THE UNIVERSITY OF ZAMBIA
SCHOOL OF ENGINEERING
DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

CEE 3211

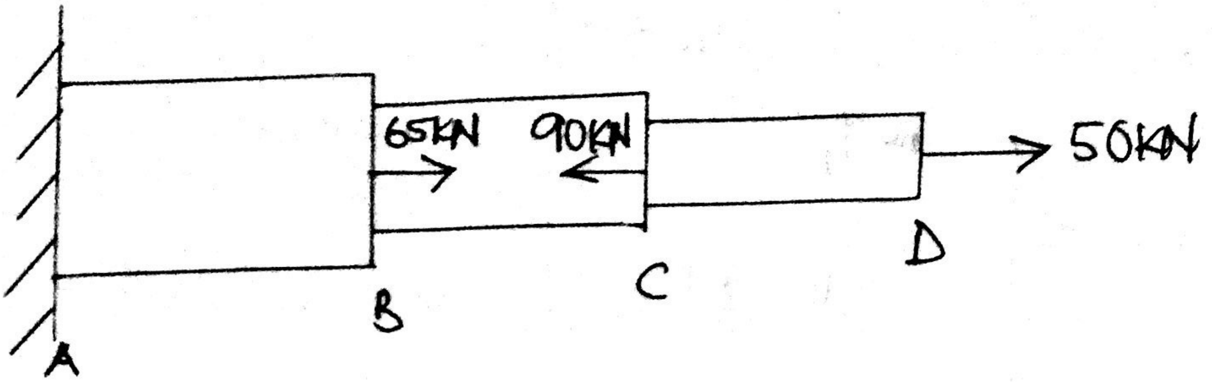
MECHANICS OF MATERIALS

Quiz 1 - 10 Solutions

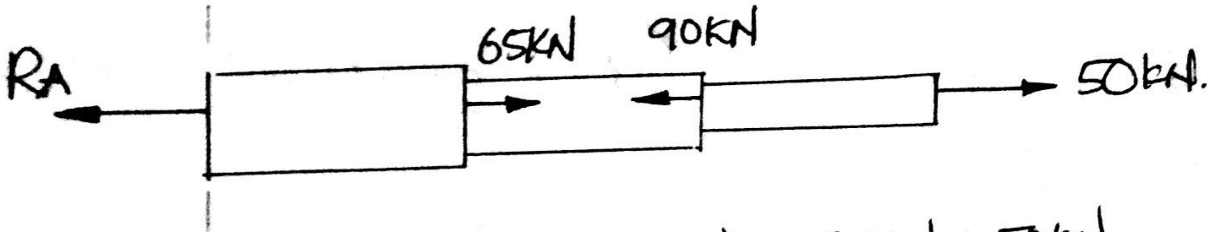
(2023)

QUIZ 1

SOLUTION



(I) Reaction of A.

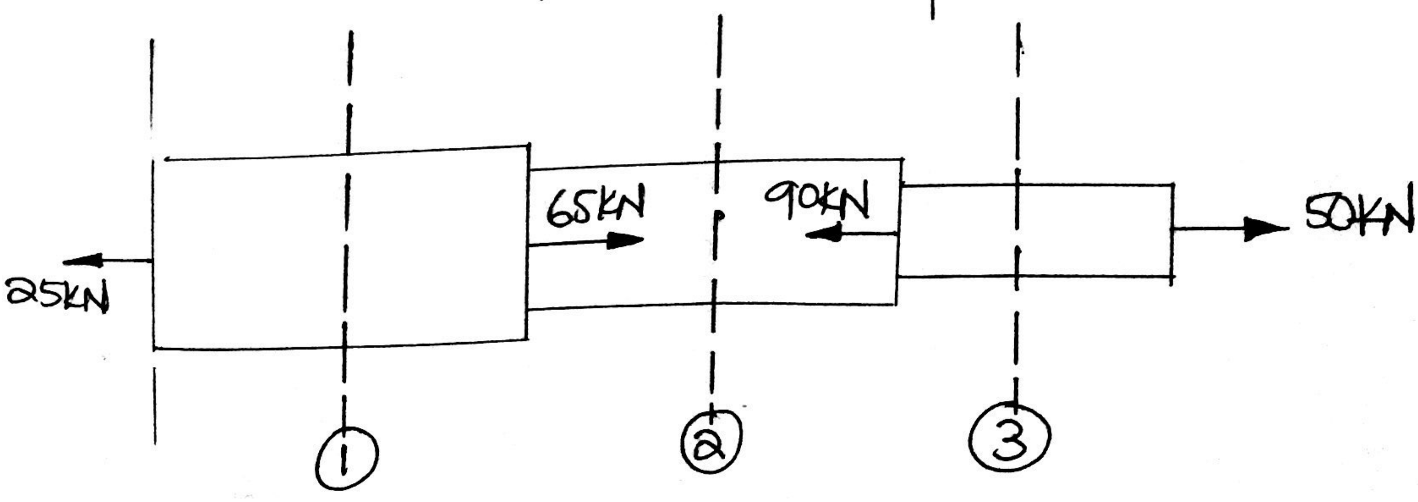


$\overset{+}{\rightarrow} \sum F_x = 0:$

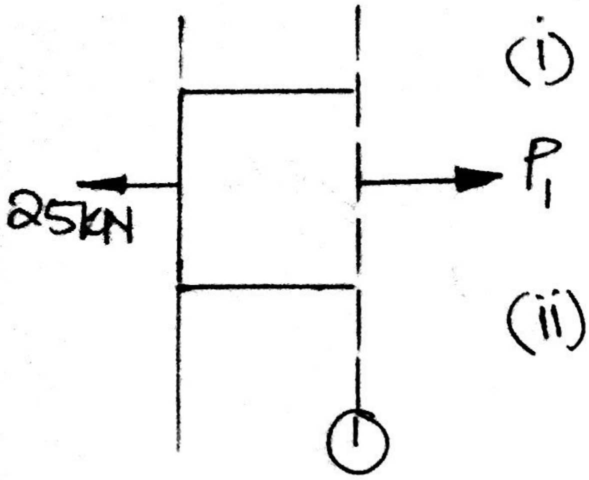
$$R_A + 90\,kN = 65\,kN + 50\,kN$$
$$R_A = (115 - 90)\,kN$$
$$R_A = \underline{25\,kN}$$

(II) GIVEN DATA:

Diameters		Length
AB = 120 mm	E = 200 GPa	30 cm = 300 mm
BC = 90 mm	E = 68.9 GPa	35 cm = 350 mm
CD = 60 mm	E = 101 GPa	45 cm = 450 mm



Segment ① : $\boxed{AB}$



(i) $\xrightarrow{+} \sum F_x = 0.$

$P_1 = 25\,kN.$

Member AB is in Tension.

(ii) Normal Stress Induced.

$$\sigma_{AB} = \frac{P_1}{A_{AB}} = \frac{25 \times 10^3\,N}{\frac{\pi}{4}(120mm)^2} = \frac{25000\,N}{11309.7\,mm^2}$$

$$\sigma_{AB} = \underline{\underline{2.21\,N/mm^2}} \quad (MPa)$$

(iii) Elongation.

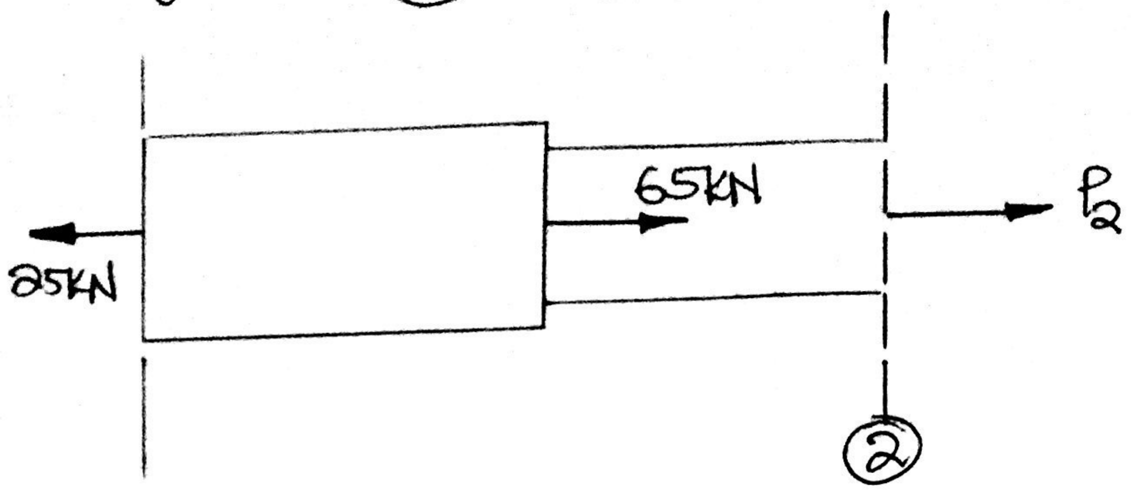
$$\delta_{AB} = \frac{PL_{AB}}{AE} = \frac{25 \times 10^3 N \times 300mm}{\frac{\pi}{4}(120mm)^2 \times (200 \times 10^3)\,N/mm^2}$$

$$= \frac{25000 \times 300}{\frac{\pi}{4}(120)^2 \times (200\,000)} \quad mm$$

$$= \underline{\underline{0.0033157\,mm}}$$

Scanned with CamScanner

Segment ② : $\boxed{BC}$



(I) $\xrightarrow{+} \Sigma F_x = 0$:

$$P_2 + 65\,KN = 25\,KN.$$
$$P_2 = (25 - 65)\,KN = -40\,KN$$
$$P_2 = 40\,KN \leftarrow$$

Member BC is in compression

(II) Normal Stress induced.

