THE UNIVERSITY OF ZAMBIA

SCHOOL OF ENGINEERING

CIVIL AND ENVIRONMENTAL ENGINEERING

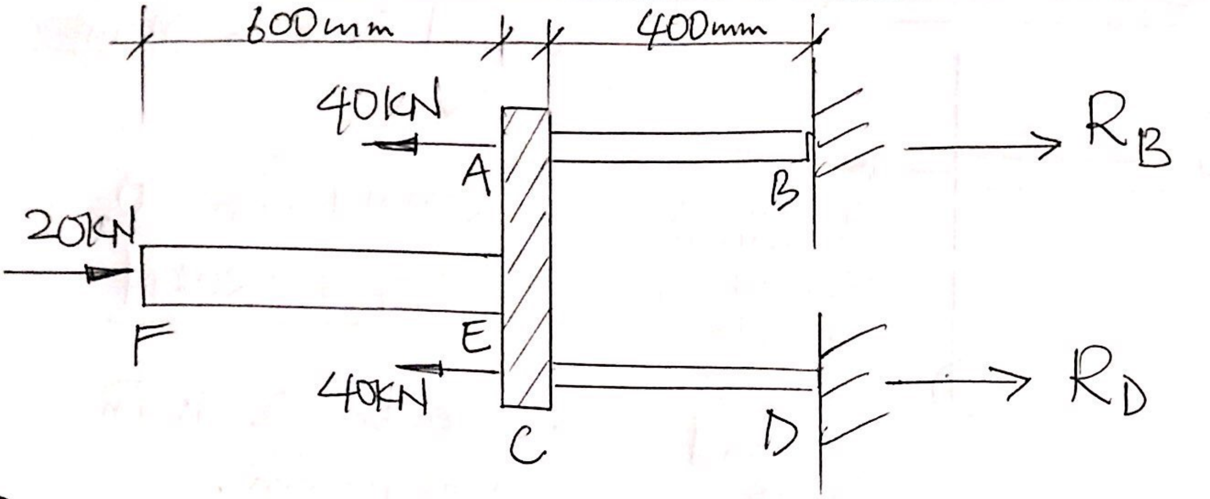
CEE 3211

MECHANICS OF MATERIALS

[TEST SOLUTIONS]

2023

QUESTION 1.



① Reaction Forces of Supports.

$\xrightarrow{+} F_x = 0$: $R_B + R_D + 20 = 40 + 40$

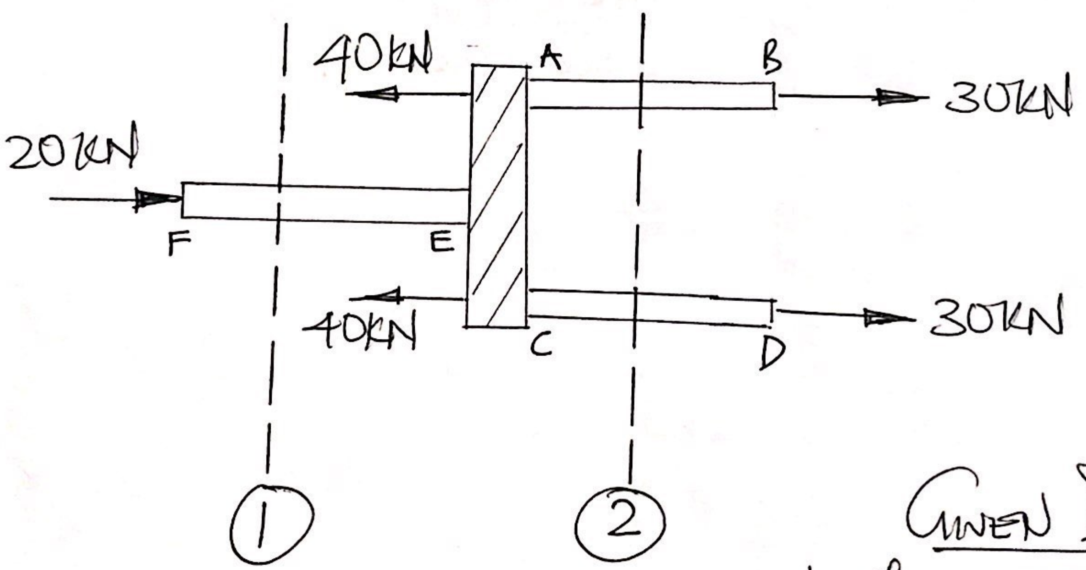
$R_B + R_D = 80 - 20 = 60 \quad \ldots ①$

Since $R_B = R_D$; (Substituting in equ①)

$R_B + (R_B) = 60$; $2R_B = 60$; $R_B = 30\text{ KN}$

Thus; $R_B = R_D = \underline{30\text{ KN}}$

② Segments

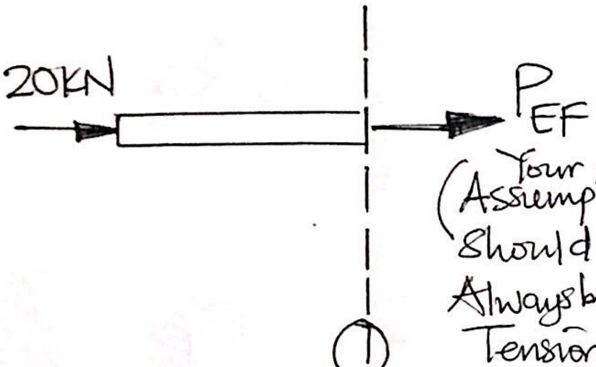


Given Data.

$\varnothing_{FE} = 25\text{mm}$

$E_{al} = 70\text{GPa}$ | $\varnothing_{AB} = \varnothing_{CD} = 15\text{mm}$

Segment (1) [FE]



(Your Assumption Should Always be Tension; For Internal Forces.)

Internal Force in Member

(1) $\xrightarrow{+} F_x = 0$:

$$20\,kN + P_{EF} = 0.$$

$$P_{EF} = -20\,kN$$

$\therefore$ Member FE is in Compression.

