THIN-WALLED PRESSURE VESSELS

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- Cylindrical or spherical pressure vessels are commonly used in industry to serve as boilers or storage tanks.

- The stresses acting in the wall of these vessels can be analyzed in a simple manner provided it has a *thin wall*, that is, the inner-radius-to-wall-thickness ratio is 10 or more ($r/t \geq 10$)

- Specifically, when $r/t \geq 10$ the results of a thin-wall analysis will predict a stress that is approximately 4% *less* than the actual maximum stress in the vessel.

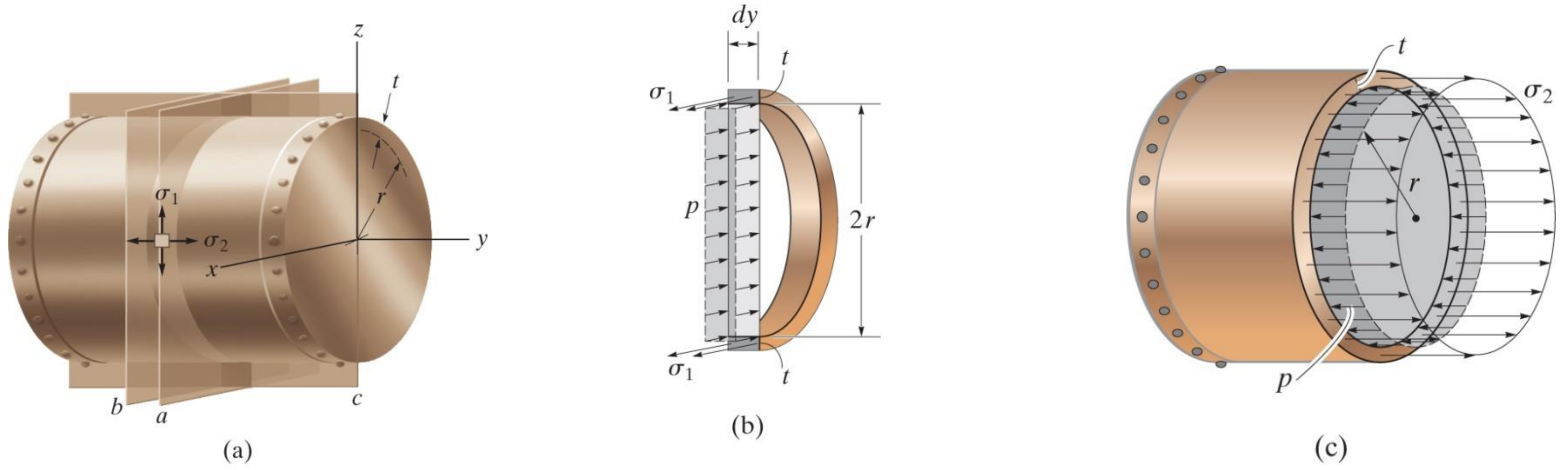
- For larger r/t ratios this error will be even smaller

THIN-WALLED PRESSURE VESSELS

Assumptions:

1. Inner-radius-to-wall-thickness ratio ≥ 10
2. Stress distribution in thin wall is uniform or constant

- **Cylindrical vessels:**

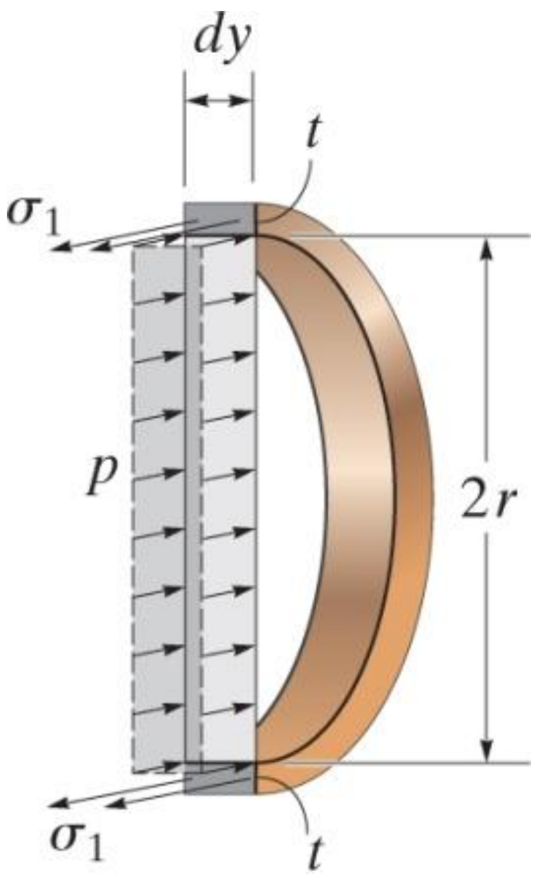


- The cylindrical vessel in Fig. *a* has a wall thickness *t*, inner radius *r*, and is subjected to an internal gas pressure *p*.
- Two types of stresses: ***circumferential*** or ***hoop stress***, & ***longitudinal stress***