$$\sigma_{BC} = \frac{P_2}{A_{BC}} = \frac{40 \times 10^3\,N}{\frac{\pi}{4}(90\,mm)^2} = \frac{40000\,N}{6361.7\,mm^2} = 6.29\,N/mm^2$$

$$\sigma_{BC} = \underline{\underline{6.29\,N/mm^2}} \quad (MPa)$$

(III) Elongation

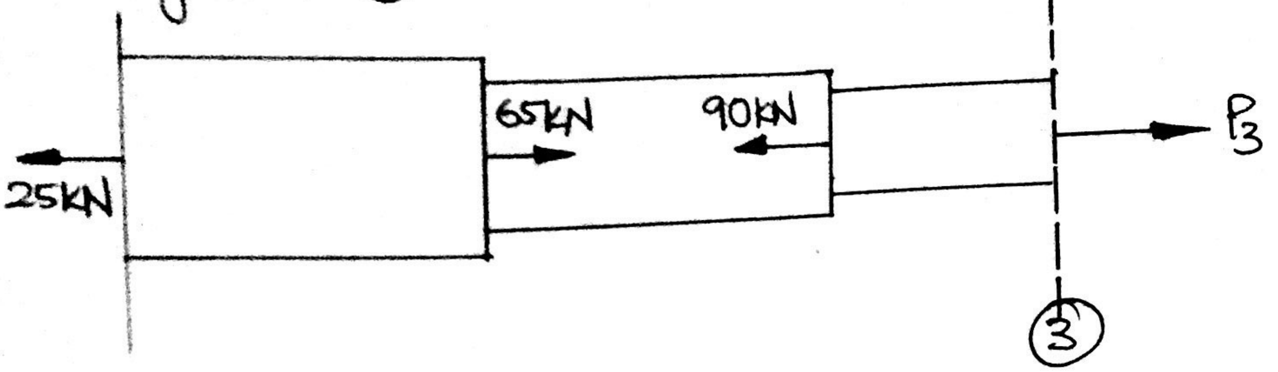
$$\delta_{BC} = \frac{PL_{BC}}{AE} = \frac{-40 \times 10^3\,N \times 350\,mm}{\frac{\pi}{4}(90\,mm)^2 \times (68.9 \times 10^3)\,N/mm^2}$$

$$= \frac{-40\,000 \times 350}{\frac{\pi}{4}(90)^2 \times (68{,}900)}\,mm = -0.0319399\,mm$$

$$= \underline{\underline{-0.0319399\,mm}}$$

Scanned with CamScanner

Segment ③ : $\boxed{CD}$



(I) $\stackrel{+}{\rightarrow}\Sigma F_x = 0$

$$P_3 + 65\,kN = 25\,kN + 90\,kN$$

$$P_3 = (25 + 90 - 65)\,kN = \underline{50\,kN}$$

Member CD is in Tension.

(II) Normal Stress induced.

$$\sigma_{CD} = \frac{P_3}{A_{CD}} = \frac{50\times10^3\,N}{\frac{\pi}{4}(60\,mm)^2} = \frac{50\,000\,N}{2827.4\,mm^2} = 17.68\,(MPa)$$

$$= \underline{\underline{17.68\,N/mm^2}}\ (MPa)$$

(III) Elongation.

$$\delta_{CD} = \frac{PL_{CD}}{AE} = \frac{50\times10^3\,N \times 450\,mm}{\frac{\pi}{4}(60\,mm)^2 \times (101\times10^3)\,N/mm^2}$$

$$= \frac{50\,000 \times 450}{\frac{\pi}{4}(60)^2 \times (101\,000)}\,mm$$

$$= \underline{0.0787896\,mm}$$

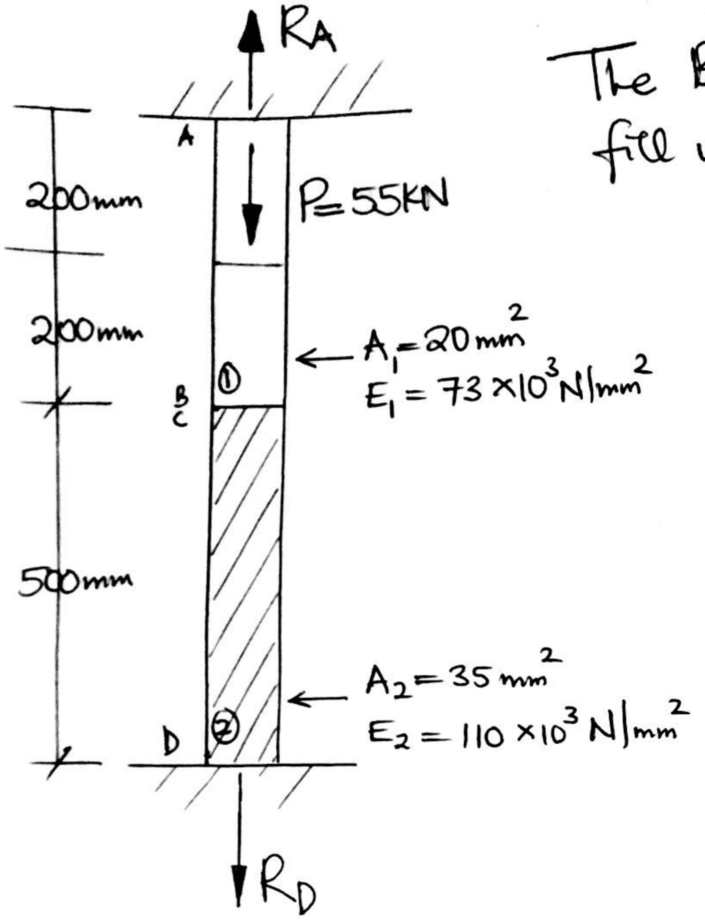
$\therefore$ Total Elongation: $\delta_T = \delta_{AB} + \delta_{BC} + \delta_{CD}$

$$\Rightarrow \delta_T = [0.0033157]\,mm + [-0.0319399]\,mm + [0.0787896]\,mm$$

$$= \underline{\underline{0.0502\,mm}}$$

QUIZ 2

SOLUTION



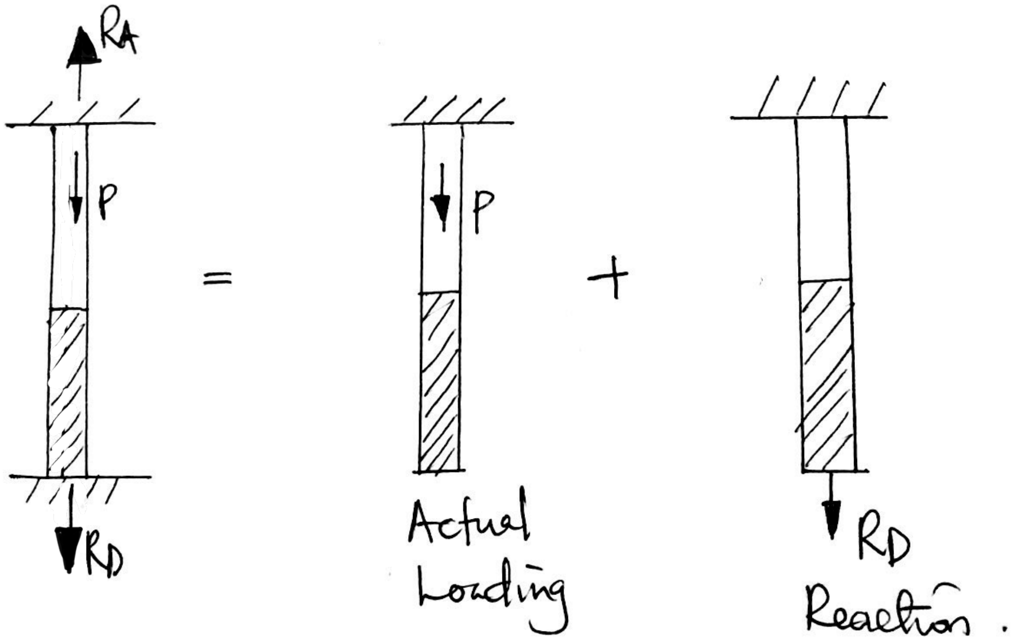
The Bars have elongated to fill up the Gap.

$$\therefore \delta_{Total} = 2.5\,mm$$

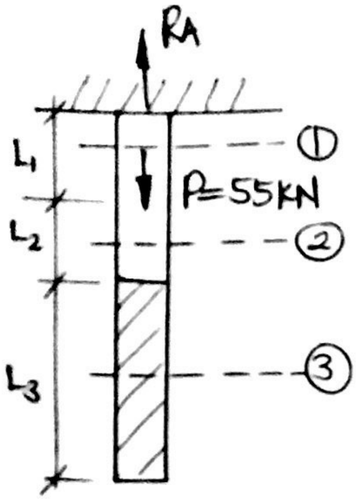
① Reactions of Fixed Supports.

$$R_A - R_D = 55\,kN \quad \ldots\ldots \text{eqn } ①$$

(Statically Indeterminate.)



① With Respect to Actual Loading.

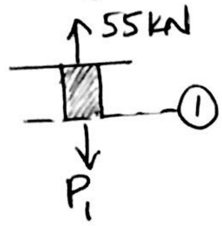


① Reaction (RA).

$+\uparrow \Sigma F_y = 0: \; R_A = P = 55\,\text{kN}.$

② Segments

① Segment ①



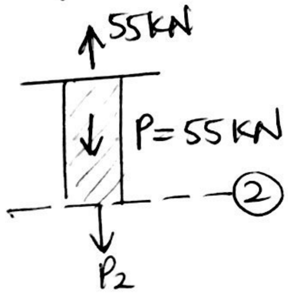
$+\uparrow \Sigma F_y = 0:$

$\underline{P_1 = 55\,\text{kN}}$

$\delta_1 = \frac{P_1 L_1}{E_1 A_1} = \frac{55\,000 \times 200}{(73 \times 10^3 \times 20)}$

$= \underline{7.534\,\text{mm}}$

⑪ Segment ②



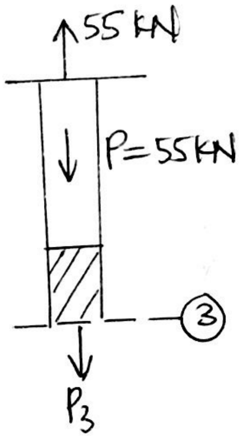
$+\uparrow \Sigma F_y = 0.$

$P_2 + 55 = 55$

$P_2 = \underline{0\,\text{kN}}$

$\delta_2 = \frac{P_2 L_2}{E_1 A_1} = \underline{0\,\text{mm}}$

⑪ Segment ③



$+\uparrow \Sigma F_y = 0:$

$P_3 + 55 = 55$

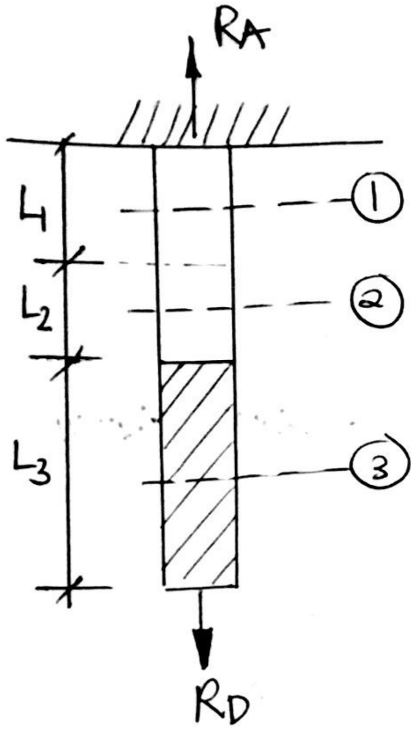
$P_3 = \underline{0\,\text{kN}}$

$\delta_3 = \frac{P_3 L_3}{E_2 A_2} = \underline{0\,\text{mm}}$

[1]

$\therefore \; \delta_{\text{Due to Actual Load}} = \delta_1 + \delta_2 + \delta_3 = 7.534 + 0 + 0 = \underline{\underline{7.534\,\text{mm}}}$

(2) With Respect to Redundant Reaction

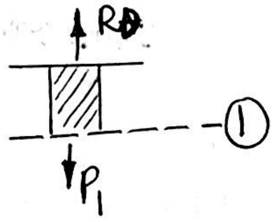


(1) Reaction (R_A)

$+\uparrow \Sigma F_y = 0$: $R_A = R_D$

(2) Segments.

