Mathematics of Cartography

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Introduction

- **The map as an interface**
  - Maps are used to *visualize geospatial data*
  - Maps help their users to *better understand* geospatial relationships
  - From maps, information on *distances*, *directions* and *area sizes* can be retrieved, *patterns* revealed, and *relations* understood
  - Since **1980s**, developments in digital geospatial data handling have gained momentum:
  - with the computer came **on-screen maps -** 3D and Animated Maps
  - **software packages** that allowed for *queries* and *analyses* of geospatial data became known as geographical information systems (GIS)
  - GIS introduced the *integration* of *geospatial data* from different kinds of sources
  - Maps no longer only the final products they used to be - the **paper map** functioned & functions, as a *medium* for *storage* & *presentation* of geospatial data.

Introduction

- **The map as an interface (cont'd)**
  - Geospatial analysis often begins with maps; maps support judging intermediate analysis results, as well as presenting final results.
  - maps play a major role in the process of geospatial analysis.
  - The rise of Internet brought the next revolution in mapping
  - Access to interactive maps is no longer limited to professionals.
  - Products such as Google Maps/Earth even allow people to add their own data to the maps and share it with others in a mouse click

Introduction

- **Geospatial data**
  - Why is Geographical Information *different* from *other information*?
  - Because of this *special characteristic* the *locations* of the objects or phenomena can be visualized, and these *visualizations* – called maps – are the key to their further study.
  - objects from the real world that can be localized in space (like houses, roads, fields, or mountains) can be *abstracted* from the real world as a digital landscape model (**DLM**), according to some predetermined criteria, and stored in GIS (as *points, lines, areas* or *volumes*)
  - and later, after being converted into a digital cartographic model (**DCM**), represented on maps (with *dots, dashes* and *patches*) and
  - Integrated in people's ideas about space.

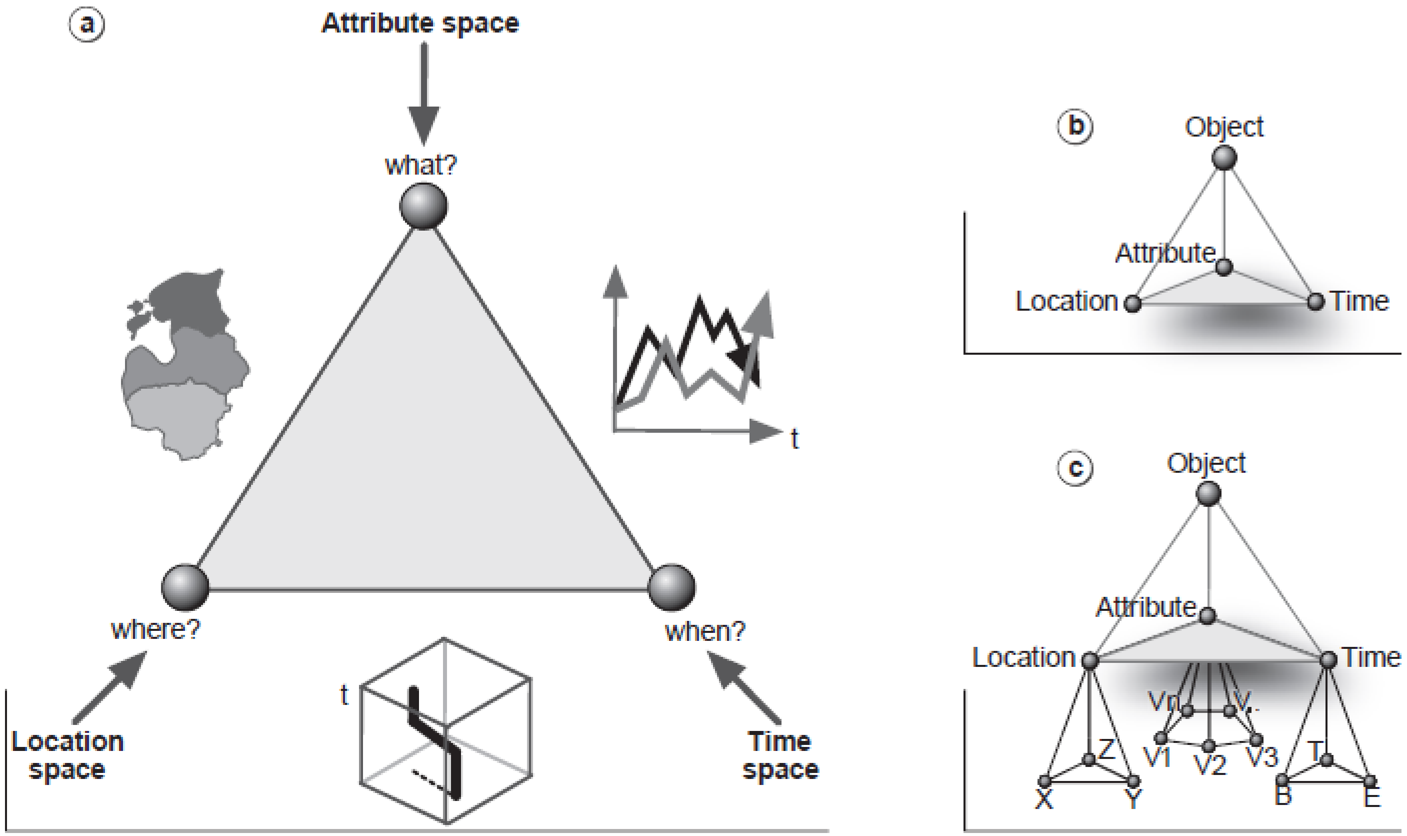


Figure 1.3 The characteristics of geospatial data: (a) its components location, attribute and time, and their related elementary questions where, what and when; (b) the object view; (c) detailed characteristics of the data components

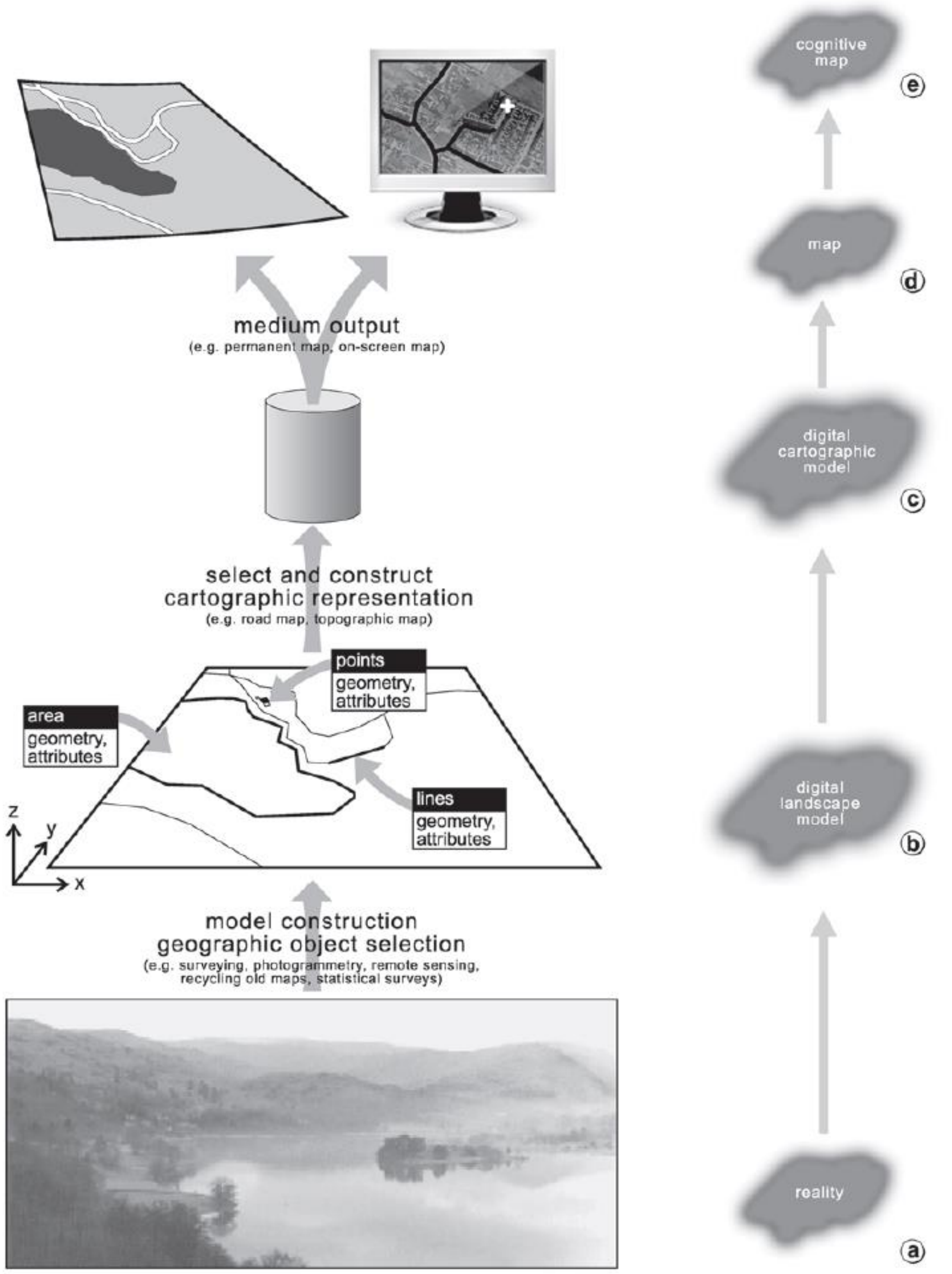


Figure 1.4 The nature of geospatial data: from reality (a), via model construction and selection to a digital landscape model (b), followed by selection and construction of a cartographic representation towards a digital cartographic model (c), presented as a map (d), which results in the user's cognitive map (e)