(11) Elongation of Member

Elongation is given by:

$$\delta = \frac{PL}{EA}$$

$$\delta_{FE} = \frac{P_{EF}\,L_{EF}}{A_{EF}\,E_{EF}}$$

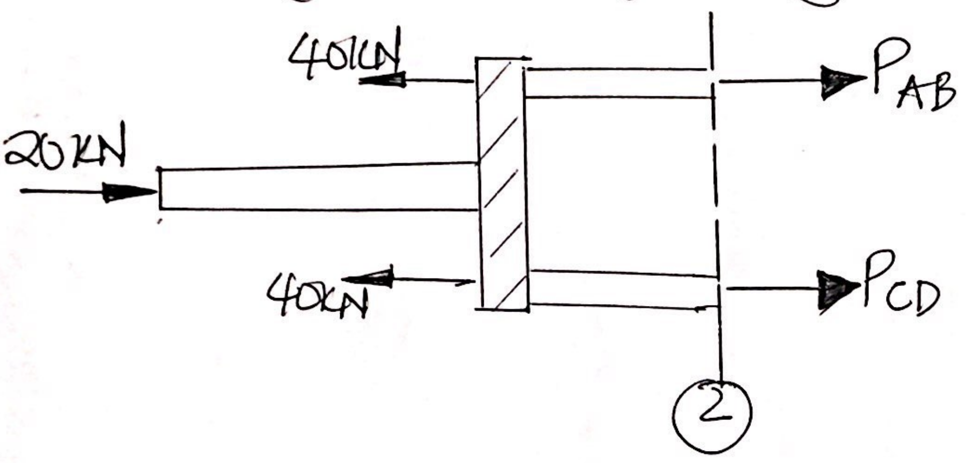
$$= \frac{-20\,kN \times 600\,mm}{\left[\frac{\pi}{4}(25\,mm)^2\right] \times 70 \times 10^3\,N/mm^2}$$

$$= -0.3492\,mm$$

The Negative Sign Implies Shortening.

Segment 2 [AB] or [CD]

Note: Elongation of AB = Elongation of CD.
(Due to Symmetry).



(Assuming Tension)

Note: $P_{AB} = P_{CB}$
(Symmetrical).

(I) Internal Force in Members

$\xrightarrow{+} \sum F_x = 0$:

$$20 + P_{AB} + P_{CD} = 40 + 40$$

$$(2P_{AB}) = 80 - 20 = 60$$

$$\underline{\underline{P_{AB} = 30\,KN = P_{CB}}}$$

∴ Since Internal Force P_{AB}/P_{CB} is Positive (as Assumed); Member AB/CB is in Tension respectively.

(II) Elongation of Member [AB]

$$\delta_{AB} = \frac{P_{AB}L_{AB}}{A_{AB}E_{AB}}$$

$$= \frac{30\,KN \times 400mm}{\left[\frac{\pi}{4}(15mm)^2\right] \times 70 \times 10^3\,N/mm^2} = \underline{0.970087mm}$$

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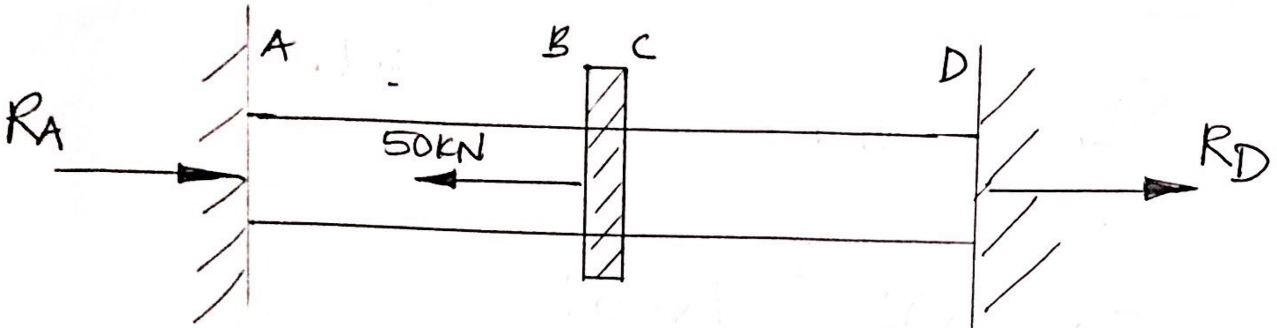
Total Elongation:

$$\delta_T = \delta_{FE} + \delta_{AB}$$

$$= -0.3492 + 0.970087$$

$$= \underline{\underline{0.621\ mm}}$$

QUESTION 2.



① Reactions at Supports.

$\xrightarrow{+} F_x = 0$: $\quad R_A + R_D = 50\,KN \; \ldots \; eqn$ ①

Since we have 2 Unknowns (R_A and R_D) and Only 1 equation (eqn ①): This is Statically Indeterminate.

Thus;

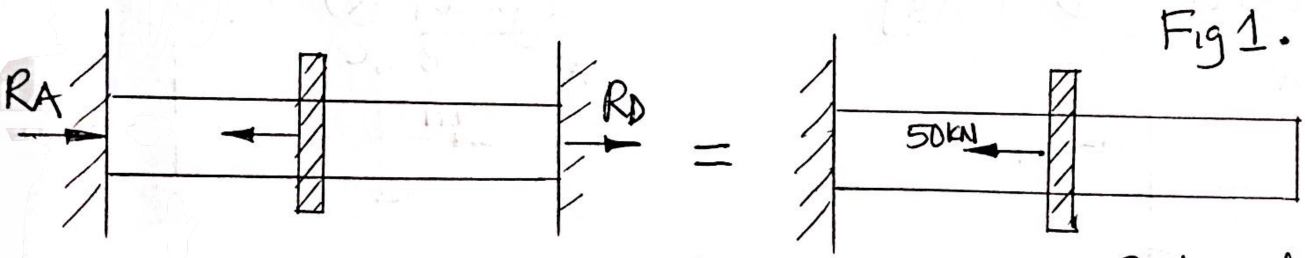
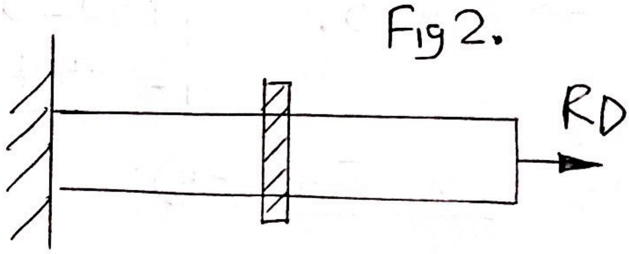


Fig 1.