(i) Segment (1)

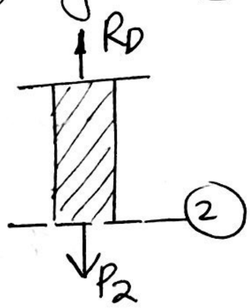


$+\uparrow \Sigma F_y = 0$:

$P_1 = R_D$

$$\delta_1 = \frac{P_1 L_1}{E_1 A_1} = \frac{R_D L_1}{E_1 A_1}$$

(ii) Segment (2)

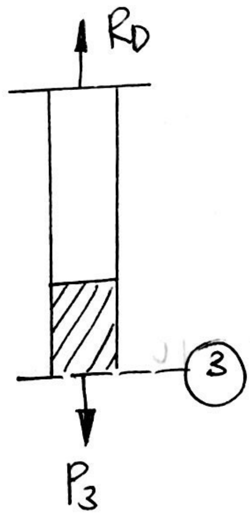


$+\uparrow \Sigma F_y = 0$:

$P_2 = R_D$

$$\delta_2 = \frac{P_2 L_2}{E_1 A_1} = \frac{R_D L_2}{E_1 A_1}$$

(iii) Segment (3)



$+\uparrow \Sigma F_y = 0$:

$P_3 = R_D$

$$\delta_3 = \frac{P_3 L_3}{E_2 A_2} = \frac{R_D L_3}{E_2 A_2}$$

[2]

$$\therefore \delta_{\text{Due to Reaction}} = \delta_1 + \delta_2 + \delta_3$$

$$= \left[\frac{R_D L_1}{E_1 A_1} + \frac{R_D L_2}{E_1 A_1}\right] + \frac{R_D L_3}{E_2 A_2}$$

$$= \left[\frac{R_D L_1}{E_1 A_1} + \frac{R_D L_1}{E_1 A_1}\right] + \frac{R_D L_3}{E_2 A_2}$$

Note : $\underline{L_1 = L_2}$

$$= \frac{2 R_D L_1}{E_1 A_1} + \frac{R_D L_3}{E_2 A_2}$$

$$= R_D \left[\frac{2L_1}{E_1 A_1} + \frac{L_3}{E_2 A_2}\right]$$

Compatibility Equation ;

$$\delta_T = \underset{[1]}{\delta_{Load}} + \underset{[2]}{\delta_{Reaction}} = 2.5\,mm$$

$$\Rightarrow R_D \left[\frac{2L_1}{E_1 A_1} + \frac{L_3}{E_2 A_2}\right] + (7.534\,mm) = 2.5\,mm$$

$$R_D \left[\frac{2E_2 A_2 L_1 + E_1 A_1 L_3}{E_1 A_1 \cdot E_2 A_2}\right] = 2.5 - 7.534$$

$$R_D = \frac{(2.5 - 7.534)\left[(73\times10^3 \times 20) \times (110\times10^3 \times 35)\right]}{\left[2(110\times10^3 \times 35 \times 200) + (73\times10^3 \times 20 \times 500)\right]}$$

$$R_D = \frac{-5.034\,(5.621\times10^{12})}{(2.27\times10^9)} = \frac{-5.034 \times 5.621 \times 10^3}{2.27}$$

$$R_D = \underline{\underline{-12.465\,KN}}$$

$$R_D = \underline{\underline{12.465\,KN \uparrow}}$$

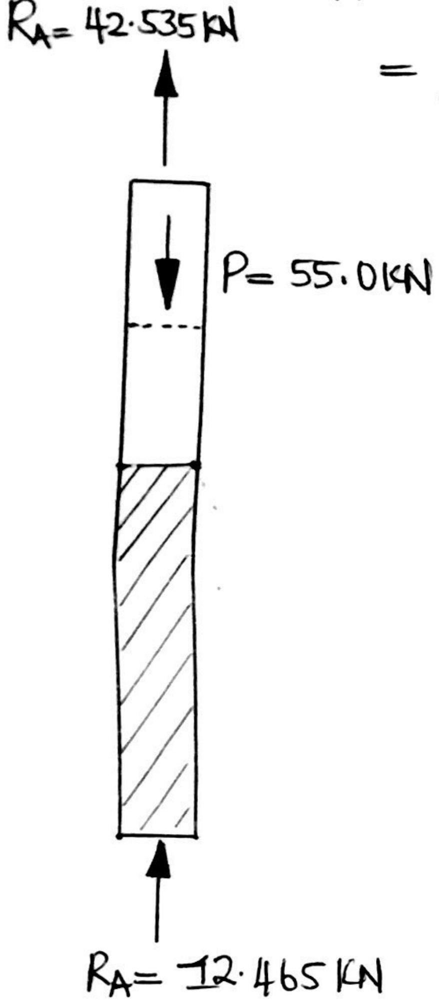
From equ (1)

$$R_A - R_D = 55\,KN$$

$$R_A = 55 + R_D$$

$$R_A = 55 + (-12.465)$$

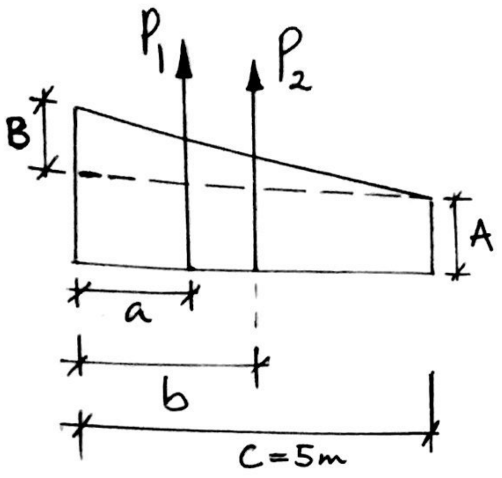
$$= \underline{\underline{42.535\,KN}}$$



QUIZ 3

SOLUTION

Converting Distributed Load into Point Load.



① Triangle (A) = $P_1 = \frac{1}{2}(C) \times (B)$

$P_1 = \frac{1}{2}(5) \times (15-7)$ KN/m

$\boxed{@\ a = \frac{5}{3}}$

$P_1 = \frac{1}{2}(5)(8) = \underline{20\,KN}$ ↑

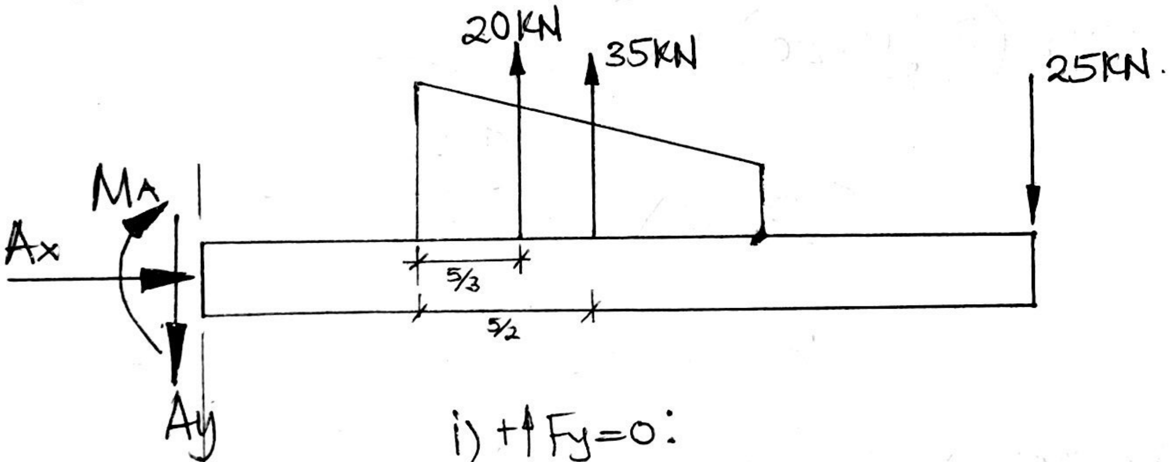
② Rectangle (A) = $P_2 = (C) \times (A)$

$P_2 = (5m) \times (7 KN/m)$

$\boxed{@\ b = 2.5}$

$P_2 = \underline{35\,KN}$ ↑

① Reactions at A.



i) $+\uparrow F_y = 0$:

$A_y + 25 = 20 + 35$

$A_y = (20+35) - 25 = \underline{\underline{30KN}}$

ii) $\xrightarrow{+} F_x = 0.$

$A_x = 0.$

iii) $\circlearrowright + M_A = 0.$

$M_A + 25(12) = 20\left(5/3 + 3\right) + 35\left(5/2 + 3\right)$

$M_A + 25(12) = 20(4.67) + 35(5.5)$

$M_A = 93.4 + 192.5 - 300$

$= \underline{\underline{-14.17\ KNm}}$

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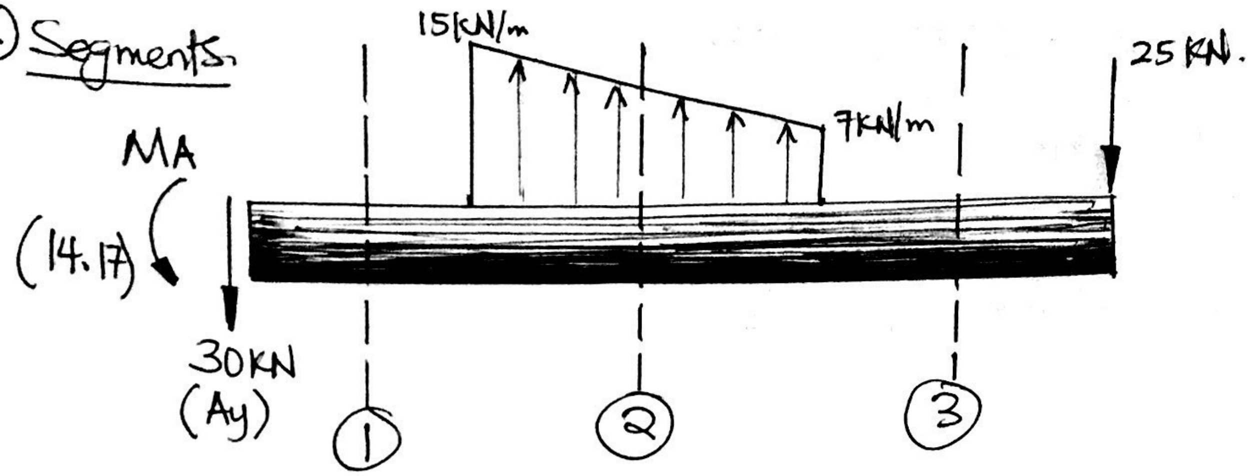
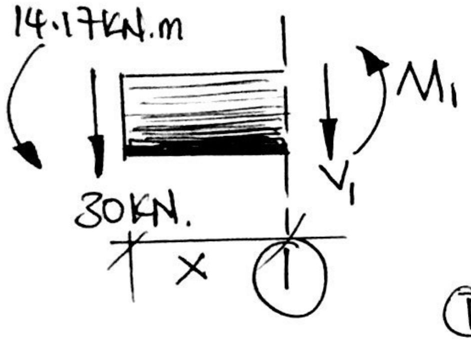
② Segments.



