TS-2-MAT2110

Q2 - n^{th} derivative

(a) $y = \sqrt{5x+1} = (5x+1)^{1/2}$

$y' = \frac{5}{2}(5x+1)^{-1/2}$

$y'' = -\frac{25}{4}(5x+1)^{-\frac{3}{2}}$

$y''' = \frac{375}{8}(5x+1)^{-\frac{5}{2}}$

$y^{iv} = -\frac{9375}{16}(5x+1)^{-\frac{7}{2}}$

$y^{v} = \frac{328125}{32}(5x+1)^{-\frac{7}{2}}$

- The index

$(5x+1)^{\frac{1}{2}-n} = (5x+1)^{\frac{1-2n}{2}}$

Key points

- Address the negative & positive sign

$(-1)^{n+1}$

- The coefficient denominator

2^n

- The coefficient numerator

$(2n-3)!!.5^n$

Take note

- double factorials depend on whether what you have is an even or odd number

eg

i) $7!! = 1 \times 3 \times 5 \times 7$

ii) $6!! = 2 \times 4 \times 6$ and so on.

$\therefore \; y^n = f^n(x) = \frac{(-1)^{n+1}\,(2n-3)!!.5^n.(5x+1)^{\frac{1-2n}{2}}}{2^n}$

Q2

(b)

$$y = \frac{1}{1-2x} = (1-2x)^{-1}$$

$$y' = (-1)(-2)(1-2x)^{-2} = 2(1-2x)^{-2}$$

$$y'' = 8(1-2x)^{-3}$$

$$y''' = 48(1-2x)^{-4}$$

$$y^{iv} = 384(1-2x)^{-5}$$

$$y^{5} = 3840(1-2x)^{-6}$$

from the above pattern

$$y^{n} = f^{n}(x) = 2^{n} n!\,(1-2x)^{-(n+1)}$$

$$y^{n} = f^{n}(x) = \frac{2^{n} n!}{(1-2x)^{n+1}}$$

Q2

(c)

$$y = \frac{1}{x} + \cos(2x) = x^{-1} + \cos(2x)$$

$$y' = -x^{-2} - 2\sin(2x)$$

$$y'' = 2x^{-3} - 4\cos(2x)$$

$$y''' = -6x^{-4} + 8\sin(2x)$$

$$y^{iv} = 24x^{-5} + 16\cos(2x)$$

$$y^{v} = -120x^{-6} - 32\sin(2x)$$

$$y^{vi} = 720x^{-7} - 64\cos(2x)$$

Key points

- Address the sign of the 1st term

  $(-1)^n$

- coefficient of the ~~the~~ first term

  $n!$

- pattern of x

  $x^{-(n+1)} = \frac{1}{x^{n+1}}$

- Addressing the 2nd term [say we are given $\cos px$]
  - when n is odd then

$$\frac{d^n}{dx^n} = y^n = f^n(x) = \pm p^n \sin(px).$$

  - when n is even then

$$\frac{d^n}{dx^n} = y^n = f^n(x) = \pm p^n \cos(px)$$

but $p = 2$

$$\therefore y^n = f^n(x) = (-1)^n \frac{n!}{x^{n+1}} + \begin{cases} \pm p^n \sin(px) \\ \pm p^n \cos(px) \end{cases}$$

$$y^n = f^n(x) = (-1)^n \frac{n!}{x^{n+1}} + \begin{cases} \pm 2^n \sin(2x) \\ \pm 2^n \cos(2x) \end{cases}$$