that consist of *i* shapes

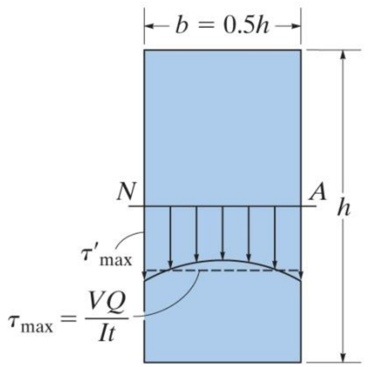
$$\boxed{Q = \sum_i y_i A_i}$$

where *yi* is the distance between the neutral axis and the centroid of shape *i* and *Ai* is the area of shape *i*.

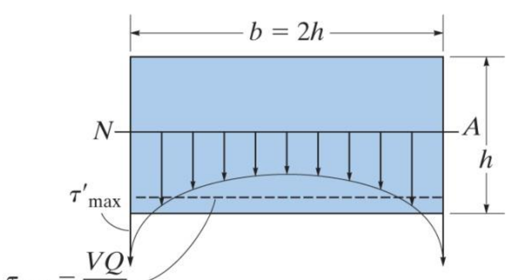
SHEAR IN BEAMS

Rectangular cross section

- Shear –stress distribution is parabolic



(a)



(b)

$$\tau = \frac{6V}{bh^3}\left(\frac{h^2}{4} - y^2\right)$$

$$\tau_{max} = 1.5\frac{V}{A}$$

SHEAR IN BEAMS (cont)

Wide-flange beam

- Shear-stress distribution is parabolic but has a jump at the flange-to-web junctions.

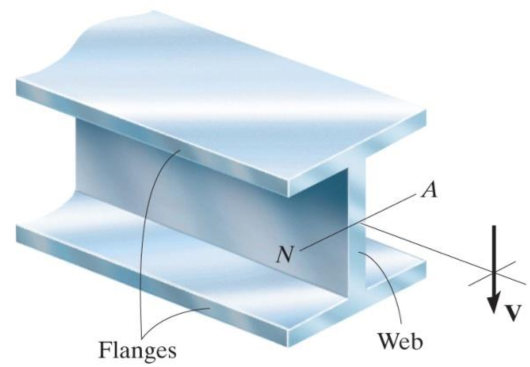


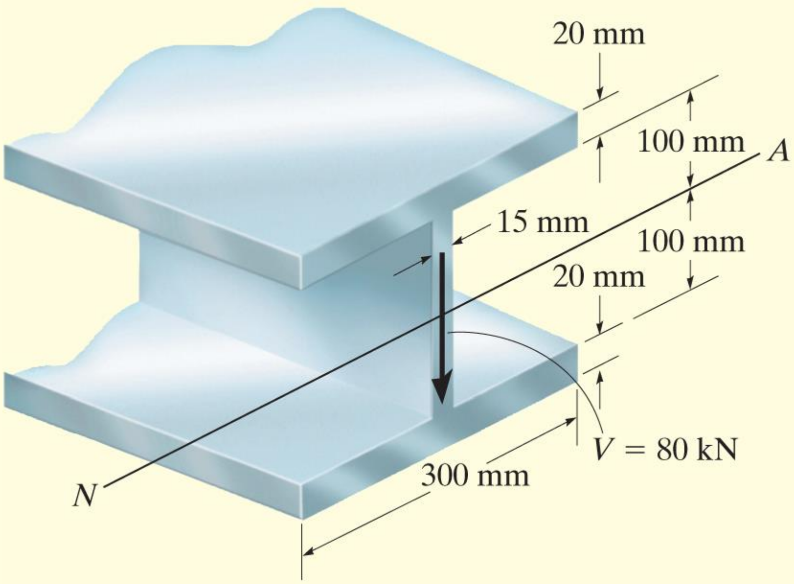
Figure: 07_07

Limitations on the use of shear formula

- Not on cross sections that are short or flat
- Not at points of sudden cross sectional changes (e.g. flange-to-web junction in wide flange beam)
- Not at a joint on an inclined boundary

EXAMPLE 1

A steel wide-flange beam has the dimensions shown in Fig. *a*. If it is subjected to a shear of V = 80kN, plot the shear-stress distribution acting over the beam's cross-sectional area.



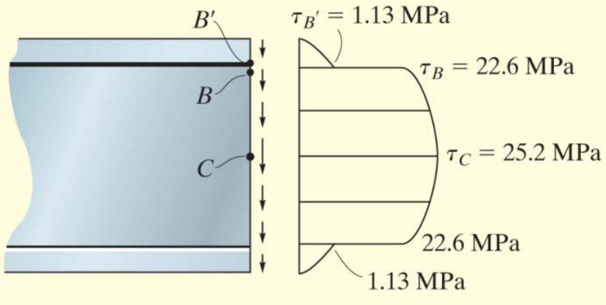
(a)

EXAMPLE 1 (cont)

Solutions

- The moment of inertia of the cross-sectional area about the neutral axis is

$$I = \left[\frac{1}{12}(0.015)(0.2^3)\right] + 2\left[\frac{1}{12}(0.3)(0.02^3) + (0.3)(0.02)(0.11^2)\right] = 155.6(10^{-6})\,\text{m}^4$$

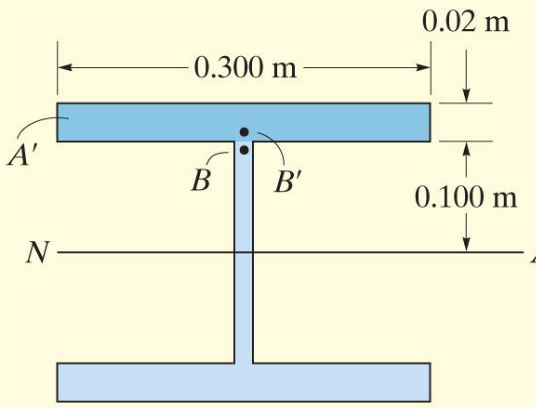


(b)