Segment 1.



① Shear Force

$$V_1 + 30\,kN = 0$$

$$V_1 = -30\,kN \quad \ldots \text{equ } ①$$

⑪ Moment.

$$\circlearrowright + \sum M_1 = 0.$$

$$M_1 + 30(x) + 14.17 = 0.$$

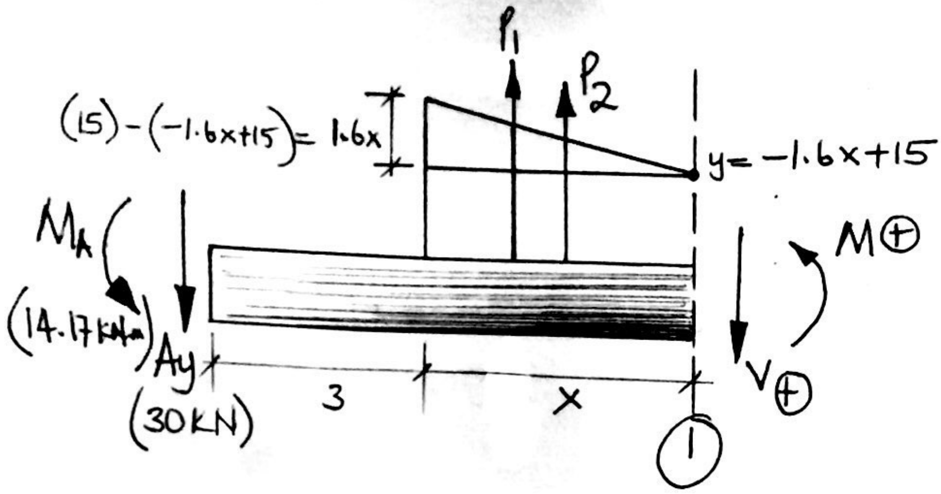
$$M_1 = -14.17 - 30x \quad \ldots \text{equ } ②$$

∴ Moment at 3 m (Span of Beam).

$$M_1 = -14.17 - 30(3)$$

$$= -14.17 - 90$$

$$= -104.17$$



① Gradient (m)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-15}{8-3} = -1.6$$

② equation of line

$$y = mx + C$$
$$y = -1.6x + 15$$

Point Loads.

① $P_1 = \frac{1}{2}bh$
$= \frac{1}{2}(x)(1.6x)$
$= 0.8x^2$

② $P_2 = Lb$
$= (x) \times (-1.6x + 15)$
$= -1.6x^2 + 15x$

(I) $+\uparrow \sum F_Y = 0$: [To evaluate shear force].

$$A_y + V = P_1 + P_2$$
$$V = (P_1 + P_2) - A_y$$
$$= [(0.8x^2) + (-1.6x^2 + 15x)] - 30$$
$$= [-0.8x^2 + 15x] - 30$$
$$V = -0.8x^2 + 15x - 30 \quad \text{.. eqn (1)}$$

(II) Moment

$$M = \int V\,dx.$$
$$= \int (-0.8x^2 + 15x - 30)\,dx$$
$$= -\frac{0.8x^3}{3} + \frac{15x^2}{2} - \frac{30x}{1}$$
$$M = -0.2667x^3 + 7.5x^2 - 30x \quad \text{... eqn (2)}$$

Since Moment (M) is Maximum at $V = 0$; we can evaluate this point Using eqn (1).

$$V = 0 = -0.8x^2 + 15x - 30.$$

$$\Rightarrow \; -0.8x^2 + 15x - 30 = 0$$

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(15) \pm \sqrt{(15)^2 - 4(-0.8)(-30)}}{2(-0.8)}$$

$$= \underline{2.276 \text{ or } 16.47} \qquad \therefore x = \underline{2.276\ m}$$

Maximum Moment ; (equ 2).

$$M = -0.2667(2.276)^3 + 7.5(2.276)^2 - 30(2.276)$$

$$= \underline{-32.57\ KN.m.}$$

1 Thus at $x = (3 + 2.276)m = \underline{5.276m}$ [where V=0]

$$\text{Moment} = M = -104.17 + (-32.57)$$

$$= \underline{-136.74\ KN.m}$$

2 $x = 5m$ (full Length of Distributed Load).

$$M = -0.2667(5)^3 + 7.5(5)^2 - 30(5)$$

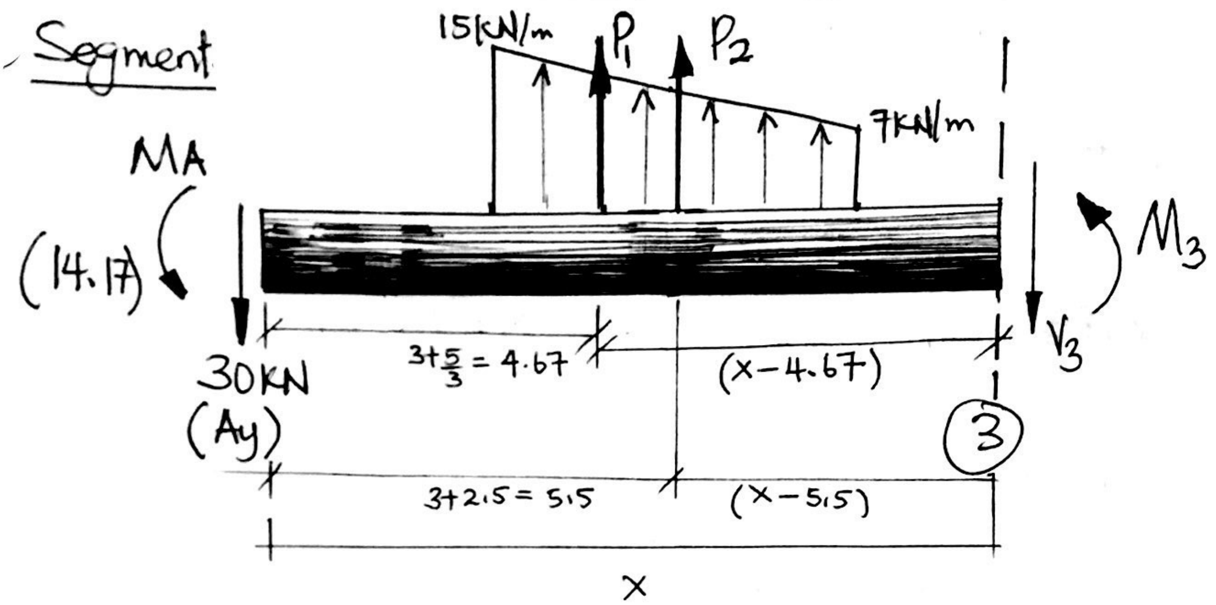
$$= \underline{4.16\ KN.m}$$

$\therefore$ At length 8 m (Span of Beam).

$$\text{Moment} = M = -104.17 + (4.16)$$

$$= \underline{-100.00\ KN.m}$$

Segment



$\circlearrowleft + M_{③} = 0$;