Real Loading.

+

Fig 2.



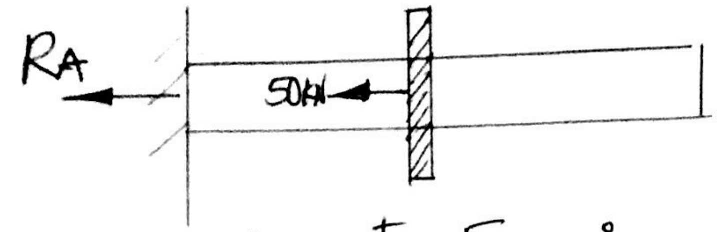
Reaction.

Compatibility Equation.

$$\delta_T = \underset{(Fig1)}{\delta_P} + \underset{(Fig2)}{\delta_{RD}} = 0$$

(i) Elongation wrt Real Loading (δ_P)

Fig1.

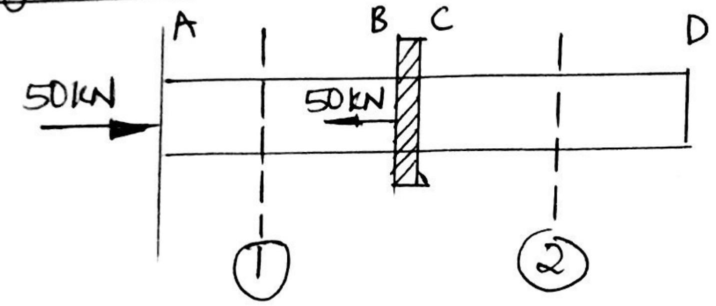


① Reaction (R_A): $\xrightarrow{+} F_x = 0$:

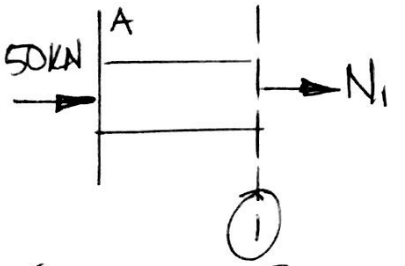
$R_A + 50 = 0$; $R_A = \underline{-50\ KN}$

$R_A = \underline{50\ KN} \longrightarrow$

② Segments



Segment ① [AB]



① Internal Force.

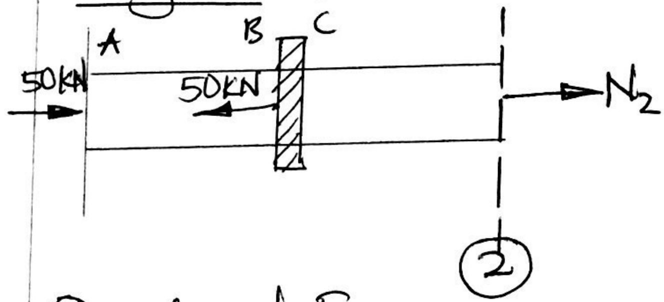
$\xrightarrow{+} \Sigma F_x = 0$: $N_1 + 50 = 0$

$N_1 = \underline{-50\ KN}$

② Elongation. (AB)

$$\delta_{AB} = \frac{N_1 L_1}{E_1 A_1} + \alpha_1 \Delta T L_1$$

Segment ② [CD]



① internal Force.

$\xrightarrow{+} \Sigma F_x = 0$:

$N_2 + 50 = 50$

$N_2 = \underline{0}\ KN$

② Elongation (CD)

$$\delta_{CD} = \cancel{\frac{N_2 L_2}{E_2 A_2}}^{\,0} + \alpha_2 \Delta T L_2$$

[A] $\therefore \delta_P = \delta_{AB} + \delta_{CD} = \left[\frac{N_1 L_1}{E_1 A_1} + \alpha_1 \Delta T L_1 + \cancel{\frac{N_2 L_2}{E_2 A_2}}^{\,0} + \alpha_2 \Delta T L_2\right]$

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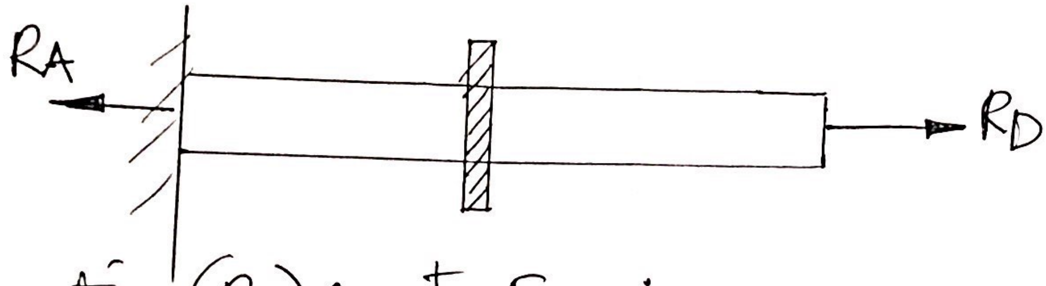
(ii) Elongation Due to Reaction (δ_{RD}).

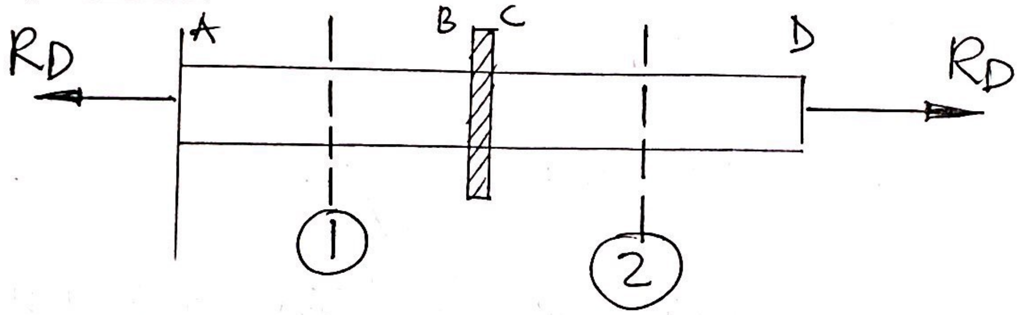
Fig2.