THIN-WALLED PRESSURE VESSELS

- To find the ***circumferential*** or ***hoop stress***, we can section the vessel by planes *a*, *b*, and *c* (in the previous slide).
- Considering only loadings in the *x*-direction: shown in fig (b) below

$$\sum F_x = 0; \qquad 2[\sigma_1(t\,dy)] - p(2r\,dy) = 0$$



Hoop direction:

$$2tdy\sigma_1 = 2prdy$$

Hoop direction: $\sigma_1 = \frac{pr}{t}$

THIN-WALLED PRESSURE VESSELS

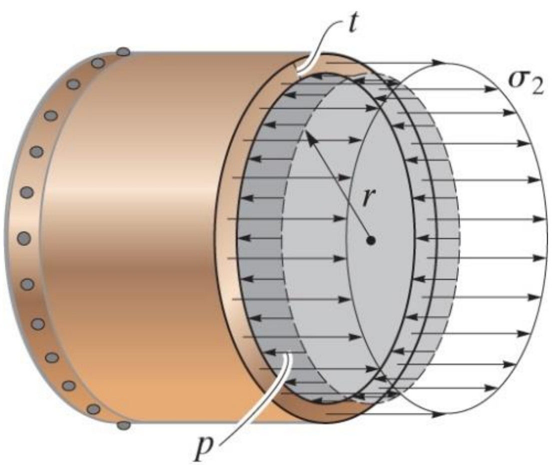
LONGITUDINAL STRESS

- In a FBD below, σ_2 a uniformly throughout the wall, and p acts on the section of the contained gas.

- Since the mean radius is approximately equal to the vessel's inner radius, equilibrium in the y direction requires:

$$\sum F_y = 0; \qquad \sigma_2(2\pi r t) - p(\pi r^2) = 0$$

$$\boldsymbol{\sigma_2 = \frac{pr}{2t}}$$



THIN-WALLED PRESSURE VESSELS (cont)

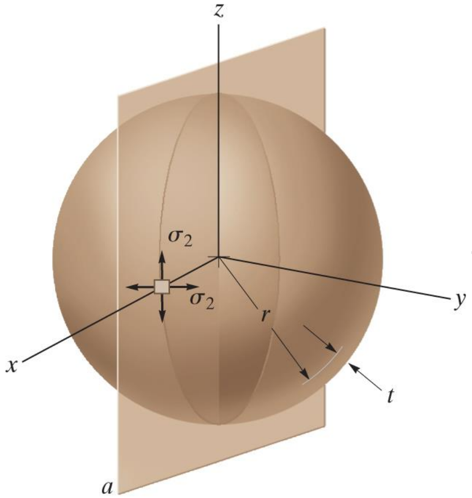
LONGITUDINAL STRESS

- Comparing the two stresses, it can be seen that the hoop or circumferential stress is *twice as large* as the longitudinal or axial stress.

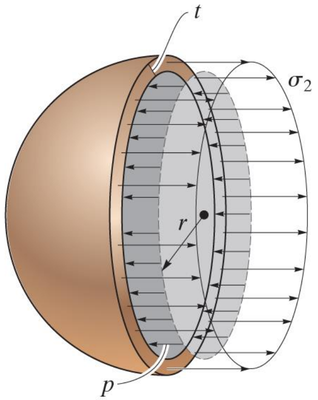
- This implies that:
  - when fabricating cylindrical pressure vessels from rolled-formed plates, it is important that the longitudinal joints be designed to carry twice as much stress as the circumferential joints.

THIN-WALLED PRESSURE VESSELS (cont)

- Spherical vessels:



$$\sum F_y = 0; \qquad \sigma_2(2\pi r t) - p(\pi r^2) = 0$$



$$\sigma_2 = \frac{pr}{2t}$$

This is the same result as that obtained for the longitudinal stress in the cylindrical pressure vessel, although this stress will be the same regardless of the orientation of the hemispheric free-body diagram

THIN-WALLED PRESSURE VESSELS (cont)

- LIMITATIONS:

- The above analysis indicates that an element of material taken from either a cylindrical or a spherical pressure vessel is subjected to ***biaxial stress***, i.e., normal stress existing in only two directions.

- Actually, however, the pressure also subjects the material to a ***radial stress***, σ_3 , which acts along a radial line. This stress has a maximum value equal to the pressure *p* at the interior wall and it decreases through the wall to zero at the exterior surface of the vessel, since the pressure there is zero.

- For thin-walled vessels, however, we will *ignore* this stress component, since our limiting assumption of $r/t = 10$ results in σ_2 and σ_1 being, respectively, 5 and 10 times *higher* than the maximum radial stress, $(\sigma_3)_{max} = p$.