- For point B’, $t_{B'}$ = 0.3m, and A’ is the dark shaded area shown in Fig.*c*

$$Q_{B'} = \bar{y}'A' = [0.11](0.3)(0.02) = 0.66(10^{-3})\,\text{m}^3$$

$$\tau_{B'} = \frac{VQ_{B'}}{It_{B'}} = \frac{80(10^3)0.66(10^{-3})}{155.6(10^{-6})(0.3)} = 1.13\,\text{MPa}$$



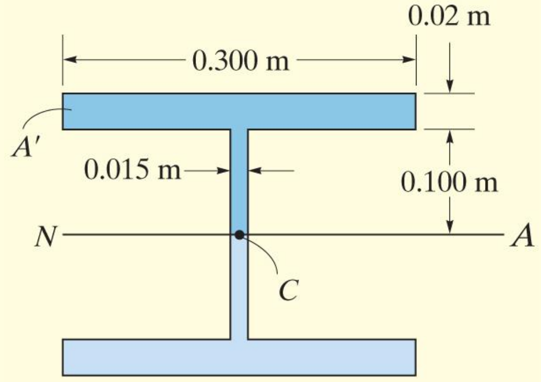
(c)

EXAMPLE 1 (cont)

Solutions

- For point B, t_B = 0.015m, and $Q_B = Q_{B'}$,

$$\tau_B = \frac{VQ_B}{It_B} = \frac{80(10^3)0.66(10^{-3})}{155.6(10^{-6})(0.015)} = 22.6\,\text{MPa}$$



(d)

- For point C, t_C = 0.015m, and A' is the dark shaded area in Fig. *d*.

- Considering this area to be composed of two rectangles,

$$Q_C = \sum \bar{y}' A' = (0.11)(0.3)(0.02) + (0.05)(0.015)(0.1) = 0.735(10^{-3})\,\text{m}^3$$

- Thus, $\tau_C = \tau_{\max} = \dfrac{VQ_c}{It_C} = \dfrac{80(10^3)(0.735)(10^{-3})}{155.6(10^{-6})(0.015)} = 25.2\,\text{MPa}$

SHEAR FLOW IN BUILT-UP BEAM

- Shear flow ≡ shear force per unit length along longitudinal axis of a beam.

$$q = \frac{VQ}{I}$$

q = shear flow
V = internal resultant shear
I = moment of inertia of the *entire* cross-sectional area
Q = moment of the area A

Figure: 07_12_

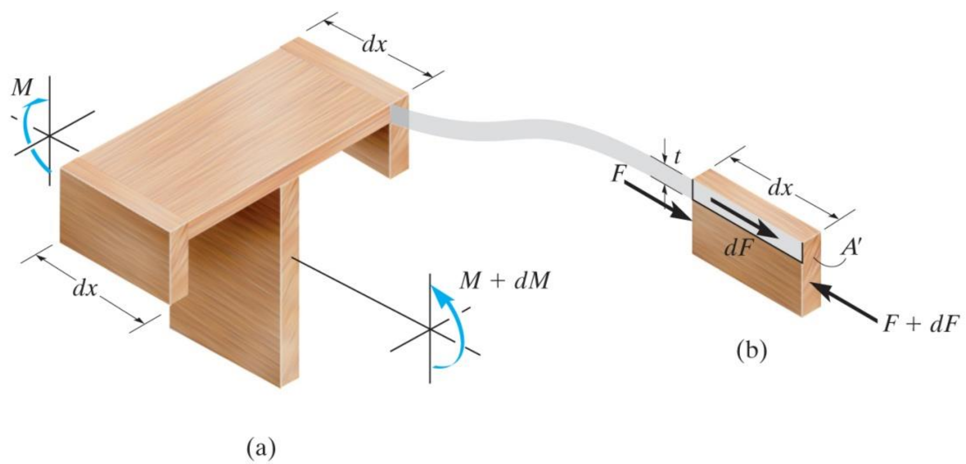
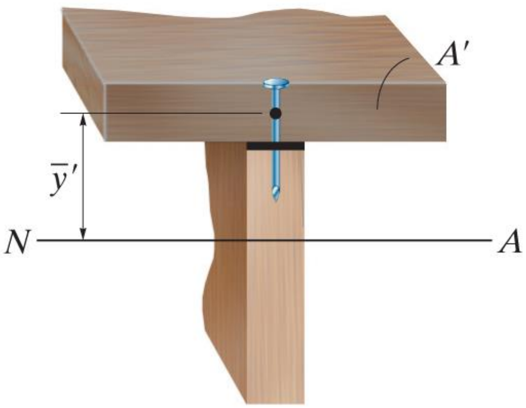
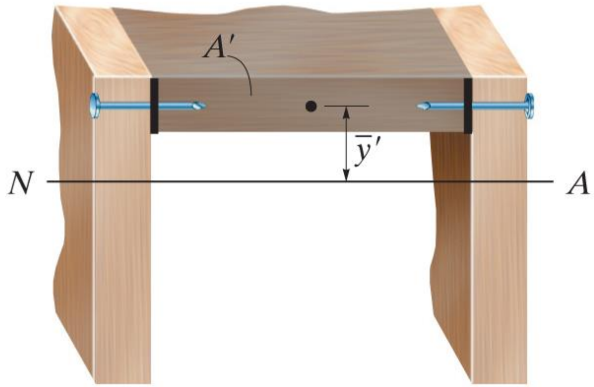


Figure: 07_13

SHEAR FLOW IN BUILT-UP BEAM (cont)

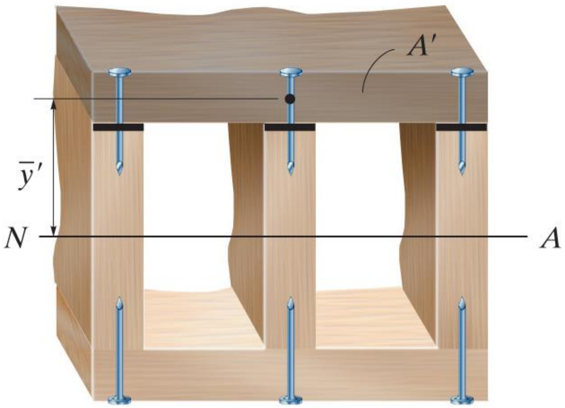


(a)