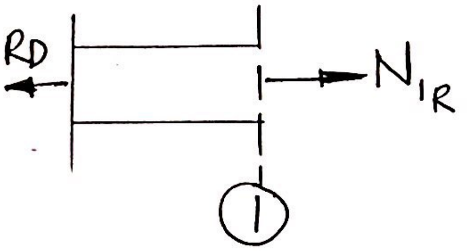
① Reaction (R_A): $\xrightarrow{+} F_x = 0$:

$$R_A = R_D$$

② Segments



Segment ① [AB]



① Internal force. (N_{1R}).

$\xrightarrow{+} \Sigma F_x = 0$:

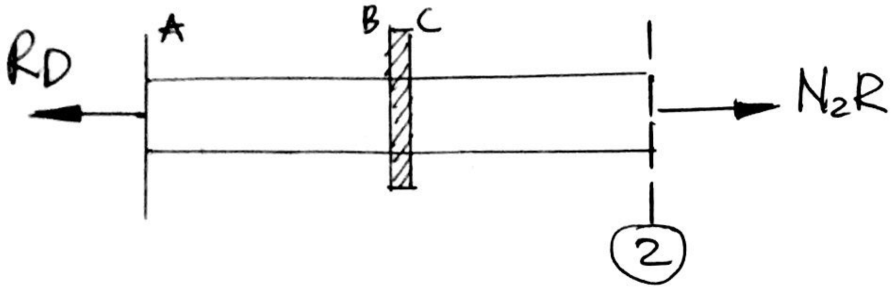
$$N_{1R} = R_D.$$

② Elongation (AB).

$$\delta_{AB} = \frac{N_{1R} L_1}{E_1 A_1}$$

$$= \frac{R_D L_1}{E_1 A_1}$$

Segment 2 [CD]



① Internal Force.

$\overset{+}{\rightarrow} \sum F_x = 0: \quad N_{2R} = \underline{R_D}$

② Elongation (CD),

$$\delta_{CD} = \frac{N_{2R} L_2}{E_2 A_2} = \frac{R_D L_2}{E_2 A_2}.$$

Total Elongation wrt the Reaction Force (R_D)

$$\boxed{B} \therefore \quad \delta_{RD} = \left[\frac{R_D L_1}{E_1 A_1} + \frac{R_D L_2}{E_2 A_2} \right]$$

Therefore ;

From Compatibility Equation ;

$$\delta_T = \delta_P^{\boxed{A}} + \delta_{RD}^{\boxed{B}} = 0$$

$$\Rightarrow \left[\frac{N_1 L_1}{E_1 A_1} + \alpha_1 \Delta T L_1 + \cancelto{0}{\frac{N_2 L_2}{E_2 A_2}} + \alpha_2 \Delta T L_2 \right] + \left[\frac{R_D L_1}{E_1 A_1} + \frac{R_D L_2}{E_2 A_2} \right] = 0$$

Note: $L_1 = L_2 = 500$ mm

(Check Dimensions).

Since $N_2 = 0 \Rightarrow \frac{N_2 L_2}{E_2 A_2}$ is cancelled out.

Therefore ;

$$\left[\frac{N_1 L_1}{E_1 A_1} + \alpha_1 \Delta T L_1 + \alpha_2 \Delta T L_2\right] + \left[\frac{R_D L_1}{E_1 A_1} + \frac{R_D L_2}{E_2 A_2}\right] = 0$$

As stated ; $L_1 = L_2$ Hence we can factor $[L_1]$. out.

$$L_1\left[\frac{N_1}{E_1 A_1} + \alpha_1 \Delta T + \alpha_2 \Delta T\right] + L_1\left[\frac{R_D}{E_1 A_1} + \frac{R_D}{E_2 A_2}\right] = 0.$$

$$R_D\left[\frac{1}{E_1 A_1} + \frac{1}{E_2 A_2}\right] = -\left[\frac{N_1}{E_1 A_1} + \alpha_1 \Delta T + \alpha_2 \Delta T\right]$$

$$R_D\left[\frac{E_2 A_2 + E_1 A_1}{E_1 A_1 \cdot E_2 A_2}\right] = \left[-\frac{N_1}{E_1 A_1} - \alpha_1 \Delta T - \alpha_2 \Delta T\right]$$

$$R_D = \left[\frac{E_1 A_1 \cdot E_2 A_2}{E_2 A_2 + E_1 A_1}\right] \times \left[-\frac{N_1}{E_1 A_1} - \alpha_1 \Delta T - \alpha_2 \Delta T\right]$$

(Since $N_1 = -50\,kN$)

$$R_A = \left[\frac{(101 \times 10^3 \times 350) \cdot (84 \times 10^3 \times 450)}{(84 \times 10^3 \times 450) + (101 \times 10^3 \times 350)}\right] \times \left[\frac{\oplus 50000}{(101 \times 10^3 \times 350)} - (17 \times 10^{-6} \times 130) - (10.8 \times 10^{-6} \times 130)\right]$$