- Finally, note that if the vessel is subjected to an *external pressure*, the resulting compressive stresses within the wall may cause the wall to suddenly collapse inward or buckle rather than causing the material to fracture.

EXAMPLE 1

A cylindrical pressure vessel has an inner diameter of 1.2 m and a thickness of 12 mm.

- Determine the maximum internal pressure it can sustain so that neither its circumferential nor its longitudinal stress component exceeds 140 MPa.
- Under the same conditions, what is the maximum internal pressure that a similar-size spherical vessel can sustain?

EXAMPLE 1 (cont)

Solutions

- The maximum stress occurs in the circumferential direction.

$$\sigma_1 = \frac{pr}{t}$$

$$140 = \frac{p(600)}{12}$$

$$p = 2.8 \text{ MPa}$$

•The stress in the longitudinal direction will be $\sigma_2 = \frac{1}{2}(140) = 70\,\text{MPa}$

•The *maximum stress* in the *radial direction* occurs on the material at the inner wall of the vessel and is

$$\sigma_{3(\max)} = p = 2.8 \text{ MPa}$$

EXAMPLE 1 (cont)

Solutions

- For sphere: The maximum stress occurs in any two perpendicular directions on an element of the vessel is

$$\sigma_2 = \frac{pr}{2t}$$

$$140 = \frac{p(600)}{2(12)}$$

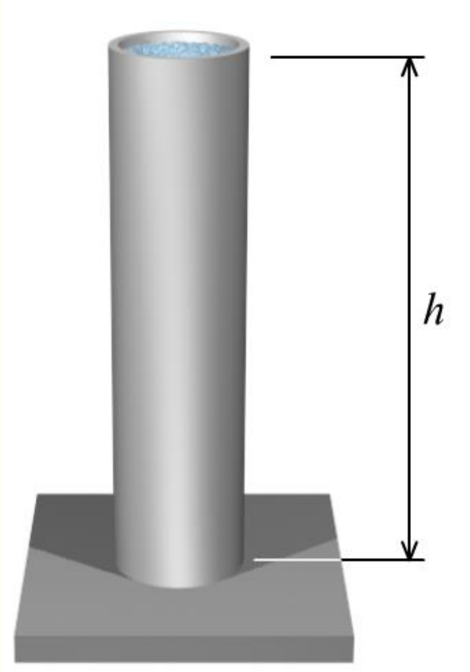
$$p = 5.6\ \text{N/mm}^2 = 5.6\ \text{MPa} \quad \text{(Ans)}$$

EXAMPLE 2

A tall open-topped standpipe below has an inside diameter of 2,750 mm and a wall thickness of 6 mm. The standpipe contains water, which has a mass density of 1,000 kg/m^3.

(a) What height ***h*** of water will produce a circumferential stress of 16 MPa in the wall of the standpipe?

(b) What is the axial stress in the wall of the standpipe due to the water pressure?



EXAMPLE 2 (cont)

Solutions

- Circumferential or hoop stress:

$$\sigma_{\text{hoop}} = \frac{pr}{t} = \frac{p(1375\text{mm})}{(6\text{ mm})} = 16\text{ Mpa}$$

$$\therefore p = 0.0698\text{ MPa}$$

(a) Height *h* of water

$$p = \rho g h = 69.818 \times 10^{-3}\text{ MPa}$$

$$\therefore h = \frac{69.818 \times 10^{3}\text{ N/m}^2}{(1{,}000\text{ kg/m}^3)(9.81\text{ m/s}^2)} = 7.122684\text{ m} = \boxed{7.12\text{ m}}$$

EXAMPLE 2 (cont)

Solutions

(b) Axial stress in the wall of the standpipe due to water pressure

Since the standpipe is open to the atmosphere at its upper end, the fluid pressure will not create stress in the longitudinal direction of the standpipe; therefore,

$$\sigma_{\text{long}} = 0$$

STRESS CAUSED BY COMBINED LOADINGS

STRESS CAUSED BY COMBINED LOADINGS

- So far, we've determined the stress in a member subjected to either an internal axial force, a shear force, a bending moment, or a torsional moment.

- Most often, however, the cross-section of a member will be subjected to several of these loadings simultaneously, and when this occurs, then the method of superposition should be used to determine the resultant stress.