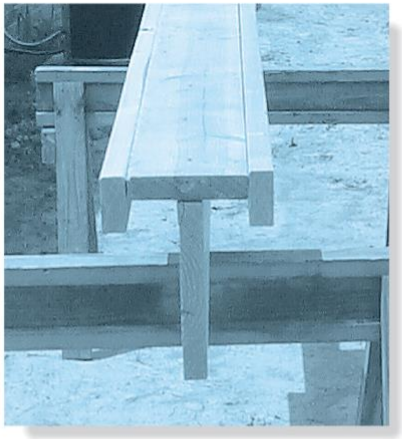
(b)

Figure: 07_14

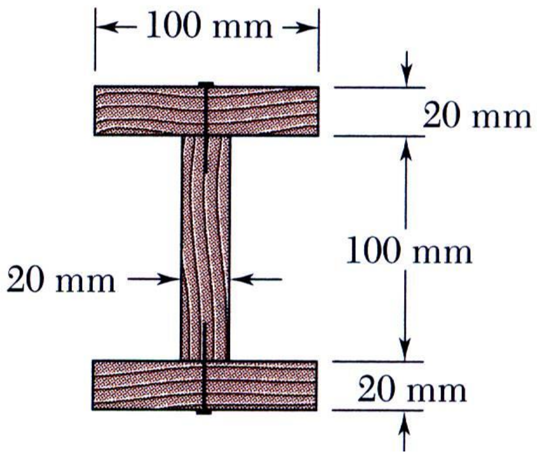


(c)

Figure: 07_14c



Example 2

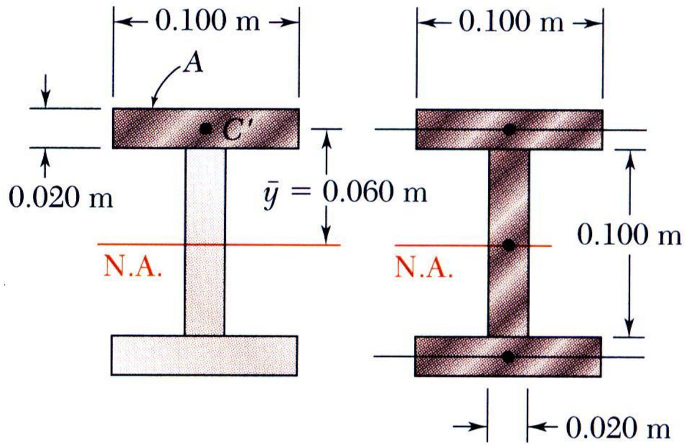


A beam is made of three planks, nailed together. Knowing that the spacing between nails is 25 mm and that the vertical shear in the beam is V = 500 N, determine the shear force in each nail.

SOLUTION:

- Determine the horizontal force per unit length or shear flow q on the lower surface of the upper plank.
- Calculate the corresponding shear force in each nail.

Example 2



$$Q = A\bar{y}$$
$$= (0.020\text{m} \times 0.100\text{m})(0.060\text{m})$$
$$= 120 \times 10^{-6}\text{m}^3$$

$$I = \tfrac{1}{12}(0.020\text{m})(0.100\text{m})^3$$
$$+ 2[\tfrac{1}{12}(0.100\text{m})(0.020\text{m})^3$$
$$+ (0.020\text{m} \times 0.100\text{m})(0.060\text{m})^2]$$
$$= 16.20 \times 10^{-6}\text{m}^4$$

SOLUTION:

- Determine the horizontal force per unit length or shear flow q on the lower surface of the upper plank.

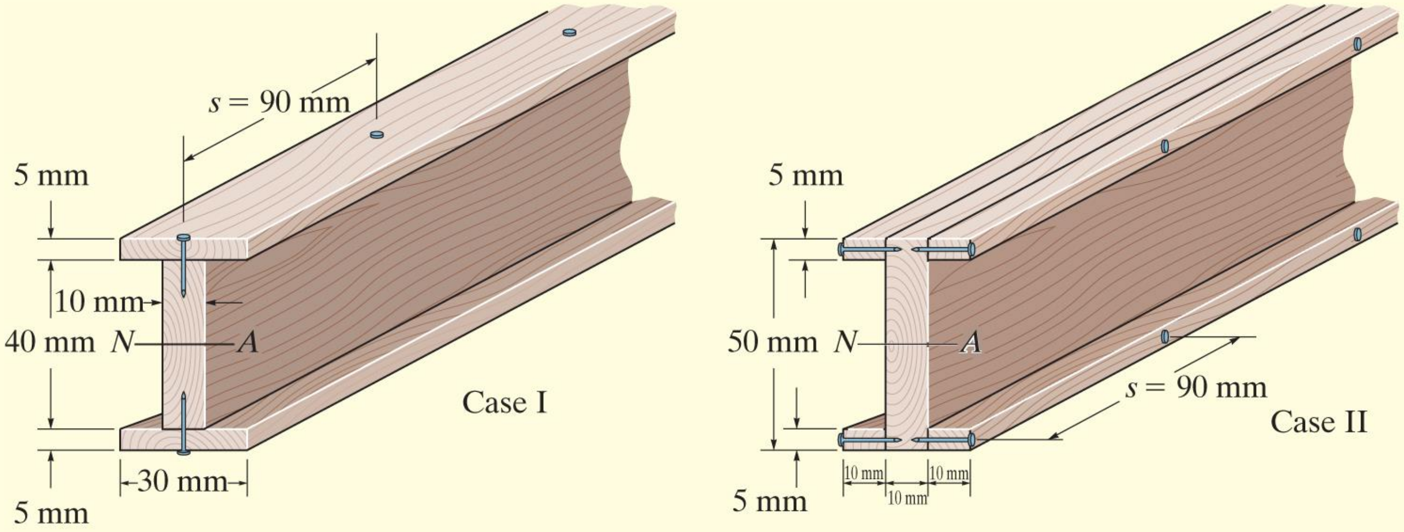
$$q = \frac{VQ}{I} = \frac{(500\text{N})(120 \times 10^{-6}\text{m}^3)}{16.20 \times 10^{-6}\text{m}^4}$$
$$= 3704 \,{}^{\text{N}}\!/_{\text{m}}$$

- Calculate the corresponding shear force in each nail for a nail spacing of 25 mm.

$$F = (0.025\text{m})q = (0.025\text{m})(3704\,N/m$$
$$\boxed{F = 92.6\text{N}}$$

EXAMPLE 3

Nails having a total shear strength of 40 N are used in a beam that can be constructed either as in Case I or as in Case II. If the nails are spaced at 90 mm, determine the largest vertical shear that can be supported in each case so that the fasteners will not fail.