$$= [18,266985.65]\,[-2.199 \times 10^{-3}]$$

$$= -40180\,N$$

$$= \underline{-40.18\,kN}$$

$$= 40.18\,kN \leftarrow$$

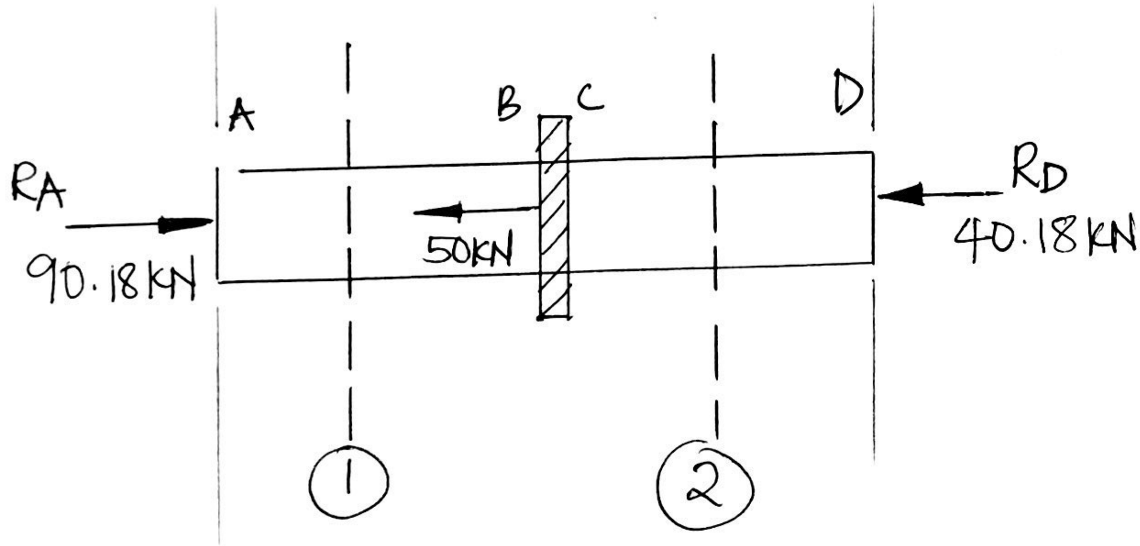
Thus ; from equ (1)

$$R_A + R_D = 50\ \text{KN}.$$

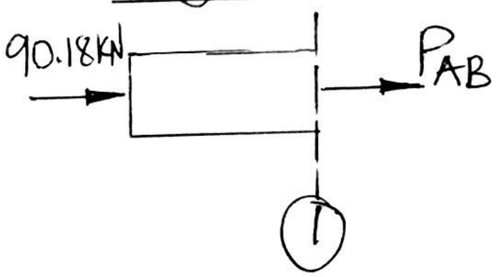
$$R_A + (-40.180) = 50\ \text{KN}$$

$$R_A = 50 - (-40.180) = \underline{\underline{90.18\ \text{KN}}}$$

Therefore : We have the following Reactions.



Segment 1



(1) internal Force

$\overset{+}{\rightarrow} F_x = 0$;

$$P_{AB} + 90.18\text{KN} = 0$$

$$P_{AB} = \underline{-90.18\ \text{KN}}$$

Member AB is in Compression.

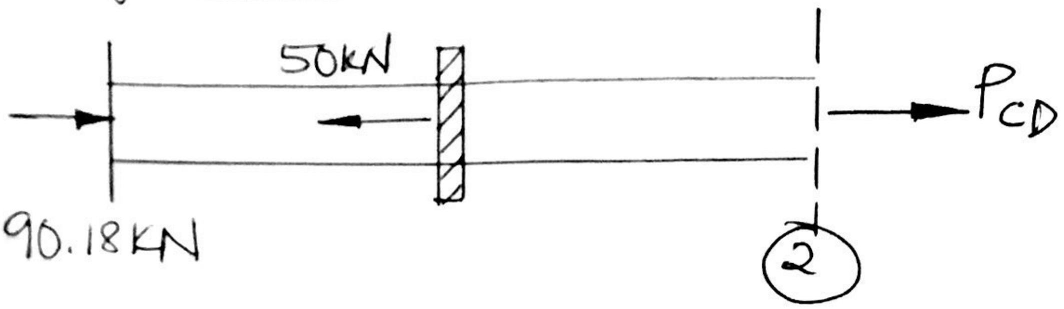
(2) Stress induced in AB

$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}}$$

$$= \frac{90.18 \times 10^3\ \text{N}}{350\ \text{mm}^2}$$

$$= 257.656\ \text{N/mm}^2$$

Segment 2.



① internal Force

$\xrightarrow{+} F_x = 0: \quad 90.18 + P_{CD} = 50$

$P_{CD} = 50 - 90.18 = \underline{-40.18\ kN}$

Member CD is in Compression.

② Stress induced in CD,

$$\sigma_{CD} = \frac{P_{CD}}{A_{CD}}$$

$$= \frac{40.18 \times 10^3\ N}{450\ mm^2}$$

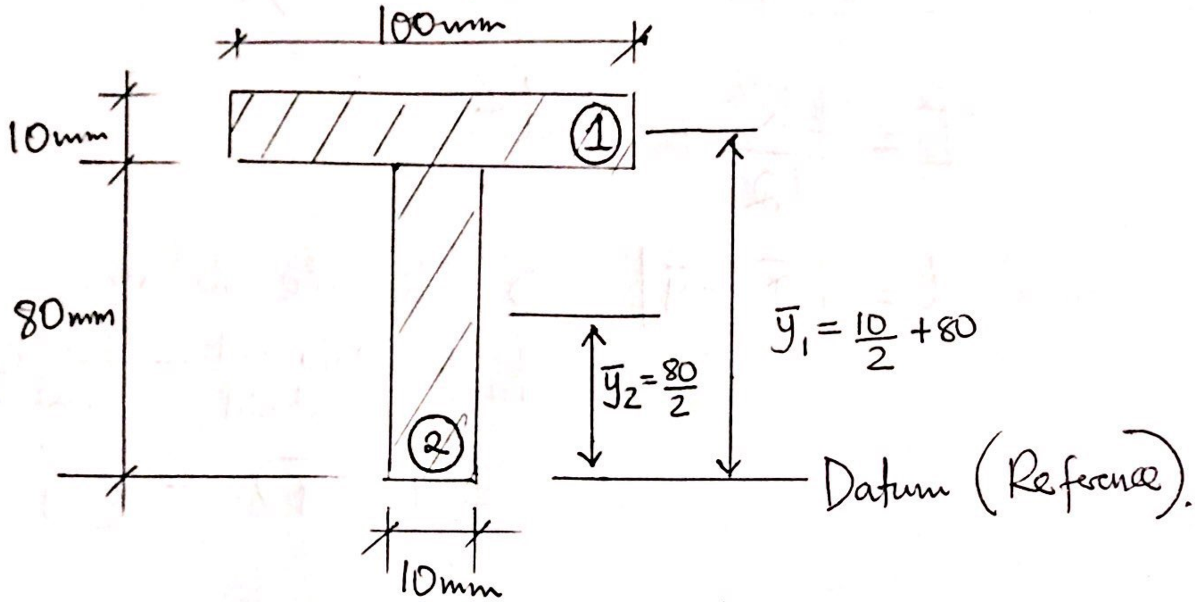
$$= \underline{\underline{89.288\ N/mm^2}}$$

$\therefore$ Stress induced in :

Bar AB $\Rightarrow \sigma_{AB} = \underline{257.656\ MPa}$

CD $\Rightarrow \sigma_{CD} = \underline{89.288\ MPa}$

QUESTION 3



(I) Centroid of Cross Section (Neutral Axis).