- The following procedure for analysis provides a method for doing this

REVIEW OF STRESS ANALYSES

- **Normal force, P** leads to:

$$uniform\ normal\ stress, \qquad \boldsymbol{\sigma = \frac{P}{A}}$$

- **Shear force, V** leads to:

$$shear - stress\ distribution, \qquad \boldsymbol{\tau = \frac{VQ}{It}}$$

- **Bending moment, M** leads to:

$$longitudinal\ stress\ distribution,$$

$$\boldsymbol{\sigma = -\frac{My}{I}} \text{ (for straight beam)}$$

REVIEW OF STRESS ANALYSES (cont)

- **Torsional moment T** leads to:

$$shear - stress\ distribution, \boldsymbol{\tau} = \frac{\boldsymbol{T\rho}}{\boldsymbol{J}} \quad \text{(for circular shaft)}$$

$$\boldsymbol{\tau} = \frac{\boldsymbol{T}}{(\boldsymbol{2A_m t})} \quad \text{(for closed thin−walled tube)}$$

- **Stresses in pressure thin-walled vessels**

Circumferential or hoop stress, $\boldsymbol{\sigma_1 = \frac{pr}{t}}$

Longitudinal or axial stress, $\boldsymbol{\sigma_2 = \frac{pr}{2t}}$

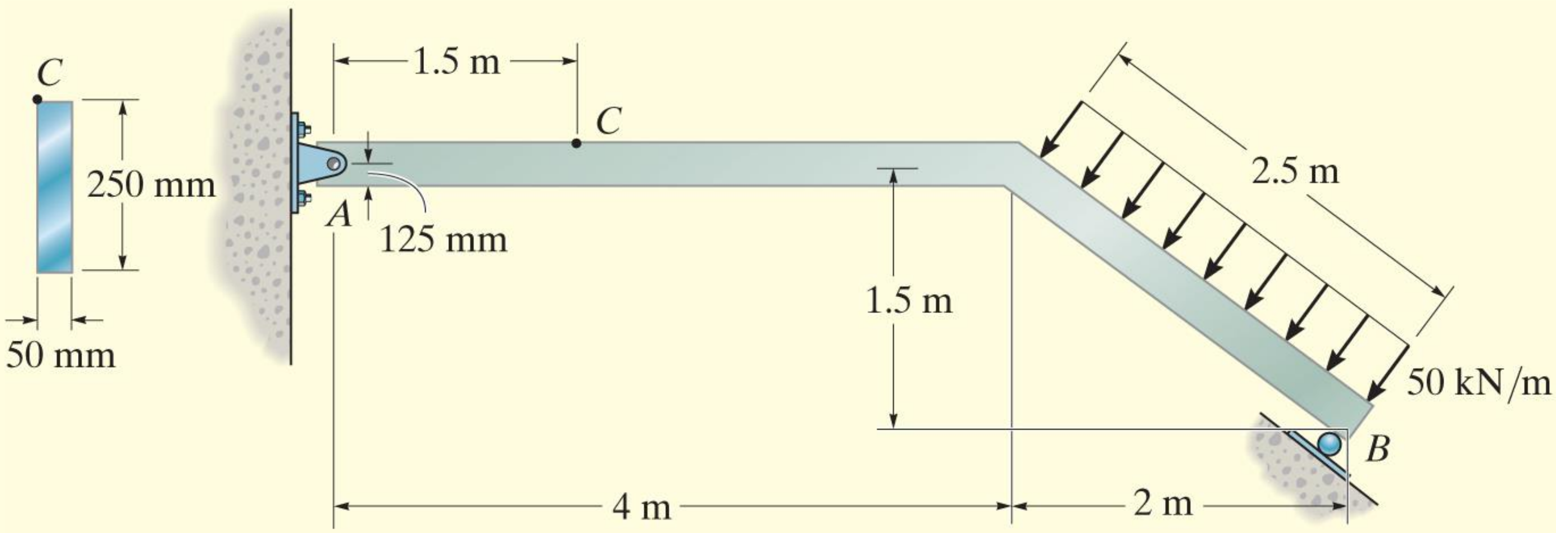
RESULTANT STRESSES BY SUPERPOSITION

Once the normal and shear stress components for each loading have been calculated, use the principal of superposition to determine the resultant normal and shear stress components.

Represent the results on an element of material located at a point, or show the results as a distribution of stress acting over the member's cross-sectional area.

EXAMPLE 1

The member shown below has a rectangular cross section. Determine the state of stress that the loading produces at point *C*.