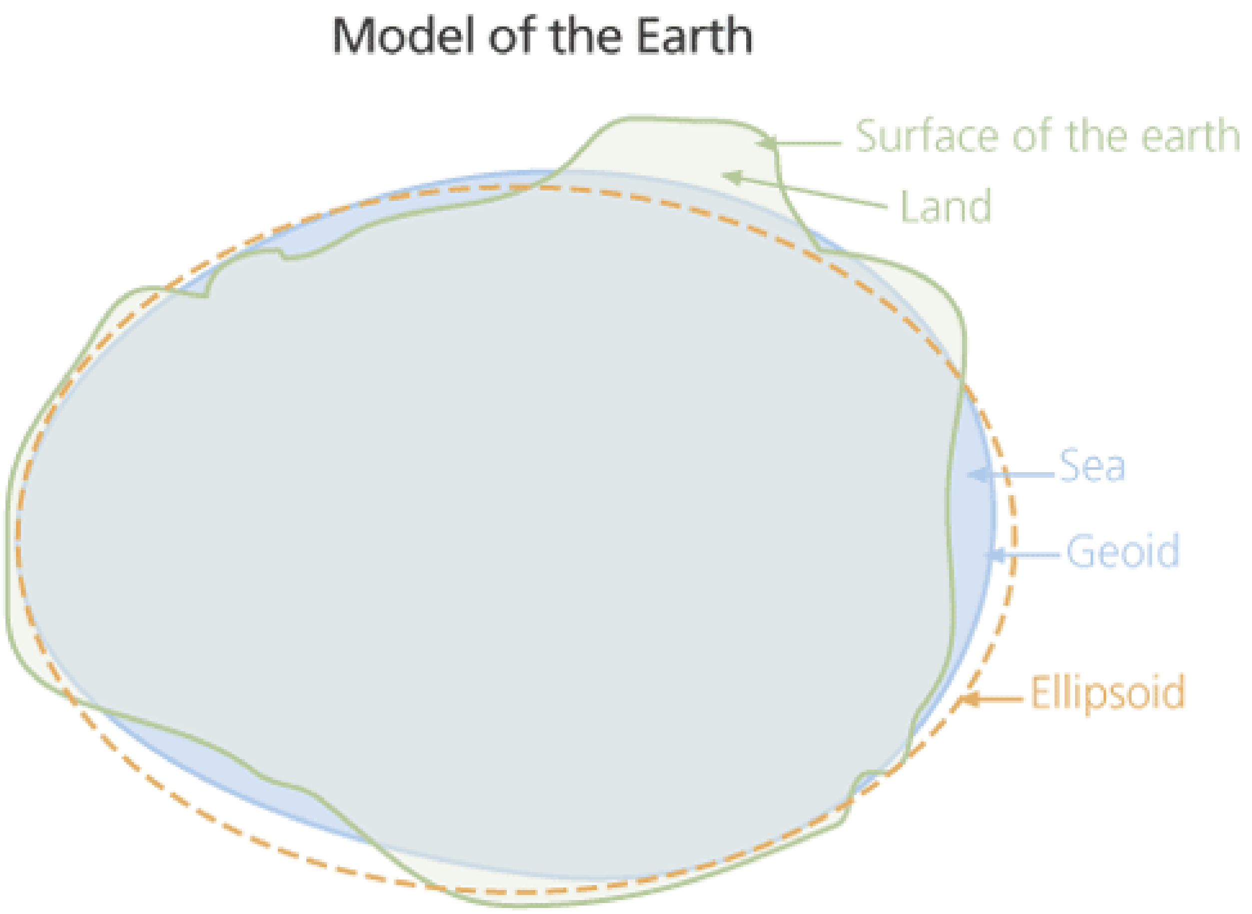
Introduction

- The surface of the earth is curved and irregular
  - To map this irregular surface a more regular surface, that closely approximates the irregular surface, has to be defined.
  - The more regular surface can then be transformed to a **plane map**.
  - Since a map is a small scale representation of the earth's surface it is necessary to apply some kind of scale reduction.
  - Meanwhile different types of coordinate systems are used to locate data on the earth's surface and on the map.

Reference Surfaces

- The earth is not a uniform and smooth globe.
  - The surface of landmasses shows large vertical variations between mountains an valleys
  - Its therefore impossible to describe the earth’s surface with a simple mathematical model.
- Geoid also called the Figure of the Earth
  - If we extend the oceans and seas below land masses the resulting water surface is only affected by gravity and is called the **Geoid**.
  - The Geoid represents an imagery sea level which is an equipotential surface where gravity is everywhere equal.

The Geoid and Ellipsoid



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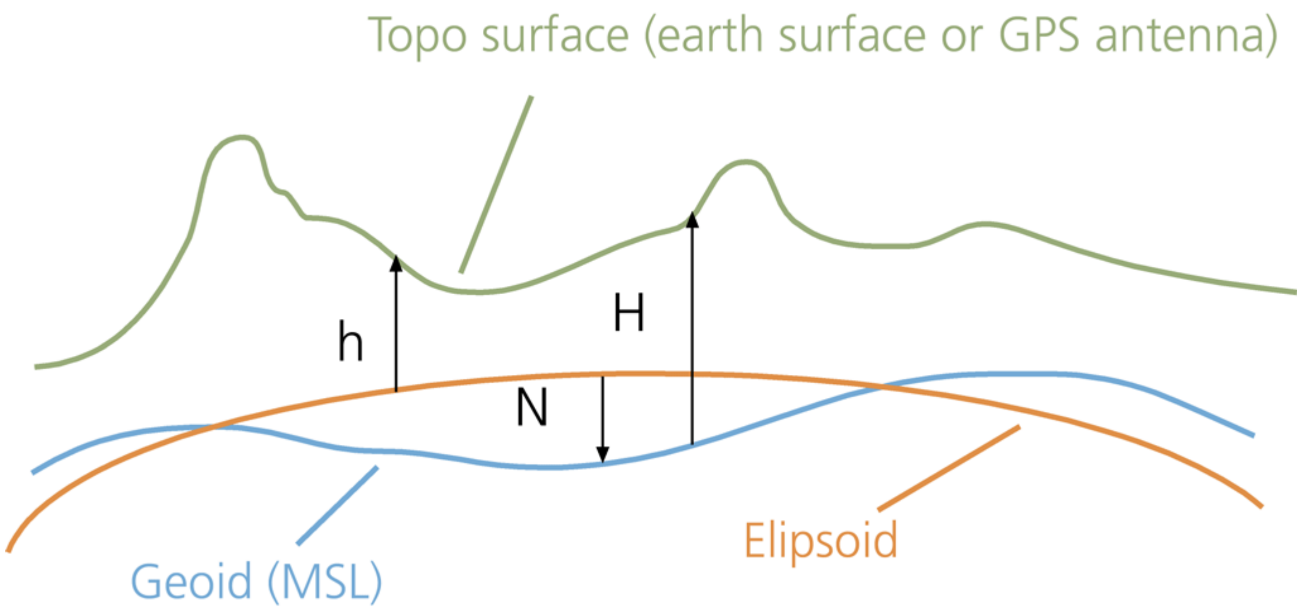
Reference Surfaces

$$h=H+N$$

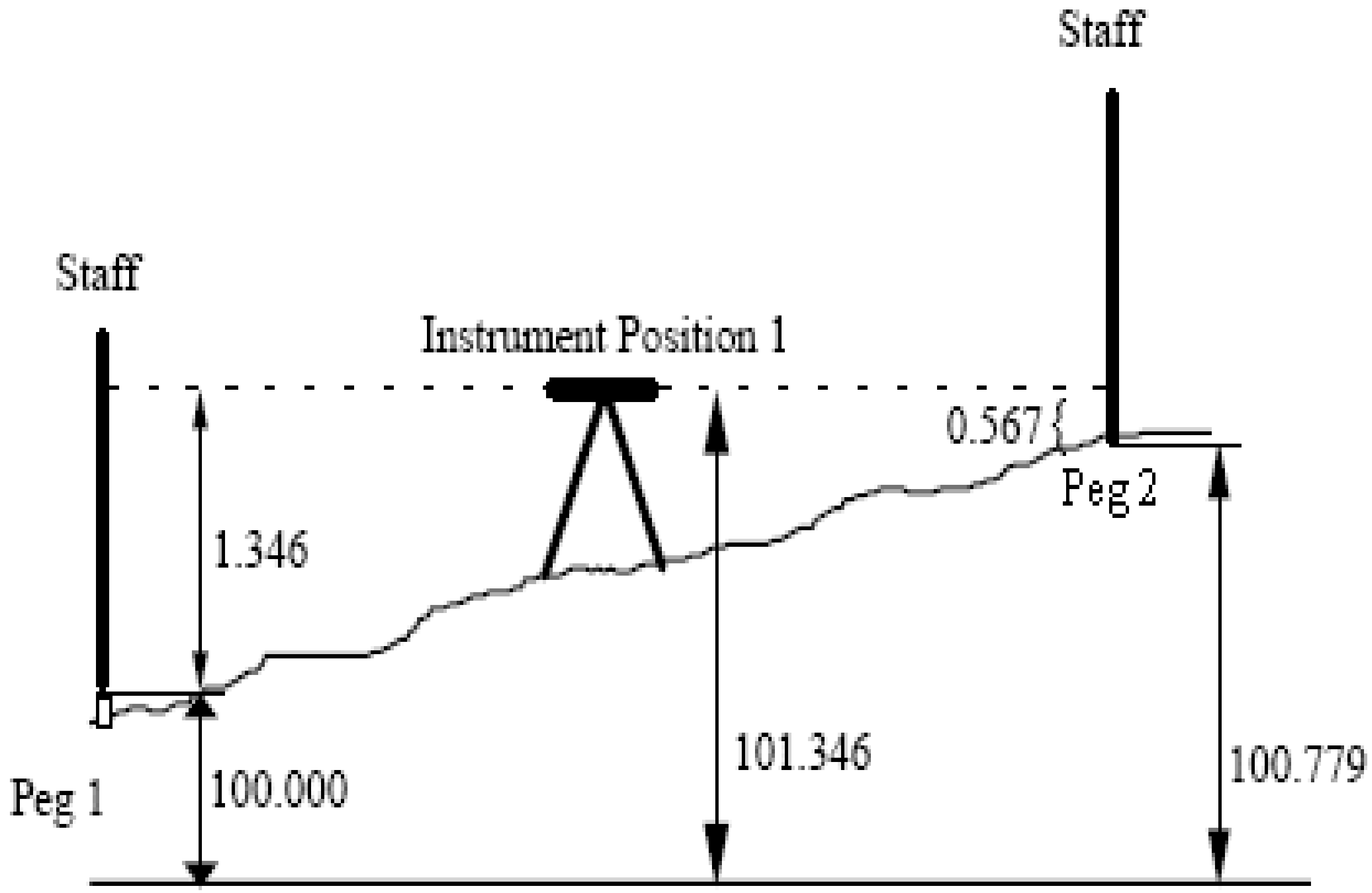


h=elipsoid height
H=orthometric height
N=geoid height

Reference Surfaces

- **Mean Sea Level**
  - In mapping the Geoid is used as a reference surface for height measurements.
  - The *oceans water level* is registered at coastal places over several years and an *average* taken
  - This *approximates the Geoid* and is called the **Mean Sea Level** (**MSL**)
  - Heights of points on the earth's surface are transferred inland by surveying methods (spirit levelling)
  - For historical, and cultural as well as political reasons nations or group of nations have established their own MSL which can differ from that of others.
    - For example, In the Netherlands heights are measured from Amstaerdam while in Belgium they are measured from Oostende
    - As a result the Netherlands heights are -2.32m lower

Spirit Levelling



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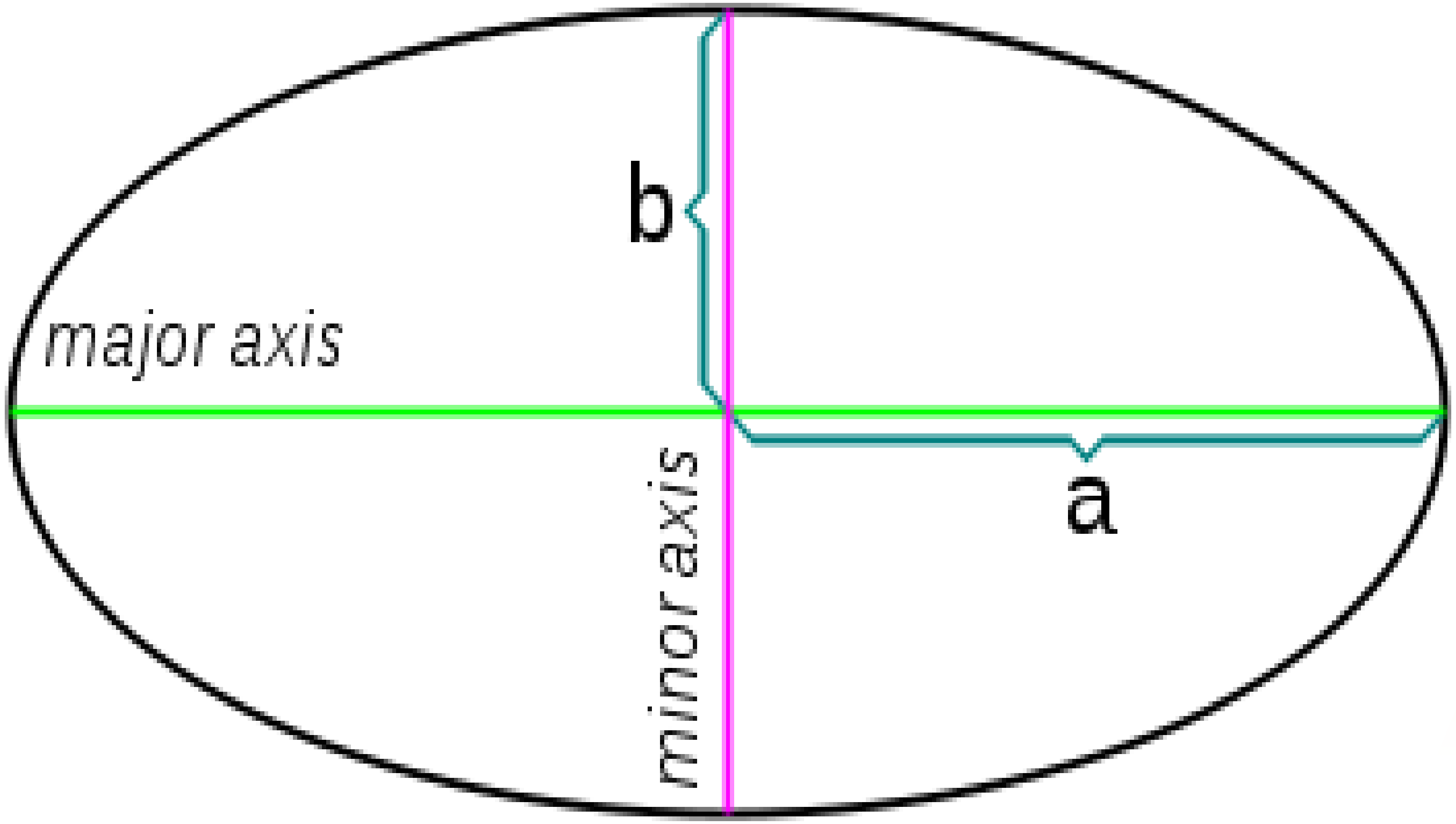
Reference Surfaces