	A	y	$A\bar{y}$ (mm^3)
Part	Area (mm^2)	Centroid $\bar{y}$ (mm)	Product of Area & Centroid
1	$10 \times 100 = 1000$	$\frac{10}{2} + 80 = 85$	$1000 \times 85 = 85000$
2	$80 \times 10 = 800$	$\frac{80}{2} = 40$	$800 \times 40 = 32000$
Σ	1800		117,000

Centroid; $\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A}$

$$\bar{Y} = \frac{117000\ mm^3}{1800\ mm^2}$$

$$\bar{Y} = 65\ mm$$

(from the Bottom).

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② Moment of Inertia (I)

$$I = \frac{bh^3}{12} + Ad^2$$

Where $d = |\bar{Y} - \bar{y}|$ ⇒ Absolute Value

$$d = \left| \left(\begin{array}{c}\text{Universal}\\\text{Centroid}\end{array} - \begin{array}{c}\text{Local}\\\text{Centroid}\\\text{from Datum.}\end{array} \right) \right|$$

$$= |(\bar{Y}) - (\bar{y})|$$

Part	A	$\bar{y}$	$A\bar{y}$	$\frac{bh^3}{12}$	$d = \lvert\bar{Y} - \bar{y}\rvert$	Ad^2	$I = \frac{bh^3}{12} + Ad^2$
①	1000	85	85000	$\frac{100 \times 10^3}{12}$ $= 8333.33$	$\lvert 65 - 85 \rvert$ $= 20$	1000×20^2 $= 400\,000$	$(8333.33 + 400\,000)$ $= 408\,333.33$
②	800	40	32000	$\frac{10 \times 80^3}{12}$ $= 426\,666.67$	$\lvert 65 - 40 \rvert$ $= 25$	800×25^2 $= 500\,000$	$(426666.67 + 500\,000)$ $= 926\,666.67$
						ΣI	1335 000

$$\therefore I = 1335\,000 \text{ mm}^4$$

$$= 1.335 \times 10^6 \text{ mm}^4$$

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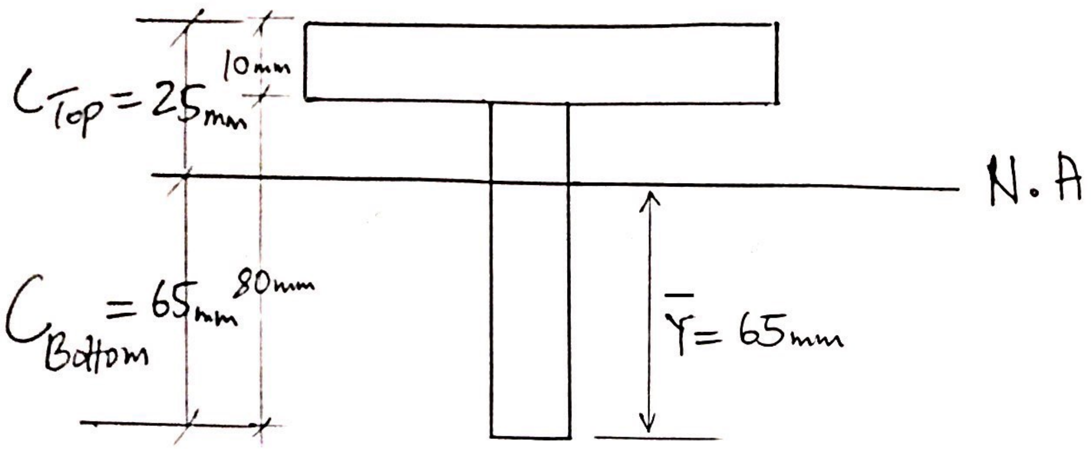


(3) Maximum Compressive Stress (Top)

$y_{top} = C_{top} = \oplus\, 25mm$ ↑ from N.A

$$\sigma_{\substack{Compression \\ (Top)}} = -\frac{M y_{top}}{I}$$

$$= -\frac{25 \times 10\ Nmm \times 25mm}{1.335 \times 10^6\ mm^4}$$

$$= -\frac{25{,}000{,}000 \times 25}{1.335 \times 10^6}\ (N/mm^2)$$

$$= -468.16\ MPa.$$

(Negative Implies Compression)

Maximum Tensile Stress (Bottom)

$y_{Bottom} = -65mm$ ↓ from N.A

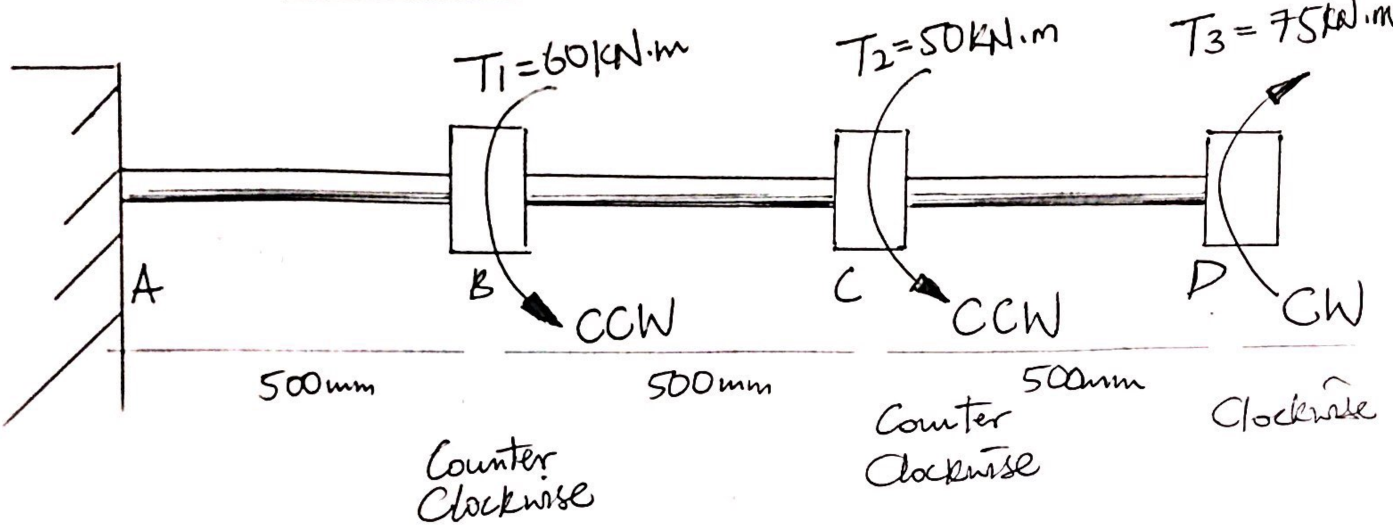
$$\sigma_{\substack{Tension \\ (Bottom)}} = -\frac{M y_{Bottom}}{I}$$