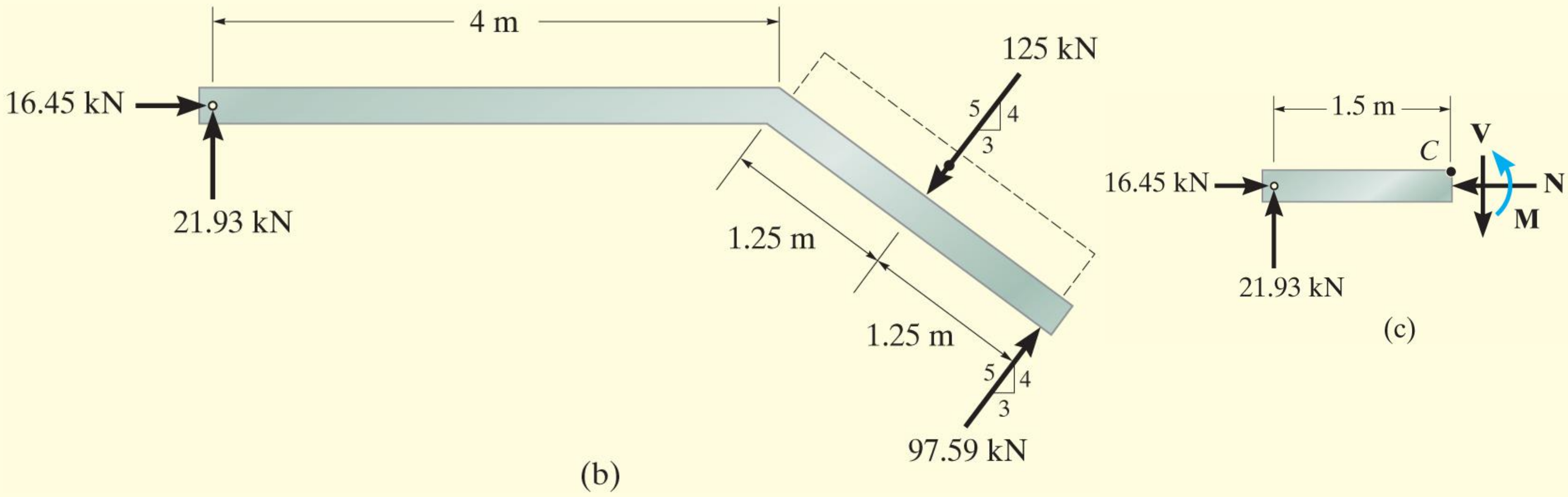
(a)

EXAMPLE 1 (cont)

Solutions

- The resultant internal loadings at the section consist of a normal force, a shear force, and a bending moment.

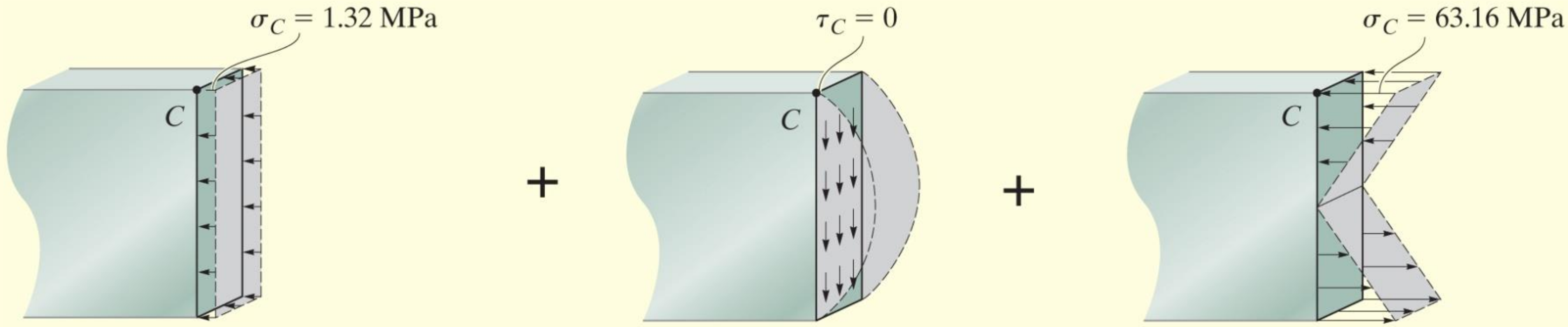
- Solving, $N = 16.45\text{ kN},\;\; V = 21.93\text{ kN},\;\; M = 32.89\text{ kN.m}$



EXAMPLE 1 (cont)

Solutions

- The uniform normal-stress distribution acting over the cross section is produced by the normal force.
- At Point C, $\sigma_c = \frac{P}{A} = \frac{16.45(10^3)}{(0.05)(0.25)} = 1.32\,\text{MPa}$
- In Fig. *e*, the shear stress is zero.