- **Ellipsoid**
  - The Geoidal surface has many discontinuities
  - As such it is not an analytic surface and thus not suitable as a reference surface for determination of locations
  - To carry out computations of positions, distances, directions, etc, a mathematical frame is required
  - The most convenient geometric reference surface is the **Oblate Ellipsoid**:
    - It fits the Geoid to a first order approximation
    - Its oblate because of the polar flattening
    - Its an ellipse of revolution

Ellipsoid



Reference Surfaces

- Geodetic Datum
  - Topographic maps are drawn with geodetic positions that are defined with respect to an ellipsoid with a *defined shape* and *size* and a *fixed position*.
  - This is called a geodetic datum
  - To produce accurate maps, different ellipsoids have been established to fit the geoid well over an area of interest.
  - **In Zambia** we use Clarke (modified) 1880 ellipsoid where:
    - **a** = 6378.249 km
    - **b** = 6356.515 km
    - **f** = 1: 293.47
  - and our geodetic datum is the **Arc1950**
  - **GPS measurements** are done relative to the World Geodetic System 1984 (WGS84) ellipsoid.

Reference Surfaces

- The Sphere
  - Using the sphere as the earth model might be sufficiently accurate for *certain small scale mapping* like the world maps in *atlases* and *wall charts*.
  - As a value of spherical radius 6371.1 km is accurate enough.

Map Projections

- The earth needs to be approximated by an **ellipsoid** or **sphere** to be mapped.
- To produce a map the curved surface of the earth has to be transformed to a flat plane by means of a **map projection**.
  - There are many ways to project the earth onto a map plane and the map maker must choose the right type of projection for a particular map.
- A **map projection** is a system which gives the relation between the position of a point on Earth and that of the same point on the map.
  - The main aim of a map projection is to transform a part of the Earth's surface from the globe onto a plane at the same time keeping distortions to a minimum.

Map Projections

- Distortions in map projections
  - It is not possible to map a curved surface onto a flat one without any distortions.
  - Every map projection is associated with distortions.
  - Imagine the earth to have a skin like an orange cut into narrow strips, the curved surface of the skin can be laid almost perfectly flat upon a plane. This kind of map projection has little deformation within the strips.
  - This leaves gaps between the strips.
  - However, it is inconvenient to show the continuous spherical surface interrupted by many gaps
  - because it is desired to map the earth continuously it is necessary to stretch each part until the intervening gaps have been filled.

Map Projections

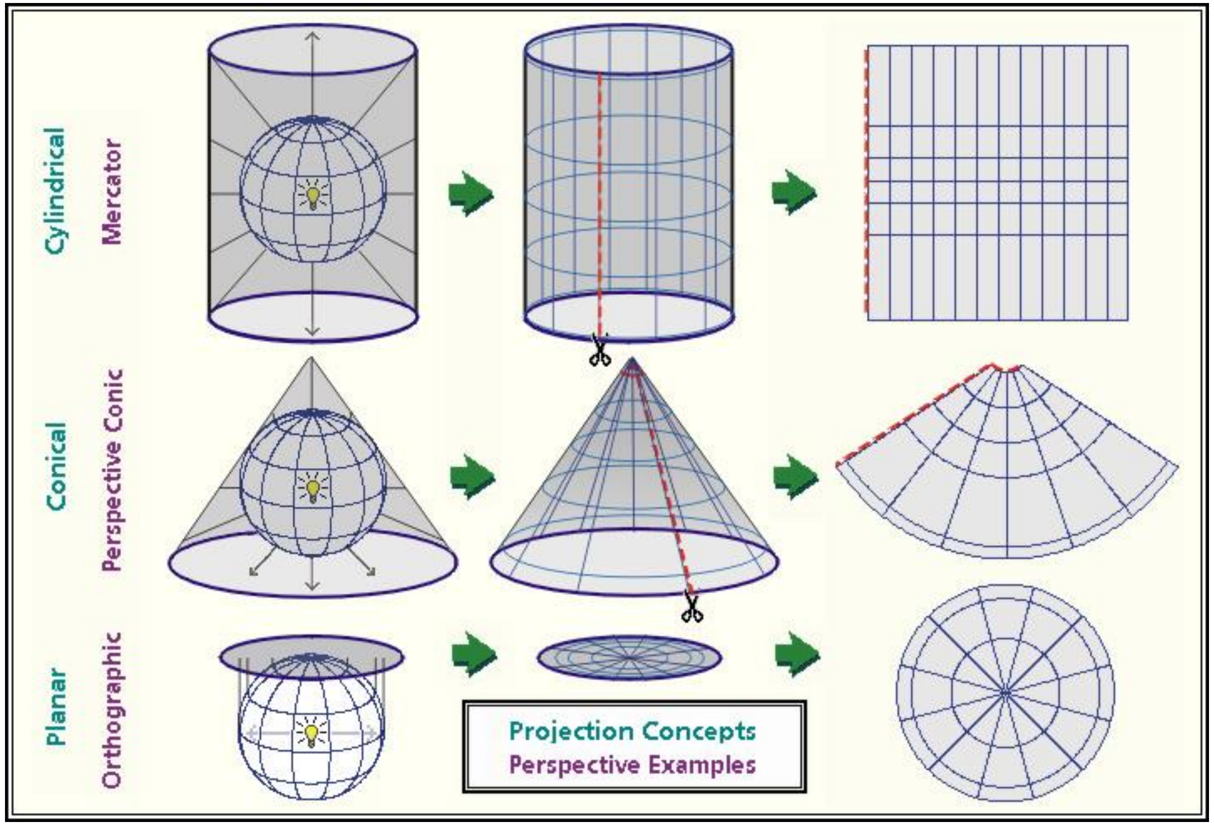
- The process of stretching alters the scale of the map and the amount by which the scale has changed increases progressively from the centre of the map towards edges.
- From the elementary definition of scale it is reasonable to make the following assumptions about distances measured on maps:
  - The scale of a map is constant:
    - *for all distances*
    - *in all parts of the map*
    - *for all directions on the map*.
  - None of these *assumptions* is correct!

Map Projections

- Distortions are present on the plane map
  - Scale varies from place to place and often in different directions at the same point on the plane map.
  - However, if we take a topographic map of 1: 50 000, for example, it is impossible to detect variations in scale with length of line, position or direction within the map sheet
    - This is because variations within such small areas (100 - 200 km^2) are so small that their effect can be ignored even though they exist.
  - This means that the elementary definition of scale is satisfactory for most kinds of map use on large and medium scale maps.
    - It becomes less reliable in the study and use of maps of scales 1: 1 mill. or smaller.

Projection Characteristics

- Projections are characterised by their *property, class* and *aspect.*
  - The three classes are azimuthal, cylindrical and conical



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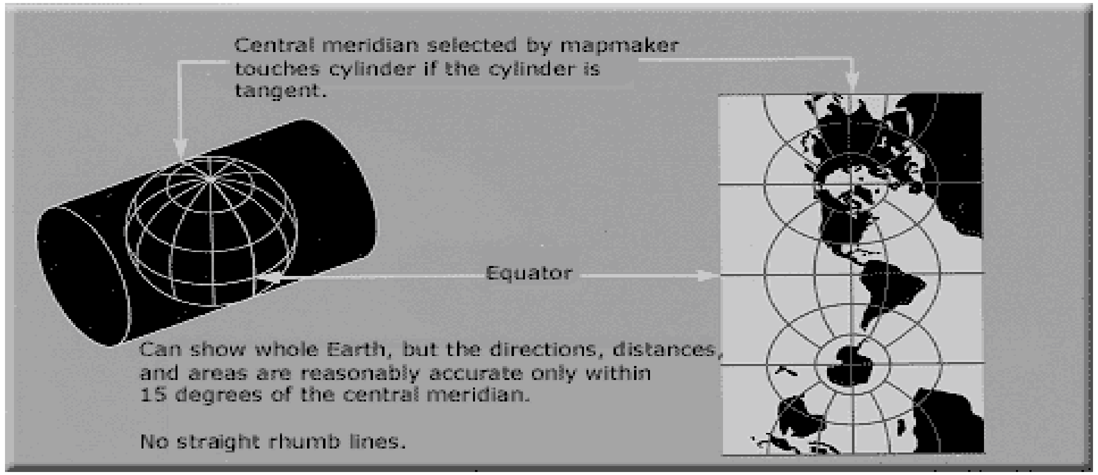
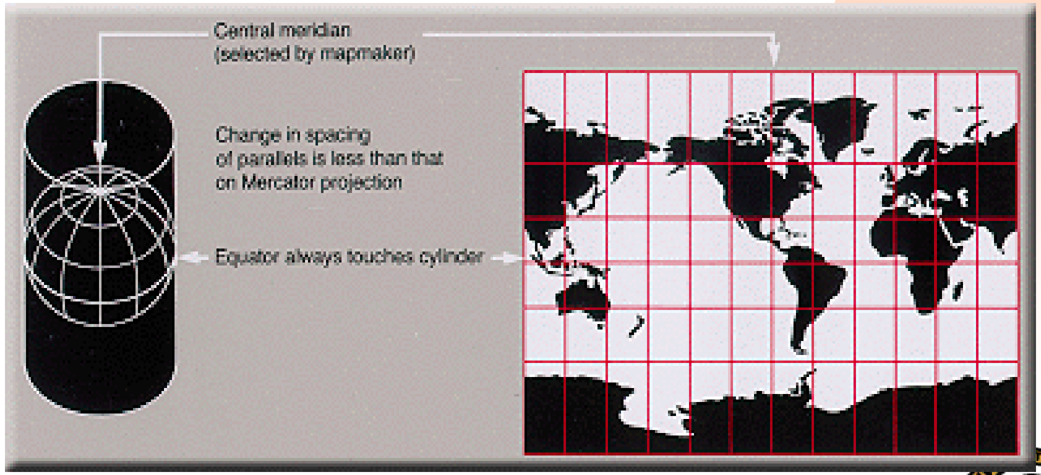
Projection Characteristics

- The orientation of the projection plane determines the **aspect** of a map project.
  - There are three orientations namely **Normal aspect**, **Oblique aspect** and **Transverse aspect**





Projection Characteristics

- The most important way to characterise a map projection is by its **property**.
- Conformal projections
  - Such projection represents angles and shapes correctly in a limited sense.
  - Shapes and angles are slightly distorted as the mapping region becomes larger.
  - Meridians and parallels intersect at *right angles*
  - This property is suitable for *topographic, sea, aeronautical* and *meteorological maps*
  - The **UTM grid** is based on a conformal projection in secant position to meet conformality and equidistance properties.