$$= \frac{-25 \times 10^6\ Nmm \times (-65mm)}{1.335 \times 10^6\ mm^4}$$

$$= 1{,}217.2\ MPa\ (N/mm^2)$$

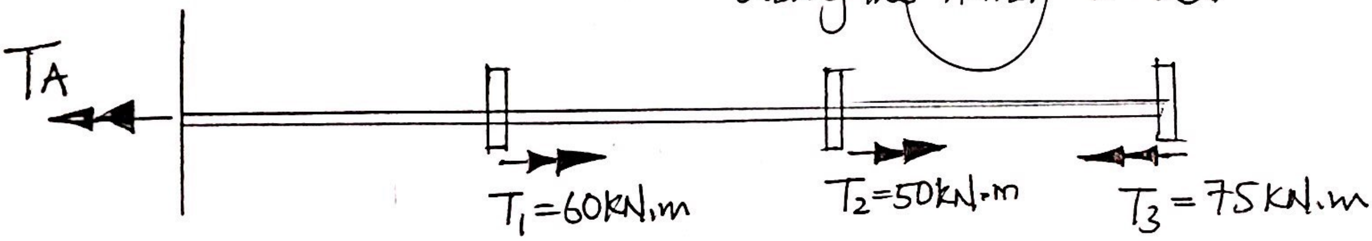
$\oplus$ Implies Tensile Stress

QUESTION 4.



① Reation at A

Using the Right Hand Rule.



Note: from 'Right Hand Rule': CCW ⇒ ⟶ and;

CW ⇒ ⟵

Treat these as "axial loads"

Thus;

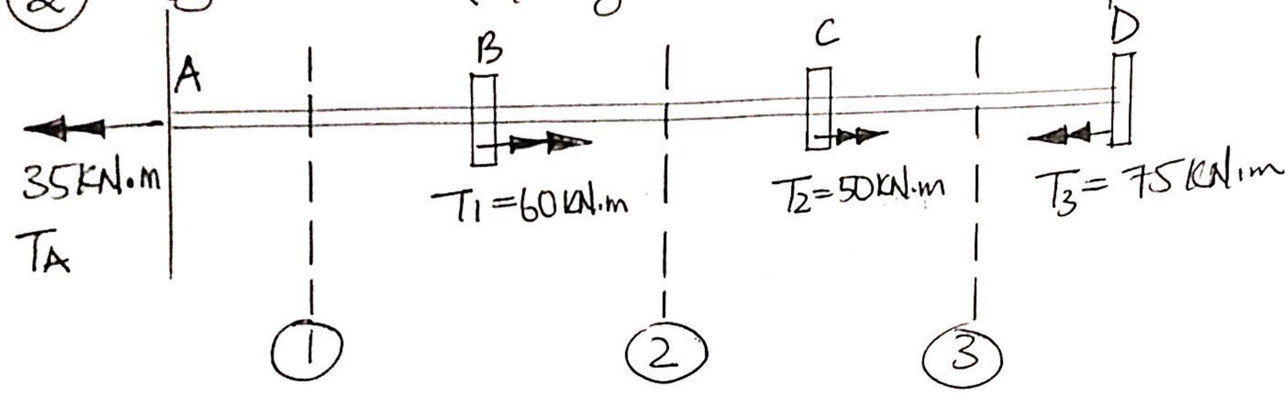
$\overset{+}{\rightarrow} F_x = 0: \quad T_A + T_3 = T_1 + T_2$

$T_A + 75 kN.m = 60 kN.m + 50 kN.m$

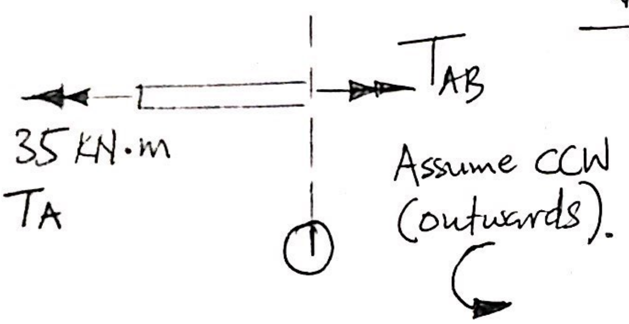
$T_A = (60 + 50) - 75 = 35 kN.m$

$\therefore \underline{\underline{T_A = 35 kN.m}}$ (Clockwise) or ⟵

② Segments: (To get internal Torque, T).