EXAMPLE 3 (cont)

Solutions

- Since the cross section is the same in both cases, the moment of inertia about the neutral axis is

$$I = \frac{1}{12}(30)(50^3) - 2\left[\frac{1}{12}(10)(40^3)\right] = 205833\,\text{mm}^4$$

Case I

- For this design a single row of nails holds the top or bottom flange onto the web.
- For one of these flanges,

$$Q = \overline{y}'A' = (22.5)(30)(5) = 3375\,\text{mm}^3$$

$$q = \frac{VQ}{I}$$

$$\frac{40}{90} = \frac{V(3375)}{205833}$$

$$V = 27.1\,\text{N} \quad \text{(Ans)}$$

EXAMPLE 3 (cont)

Solutions

Case II

- Here a single row of nails holds one of the side boards onto the web.
- Thus,

$$Q = \overline{y}'A' = (22.5)(10)(5) = 1125\,\text{mm}^3$$

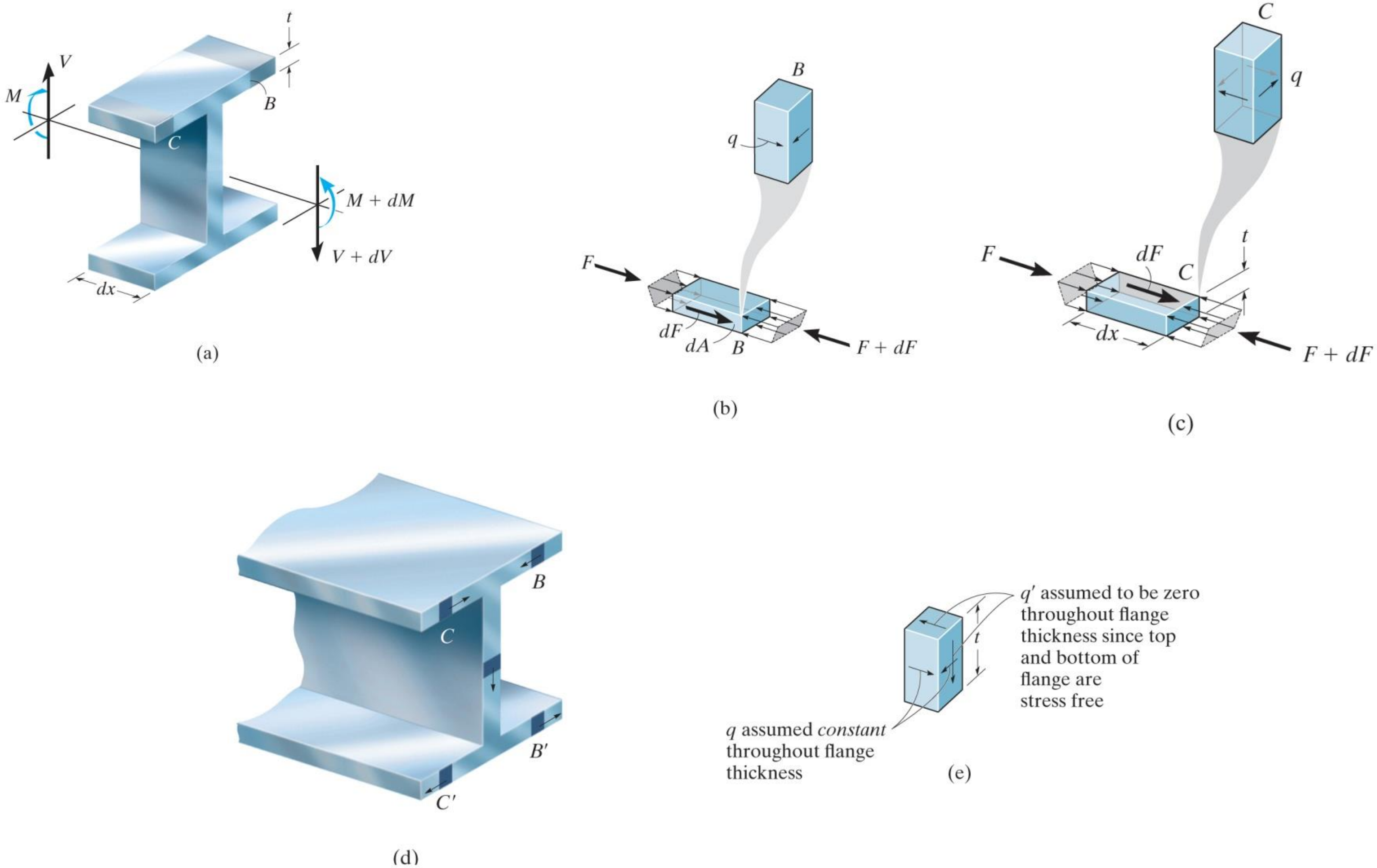
$$q = \frac{VQ}{I}$$

$$\frac{40}{90} = \frac{V(1125)}{205833}$$

$$V = 81.3\,\text{N} \quad (\text{Ans})$$

SHEAR FLOW IN THIN-WALLED BEAM

- Approximation: only the shear-flow component that acts parallel to the walls of the member will be counted.