Projection Characteristics

- **Equivalent projections**
  - These are equal area projections which correctly represent areas on a map.
  - They are suitable for *distribution maps*
  - When used on large regions the distortion to angles and shapes is considerable
- **Equidistant Projections**
  - These correctly represent distances in a limited sense.
  - Distances measured on the map are only correct on one or two points in a *certain direction*
    - These are the only points in which the nominal scale is true
    - Points or lines of zero distortion also called **standard lines** or **points**

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Projection Characteristics

- Conformal projections
  - A map equidistant along meridians is one with the correct scale along meridians
  - An equidistance map is a useful compromise between conformal and equal area maps since shape and area distortions are moderate
- A projection can therefore not be both conformal and equivalent
- Selection of a projection should be made on the basis of *position, shape* and *size of the area* to be mapped and the *purpose of the map*.

Coordinate Systems

- The most fundamental principle of cartography is the establishment of a coordinate system on the Earth to which each point can be related.
- Different kinds of coordinate systems are used to position data in space.
- There are two (2) types of coordinate systems:
  - TERRESTRIAL COORDINATE SYSTEMS
    - Geographical coordinate system
    - Three-dimensional (3D) Cartesian coordinate system or geocentric
  - PLANE COORDINATE SYSTEMS
    - Plane Cartesian coordinate system
    - Raster coordinates system
    - Plane Polar coordinate system
    - Plane Grid system.

Geographical Coordinate Systems

- The best known method of referring to positions on the Earth's surface is by means of two angles,
  - Latitude and Longitude
  - which together form the system of *Geographical Coordinates*.
- The Latitude
  - Latitude is the angle measured at the centre of the Earth between the plane of the Equator and the radius drawn to a point (P) on the surface of the earth.
  - The Equator is the datum for measurement of latitude and is therefore assigned the value of 0°.
  - Northwards and southwards from the datum the latitude increases until it is 90° North at the North Geographical Pole and 90° South at the South Geographical Pole.

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Geographical Coordinate Systems

- The Latitude
  - In calculations which are made using geographical coordinates
  - N - latitudes are reckoned **+ve** and
  - S - latitudes are reckoned **-ve**
  - Latitude is denoted by the Greek letter φ.
- The Length of a Degree of Latitude
  - Degrees of latitude are closely the same length, but not quite because the Earth approximates an oblate spheroid
  - a N-S line (a meridian) has more curvature near the Equator and less near the poles.
    - the degrees of north-south arc on the Earth vary from about 110.6 km near the Equator to about 111.7 km near the poles.
    - The difference of 1.1 km is of little significance in small scale maps, but it is important on large and medium scale maps.

Geographical Coordinate Systems

The Longitude

- Longitude is the angle, measured at the centre of the Earth, between the plane containing the point (P) and the datum plane.
- This angle is measured eastwards and westwards from the datum plane and is recorded as East or West Longitude
- In calculations,
  - E - longitude is **+ve**
  - W- longitude is **-ve**
- The angle is denoted by the Greek letter λ.
- The datum from which longitude is measured may be chosen as the plane passing through the origin of a national survey.

Geographical Coordinate Systems

The Longitude

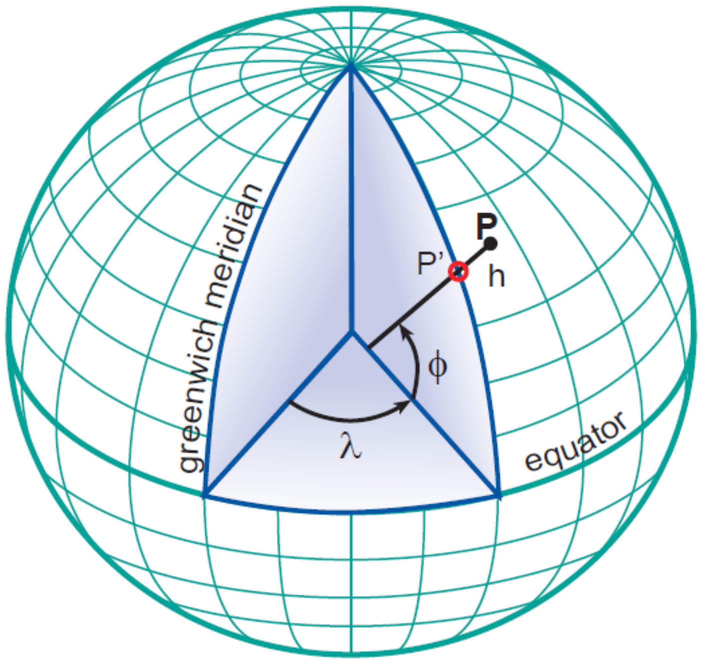
- However, it is better to use a single, **internationally recognised datum**.
- During the last century many nations began to accept the longitude of the Royal Observatory at Greenwich, near London in England, as 0°. In **1884** it was agreed on at an international conference.
- This is known as the Prime Meridian or Greenwich Meridian

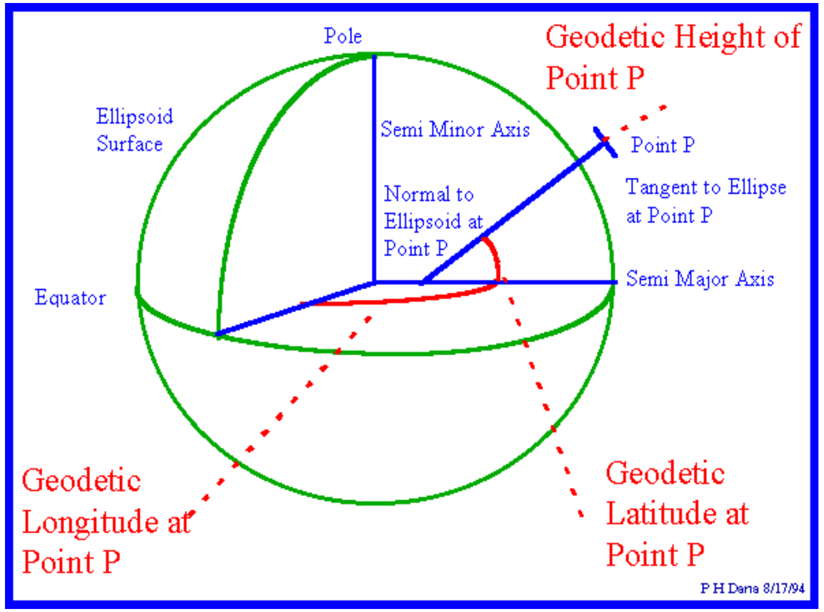
Geographical Coordinate Systems

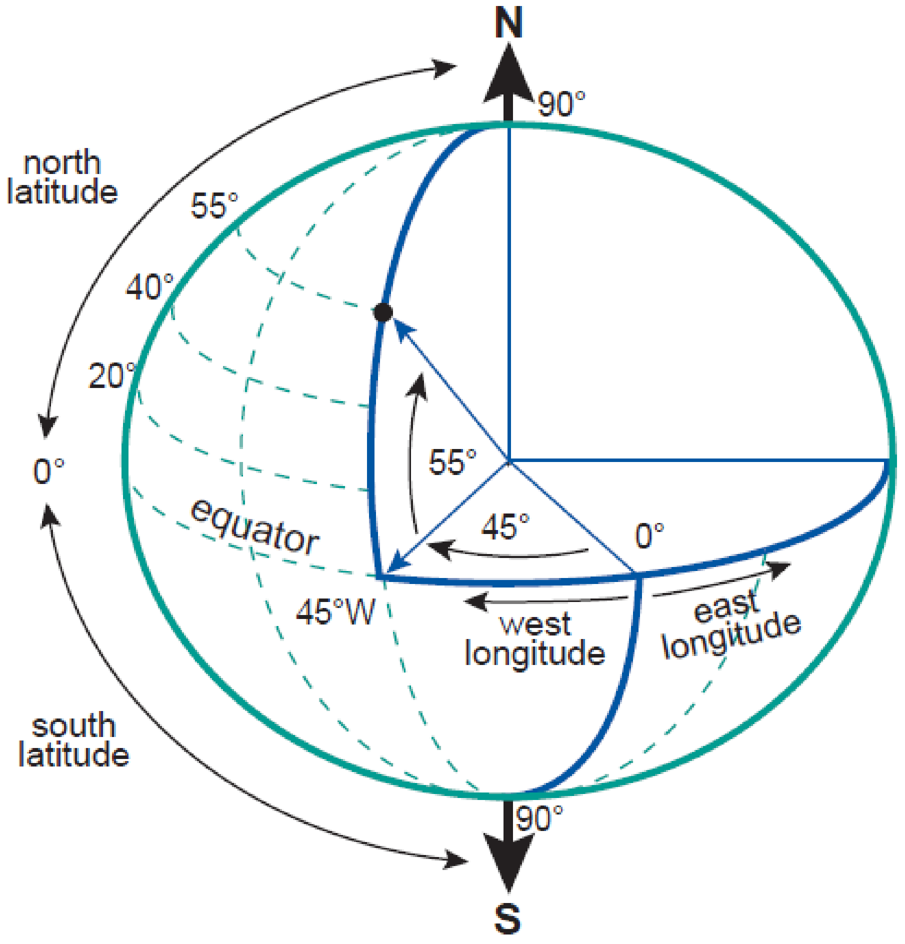
The Length of a Degree of Longitude

- The length of the Equator is very nearly the same as the length of a meridian circle. As we go towards the poles all other parallels become smaller and smaller circles until you have zero radius at the poles.
- E-W degree of longitude becomes shorter with increasing latitude and is finally reduced to NIL at the poles.
- The length of degrees of latitude is from 111.3km at the equator to 0km at the poles

Geographical Coordinate Systems





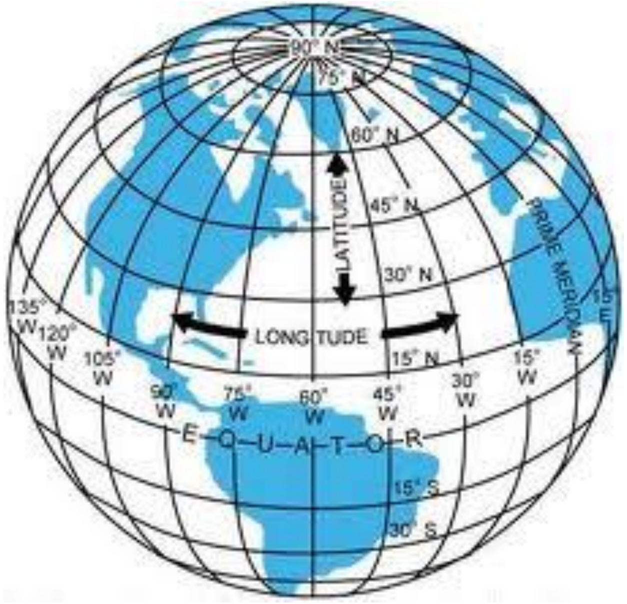


Parallels and Meridians

- **The Parallel**
  - The locus of all points having the same latitude traces a circle upon the spherical or spheroidal surface.
  - The plane containing this circle is parallel to the Equator and therefore its circumference is called a Parallel of Latitude or just a Parallel.
  - The radius of a parallel in latitude φ is determined as follows:
    - $r = R*\cos\varphi$.