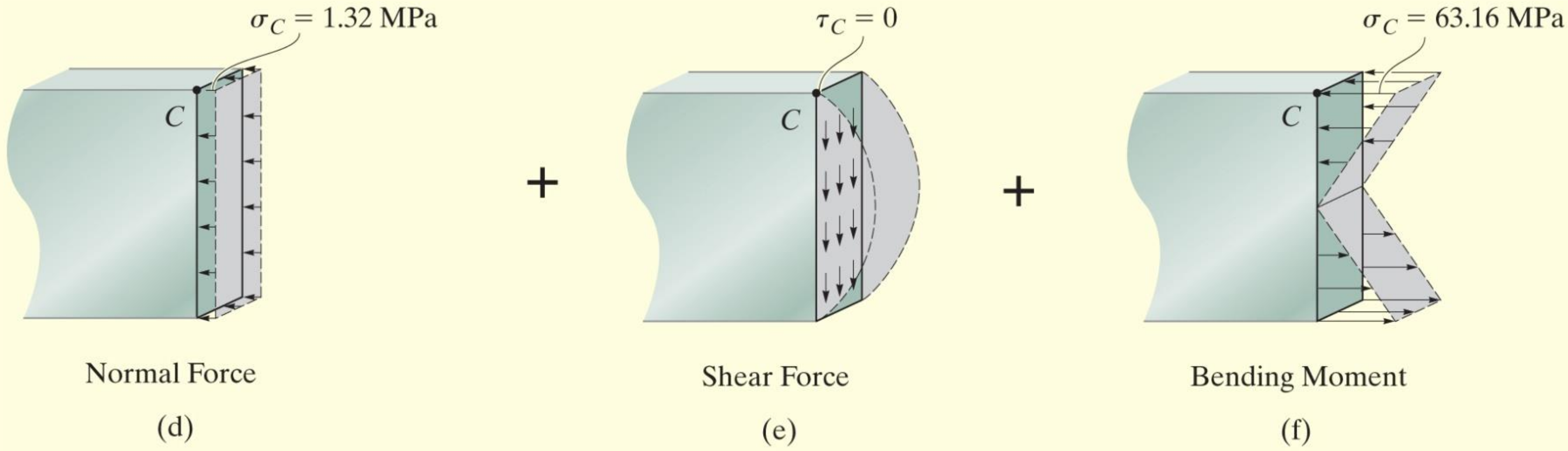
Normal Force (d) | Shear Force (e) | Bending Moment (f)

EXAMPLE 1 (cont)

Solutions

- Point *C* is located at y = c = 0.125m from the neutral axis, so the normal stress at *C*, Fig. *f*, is

$$\sigma_c = \frac{Mc}{I} = \frac{\left(32.89\left(10^3\right)\right)\left(0.125\right)}{\left[\frac{1}{2}\left(0.05\right)\left(0.25\right)^3\right]} = 63.16\,\text{MPa}$$



EXAMPLE 1 (cont)

Solutions

- The shear stress is zero.

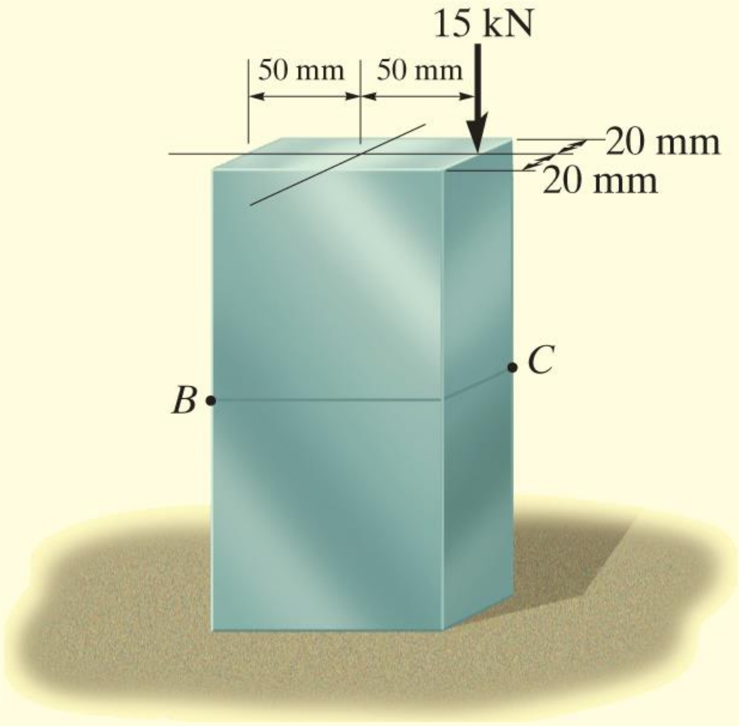
- Adding the normal stresses determined above gives a compressive stress at *C* having a value of

$$\sigma_c = \frac{Mc}{I} = 1.32 + 63.16 = 64.5\,\text{MPa}$$

(g)

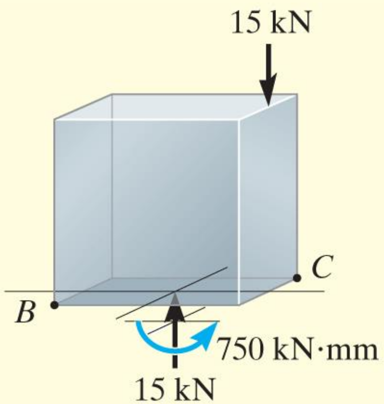
EXAMPLE 2

A force of 15 kN is applied to the edge of the member shown below. Neglect the weight of the member and determine the state of stress at points *B* and *C*.