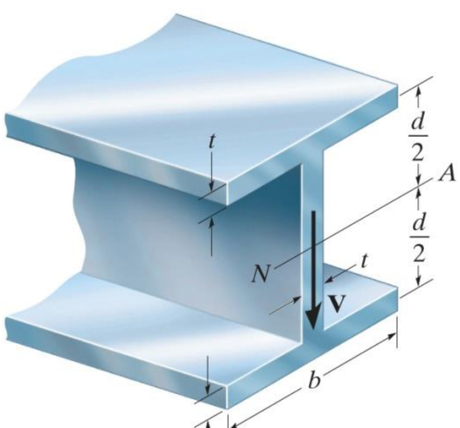
SHEAR FLOW IN THIN-WALLED BEAM (cont)

- In horizontal flanges, flow varies linearly,

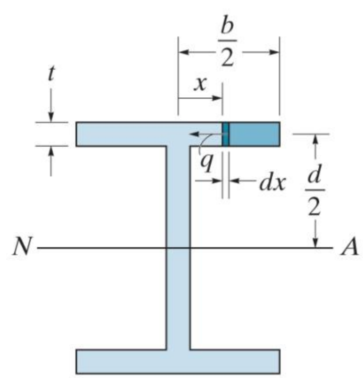
$$q=\frac{VQ}{I}=\frac{V[d/2]((b/2)-x)t}{I}=\frac{Vtd}{2I}\left(\frac{b}{2}-x\right)$$

- In vertical web(s), flow varies parabolically,

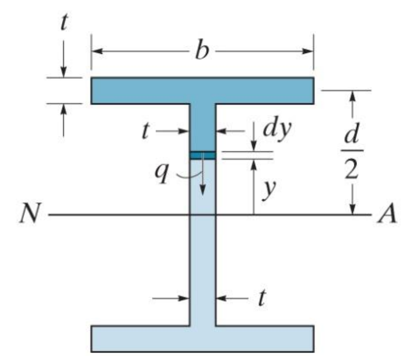
$$q=\frac{VQ}{I}=\frac{Vt}{I}\left[\frac{db}{2}+\frac{1}{2}\left(\frac{d^2}{4}-y^2\right)\right]$$



(a)



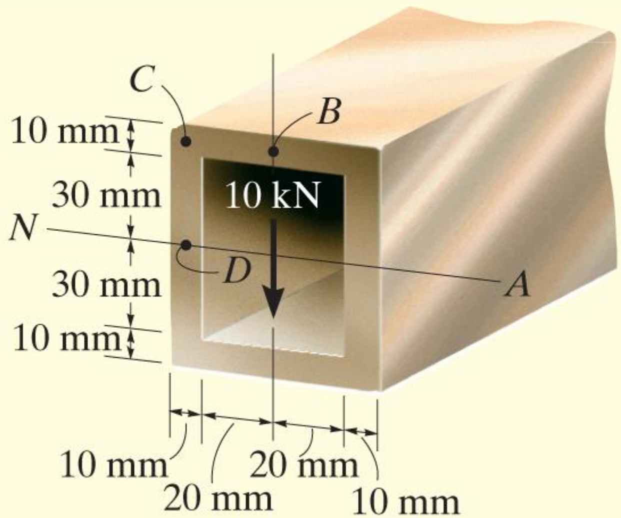
(b)



(c)

EXAMPLE 4

The thin-walled box beam in Fig. *a* is subjected to a shear of 10 kN. Determine the variation of the shear flow throughout the cross section.

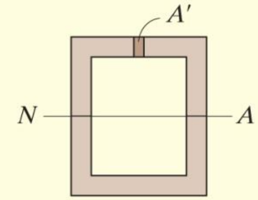


(a)

EXAMPLE 4 (cont)

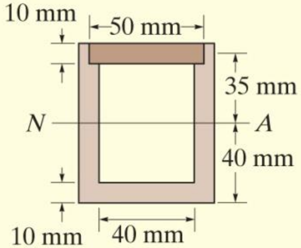
Solutions

- The moment of inertia is $I = \frac{1}{12}(6)(8)^3 - \frac{1}{12}(4)(6)^3 = 184\,\text{mm}^4$



(b)

- For point *B*, the area $A' \approx 0$ thus $q'_B = 0$.

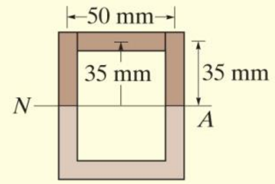


(c)

- Also,

$$Q_C = \bar{y}A' = (3.5)(5)(1) = 17.5\,\text{cm}^3$$
$$Q_D = \sum \bar{y}A' = 2(2)(1)(4) = 30\,\text{cm}^3$$

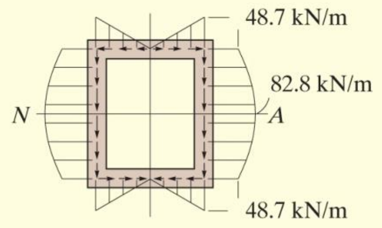
- For point *C*,



(d)

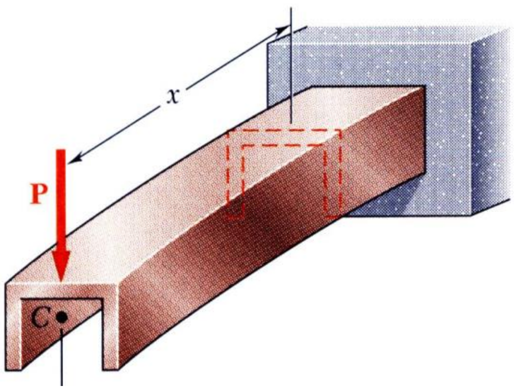
$$q_C = \frac{VQ_C}{I} = \frac{10(17.5/2)}{184} = 0.951\,\text{kN/cm} = 91.5\,\text{N/mm}$$

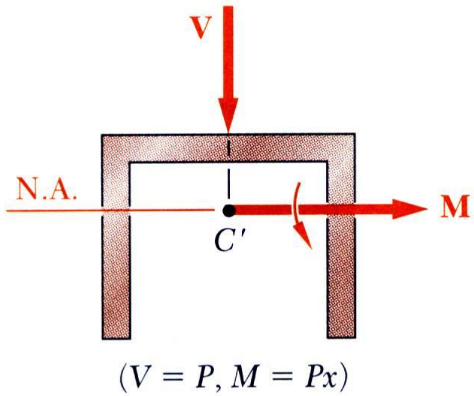
- The shear flow at *D* is



(e)