Parallels and Meridians

- The Meridian
  - The locus of all points having the same longitude lies within the same plane which traces a semi-circle on the surface of a sphere or a semi-ellipse upon a spheroid.
  - Since the plane passes through the centre of the Earth, it is the arc of a great circle and is known as a Meridian.
  - All meridians intersect at the poles



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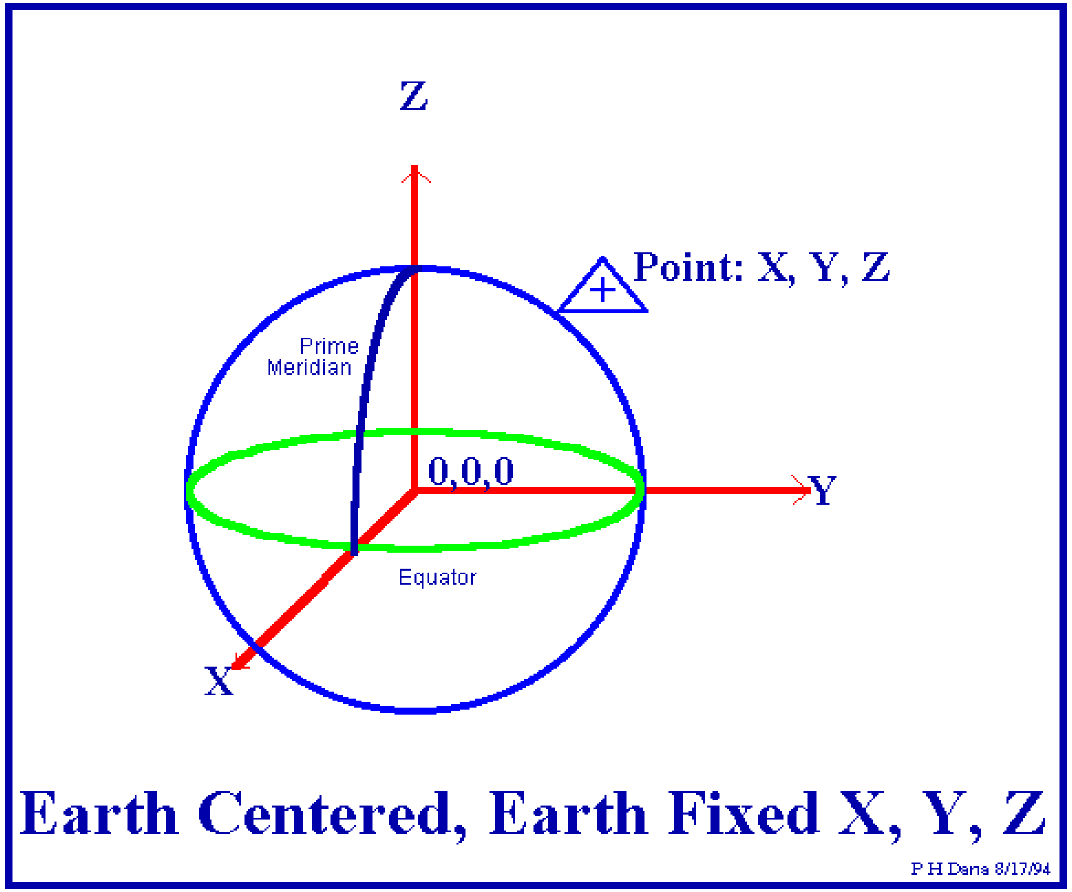
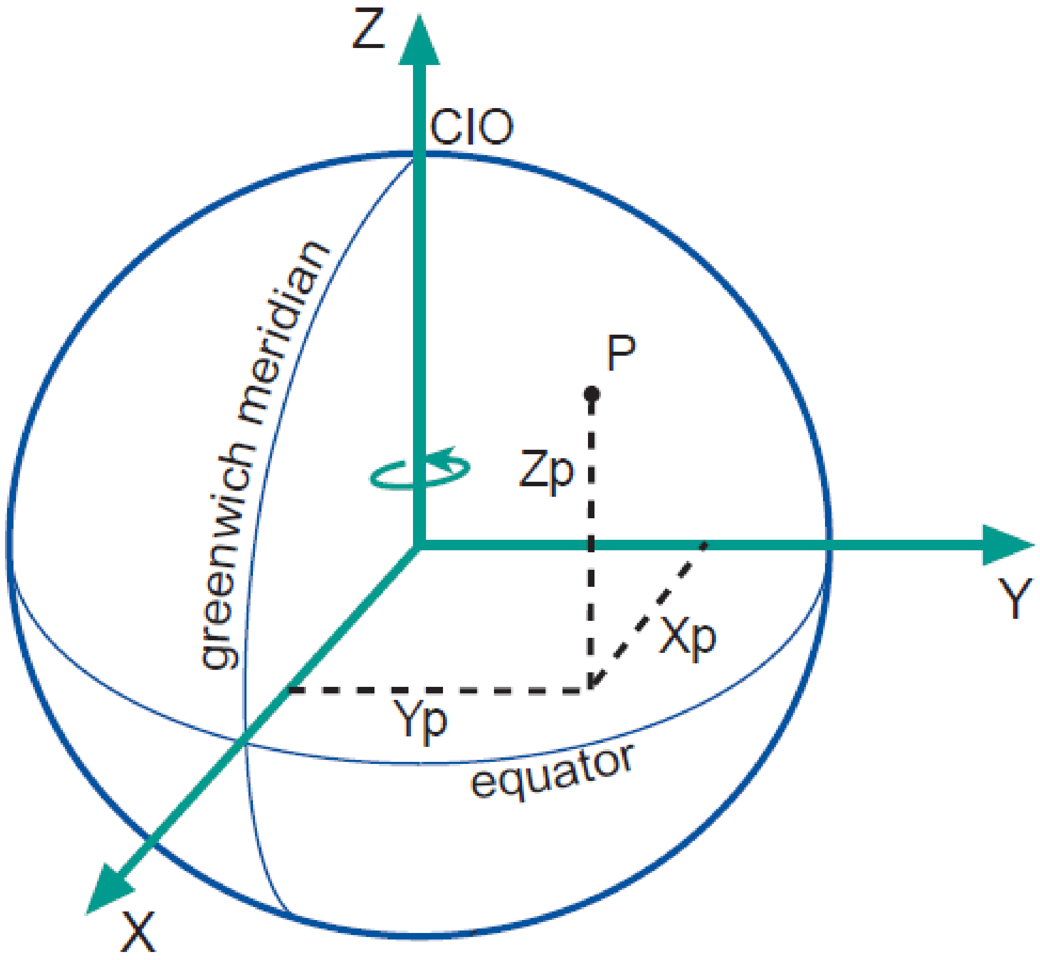
Three Dimensional (3D) Cartesian Coordinate System

- Earth Centered, Earth Fixed Cartesian coordinates used to define three dimensional positions.
- It uses X, Y, and Z Cartesian coordinates (XYZ) to define three dimensional positions with respect to the centre of mass of the reference ellipsoid.
  - The Z-axis points toward the North Pole.
  - The X-axis is defined by the intersection of the plane defined by the prime meridian and the equatorial plane.
  - The Y-axis completes a right handed orthogonal system by a plane 90 degrees east of the X-axis and its intersection with the equator

Three Dimensional (3D) Cartesian Coordinate System





PLANE COORDINATE SYSTEMS

- In order to plot the mathematical framework of a map it is desirable to use some kind of plane coordinate system.
- There are two (2) such systems in common use:
  - Plane polar coordinates
  - Plane rectangular Cartesian coordinates
  - For practical work of plotting a map, rectangular Cartesian coordinates are mostly used.

Plane Coordinate Systems

- Key words:
  - Polar coordinates
  - Plane rectangular Cartesian coordinates
  - Origin
  - Radius vector
  - Vectorial angle
  - Ordinate
  - Abscissa
  - Easting
  - Northing

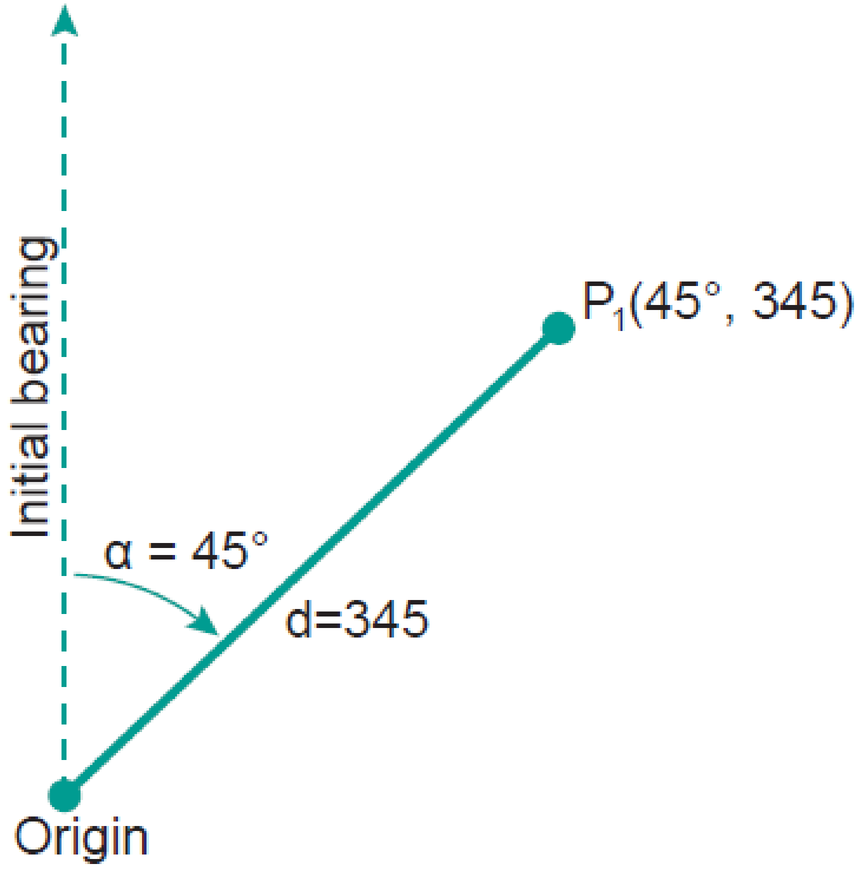
Plane Polar Coordinates

- The point O is selected as the origin from which measurements are made.
- The line OA is chosen as the axis or initial line.
- The position of any point P may be referred to this origin and axis by means of the **radius vector**, or straight line distance OP = r, and **vectorial angle** AOP = **θ** (**Theta**).
- The position of P is recorded by the two quantities **r** and **θ.**

Note:

- In **mathematics** the angle θ is measured **counter clockwise**.
- In **surveying**, **navigation** and **cartography**, angles are measured in a **clockwise** direction.