$$M_3 + M_A + A_y(x) = P_1(x-4.67) + P_2(x-5.5)$$

$$M_3 + 14.17 + 30x = 20(x-4.67) + 35(x-5.5).$$

$$M_3 + 14.17 + 30x = 20x - 93.4 + 35x - 192.5$$

$$M_3 + 14.17 + 30x = 55x - 285.9$$

$$M_3 = (55-30)x + (-285.9 - 14.17)$$

$$\underline{M_3 = 25x - 300.0 \quad \ldots \text{equ } ③}$$

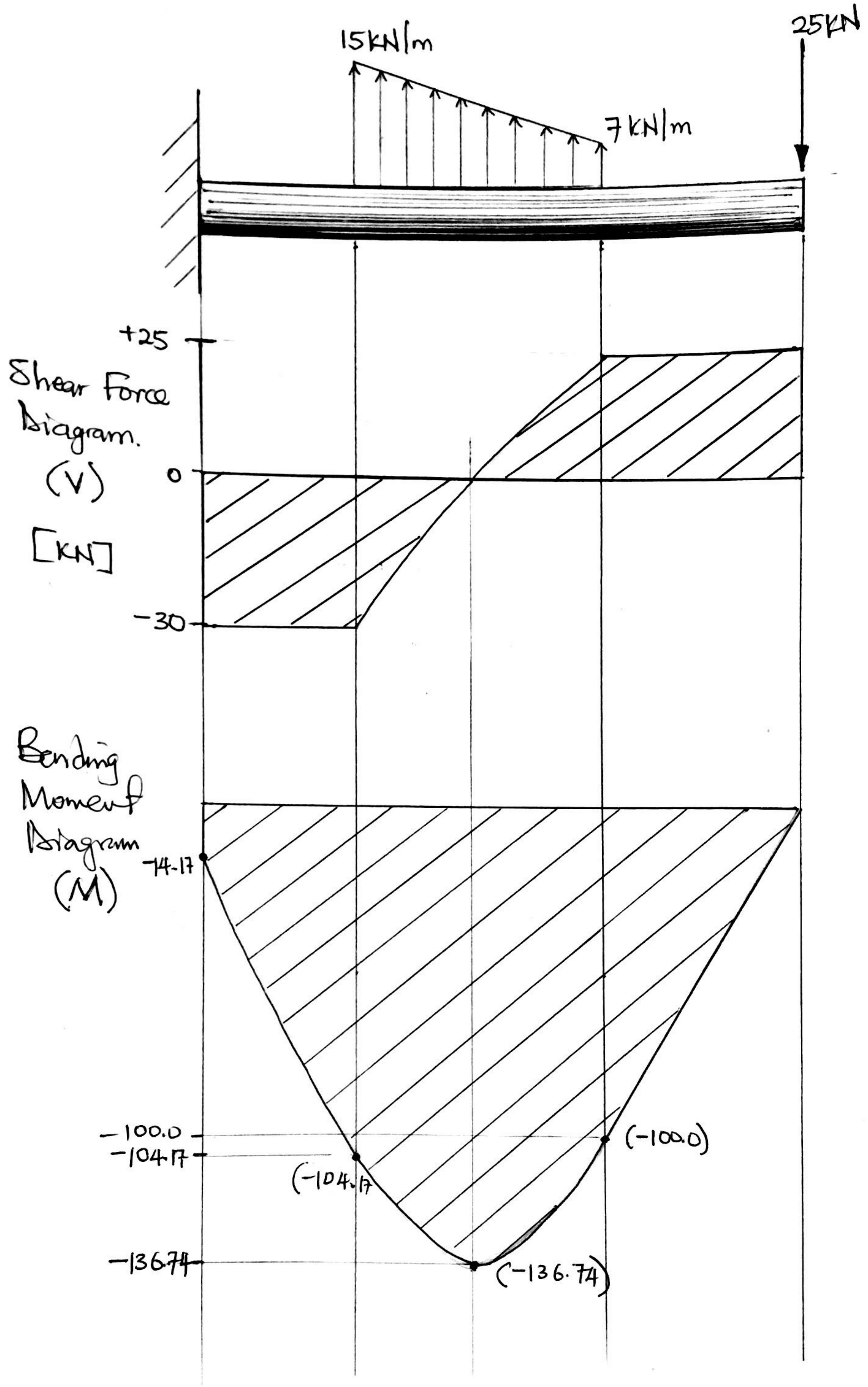
<u>At x = 12 m (full span of Beam).</u>

[eqn ③]

$$M_3 = 25(12) - 300$$
$$= (300-300)\ \text{kN.m}$$
$$= \underline{0\ \text{kN/m}}$$

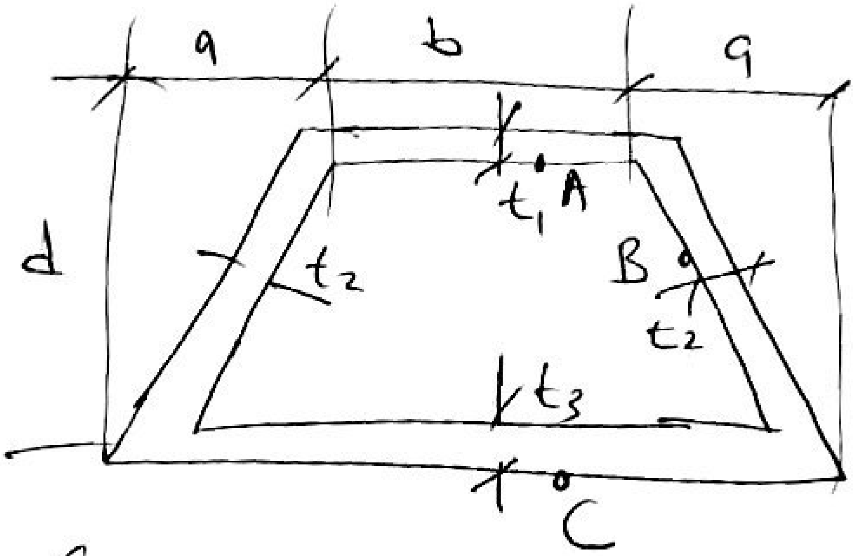
Hence we have:

	Length (m)	Moment	Shear Force
A	0	−14.17	−30
B	3	−104.17	−30
Max	5.276	−136.74	0
C	8	−100.00	+25
D	12	0	0


15kN/m
7kN/m
25kN
+25
Shear Force Diagram.
(V)
0
[kN]
-30
Bending Moment Diagram
(M)
-14.17
-100.0
-104.17
(-104.17
(-100.0)
-136.74
(-136.74)

QUIZ 4

SOLUTION



GIVEN DATA

$T = 4200$ kN.m

$a = 550$ mm

$b = 1150$ mm

$d = 1750$ mm

$t_1 = 18$ mm

$t_2 = 9$ mm

$t_3 = 15$ mm

(a) CROSS-SECTION - Mean Area

$$A_m = \frac{(2a+b)+b}{2} * d = (a+b)d$$

$$= (550 + 1150)1750$$

$$A_m = \underline{\underline{2.975 \times 10^6 \text{ mm}^2}}$$

(b) The average Shear stress @ Point A

$$\tau_A = \frac{T}{2A_m t_1}$$

$$= \frac{4200 \times 1000 \times 1000 \text{ N.mm}}{2 \times 2.975 \times 10^6 \text{ mm}^2 \times 18 \text{ mm}}$$

$$= \underline{39.22 \text{ N/mm}^2} \quad (\text{MPa})$$

(c) Shear Stress @ Point B

$$\tau_B = \frac{T}{2A_m t_2} = \frac{4200 \times 10^6 \text{ N.mm}}{2 \times 2.975 \times 10^6 \times 9 \ (\text{mm}^2 \cdot \text{mm})}$$

$$= \underline{78.43 \text{ MPa}}$$

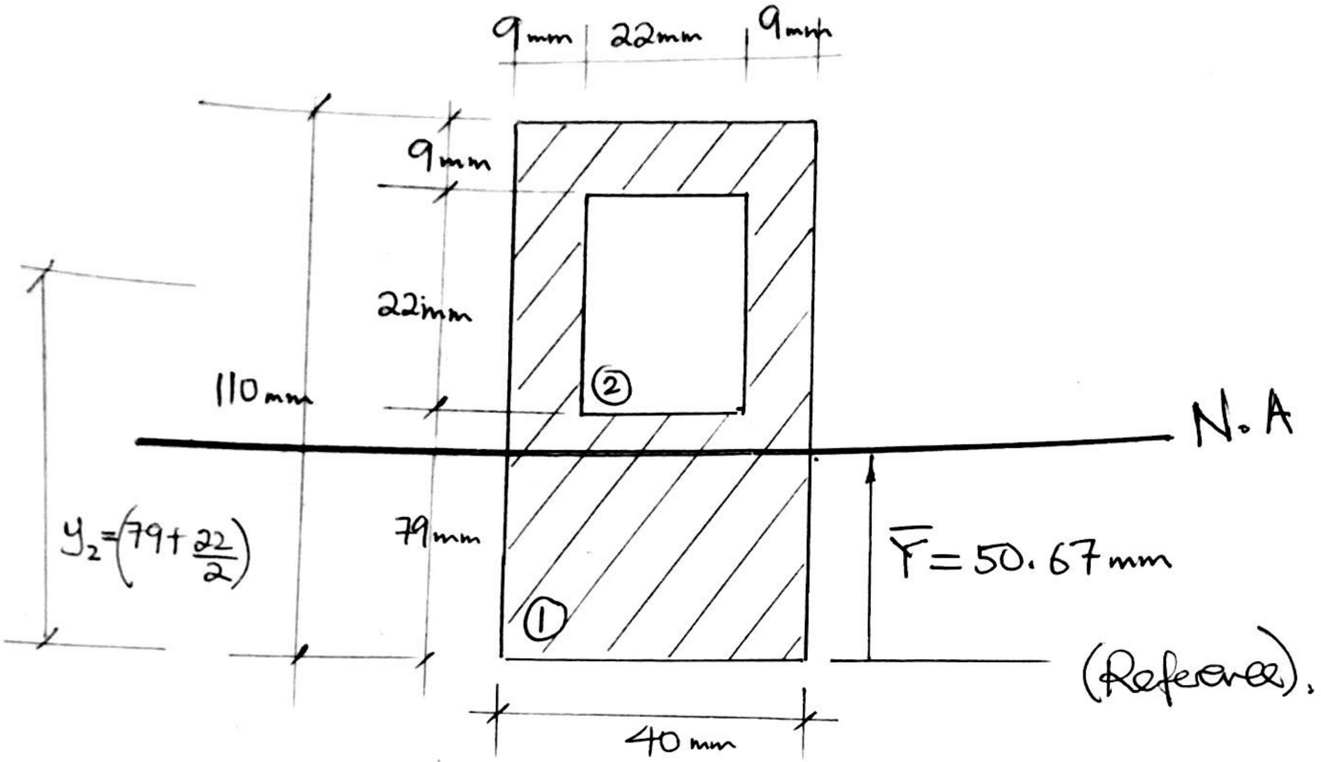
(d) Shear Stress @ Point C

$$\tau_C = \frac{T}{2A_m t_3} = \frac{4200 \times 10^6 \text{ N.mm}}{2 \times 2.975 \times 10^6 \times 15 \ (\text{mm}^2 \cdot \text{mm})}$$

$$= \underline{47.06 \text{ MPa}}$$

QUIZ 5

SOLUTION



Part	A (mm^2)	$\bar{y}$	$A\bar{y}$ (mm^3)	$bh^3/12$	d	$I = \frac{bh^3}{12} + Ad^2$
1	4400	55	242,000	4,436,666.67	4.33	4,519,162.83
2	−484	90	−43,560	−19,521.33	39.33	−768,196.20
	3916		198,440			3,750,966.63

① Centroid of Cross Section.