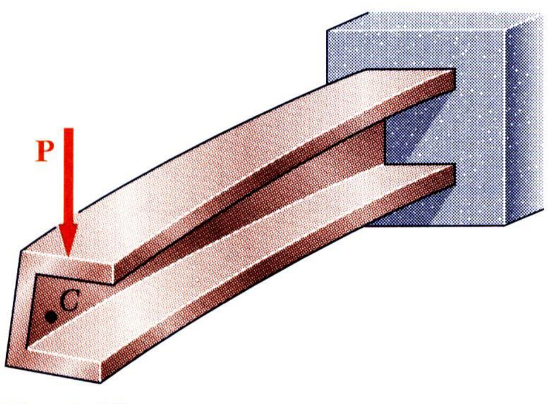
$$q_D = \frac{VQ_D}{I} = \frac{10(30/2)}{184} = 1.63\,\text{kN/cm} = 163\,\text{N/mm}$$

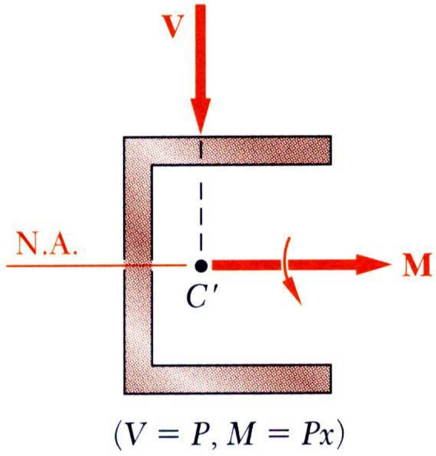




- Beam loaded in a vertical plane of symmetry deforms in the symmetry plane without twisting.

$$\sigma_x = -\frac{My}{I} \qquad \tau_{ave} = \frac{VQ}{It}$$

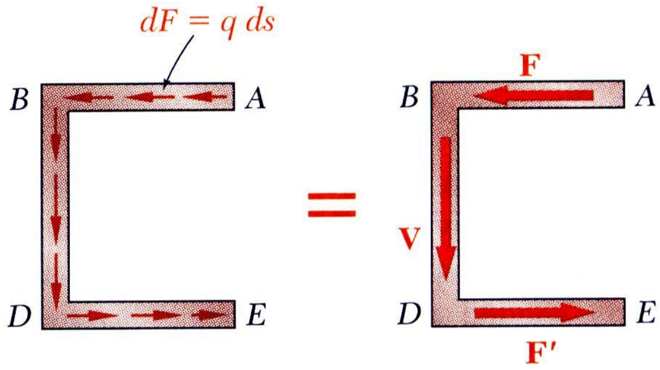


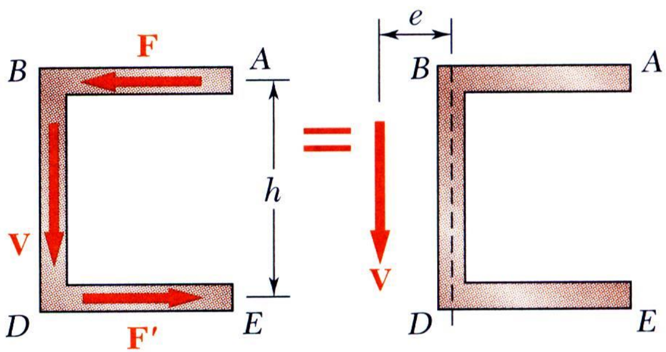


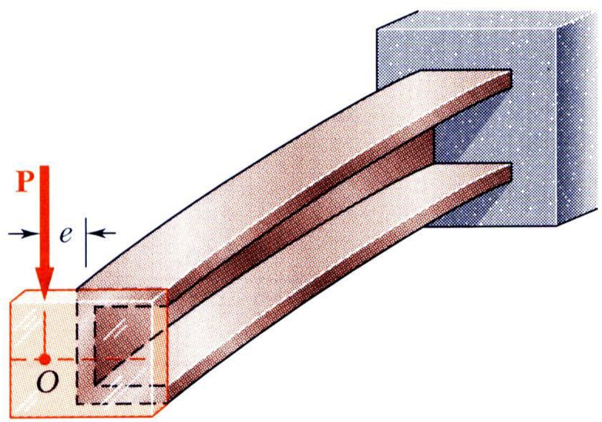
- Beam without a vertical plane of symmetry bends and twists under loading.

$$\sigma_x = -\frac{My}{I} \qquad \tau_{ave} \neq \frac{VQ}{It}$$

Unsymmetric Loading of Thin-Walled Members







- If the shear load is applied such that the beam does not twist, then the shear stress distribution satisfies

$$\tau_{ave} = \frac{VQ}{It} \quad V = \int_B^D q\,ds \quad F = \int_A^B q\,ds = -\int_D^E q\,ds = -F'$$

- F and F' indicate a couple Fh and the need for the application of a torque as well as the shear load.

$$Fh = Ve$$

- When the force P is applied at a distance e to the left of the web centerline, the member bends in a vertical plane without twisting.

SHEAR CENTRE

Shear center is the point through which a force can be applied which will cause a beam to bend and yet not twist

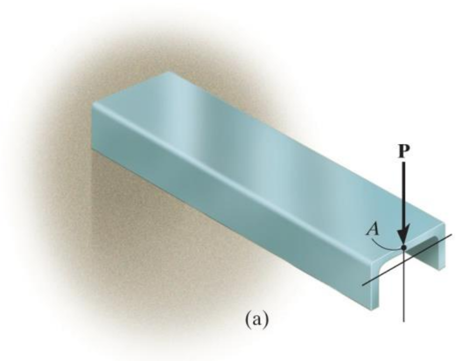
The location of the shear center is only a function of geometry of the cross section and does not depend upon the applied load.

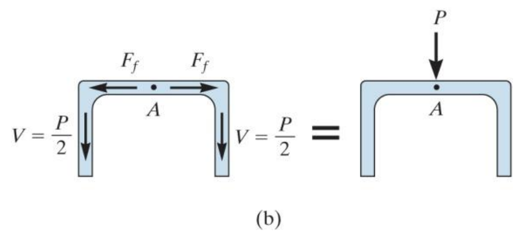
If this force is applied through the *centroid C (see next page)* of the cross section, the channel will not only bend downward, but *it will also twist* clockwise as shown.