Segment 1 [AB]



① internal Torque

$\overset{+}{\rightarrow} \sum F_x = 0$:

$$35 \text{ KN.m} = T_{AB}$$

∴ Member AB is twisted in Counter Clockwise Direction

⑪ Shear Stress in AB

$$\tau_{AB} = \frac{T_{AB}\,\rho}{J}$$

$$= \frac{35 \times 10^6 \text{ Nmm} \times 25\text{mm}}{\frac{\pi}{2} \times (25\text{mm})^4}$$

$$= \underline{1462 \text{ MPa}} \quad (\text{N/mm}^2)$$

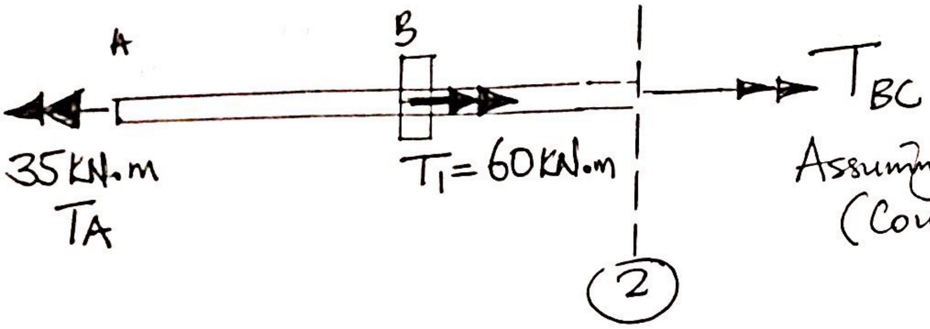
⑪ Angle of Twist.

$$\phi_{AB} = \frac{T_{AB}\,L_{AB}}{J_{AB}\,G_{AB}}$$

$$= \frac{35 \times 10^6 \text{ N mm} \times 500\text{mm}}{\left[\frac{\pi}{2} \times (25\text{ mm})^4\right] \times 26 \times 10^3 \text{ N/mm}^2}$$

$$= \underline{1.0969 \text{ Rads}}$$

Segment 2 [BC]



Assuming Outwards Torque.
(Counter-Clockwise Always!)
(+)

(1) Internal Torque in Member (BC)

$\overset{+}{\rightarrow} F_x = 0$: $35\ \text{KN.m} = T_1 + T_{BC}$

$35\ \text{KN.m} = 60\ \text{KN.m} + T_{BC}$

$T_{BC} = -25\ \text{KN.m}$

This member BC is twisted in the Clockwise Direction

(2) Shear Stress (BC)

$$\tau_{BC} = \frac{T_{BC}\,\rho}{J} = \frac{-25 \times 10^6\ \text{Nmm} \times 25\ \text{mm}}{\left[\frac{\pi}{2} \times (25\text{mm})^4\right]}$$

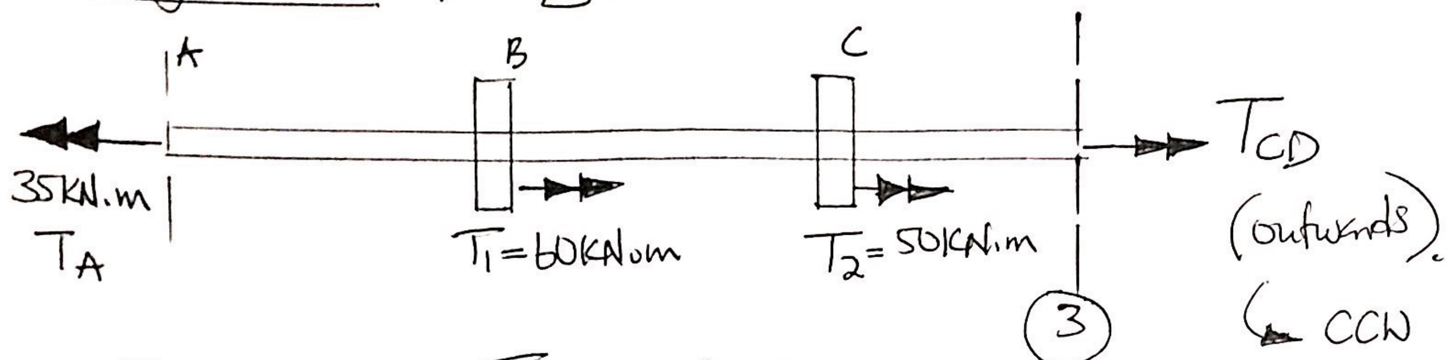
$$= -1018.6\ \text{MPa}$$

(3) Angle of Twist

$$\phi_{BC} = \frac{T_{BC}\,L_{BC}}{J_{BC}\,G_{BC}} = \frac{-25 \times 10^6\ \text{Nmm} \times 500\ \text{mm}}{\left[\frac{\pi}{2} \times (25\text{mm})^4\right] \times 37 \times 10^3\ \text{N/mm}^2}$$

$$= -0.5464\ \text{Rads}$$

Segment 3 [CD].



① Internal Torque. (T_{CD}).

$\xrightarrow{+} F_x = 0$: $\quad 35\,KN.m = T_1 + T_2 + T_{CD}$

$$35 = 60 + 50 + T_{CD}$$

$$T_{CD} = 35 - 110 = \underline{\underline{-75\,KN.m}}$$

∴ Member is twisted in the CW Direction

② Shear Stress in CD.

$$\tau_{CD} = \frac{T_{CD}\,\rho}{J} = \frac{-75 \times 10^6\,Nmm \times 25\,mm}{\left[\frac{\pi}{2} \times (25\,mm)^4\right]}$$

$$= \underline{\underline{-3055.8\ MPa}} \quad (N/mm^2)$$

③ Angle of Twist.

$$\phi_{CD} = \frac{T_{CD}\,L_{CD}}{J_{CD}\,G_{CD}} = \frac{-75 \times 10^6\,Nmm \times 500\,mm}{\left[\frac{\pi}{2}(25\,mm)^4\right] \times 76 \times 10^3\,N/mm^2}$$

$$= \underline{-0.8042\ Rads}$$

Thus;

Angle of Twist at end D

$$\phi_T = \phi_{AB} + \phi_{BC} + \phi_{CD}$$

$$= 1.0969 + (-0.5464) + (-0.8042)$$

$$= -0.2536 \text{ Rads}$$

$\Rightarrow$ Converting to Degrees: $\pi \text{ rad} = 180^\circ$.

π rad — 180° Direct proportion.

-0.2536 — x

$$x = 180^\circ \left(\frac{-0.2536 \text{ rad}}{\pi \text{ rad}} \right) = \underline{\underline{-14.53^\circ}}$$

The Torque Diagram

[from the calculated Internal Torques in members.]