EXAMPLE 2

- For equilibrium at the section there must be an axial force of 15 000 N acting through the centroid and a bending moment of 750 000 N.mm about the centroidal or principal axis.



(b)

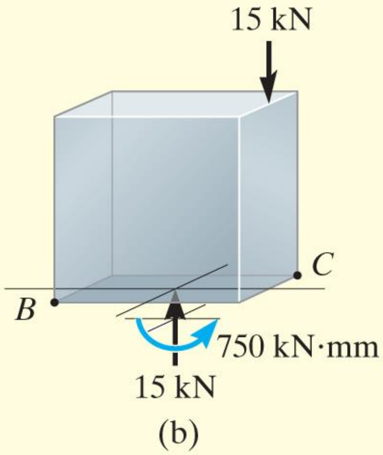
EXAMPLE 2 (cont)

Solutions

- For equilibrium at the section there must be an axial force of 15 000 N acting through the centroid and a bending moment of 750 000 N.mm about the centroidal or principal axis.

Normal force:

$$\sigma = \frac{P}{A} = \frac{15000}{(100)(40)} = 3.75\,\text{MPa}$$



Bending Moment:

- The maximum stress is

$$\sigma_{\max} = \frac{Mc}{I} = \frac{75000(50)}{\frac{1}{12}(40)(100)^3} = 11.25\,\text{MPa}$$

EXAMPLE 2 (cont)

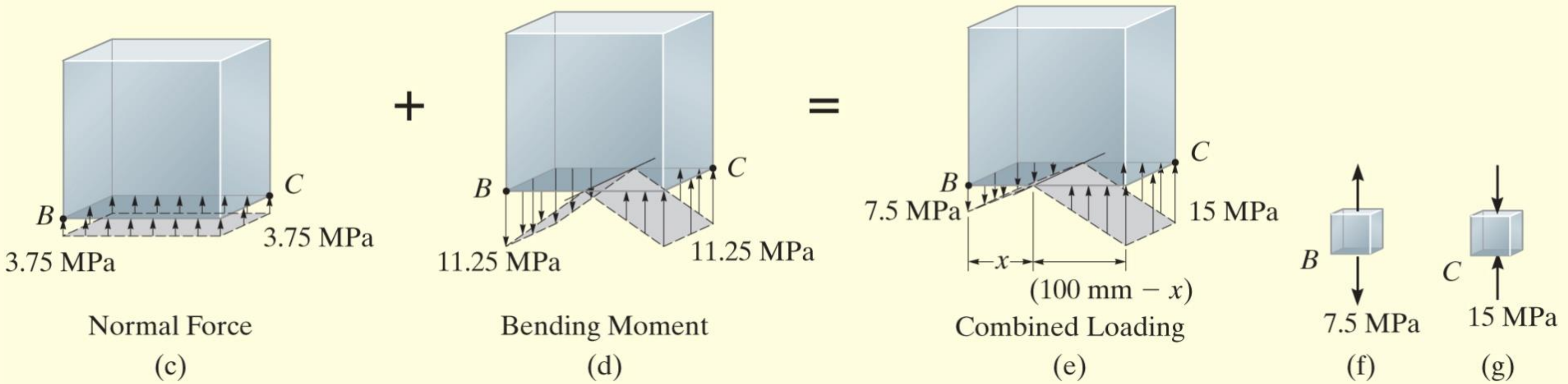
Superposition:

Elements of material at B and C are subjected only to normal or uniaxial stress

$$\sigma_B = -3.75 + 11.25 = 7.5 \text{ MPa} \quad \text{(tension)} \quad \text{(Ans)}$$
$$\sigma_C = -3.75 - 11.25 = -15 \text{ MPa} \quad \text{(compression)} \quad \text{(Ans)}$$

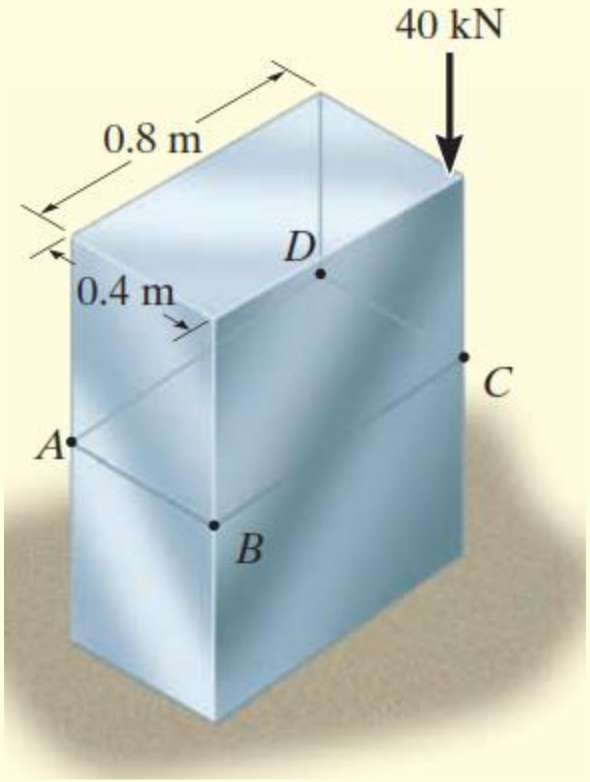
The location of the line of zero stress can be determined by proportional triangles