Plane Polar Coordinates (radius vector ~ d) and (theta ~ α)



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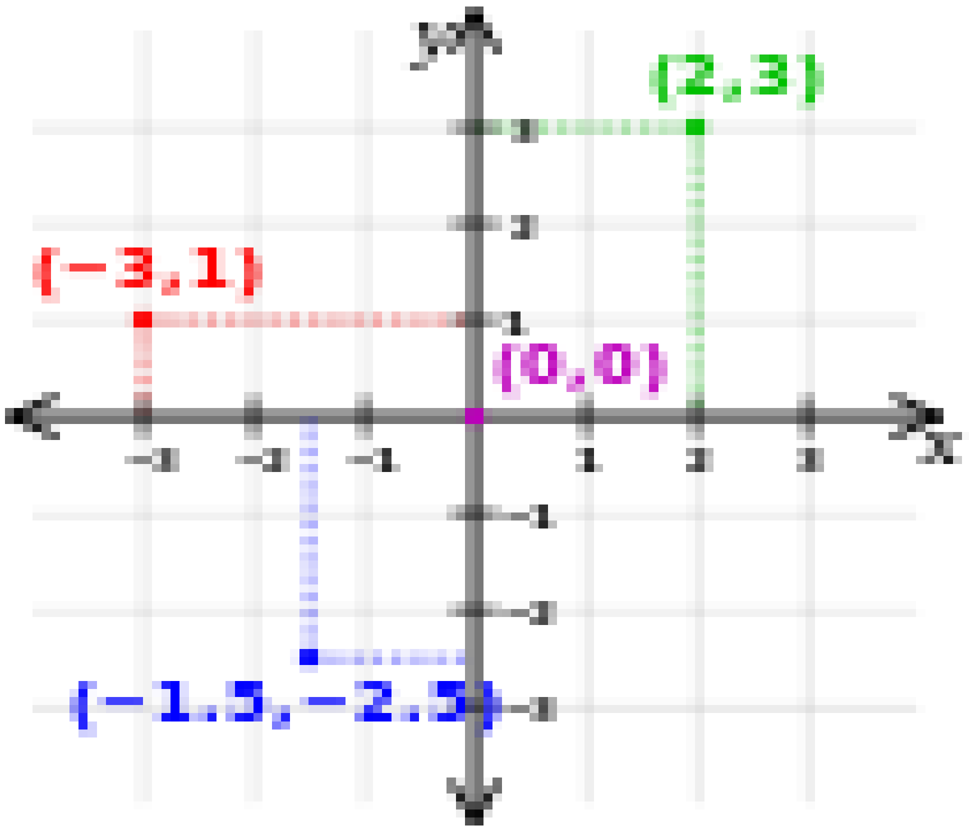
Plane Rectangular Cartesian Coordinates

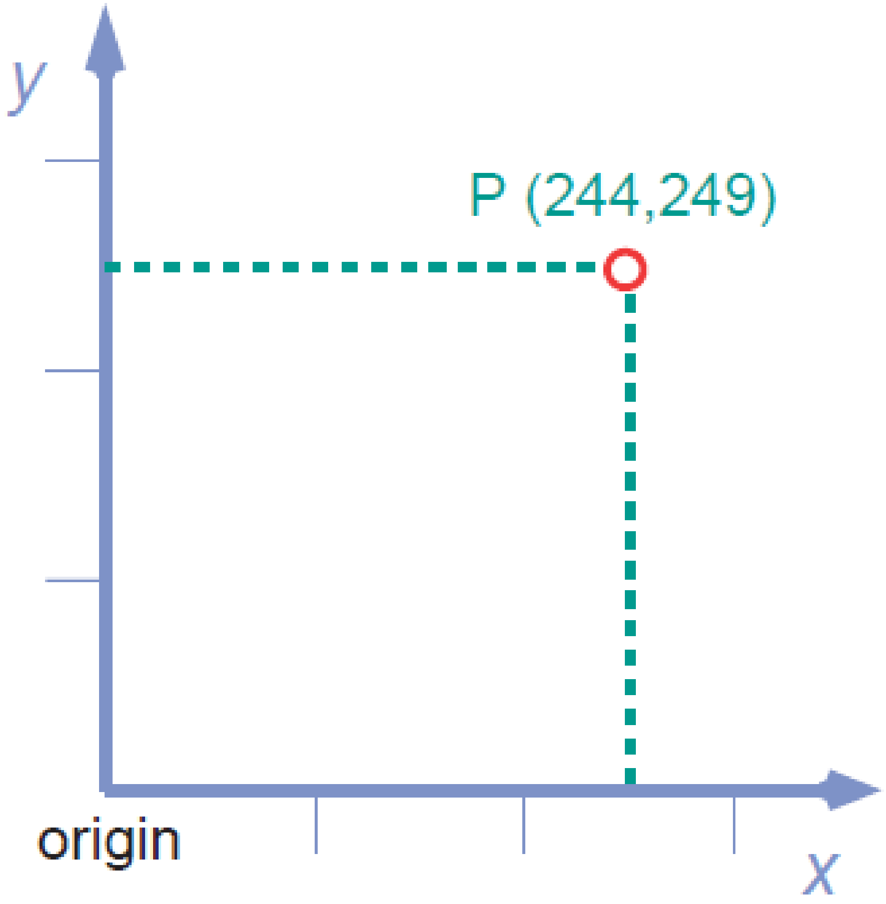
- These may also be called Cartesian Coordinates or just Rectangular Coordinates
- On a limitless plane surface there is no natural reference point.
- So an arbitrary system of location on a plane surface has long been used:
  - By establishing a ‘point of origin’ at the intersection of two conveniently located, perpendicular ‘axes’.
  - The plane is then divided into a grid by an infinite number of equally spaced lines parallel to each axes.

Plane Rectangular Cartesian Coordinates

- The customary mathematical convention is to call
  - the abscissa, the x-axis and refer to linear distances as x.
  - the ordinate is called the y-axis and distances respectively as y.
  - the position of point is defined as (x, y).
  - angles are measured anti-clockwise *from the x-axis*.
- In cartography, angles are measured clockwise.
  - Therefore it is more convenient to transpose the meaning of the two axes so that the ordinate is labelled the x-axis and the abscissa becomes y.
  - Because of the confusion which may arise from using the letters x and y for both systems, it is better to use
    - the letter **E** (= Easting) for y
    - the letter **N** (= Northing) for x.
    - Point P has now coordinates (E, N).

Cartesian Coordinate System (2D – X, Y)





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Rectangular Grid System

- A grid is simply a plane Cartesian reference system which satisfies the following rules:
  - The origin of the grid is defined as a particular point on the earth's surface.
  - The orientation of the ordinate is normally the direction of the meridian passing through the origin.
  - Consequently the abscissa measures distances East or West of the origin.
    - Since the sign convention indicates positive coordinates to the right of the origin, the term Easting is used for these measurements.
    - The corresponding measurements along the ordinate are Northings.
  - The unit of measurement is usually the *metre*.

Rectangular Grid System

- In order to overcome the disadvantage of having any negative coordinates for points which lie South or West of the origin, it is usual to re-number the axes to ensure that all coordinates are positive.
  - For this purpose a False Origin is created for the grid.
  - This point is located beyond the south-west extremity of the country to be mapped on the grid.

Rectangular Grid System

- The **Geographical Coordinate System** is useful for *large areas* at *small scale maps* but for *small areas* at *medium and large scale maps* the **Rectangular Coordinate System** is **better**.
  - The use of rectangular grid systems has become widespread during the last century.
  - Therefore **Geographical coordinates** have to be transformed as follows:
    - Transforming the spherical surface to a plane (by a system of map projection)
    - Preparing the map on a plane and
    - Placing a rectangular plane coordinate grid over the map.
  - To locate a position we need only specify the **E** and **N** coordinates to whatever degree of precision we desire.

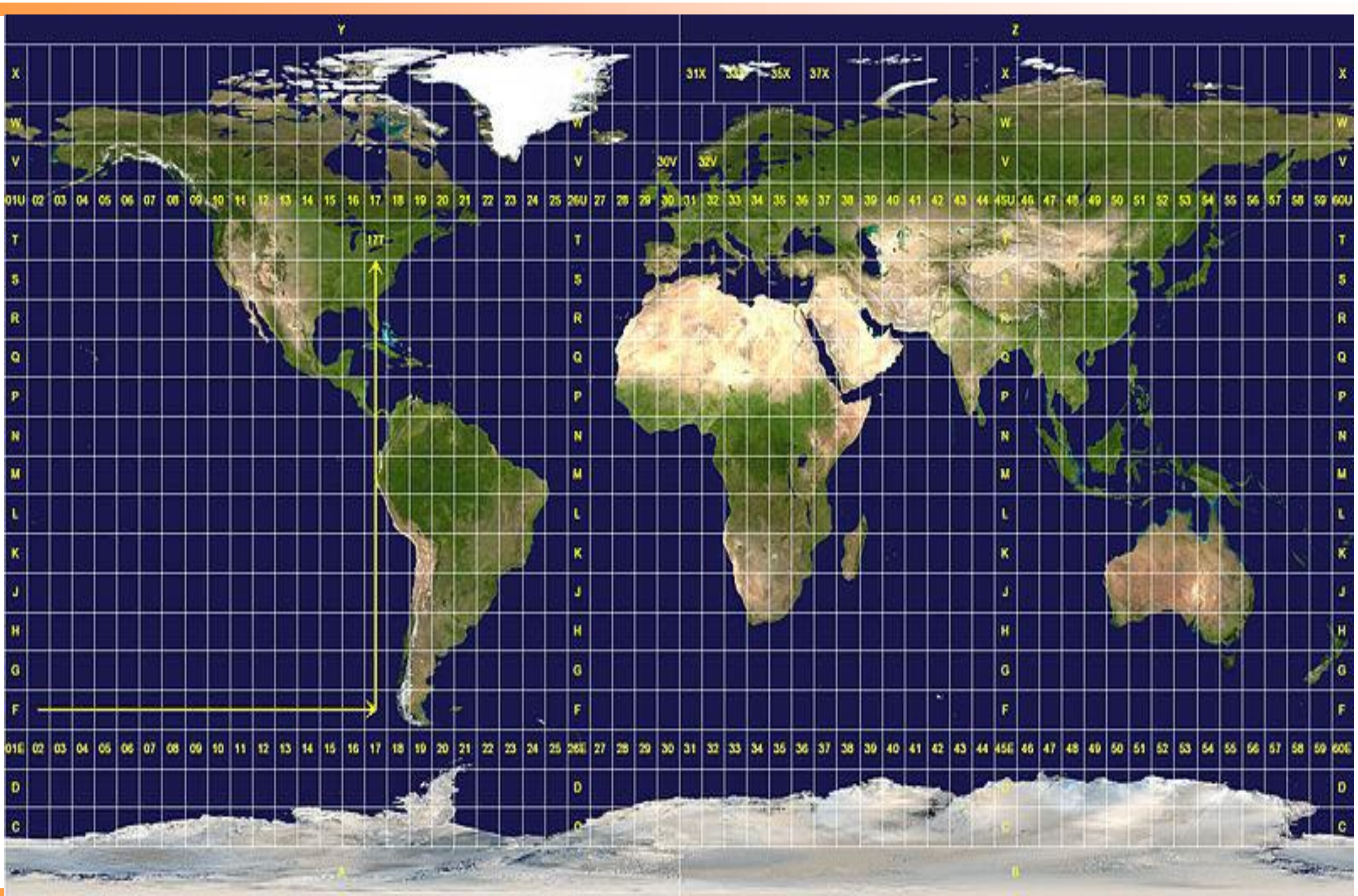
The UTM Grid System

- Individual countries may develop their own grid systems suitable for their needs, but there is one system that is commonly used in the world and also in Zambia.
- This is the Universal Transverse Mercator (UTM) grid system.
- The UTM grid system and the projection (Transverse Mercator) on which it is based have been widely adopted especially for topographic maps the world over.

The UTM Grid System

- In the UTM grid system:
  - The area of the Earth between 80°S and 84°N latitude is divided into north-south columns 6° of longitude wide called zones.
    - These are numbered from 1 to 60 eastward, beginning at the 180°W meridian.
  - Each column is divided into quadrilaterals 8° of latitude high with the exception of one row which is 12° latitude high extending from 72°N to 84°N.
    - The rows of quadrilaterals are assigned letters from C to X consecutively (I and O omitted) beginning at 80° S latitude.

The UTM Zones



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The UTM Grid System

- Each quadrilateral is assigned a number-letter combination.
- Within each zone the meridian in the centre is given an Easting value of 500,000 m.
- The Equator is designated as having a Northing value of 0 for the northern hemisphere coordinates and an arbitrary Northing value of 10 million metres for southern hemisphere is given.