(a)

SHEAR CENTRE

To *prevent* this twisting and therefore cancel the unbalanced moment, it is necessary to apply **P** at a point *O* located an eccentric distance *e* from the web,





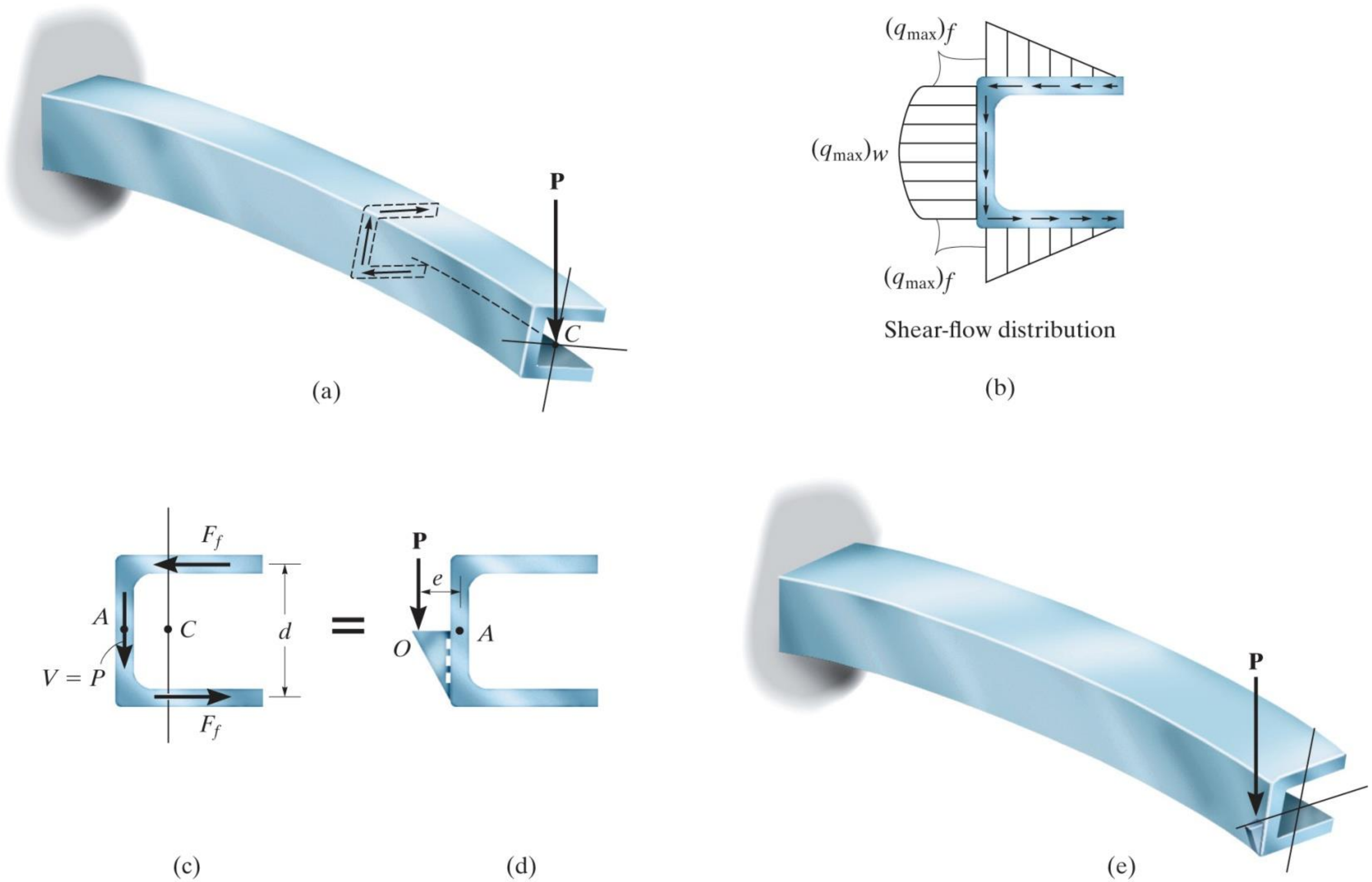
$$e = \frac{F_f d}{P}$$

- From this analysis, it should be noted that ***the shear center will always lie on an axis of symmetry*** of a member’s cross-sectional area.

- For example, if the channel is rotated 90° and **P** is applied at *A*, Fig.*a*, no twisting will occur since the shear flow in the web and flanges for this case is *symmetrical*, and therefore the force resultants in these elements will create zero moments about *A*, Fig. *b*.

- Obviously, if a member has a cross section with *two* axes of symmetry, as in the case of a wide-flange beam, the shear center will coincide with the intersection of these axes (the centroid)

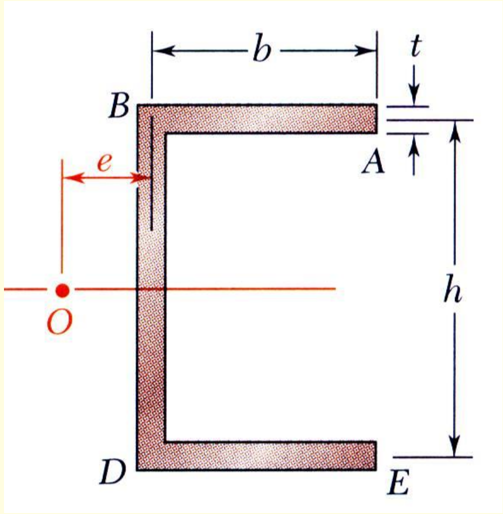
SHEAR CENTRE (cont)