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{198\,440\ (mm^3)}{3916\ (mm^2)} = \underline{\underline{50.67 mm}}$$

② Moment of Inertia

$$I = \Sigma\left(\frac{bh^3}{12} + Ad^2\right)$$

$$= 3,750,966.63\ mm^4$$

$$= \underline{\underline{3.751 \times 10^6\ mm^4}}.$$

③ Tensile Stress

$$\sigma = -\frac{My}{I} < \sigma_{all}$$

[Tensile stress less than Allowable]

$$-\frac{My}{I} \leq \sigma_{all}.$$

$$M \geq -\frac{\sigma_{all} I}{y_{top}}$$

$y_{top} = 110 - 50.67$
$= \underline{59.33\,mm}$

$y_{bottom} = \underline{50.67\,mm}$

$$\Rightarrow M \geq -\frac{\sigma_{all} I}{y_{top}} = -\frac{16 \times (3{,}750{,}966.63)}{(+)59.33}$$

$$M \geq \underline{\underline{1{,}011{,}553.4\,Nmm}}$$

upwards from N.A (+)

N.A

④ Compressive Stress

$$\sigma = -\frac{My}{I} \leq \sigma_{all}$$

$$-\frac{My}{I} \leq \sigma_{all}$$

$$M \geq -\frac{\sigma_{all} I}{y_{bottom}} = \frac{-(-12)(3{,}750{,}966.63)}{(-)50.67}$$

$$M \geq \underline{888{,}328\,N.mm}$$

downwards from N.A

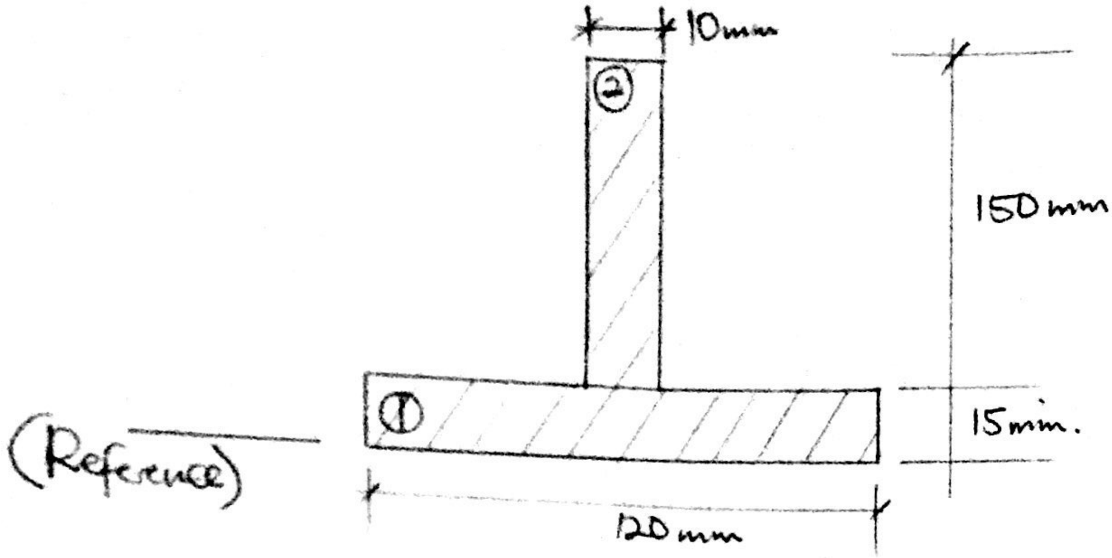
Hence Negative (−)

Note: Largest Moment M is the SMALLEST value of the two values obtained.

$$\text{i.e } M = \underline{\underline{888\,328\,Nmm}}$$

QUIZ 6

SOLUTION



1 Centroid of Cross section ($\bar{Y}$)

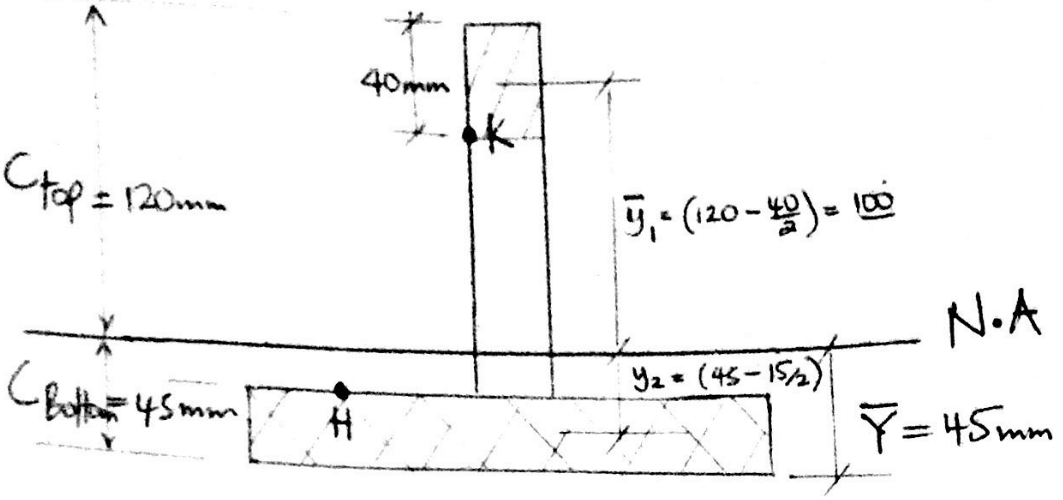
Part	A(mm²)	$\bar{y}$ (mm)	$A\bar{y}$
1	$120 \times 15 = 1800$	$\frac{15}{2} = 7.5$	13,500
2	$10 \times 150 = 1500$	$15 + \frac{150}{2} = 90$	135,000
	3,300		148,500

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{148{,}500}{3300} = \underline{\underline{45\,mm}}$$ (from Bottom).

2 Moment of Inertia

	A	$\bar{y}$	$A\bar{y}$	$bh^3/12$	$d = \lvert\bar{Y} - \bar{y}\rvert$	Ad^2	I
1	1800	7.5	13500	$\frac{(120 \times 15^3)}{12} = 33750$	$\lvert 45 - 7.5 \rvert = 37.5$	2.53125×10^6	2.565×10^6
2	1500	90	135,000	$\frac{(10 \times 150^3)}{12} = 2.8125 \times 10^6$	$\lvert 45 - 90 \rvert = 45$	3.0375×10^6	5.850×10^6
	3300		148 500				8.415×10^6

$$\therefore I = \underline{8.415 \times 10^6\,mm^4}.$$



③ Section Modulus.

$$S = \frac{I}{C_{top}} = \frac{8.415 \times 10^6 \ mm^4}{120 \ mm} = 70,125 \ mm^3$$

④ Shear Stress.

Data
$V = 80 \ kN.$
$Q = \bar{y}A = (40 \times 10 \times 100)$
$\bar{y}_1 \Rightarrow C_{top} - 40/2$
$\Rightarrow 120 - 40/2 = 100$

At K: $\tau_K = \frac{VQ}{It}$

$$\tau_K = \frac{80000 N \times (40 \times 10) \times (100) \ mm^3}{8.415 \times 10^6 \ mm^4 \times (10) mm}$$

$= 38.03 \ MPa$

At H: $\tau_H = \frac{VQ}{It}$

$$\tau_H = \frac{80000 N \times (15 \times 120) \times (37.5) \ mm^3}{8.415 \times 10^6 \ mm^4 \times (120) mm}$$

$= 5.35 \ MPa$

QUIZ 7

SOLUTION

Given Data

$r = 12.5m$ $(d = 2.5m)$.

Ext $P = 4MPa$

$\sigma_y = 600MPa$

Factor of Safety $(F.S) = 3.0$

(a) Allowable Stress

$$\text{Factor of Safety} = \frac{\sigma_y}{\sigma_{all}} \quad (\text{max.})$$

$$\sigma_{all} = \frac{\sigma_y}{F.S} = \frac{600}{3} = \underline{\underline{200 MPa}} \quad (N/mm^2).$$

$\Rightarrow$ Required thickness, t

$$\sigma_{all} = \frac{Pr}{2t}$$

$$t = \frac{Pr}{2\sigma_{all}} = \frac{4 N/mm^2 \times 12500 mm}{2 \times (200 N/mm^2)}$$

$$t = \underline{\underline{125 mm}}$$

(b) If $t = 90mm$; Solve P_{max}

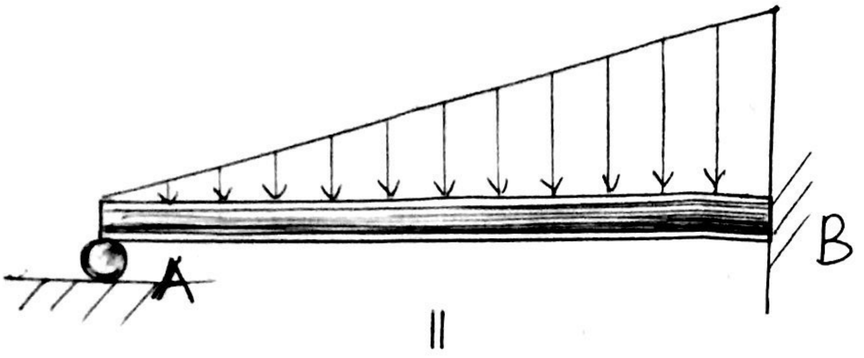
$$\sigma_{all} = \frac{Pr}{2t} = \frac{P(d/2)}{2t}.$$

$$(2t)\sigma_{all} = P\left(\frac{d}{2}\right)$$

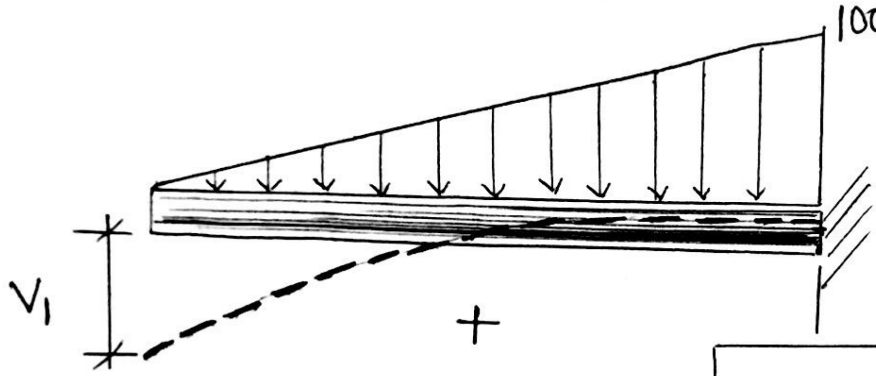
$$P_{max} = \frac{(4t)\sigma_{all}}{d} = \frac{(4 \times 90) \times 200}{25000} = \underline{\underline{2.88 MPa}}$$

QUIZ 8

SOLUTION

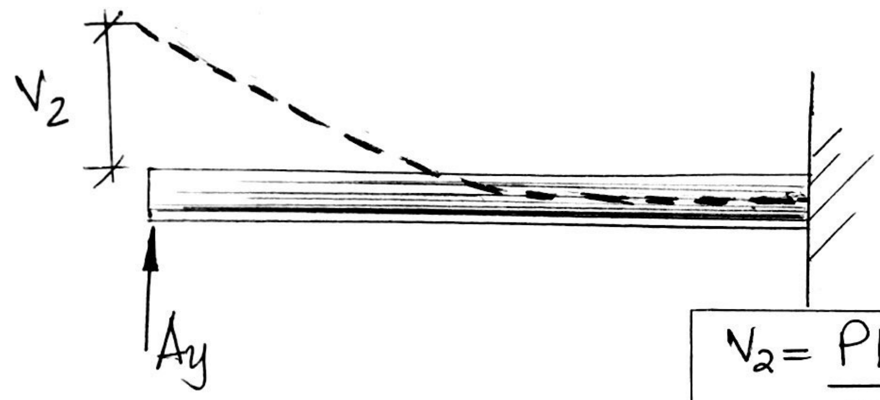


=



+

$$\boxed{V_1 = -\frac{\omega_o L^4}{30IE}}$$



$$\boxed{V_2 = \frac{PL^3}{3EI}}$$

Note: $P = Ay$.