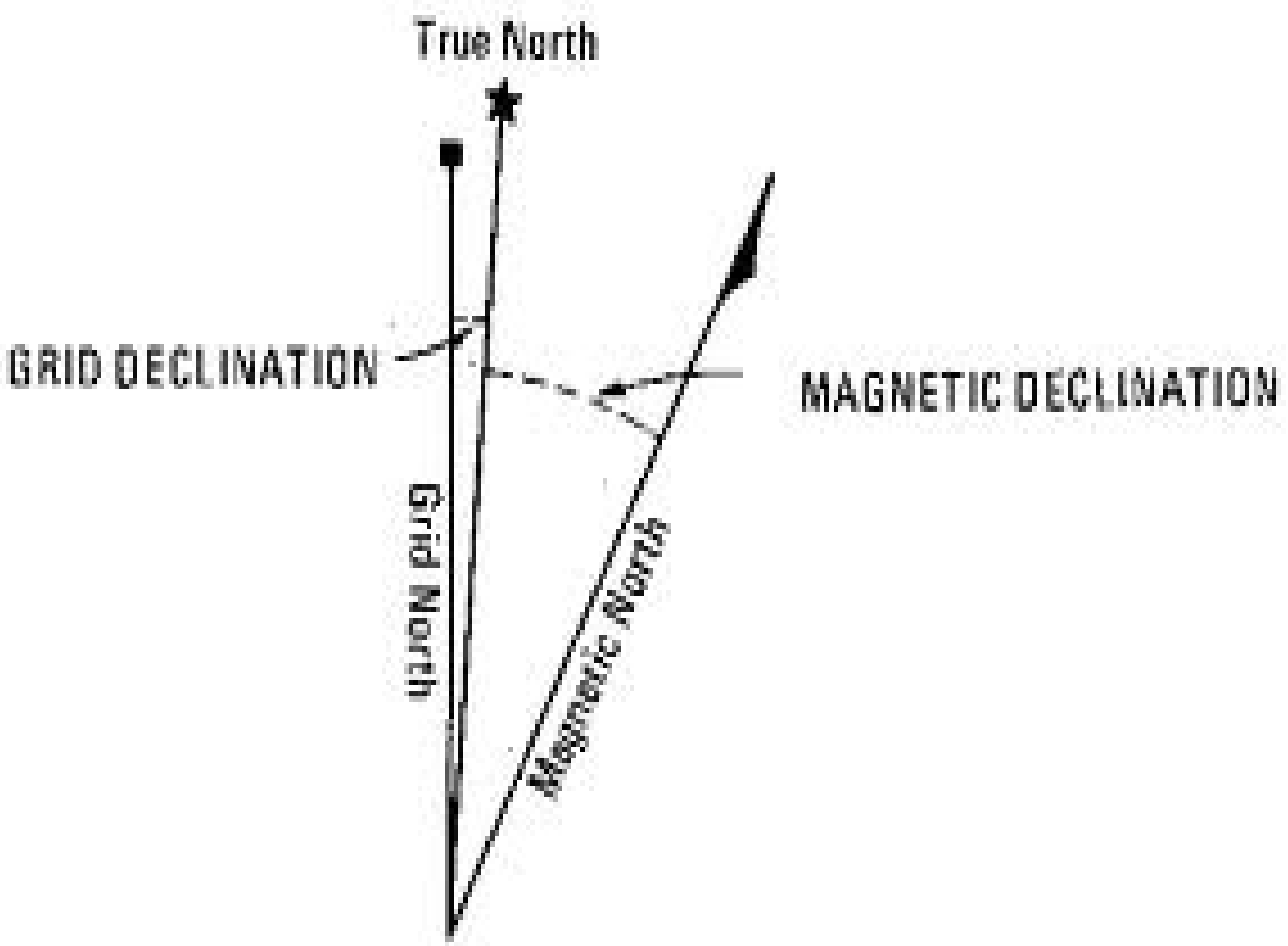
Directions

- By definition, north-south is along any meridian and east-west is along any parallel.
- The directions determined by the orientation of the graticule are called geographic or true directions
- The meridian through any point is directed to True North (TN)
- When a rectangular grid is placed over the graticule of a map, the vertical grid lines point to Grid North (GN)

Directions

- In most places the north direction of the grid will not coincide with true north.
- Therefore, primary topographic maps specify the *discrepancy* in degrees and minutes between grid north and true north at the centre of the sheet.
- This angle is called Grid Declination or Convergence
- The needle of the magnetic compass aligns itself with the total field of magnetic force and points to the Magnetic North (MN).

The Declination Diagram



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Directions

- In most parts of the earth MN is not parallel with the meridian as there is usually a difference between TN and MN.
- This angle is called Magnetic Declination or Compass Variation.
- Declination Diagram:
  - The difference between TN, GN and MN is usually shown with a diagram on primary topographic maps.
  - Furthermore, the magnetic field changes slowly so that the variation value is likely to be correct only for the date the map was issued.
  - Often a statement of the amount of annual change is included as is the case in Zambia.

Directions

- Directions are quite commonly measured from a map and transposed by compass to a terrain and vice versa.
- The direction of a line on the Earth can be called:
  - Azimuth
  - Bearing
  - Their meaning is essentially the same, differing largely in the context in which they are used.
  - The two of importance in cartography are azimuth and bearing.

Directions

- **Azimuth**
  - This direction is reckoned by observing the angle between the Meridian of the starting point and the arc of the Great Circle.
  - Since arcs of great circles are the shortest courses between points on Earth, movement along them is of major importance. Hence many maps are constructed so that directional relations are maintained as far as possible.
  - The angle is given in degrees (0° to 360°), reading clockwise from true north (TN).
  - The computation of azimuths in the geographical coordinate system is normally needed only in geodetic work.

Directions

- **Bearing**
  - A bearing is the direction from one point to another, usually expressed in relation to the compass.
  - Measuring a direction of the route AB from a topographic map by compass we measure an angle which is read clock-wise from Grid North and is called the Grid Bearing.
  - The bearing must be corrected by Total Correction, which is the angle between grid north (GN) and magnetic north (MN).
  - The total correction is added when the direction is measured from the map and transferred to the ground, and vice versa.

Measuring on Maps

- In land surveying the following ‘rules’ should be applied:
- The unit of measure for area is the:
  - hectare or square metre.
- When the area is smaller than 10 000 m^2
  - it should be expressed in m^2.
- When the area is greater than 10 000 m^2
  - it should be expressed in ha.

Measuring on Maps

- The use of maps is not only a question of map reading and interpretation.
- There is also often a need to get numerical values of points, distances, areas, height differences, profiles and slopes

Measuring on Maps

Units of Length

- The Basic SI-Unit is the metre (m)
  - Derived units are:
    - 1 kilometre(km) = 1 000 m
    - 1 decimetre (dm) = 0.1 m
    - 1 centimetre (cm) = 0.01 m
    - 1 millimetre (mm) = 0.001 m
- The British system:
  - 1 chain = 66 English feet
  - 1 Engl. feet (ft) = 0.3048 m
  - 1 yard = 0.9144 m
  - 1 miles (US) = 1.609 344 km
  - 1 miles (nautical) = 1.852 km

Measuring on Maps

Units of Area

- The SI-Units are:
  - 1 square metre = 1 m^2 = 1 m * 1 m
  - 1 hectare = 1 ha = 100 m * 100 m = 10 000 m^2
- The British system:
  - 1 acre = 0.40468 ha
  - 1 square mile = 640 acres

Measuring on Maps

Errors

- In any technology concerned with measurements there are the following type of errors:
  - gross errors
  - systematic errors
  - random errors
- Gross Errors:
- These are **blunders** made usually
  - in reading the scale incorrectly
  - in copying the results of a measurement incorrectly
  - in mishandling instruments
- The best way to avoid gross errors is to make at least <u>one extra measurement</u> than is theoretically necessary.
- The first fundamental principle is eliminate the gross errors

Measuring on Maps

Systematic Errors

- Systematic errors are normally small errors but they are cumulative and therefore grow into large errors when they are repeated.
- Causes for systematic errors are often
  - Use of unsuitable method
  - inaccurate setting of instruments
  - e.g. measuring distances with a pair of dividers instead of a scale rule. The measured distance will not be exact because of the inaccurate setting of the dividers.
- The second fundamental principle is to Eliminate systematic errors

Measuring on Maps

- Random Errors
  - In mapping the most significant single cause of random errors is the uncertainty of estimating distance in the engraved sub-division of the scale.
  - Small errors are more common than large ones.
  - Positive and negative errors are likely to occur.
  - If we repeat a single measurement many times, the frequency of the result has a bell shaped outline known as a Normal Distribution
  - The small errors cancel off
  - In estimating distances on maps we may expect to get a Graphical Accuracy of 0.2 mm.

Measuring on Maps

Measuring of Coordinates

- Measuring of coordinates can have reference to the geographical coordinates (φ,λ) but normally rectangular coordinates are determined.
- In case of shrinkage the results ought to be adjusted.
- The determination of coordinates can be done:
  - manually by use of a scale and divider or set-squares
  - by use of a rectangular coordinatograph
  - by use of digitizer in combination with a desk computer

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Measuring on Maps

Measuring of Distances

- to obtain the distance between two points on a map or plan, simply mark off the distance between the points
  - with a pair of dividers or
  - with the straight edge of a piece of paper
  - transfer measurements to the bar scale and read off the distance.
- to measure curvilinear lines, e.g. roads or other irregular lines
  - step the distance with a pair of dividers
  - follow the curvilinear feature with a piece of paper
  - follow the line with a special curvilinear measuring instrument.

Measuring on Maps

- Areas can be determined on maps in many ways but three most important methods are:
  - Mathematical method
  - Use of Planimeter
  - Use of Area Measurement Overlays
- Mathematical Method
  - Areas of closed polygons can be calculated numerically from coordinates by using the Gauss Formula

$$2A = \sum_{i=1}^{i=n} x_i \, (y_{i+1} - y_{i-1})$$

  - In this formula the points are taken in clockwise order but should they be taken in an anticlockwise direction you will get the same answer but with a negative sign.

Measuring on Maps

Planimeter Method

- An area of an irregular figure can be determined by use of a planimeter.
- The outline is followed with the tracing point and the vernier is read.
- The difference between the initial and the final readings is proportional to the area measured.
- The measuring process has to be repeated until at least three consistent readings are obtained.
- The mean of the readings is multiplied by the scale factor to produce the area.
- Usually the scale factor is available in the planimeter case or the following formula: A = R * (S/1000)□, where A = Area in hectares, R = Reading result, S = Scale denominator

Measuring on Maps

Area Measurement Overlays

- A very simple way of making a rough check of the calculated or measured area is to use the area measurement overlays such as:
  - linear template,
  - square template or
  - dot template.

Measuring on Maps

The Use of a Dot Template

- A **dot template** is a transparent template where each dot represents a specific area dependent on the scale
- The overlay is handled as follows:
  - Put the overlay on the figure to be measured. If the overlay is smaller than the figure, subdivide the figure
  - Count the dots which are inside the area. Dots just on the boundary are counted as ½ points.
  - Multiply the number of the dots by the area each dot represents.
  - To get a good result the overlay should be placed randomly on the figure. The dots must be counted at least twice with a rotation of the overlay in between the two counting.
  - The procedure has to be repeated until at least two consistent results are obtained. The mean of the consistent readings is the measured area.

Measuring on Maps

Measuring of Heights, Profiles, etc.