$$\frac{7.5}{x} = \frac{15}{(100 - x)} \longrightarrow x = 33.3 \text{ mm}$$



EXAMPLE 3

The rectangular block below of negligible weight is subjected to a vertical force of 40 kN, which is applied to its corner. Determine the largest normal stress acting on a section through *ABCD*.



EXAMPLE 3 (cont)

Solutions

- For uniform normal-stress distribution the stress is

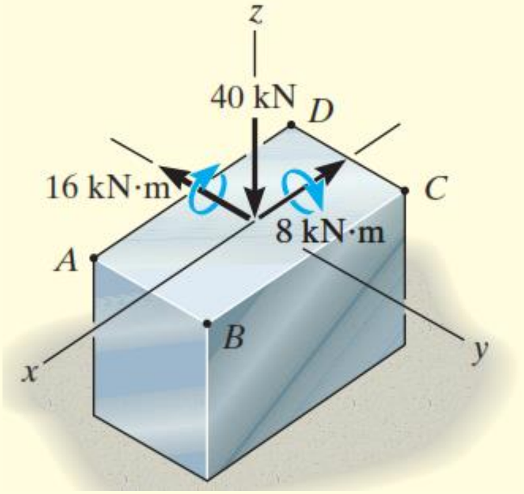
$$\sigma = \frac{P}{A} = \frac{40}{(0.8)(0.4)} = 125\,\text{kPa}$$

- For 8 kN.m, the maximum stress is

$$\sigma_{\max} = \frac{M_x c_x}{I_x} = \frac{8(0.2)}{\left[\frac{1}{12}(0.8)(0.4)^3\right]} = 375\,\text{kPa}$$

- For 16 kN.m, the maximum stress is

$$\sigma_{\max} = \frac{M_y c_x}{I_y} = \frac{16(0.4)}{\left[\frac{1}{12}(0.4)(0.8)^3\right]} = 375\,\text{kPa}$$



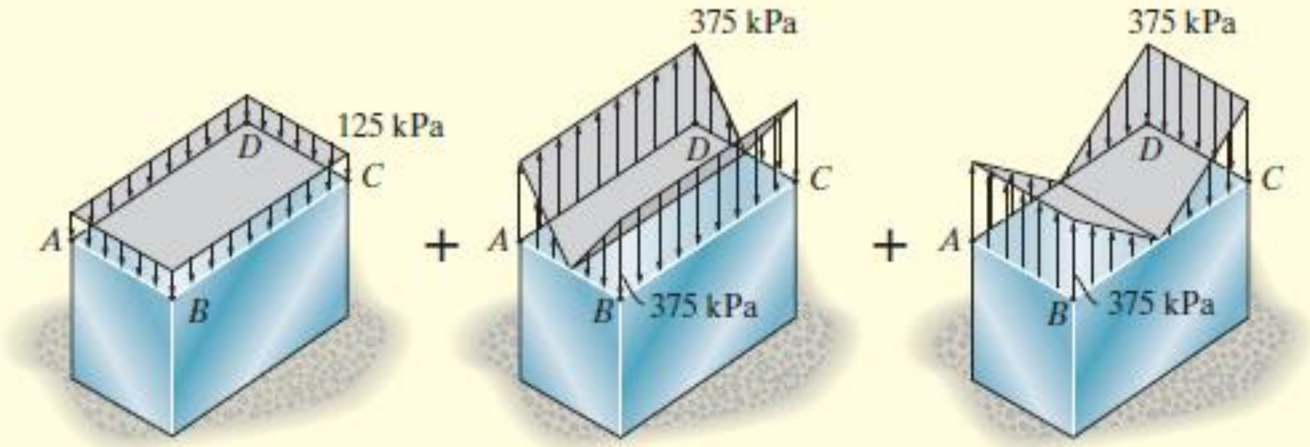
Equilibrium of the bottom segment

EXAMPLE 3 (cont)

Solutions

- By inspection the normal stress at point *C* is the largest since each loading creates a compressive stress there

$$\sigma_c = -125 - 375 - 375 = -875\,\text{kPa} \quad (\text{Ans})$$



Normal force (40 kN)

(c)

Bending moment (8 kN·m)

(d)

Bending moment (16 kN·m)

(e)