$$V_1 + V_2 = 0$$

$$-\frac{\omega_o L^4}{30IE} + \frac{PL^3}{3EI} = 0$$

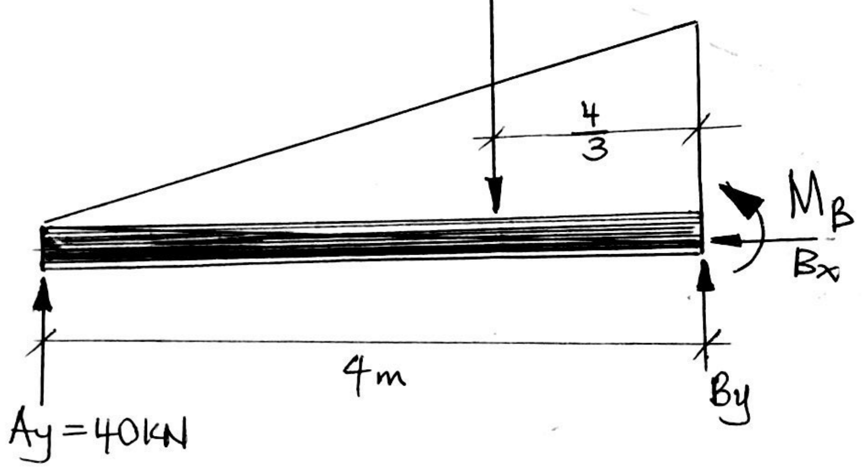
$$\frac{(A_y) L^3}{3EI} = \frac{w_o L^4}{30EI}$$

$$\frac{A_y L^3}{3} = \frac{w_o L^4}{30}$$

$$A_y = \left(\frac{3}{L^3}\right) \frac{w_o L^4}{30} = \frac{w_o L}{10}$$

$$A_y = \frac{(100\,\text{kN/m}) \times (4\,\text{m})}{10} = \underline{40\,\text{kN}}.$$

$$F = A = \frac{1}{2} bh = \frac{1}{2}(4 \times 100) = \underline{200\,\text{kN}}$$



① $+\uparrow F_Y = 0$:

$$A_y + B_y = F$$
$$40 + B_y = 200$$
$$B_y = 200 - 40 = \underline{\underline{160\,\text{kN}}}.$$

② $\circlearrowleft\!+ M_B = 0$:

$$M_B + 200\left(\frac{4}{3}\right) = 40\,(4)$$
$$M_B + \frac{800}{3} = 160$$
$$M_B = 160 - \frac{800}{3} = \underline{\underline{-106.67\,\text{kN.m}}}$$

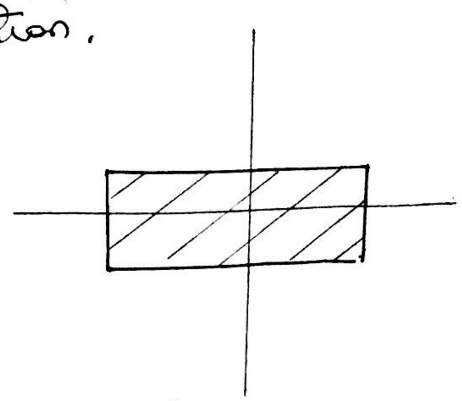
③ $\xrightarrow{+} F_x = 0$:

$$B_x = \underline{\underline{0\,\text{kN}}}$$

QUIZ 9

SOLUTION

Section Properties for steel section.



$A = 2770 \ mm^2$

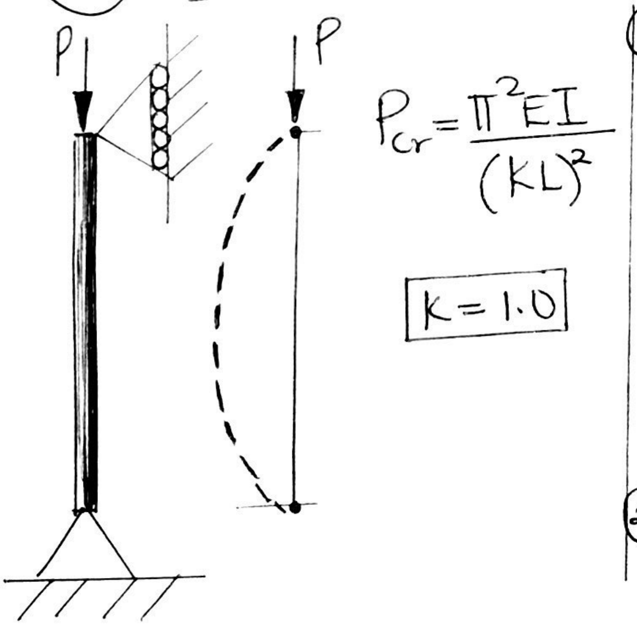
$I_x = 8.70 \times 10^6 \ mm^4$

$I_y = 4.62 \times 10^6 \ mm^4$.

Note: Columns buckle (laterally bend) in the Direction corresponding to minimum Value of the Moment of Inertia (I).

$\therefore$ from our given Values: $I_y = 4.62 \times 10^6 \ mm^4$ is less than I_x.

(a) Pinned – Pinned Column.



$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$\boxed{K = 1.0}$

(1) $P_{cr} = \dfrac{\pi^2 EI}{(KL)^2}$ [Critical Buckling Load]

$$= \frac{\pi^2 (200 \times 10^3 \ N/mm^2)(4.62 \times 10^6 \ mm^4)}{[(1.0) \times (6000 mm)]^2}$$

$= 253,320 N = \underline{\underline{253.320 \ kN}}$

(2) Factor of Safety $= \dfrac{P_{critical} \ (max)}{P_{allow}}$

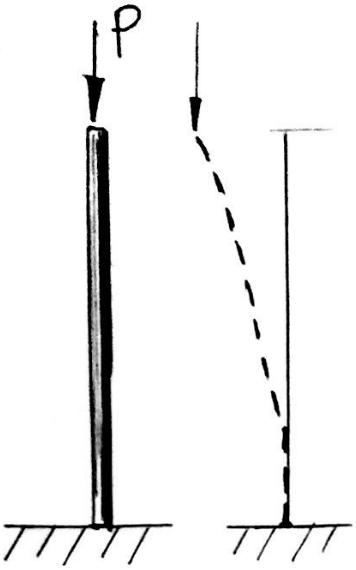
$$P_{all} = \frac{P_{cr}}{F.S}$$

$$P_{all} = \frac{253.320 kN}{2}$$

$= \underline{\underline{126.66 kN}}$

Length of Column $= L = 6m$

(b) Fixed - Free Column



$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$\boxed{K = 2}$$

(1) $P_{cr} = \dfrac{\pi^2 EI}{(KL)^2}$

$$= \frac{\pi^2 (200 \times 10^3 \, N/mm^2)(4.62 \times 10^6 \, mm^4)}{[(2) \times (6000 \, mm)]^2}$$

$$= 63{,}330 \, N = \underline{\underline{63.330 \, kN}}$$

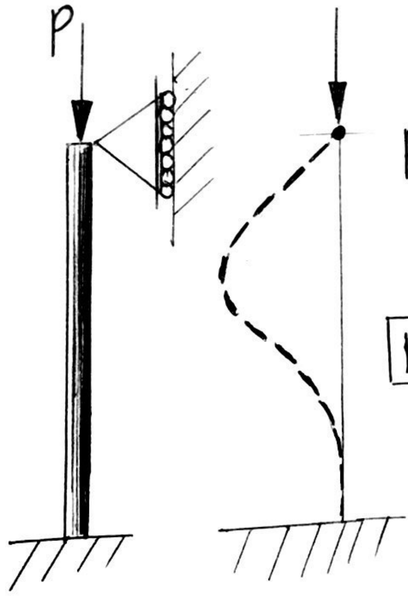
(2) $\text{Factor of Safety} = \dfrac{P_{cr}}{P_{all.}}$

$$P_{all} = \frac{P_{cr}}{F.S}$$

$$P_{all} = \frac{63.330 \, kN}{2}$$

$$= \underline{\underline{31.665 \, kN}}$$

(c) Fixed - Pinned Column.



$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$\boxed{K = 0.7}$$

(1) $P_{cr} = \dfrac{\pi^2 EI}{(KL)^2}$ Critical Load.

$$= \frac{\pi^2 (200 \times 10^3 \, N/mm^2)(4.62 \times 10^6 \, mm^4)}{[(0.7) \times (6000 \, mm)]^2}$$

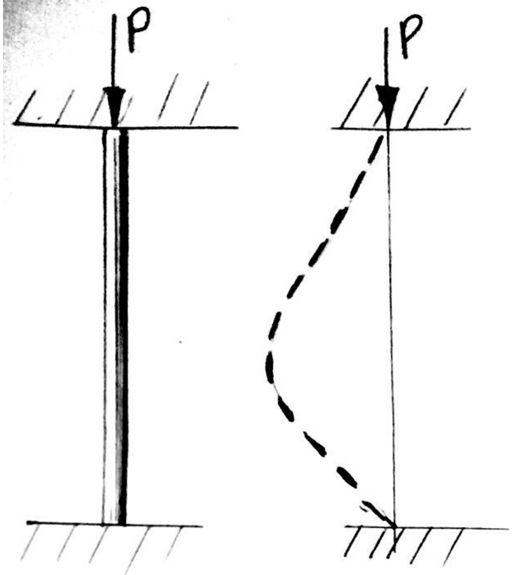
$$= 516{,}979 \, N = \underline{\underline{516.979 \, kN}}$$

(2) $\text{Factor of Safety} = \dfrac{P_{cr}}{P_{all.}}$

$$P_{all} = \frac{P_{cr}}{F.S} = \frac{516.979}{2}$$

$$= \underline{\underline{258.490 \, kN}}$$

(d) Fixed – Fixed Column.



$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$\boxed{K = 0.5}$$

(1) Critical Buckling Load (P_{cr}).