- Measurable heights on a map can be shown as:
  - Spot heights
  - Contour lines
- The spot heights:
  - are heights of single points given numerically determined by:
  - geometric levelling with the accuracy of cm
  - trigonometric levelling " dm
  - barometric heighting or by " m
  - photogrammetric methods " dm - m

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Measuring on Maps

Contour lines

- Contour lines are lines of equal height determined by:
  - plane table survey
  - tacheometric methods
  - surfaced levelling method
  - stereo interpolation or by
  - stereoplotting
- Whatever the surveying method of contour lines they are relatively inaccurate in comparison to other details

Contours and Spot Heights

Contours and Spot Heights

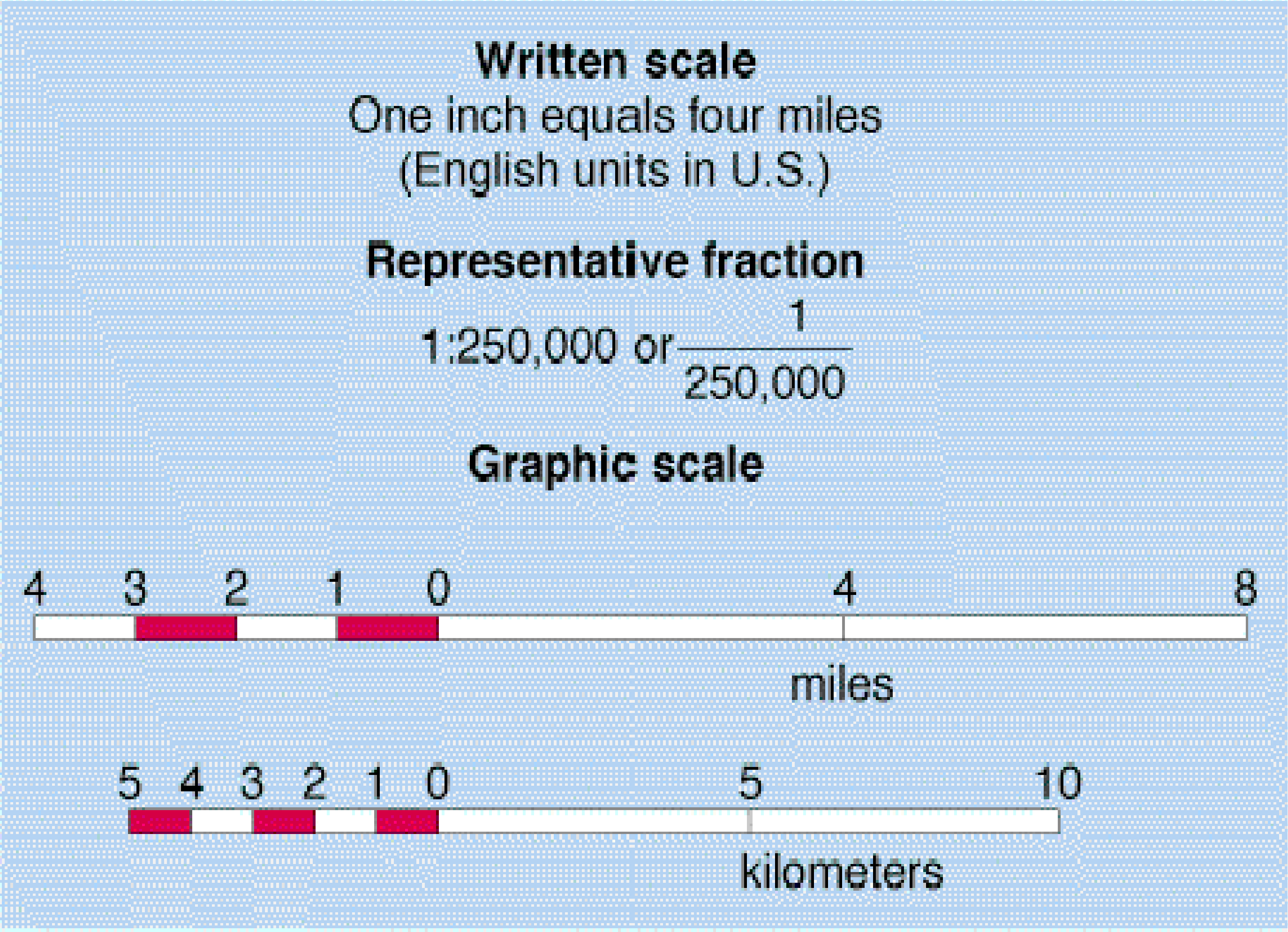
The Geometrical Framework of a Map

- The geometrical framework of a map is described by the following terms:
  - Scale
  - Graticule
  - Grid lines
  - Neat lines
  - Sheet numbering systems

Scale of a Map

- This is the ratio between distance measured on a map and corresponding distance measured on the reference surface – the ground.
  - Small Scale
    - shows large area
    - E.g. 1:10,000,000
  - Large Scale
    - shows small area
    - E.g. 1:50,000

Scale types



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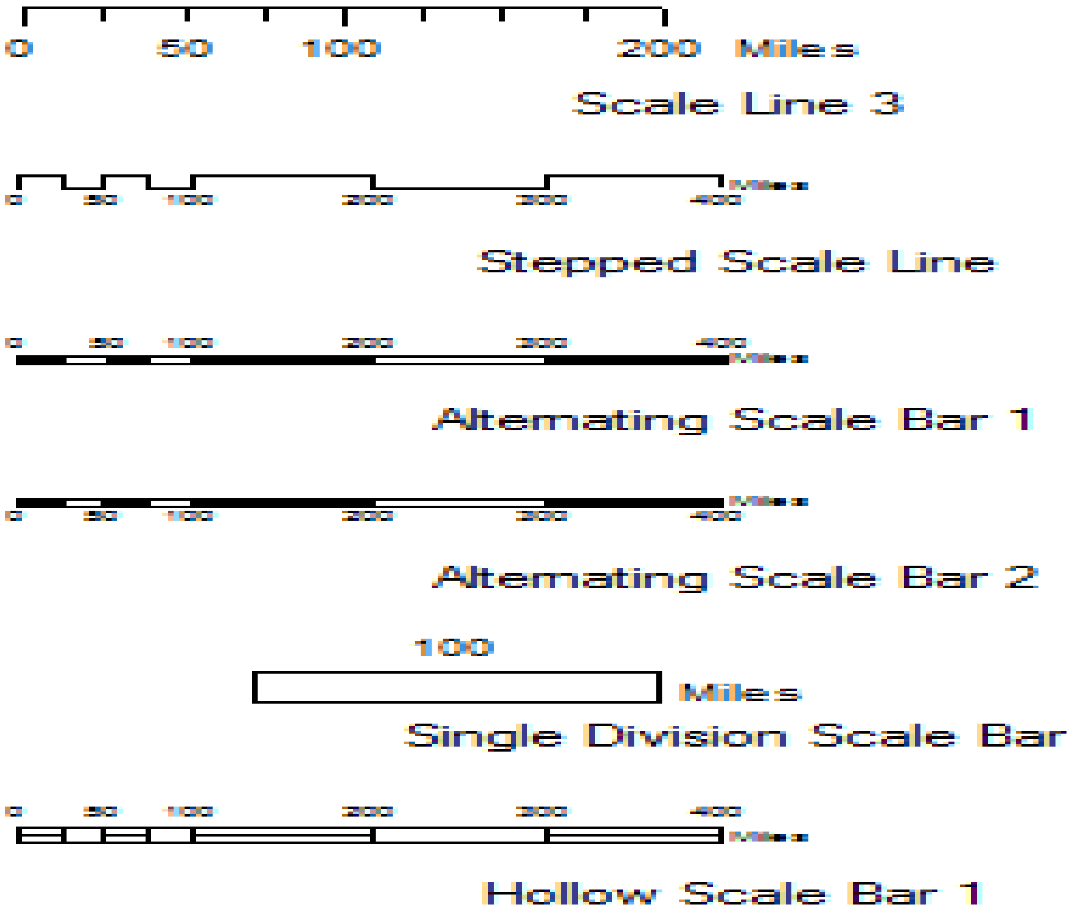
Scale Bars

- Scale bars provide:
  - a visual indication of the size of features
  - distance between features on the map
  - A scale bar is a line or bar divided into parts and labelled with its ground length, usually in multiples of map units such as tens of kilometres or hundreds of miles.
  - If the map is enlarged or reduced, the scale bar remains correct.

Scale Bars



Scale Bars

- Scale Bars make the map a useful and functional tool.
- Use a scale bar rather a verbal scale such as 1 inch = 1 mile. Maps get photocopied and enlarged or shrunk. A bar scale will always work even when the size of the paper has changed.

Scale and map content

Scale of a Map

Not to be confused with the scale bar, but related. Be aware of the scale of a map and the space on the page. A small scale map can have too much information and be difficult to "read". A large scale map may not have enough information to make it useful map.

The Graticule

- This is the network of meridians and parallels
- Each graticule is based upon a particular map projection.
- It represents the projected position of selected meridians and parallels.
- At larger scales greater than 1:10 000 the framework used on the map is almost invariably a grid.
- At scales smaller than 1:1 000 000 only the graticule is shown
- On medium scale maps grids are shown while the graticule is represented by tics along the map border

The Grid

- A grid is simply a Cartesian Reference System using distances measured on chosen projection.
- A grid on a map is a system of straight lines intersecting one another at right angles.
- It represents a method of defining position on the ground by means of distances measured upon a plane surface which is assumed to correspond to a portion of the Earth's surface.
- Arbitrary grids are possible and are often used on tourist maps.
- There must be a known relationship between graticule and grid
  - to facilitate conversion of geographical coordinates to grid coordinates (on plane).

The Neat Lines

- The neat lines of a map are those which enclose all the map details and therefore define the limits of the area mapped.
- Three kind of neat lines may be encountered
  - Grid neat lines
  - Graticule neat lines
  - Arbitrary neat lines

The Neat Lines

- **Grid neat lines**
  - On large and medium scale maps the neat lines are grid lines.
  - Consequently the format of the map is always square or rectangular.
- **Graticule neat lines**
  - On small scale maps and few medium scale maps the neat lines are formed by two parallels and two meridians of the graticule.
  - These may be straight or curved lines
  - the edge of the map closer to the pole is shorter than the side nearer the Equator.
  - Example: The Basic Zambian topographic Maps 1: 50 000

The Neat Lines

Arbitrary neat lines

- The neat lines are arbitrary straight lines which have no relationship to either grid or graticule.
- They merely serve to subdivide the area to be mapped into a series of rectangular maps of similar dimensions.
- Example: City maps in Zambia.

Sheet Numbering Systems

- The majority of maps form part of a series.
- Each map is one from hundreds or even thousands of topographic maps needed to cover the whole country.
- In order to make identification easier, each map sheet is numbered and may also have a name.
- Generally the name given to a map sheet is descriptive of the area as a whole (country) or it is the name of the most important feature (town, mountain, lake) on a particular sheet.

Sheet Numbering Systems

- Three different systems of sheet numbering may be used depending upon the nature of the neat lines used for the whole series:
  - Grid reference designation
  - Graticule reference designation and
  - Arbitrary numbering system
- The Sheet Numbering System of IMW
  - The most common, world-wide numbering system which uses graticule reference designation, is adopted for the International Map of the World (IMW) at 1: 1 Million.

Sheet Numbering Systems

- Sheet designation is made by using the following letters and numbers
  - The letter **N** or **S** indicate whether the sheet lies in the northern or southern hemisphere
  - The latitude zones of the map are indicated by letters from **A** to **V**.
    - Each map extends **4°** in latitude, therefore **A** represents the zone 0°-4°, **B** represents the zone 4°-8°, **D** is the zone 12°-16°, etc.
  - The numbers from **1** to **60** are used to denote the longitude zones
    - each of measures **6°** and corresponds to the west-east extent of a single map.
    - The numbering of the zones is eastwards from 180°W longitude so that number **1** represents the zone 180°W-174°W.
  - Sheet identification is composed of these three elements in the order above. E.g. **SD 35** and **NP 35**

Sheet Numbering Systems in Zambia

- In Zambia the numbering system of 1:50 000 and 1:250 000 topographic map series also uses the graticule reference designation.
- The 1:50 000 Topographic Maps:
  - The map area is 15′*15′
  - the sheet numbering system is based upon the geographical coordinates of the north-west corner of the map.
  - E.g. 1528A4

Sheet Numbering Systems in Zambia

- The 1: 250 000 Topographic Maps
  - The map area is 1°*1°30′
  - The sheet numbering system is based upon the sheet numbering system of IMW, where each 6°-zone is numbered eastwards from 180°W.
  - Within each zone, e.g. SD35, there are 16 of such maps numbered 1 - 16
  - Example: SD35-15 for Lusaka