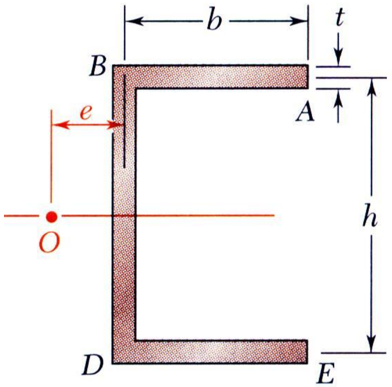
$$e = \frac{F_f d}{P}$$

EXAMPLE 4

Determine the location for the shear center of the channel section with b = 100 mm, h = 150 mm., and t = 4 mm



Example 4



$$e = \frac{F\,h}{I}$$

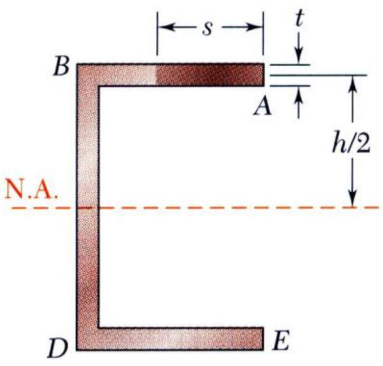
- where

$$F = \int_0^b q\,ds = \int_0^b \frac{VQ}{I}\,ds = \frac{V}{I}\int_0^b st\frac{h}{2}\,ds$$

$$= \frac{Vthb^2}{4I}$$

$$I = I_{web} + 2I_{flange} = \frac{1}{12}th^3 + 2\left[\frac{1}{12}bt^3 + bt\left(\frac{h}{2}\right)^2\right]$$

$$\cong \frac{1}{12}th^2(6b+h)$$



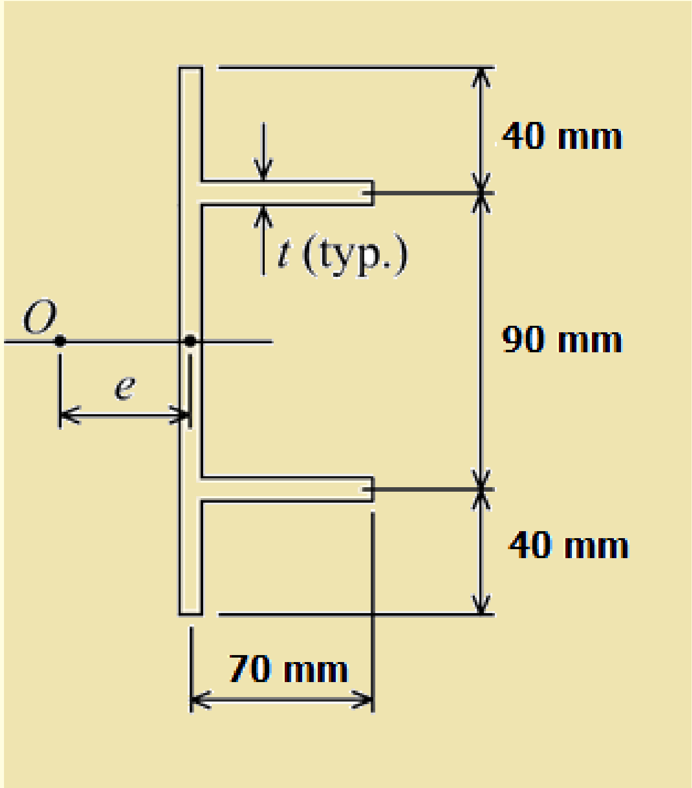
- Combining,

$$e = \frac{b}{2+\frac{h}{3b}} = \frac{100}{2+\frac{150}{3(100)}}$$

$$\boxed{e = 40\text{ mm}}$$

EXAMPLE 5

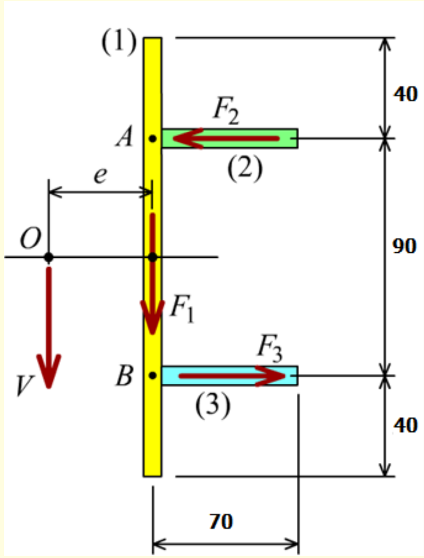
Determine the location of the shear centre O for the cross section shown below. Assume a uniform thickness of $t = 4$ mm for all portions of the cross section.



EXAMPLE 5 (cont)

Solutions

a) Moment of inertia about the neutral axis: Recognizing that the wall thickness is thin, the moment of inertia for the shape can be calculated as:



$$I_{NA} = \frac{(4\text{ mm})(170\text{ mm})^3}{12} + 2\left[(70\text{ mm})(4\text{ mm})\left(\frac{90\text{ mm}}{2}\right)^2\right]$$

$$= 1,637,666.67\text{ mm}^4 + 1,134,000\text{ mm}^4 = 2,771,666.67\text{ mm}^4$$

Shear flow in area (1): We will choose to sum moments about point B so that we need only determine one force: F_2. Begin by calculating the shear flow at point A.

$$Q_A = (70\text{ mm})(4\text{ mm})\left(\frac{90\text{ mm}}{2}\right) = 12,600\text{ mm}^3$$

EXAMPLE 5 (cont)

Solutions

Let the shear force V be equal to the moment of inertia I_{NA}, and express the shear flow q at point A as.

$$q_A = \frac{VQ_A}{I} = \frac{(2,771,666.67 \text{ N})(12,600 \text{ mm}^3)}{2,771,666.67 \text{ mm}^4} = 12,600 \text{ N/mm}$$

Calculate the horizontal force F_2 as:

$$F_2 = \frac{q_A}{2}(70 \text{ mm}) = \frac{12,600 \text{ N/mm}}{2}(70 \text{ mm}) = 441,000 \text{ N}$$

Shear Center

$$Pe = (90 \text{ mm})F_2$$

$$e = \frac{(90 \text{ mm})(441,000 \text{ N})}{2,771,666.67 \text{ N}} = \boxed{14.32 \text{ mm}}$$