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2 (200 \times 10^3 \, N/mm^2)(4.62 \times 10^6 \, mm^4)}{[(0.5) \times (6000 \, mm)]^2}$$

$$= 1{,}013{,}279 \, N = \underline{\underline{1{,}013.279 \, KN}}$$

(2) Factor of Safety $= \frac{P_{cr}}{P_{all}}$ (Maximum Load)

$$P_{all} = \frac{P_{cr}}{F.S}$$

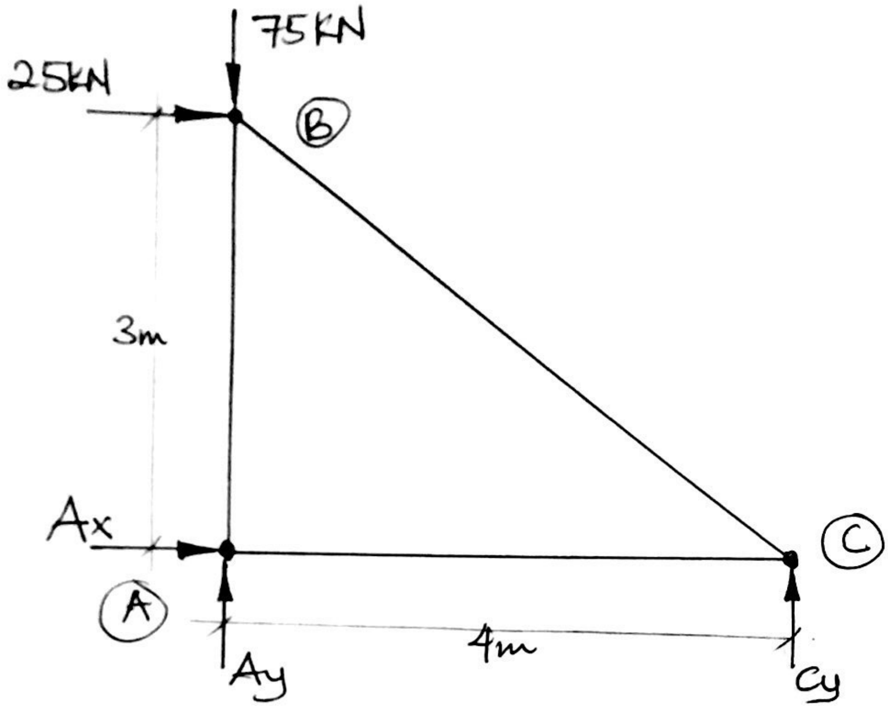
$$= \frac{1{,}013.279 \, KN}{2}$$

$$= \underline{\underline{506.640 \, KN}}$$

QUIZ 10

SOLUTION

Actual Loading



(1) Reactions

(i) $\xrightarrow{+} \Sigma F_x = 0$: $\quad A_x + 25 = 0$

$A_x = -25\,KN = \underline{25\,KN} \leftarrow$

(ii) $+\uparrow F_y = 0$: $\quad A_y + C_y = 75\,KN \quad \cdots$ eqn (1)

(iii) $\circlearrowleft + M_A = 0$: $\quad C_y(4) = 25(3)$

$C_y = 25\left(\frac{3}{4}\right) = \underline{18.75\,KN}$

$\Rightarrow A_y + C_y = 75$

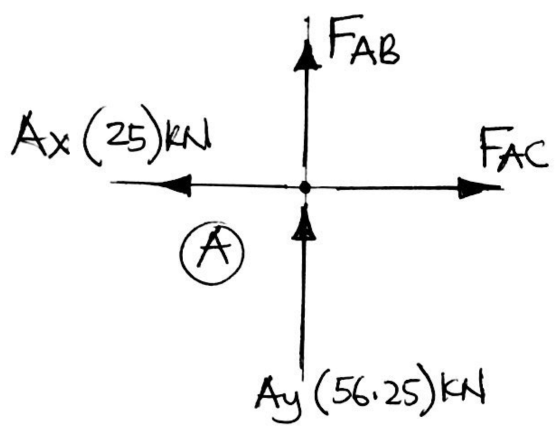
$A_y + (18.75) = 75$

$A_y = 75 - 18.75 = \underline{56.25\,KN}$

② Internal Forces in Members (N)

Method of Joints

Joint A



① $\xrightarrow{+} F_x = 0$;

$F_{AC} = \underline{\underline{25 KN}}$ (T)

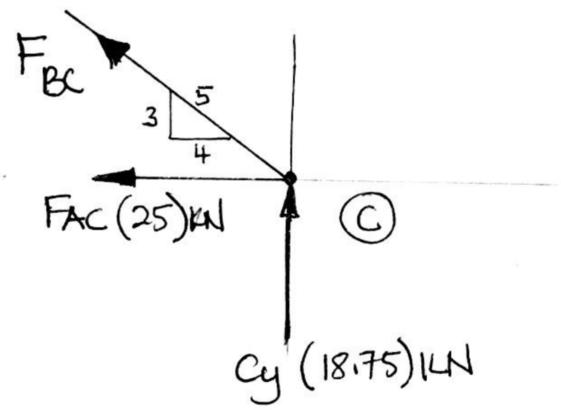
② $+\uparrow F_Y = 0$;

$F_{AB} + 56.25 = 0$

$F_{AB} = -56.25$ KN

$= \underline{\underline{56.25 KN}}$ (C)

Joint C



① $\xrightarrow{+} F_x = 0$;

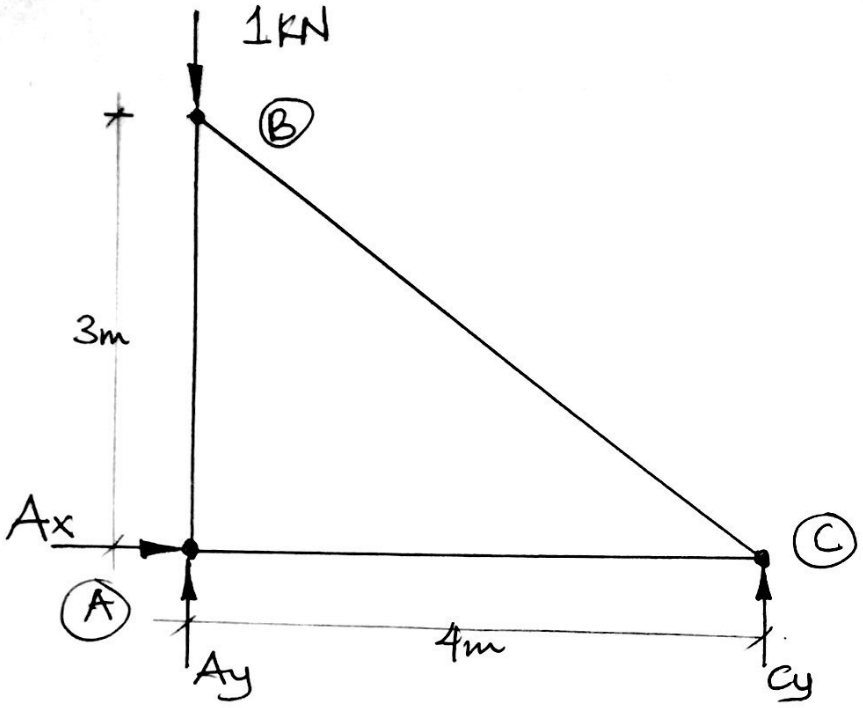
$\frac{4}{5} F_{BC} + 25 = 0$

$F_{BC} = -25\left(\frac{5}{4}\right) = -31.25$ KN

$= \underline{\underline{31.25 KN}}$ (C)

Scanned with CamScanner

② Virtual Loading



① Reactions

(i) $\xrightarrow{+} \Sigma F_x = 0$:

$$A_x + 0 = 0$$
$$A_x = \underline{0\ KN}$$

(ii) $+\uparrow F_y = 0$: $A_y + C_y = 1\ KN$ .... eqn ①

(iii) $\circlearrowleft + M_A = 0$:

$$C_y(4) = 1(0)$$
$$C_y = \underline{0\ KN}$$

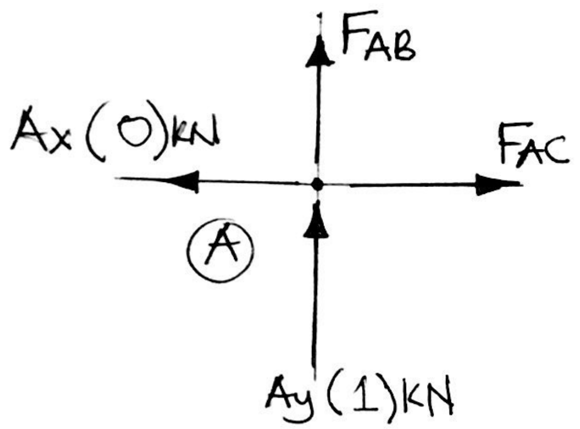
$$\Rightarrow A_y + C_y = 1\ KN$$
$$A_y + (0) = 1\ KN$$
$$A_y = \underline{1\ KN}$$

Scanned with CamScanner

② Internal Forces in Members (n)

Method of Joints

Joint A



① $\xrightarrow{+} \sum F_x = 0$;

$F_{AC} = \underline{\underline{0 \text{ kN}}}$

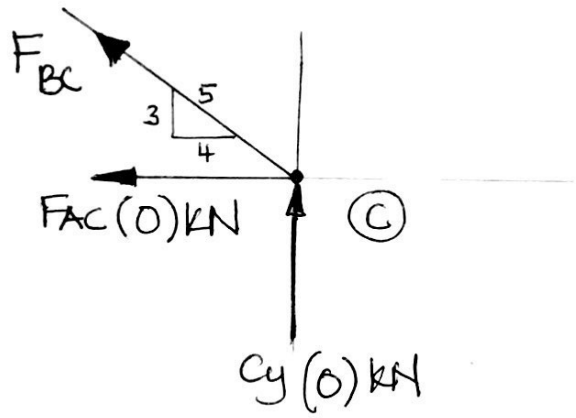
② $+\uparrow \sum F_y = 0$;

$F_{AB} + 1\text{kN} = 0$

$F_{AB} = -1\text{kN}$

$= \underline{\underline{1\text{kN (C)}}}$

Joint C



① $\xrightarrow{+} F_x = 0$!

$\frac{4}{5} F_{BC} + 0 = 0$

$F_{BC} = 0\left(\frac{5}{4}\right) = \underline{\underline{0 \text{ kN}}}$

	n	N	L	nNL	E	A	EA	$\frac{nNL}{EA}$
AB	-1	-56.25	3	168.75	210	150	31500	0.005357
AC	0	25.00	4	0.00	210	150	31500	0
BC	0	-39.25	5	0.00	210	150	31500	0
							Σ	0.005357

1. $\Delta = \frac{\sum nNL}{EA}$

$\Delta = 0.005357 \text{ m}$

$= \underline{\underline{5.357 \text{ mm}}}$

The End